

Transitivity and degree assortativity explained: The bipartite structure of social networksDemival Vasques Filho^{*}*Leibniz-Institut für Europäische Geschichte, Alte Universitätsstraße 19, 55116 Mainz, Germany*Dion R. J. O’Neale[†]*Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand
and Te Puāhā Matatini, University of Auckland, Private Bag 92019, Auckland, New Zealand*

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Dynamical processes, such as the diffusion of knowledge, opinions, pathogens, “fake news,” innovation, and others, are highly dependent on the structure of the social network in which they occur. However, questions on why most social networks present some particular structural features, namely, high levels of transitivity and degree assortativity, when compared to other types of networks remain open. First, we argue that every one-mode network can be regarded as a projection of a bipartite network, and we show that this is the case using two simple examples solved with the generating functions formalism. Second, using synthetic and empirical data, we reveal how the combination of the degree distribution of both sets of nodes of the bipartite network—together with the presence of cycles of lengths four and six—explain the observed values of transitivity and degree assortativity coefficients in the one-mode projected network. Bipartite networks with top node degrees that display a more right-skewed distribution than the bottom nodes result in highly transitive and degree assortative projections, especially if a large number of small cycles are present in the bipartite structure.

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Real-world networks of distinct nature (e.g., social, biological, technological) usually display common topological features, such as heterogeneous degree distribution and small average path length. However, it has been observed that social networks tend to present higher levels of transitivity and degree assortativity than other nonsocial networks [1,2]. These two structural properties have been extensively addressed in the fields of network theory and social network analysis due to their importance, for instance, in studies investigating the spreading of social phenomena. Nonetheless, the reasons that make social networks different are still not fully understood.

Previous works have conjectured that high values for the transitivity and assortativity coefficients are a result of building most social networks using group-based methods [3,4]—which we can directly translate to bipartite networks, with the agents (bottom nodes) belonging to groups (top nodes). Newman and Park [3] proposed a model and solved it using the generating functions approach to support this idea, while recently, Fisher *et al.* [4] used empirical evidence to do so. The latter gathered information from 88 networks of several types and classified them into three categories: “direct” or naturally one-mode social networks (i.e., those with only one set of nodes, not generated from an underlying bipartite network), group-based social networks (i.e., social networks that are projections of a bipartite network), and other (nonsocial) networks. Fisher *et al.* [4] observed that “...while social

networks are more assortative than nonsocial networks, only when they are built using group-based methods do they tend to be positively assortative.”

Here, we build on works by Newman and Park [3] and Estrada [5] and on a previous work of ours [6] to reveal the effects of the bipartite structure on the transitivity and degree assortativity coefficients of projections. Transitivity is always positive in projected networks—with one exception (neutral value), which we will see later—with levels that depend on the topology of the bipartite network. Concerning assortativity, we provide an answer to the question raised in the conclusions of Larremore *et al.* [7], “...whether assortativity is due to properties of social networks or due to implicitly projecting from bipartite data ...,” showing that degree assortativity is indeed a property of social networks. Moreover, we explain the reasons for the aforementioned observation of Fisher *et al.* [4]—that social networks are more degree assortative than other networks—by arguing that social networks have an inherent group-based structure.

We use empirical networks constructed from the arXiv and the APS (American Physical Society) datasets, and data collected from Norwegian boards of directors [8], with a corresponding bipartite configuration model (BiCM) as the null model. Moreover, with the generating functions formalism, we analytically show that every network can be regarded as having an original bipartite structure, even those we refer to as naturally one-mode networks. We will see that the latter is just a projection of a trivial case of bipartite networks. The concept of social networks being originally bipartite is not new. Newman and Park [3] proposed that social networks inherently have a bipartite structure. They were followed by Guillaume and Latapy [9], who went further to propose that all networks

^{*}vasquesfilho@ieg-mainz.de[†]d.oneale@auckland.ac.nz

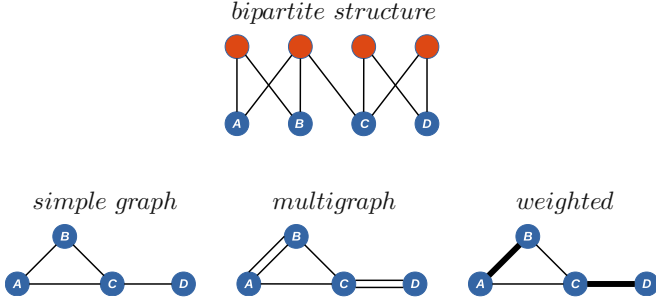


FIG. 1. Schematic drawing of the three projection methods we discuss here: simple graph, multigraph, and weighted graph projections.

can be modeled as such and to develop a model to recover the bipartite structure from any given one-mode network.

The remainder of this paper is as follows: In Sec. II, we briefly introduce network definitions and notations, and we explore the underlying bipartite structure of social networks. Section III presents the structural properties of our interest and provides the beginning of the discussion on how these properties are related. We further this discussion using empirical data in Sec. IV and conclude this work with Sec. V.

II. LATENT BIPARTITE STRUCTURE OF ALL NETWORKS

A bipartite network is a graph $B = \{U, V, E\}$, where U and V are disjoint sets of nodes and $E = \{(u, v) : u \in U, v \in V\}$ is the set of links connecting nodes of the different sets. We refer to the sets U and V as the bottom and top node sets, respectively. A projection onto the nodes U (a so-called bottom projection) results in a one-mode network, where node u is connected to u' , $\{u, u'\} \in U$, only if there exists at least one common neighbor $v \in V$, that is, at least a pair of edges (u, v) and $(u', v) \in E$. The converse is also valid: every analysis for projected networks onto the bottom set of nodes U works also for top projections onto V . Hence, for the sake of simplicity, we treat all projections as bottom projections. That is, we define a projected network as a graph $G = \{U, L\}$, where L is the set of links in the projection, such that $L = \{(u, u') : \exists v \in V, (u, v) \in E \text{ and } (u', v) \in E\}$. Projections have different flavors. Here, we mention simple graph G_s , (simple) weighted graph G_w , and multigraph G_m projections [6], as shown in Fig. 1. Then, for a bipartite network we have k_u , degree of bottom node $u \in U$; d_v , degree of top node $v \in V$; $P_b(k)$, bottom node degree distribution; and $P_t(d)$, top node degree distribution. For the projected network, we have q_u , degree of node $u \in U$, and $P(q)$, projected node degree distribution.

The relation between node degrees in B and G is given by [6]

$$q_u \leq \sum_{j=1}^{k_u} (d_{v_j} - 1), \quad (1)$$

where the equality holds for q_u^m , the degree of u , when the projection is a multigraph projection. The multigraph degree is also known as the node strength [10], s_u , such that $q_u^m = s_u = \sum_{u'} w_{uu'}$, where $w_{uu'}$ is the weight of the link connecting

nodes u and u' , in a simple weighted projection (for other weighted projection methods, see Ref. [11]).

In Ref. [6] we have shown that using the generating functions formalism, we can predict the node strength (multigraph degree) distribution of a projected network, by knowing the degree distribution of both sets of nodes of the bipartite network. To illustrate our approach in that paper, we solved two very simple cases of degree distributions for bipartite networks, which we recall here. For both examples, we assume that the degree distribution of top nodes, $P_t(d)$, follows a delta function distribution; that is, every top node has the same degree $d_v = d^*$. In the first case, the bottom degree distribution, $P_b(k)$, follows a Poisson distribution. Then, according to Eq. (31) of Ref. [6], the node strength distribution $P(q^m)$ of the projected network is calculated according to

$$P(q^m) = \begin{cases} \frac{\langle k \rangle \binom{q^m}{d^*-1} e^{-\langle k \rangle}}{\binom{q^m}{d^*-1}!} & \text{if } (d^* - 1) | q^m, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where $(d^* - 1) | q$ means q is divisible by $d^* - 1$. For the second case, the bottom node degrees are exponentially distributed. The projected strength distribution is

$$P(q^m) = \begin{cases} \lambda e^{-\lambda \frac{q^m}{d^*-1}} & \text{if } (d^* - 1) | q^m, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

which is Eq. (34) of Ref. [6], and where $\lambda = \langle k \rangle^{-1}$.

We can see from Eqs. (2) and (3) that the expected multigraph degree distributions for the projected networks also follow Poisson and exponential distributions, respectively (as the bottom nodes), modulated by $d^* - 1$. If we take $d^* = 2$, then we have the exact bottom degree distribution. In other words, even naturally one-mode networks can be viewed as projections of a trivial bipartite network, in which all top nodes of B have degree $d^* = 2$. We can deem the top nodes as events that create links in a network as, for instance, the exchange of a letter between two persons, the acceptance of a friendship request on social media, or a trade agreement between two countries.

III. STRUCTURE OF BIPARTITE AND PROJECTED NETWORKS

In the examples of the previous section, we have chosen Poisson and exponential distributions as bottom degree distributions due to their simple analytical solutions via generating functions. However, this reasoning can be extended to any other distribution, as well as to simple and weighted projections. In these cases, the resulting degree distribution of the projected network depends also on a feature frequently observed in real-world bipartite networks: four-cycles, that is, pairs of bottom nodes that share more than one (top node) common neighbor.

Four-cycles are a motif in B that affect the degree of nodes in simple and weighted projections [12]. The degree of u is the same in these two types of projection (i.e., $q_u^s = q_u^w$) and represents the number of neighbors $|N(u)|$ of u in G . The inequality of Eq. (1) depends on the number of these cycles of which u is part and on which nodes form the cycles together with u . In other words, it is not possible to solve the inequality

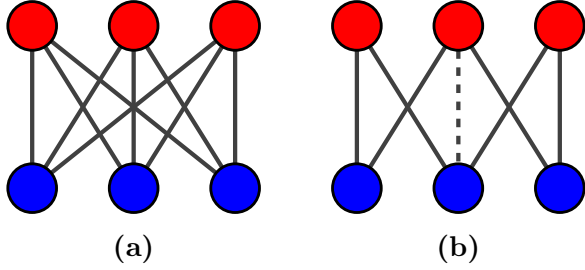


FIG. 2. Diagram with examples of structures that inhibit the formation of (simple) triangles. The six-cycles are embedded in subgraphs containing top nodes with degree $d \geq 3$. In panel (a) we have a complete subgraph with an overlapping of nodes with $d_v = 3$ that results in only one triadic closure in the projected network, for instance, three papers coauthored by the same three authors. In panel (b) the addition of the dashed link breaks the six-cycle into four-cycles.

of Eq. (1) just by counting the number of four-cycles in the bipartite network. This type of motif is directly related to the concept of redundancy [13]. Four-cycles generate redundant links when creating a simple graph projection (or add to the weight of the link, when the projection is weighted).

From now on, we treat all projections as a simple weighted projection, unless otherwise stated. We use q_u^s as the node degree and q_u^m as the node strength, as a reminder of the simple graph degree and the multigraph degree, respectively.

Another motif relevant for the topology of bipartite networks, and the resulting topology of projections, is the cycle of length six, which we refer to as six-cycle. While four-cycles create redundancy of links, six-cycles result in triadic closure in the projected network [12,14]. That is, one-mode projections of bipartite networks have two types of triangles: those induced by top nodes with degree $d \geq 3$ and those induced by six-cycles [12,14]. Then, the number of triangles

in a projected network is given by

$$|\Delta| \leq \sum_{\substack{v=1 \\ d_v > 2}}^{|V|} \binom{d_v}{3} + |\bowtie|, \quad (4)$$

where \bowtie is the set of six-cycles in the bipartite network.

Let us recall that the level of transitivity (global clustering coefficient) in networks is calculated as the ratio of the number of triangles to the number of triplets in the network, according to

$$C = \frac{\text{No. closed triplets}}{\text{No. triplets}} = \frac{3 \times |\Delta|}{|\wedge|}, \quad (5)$$

where Δ is the set of triangles and \wedge is the set of triplets (open and closed) in the network. The higher the presence of six-cycles in the bipartite network is, the more triangles can be formed in the projection, increasing transitivity. However, quantifying this relationship is not a simple task and depends on a combinations of factors, similarly to the relationship between four-cycles and the projected degree distribution.

The inequality of Eq. (4) is due to several patterns that emerge in real-world bipartite networks as, for instance, complete subgraphs and the overlapping of four-cycles with six-cycles (Fig. 2). Six-cycles are embedded in such patterns, resulting in a smaller number of triangles in simple projection clustering calculations (i.e., those without weighted or multiple links) than we would expect.

Yet, Eq. (4) helps in explaining why the transitivity coefficient reaches high values in most social networks. The number of triangles is highly dependent on the node degree distribution $P_t(d)$. The broader the top degree distribution is, the higher the number of triangles is, and therefore the larger the values of transitivity are. However, the level of transitivity in a network also depends on the total number of triplets [Eq. (5)]. The latter, in turn, is related to $P_b(k)$. The broader

TABLE I. Mean (standard deviation) values of properties for BiCM networks and their projections, over 100 runs. These networks are created with the same methods as in Ref. [6], where the degree sequences are drawn from probability distributions. Here, we use delta, Poisson, and exponential distribution for both bottom and top nodes. We use $|U| \approx |V| \approx 1000$ and $\langle k \rangle = \langle d \rangle = 5$. The upper bound number of triangles is the sum of the first and second columns, according to Eq. (4). Here $|\Delta|$ is the actual number of triangles, $|\wedge|$ is the total number of triplets, and C is the transitivity of the projection. Note that as $P_t(d)$ becomes broader, with the same $P_b(k)$, the increase in the number of triangles prevails over the increase in the number of open triplets, and the transitivity consequently increases. Otherwise, as $P_b(k)$ becomes broader, keeping $P_t(d)$ the same, the increase of open triplets prevails, and the transitivity decreases.

$P_b(k)$	$P_t(d) = \text{Delta}$				
	$\sum_v \binom{d_v}{3}$	$ \bowtie $	$ \Delta $	$ \wedge $	C
Delta	9956 (15)	673 (26)	10 622 (29)	186 546 (448)	0.171 (0.001)
Poisson	9834 (97)	1298 (68)	11 094 (146)	220 181 (4,628)	0.151 (0.001)
Exponential	9750 (166)	5114 (717)	14 105 (628)	313 217 (14,386)	0.135 (0.001)
$P_b(k)$	$P_t(d) = \text{Poisson}$				
	$\sum_v \binom{d_v}{3}$	$ \bowtie $	$ \Delta $	$ \wedge $	C
Delta	20 316 (955)	1287 (75)	21 587 (1,017)	299 836 (12 276)	0.216 (0.002)
Poisson	20 517 (1,069)	2492 (260)	22 917 (1,305)	357 399 (22,096)	0.192 (0.002)
Exponential	20 100 (936)	9705 (1,431)	28 080 (1,657)	488 607 (28 643)	0.173 (0.002)
$P_b(k)$	$P_t(d) = \text{Exponential}$				
	$\sum_v \binom{d_v}{3}$	$ \bowtie $	$ \Delta $	$ \wedge $	C
Delta	76 420 (10,542)	5105 (709)	81 406 (11,193)	818 619 (91,143)	0.299 (0.011)
Poisson	74 837 (8,724)	9556 (1,349)	83 836 (9,803)	929 043 (93,213)	0.27 (0.01)
Exponential	77 457 (11,075)	37 490 (11,084)	105 281 (17,670)	1 263 366 (183 203)	0.249 (0.008)

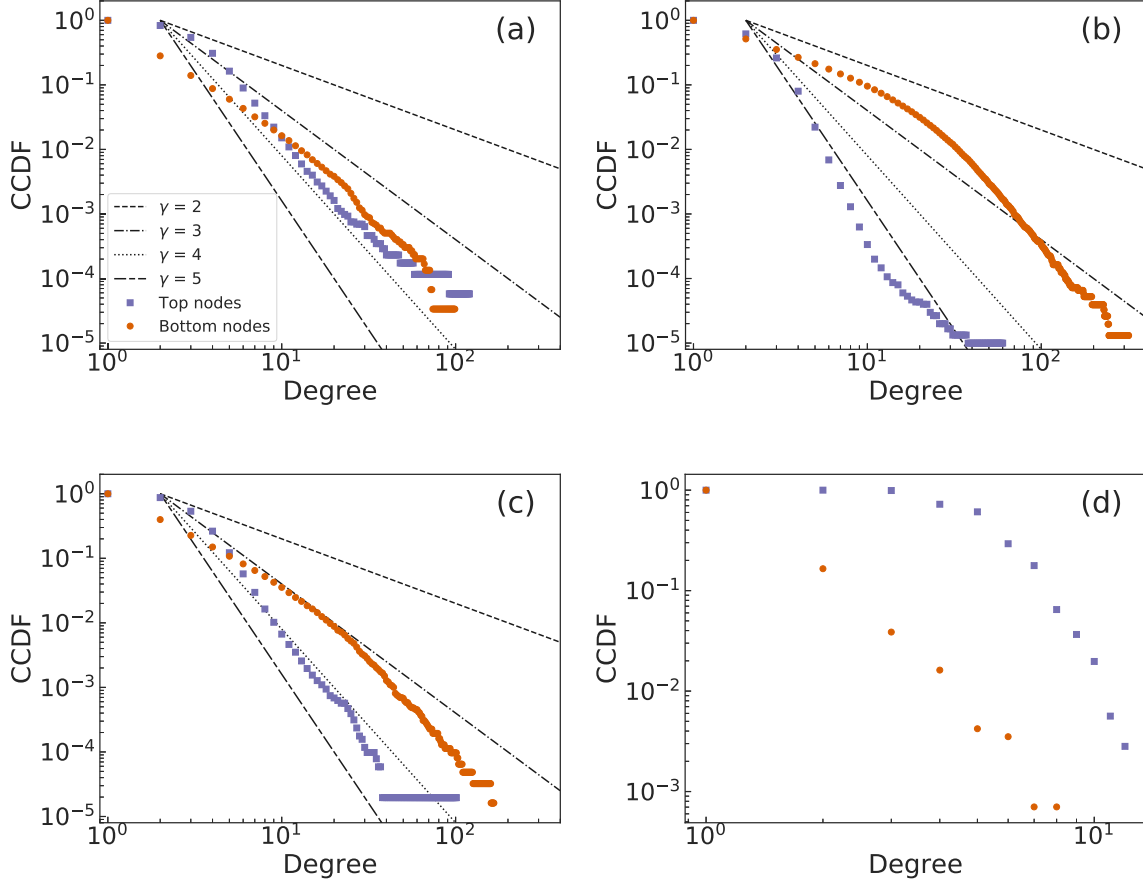


FIG. 3. Degree distributions (CCDF, complementary cumulative distribution function) of top and bottom node sets for (a) arXiv Biology, (b) arXiv Mathematics, (c) Physical Review E (PRE), and (d) Norwegian directors. The heavy-tailed top degree distributions of arXiv Biology, and PRE, supported by the high presence of small cycles (Table II), result in high projected transitivity and degree assortativity. This is so even with both cases displaying heavy-tailed bottom distributions, which create large numbers of open triplets and high relative branching. On the other hand, peaked distributions of the Norwegian board-of-directors network result in comparatively small projected degree assortativity, but high transitivity as the top distribution is more shifted to the right than the bottom distribution. In this case, transitivity is driving assortativity.

the bottom degree distribution is, the more open triplets can be found in the projection. Table I shows the number of triangles and triplets in projections of several synthetic BiCM networks. While top nodes create cliques in the projected network, bottom nodes connect these cliques together. Then, networks that are naturally one-mode, if modeled as projections of bipartite networks [i.e., $P_t(d)$ is a delta distribution with $d^* = 2$], would have triangles created exclusively by six-cycles. In a case where $d^* = 2$ and $|\mathbb{X}\mathbb{X}| \rightarrow 0$, we have the special case of bipartite networks that result in one-mode projections with $C \rightarrow 0$.

In Ref. [5], Estrada has shown that “The assortativity of a network depends on the balance between three structural factors: transitivity (clustering), intermodular connectivity, and relative branching.” The first two contribute positively to the increase of assortativity, while the third contributes negatively. The degree assortativity r , in terms of such structural features (Eq. (6) of Ref. [5]), is given by

$$r = \frac{|P_2|(|P_{3/2}| + C - |P_{2/1}|)}{3|S_{1,3}| - |P_2|(|P_{2/1}| - 1)}, \quad (6)$$

where $S_{1,3}$ is a star subgraph of four nodes, $|P_2|$ is the number of paths of size two, C is the transitivity, $|P_{3/2}|$ is the ratio of the number of paths of size three by the number of paths of size two—the so-called intermodular connectivity, and $|P_{2/1}|$ is the relative branching. For a more fundamental explanation of these concepts, see Ref. [5].

In the next section, we use empirical data to further the discussion on how these structural properties mentioned so far are entangled.

IV. EMPIRICAL OBSERVATIONS

If the degree distributions and six-cycles of B determine transitivity, they certainly play a part in the projected assortativity. Similarly, if four-cycles affect the projected degree distribution, it is reasonable to assume that they might affect the degree assortativity as well. In Ref. [6], we have already shown that BiCM can generate degree assortative, neutral, and dissortative projected networks, different to what was previously believed [15], depending on the combination of $P_t(d)$ and $P_b(k)$. However, the BiCM is not enough to fully understand degree assortativity, precisely due to the absence

TABLE II. Summary of several structural properties related to transitivity and degree assortativity for the empirical networks and their respective configuration model—mean (standard deviation) over 100 runs. The BiCM breaks the particular patterns found in empirical social networks. The result is a much larger number of open triplets and a higher level of relative branching, decreasing both projected transitivity and degree assortativity, respectively, when compared to the empirical projections. As the number of four- and six-cycles is relatively low in the Norwegian Board network, the differences between the projected empirical and configuration model networks are much more subtle than for the other cases.

Properties	arXiv Biology		arXiv Mathematics		Physical Review E		Norwegian Board	
	Empirical	Conf. model	Empirical	Conf. model	Empirical	Conf. model	Empirical	Conf. model
r_B	−0.09	0.00 (0.00)	−0.10	−0.09 (0.00)	−0.07	−0.05 (0.00)	−0.65	−0.63 (0.01)
$ \mathbb{X} $	34 659	85 (10)	515 144	148 (13)	131 544	150 (13)	92	1.5 (1.2)
$ \mathbb{X}\mathbb{X} $	5272	943 (91)	263 487	2418 (75)	47 877	1895 (99)	7	2.4 (1.7)
r_{G_s}	0.88	0.37 (0.02)	0.31	0.03 (0.00)	0.50	0.10 (0.01)	0.22	0.14 (0.02)
r_{G_m}	0.73	0.36 (0.02)	0.18	0.03 (0.00)	0.314	0.10 (0.01)	0.23	0.14 (0.02)
$ \Delta $	639 571	645 541 (4,746)	409 855	443 998 (965)	459 726	499 535 (2,687)	5606	5634 (17)
$ \wedge $	2 289 465	3 915 117 (63 604)	3 516 171	12 865 053 (64,747)	2 691 848	6 516 593 (62 436)	24 590	26 484 (394)
C	0.84	0.49 (0.01)	0.35	0.10 (0.00)	0.51	0.23 (0.00)	0.68	0.64 (0.01)
$ P_{2/1} $	24.5	35.9 (0.6)	11.5	27.2 (0.1)	16.2	29.2 (0.3)	6.4	6.7 (0.1)
$ P_{3/2} $	72.4	62.7 (1.3)	20.4	29.8 (0.4)	34.5	36.6 (0.8)	6.4	6.6 (0.2)

of large numbers of four- and six-cycles in random networks when compared to real-world networks [12]. Thus, let us now explore the topology of several empirical bipartite networks (i.e., bipartite assortativity, degree distributions, and four- and six-cycles) and its relation with the transitivity and the degree (and strength) assortativity of their projections (i.e., triangles, triplets, relative branching, and intermodular connectivity).

For that, we create three large scientific networks—connecting authors to papers, for every paper published in arXiv Biology, arXiv Mathematics, and Physical Review E (PRE), since their beginning in the 1990s until 2015—and a board-affiliation network, which is a snapshot of directors

sitting on the board of public limited liability companies in Norway, in 2011 [8]. The bipartite degree distributions of these networks are shown in Fig. 3. We also create their corresponding BiCM, keeping the same degree sequence as the original networks, but rewiring the links at random, following the method we use in Ref. [16]. The structural properties of our interest, of both empirical and BiCM networks, are listed in Table II.

In these four networks, the bipartite assortativity r_B is negative and unrelated to the projected degree assortativity r_{G_s} and the node strength assortativity r_{G_m} , which are both positive. Assortativity values are calculated as the Pearson coefficient of the degree-degree correlation [1]. In addition,

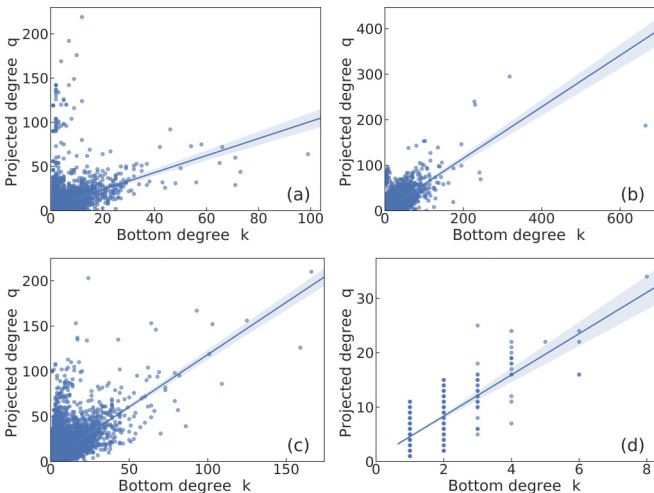


FIG. 4. Correlation between the bottom degree k and the projected degree q for (a) arXiv Biology with Person's coefficient $r = 0.23$, (b) arXiv Mathematics with $r = 0.68$ (c) – Physical Review E with $r = 0.56$, and (d) Norwegian directors with $r = 0.73$. Although many low degree bottom nodes have high projected degree (i.e., connected to high top degree nodes), especially in panel (a), we see a positive correlation between k_u and q_u^s . That is, the higher k_u is, the more likely that u will have high q_u^s .

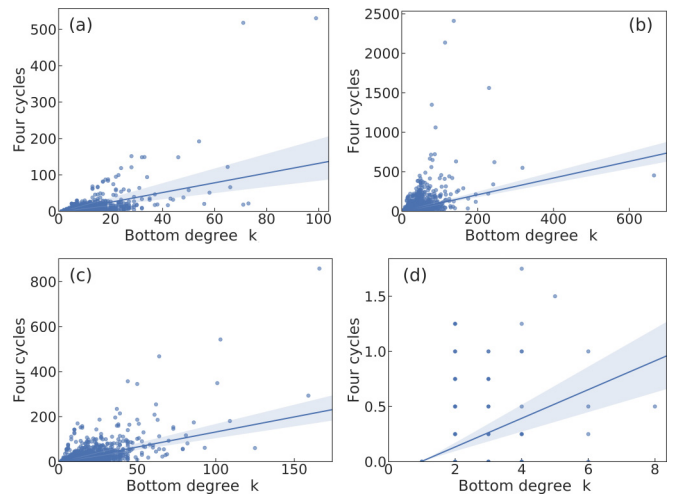


FIG. 5. Correlation between the bottom degree k and the number of four-cycles in B a node is part of for (a) arXiv Biology with Person's coefficient $r = 0.61$, (b) arXiv Mathematics with $r = 0.51$, (c) – Physical Review E with $r = 0.66$, and (d) Norwegian directors with $r = 0.52$. The positive correlations mean that high degree bottom nodes are more likely to be part of several four-cycles (to have several common neighbors with other bottom nodes).

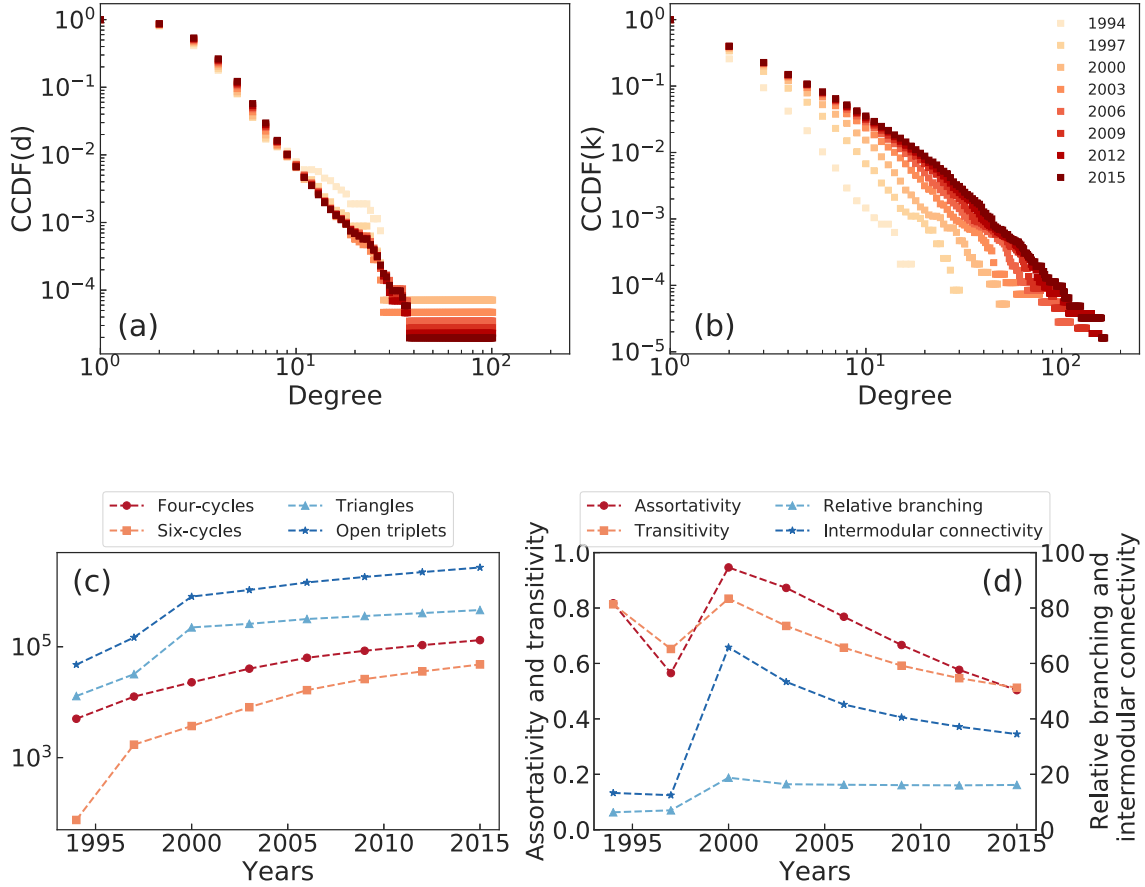


FIG. 6. Evolution of PRE network topology until 2015. (a) Top and (b) bottom degree distribution; (c) four- and six-cycles, triangles, and triplets; and (d) transitivity, assortativity, relative branching, and modular connectivity. The highest-degree top node enters the network (snapshot of the year 2000), increasing transitivity and assortativity as it favors triangles and intermodular connectivity, respectively. As only the bottom degree distribution gets shifted to the right with time, open triplets and relative branching are favored, reducing transitivity and assortativity as the network grows.

r_{G_s} is also calculated using Eq. (6), which gives the same results, as expected.

From this first observation, we can say that, as top and bottom node degrees are uncorrelated, the bottom degree k_u of node u plays a major role in the resulting projected degree (and strength). That is, u is more likely to have high q_u^s (and q_u^m) if it has high k_u in B (Fig. 4). However, high degree bottom nodes are also more likely to be part of more four-cycles in the bipartite network (Fig. 5), increasing the difference between q_u^s and q_u^m , in the projection. As a consequence, the node strength distribution is more right-skewed than the degree distribution [12]. We conjecture that this is why degree assortativity is higher than the node strength assortativity in the projections of bipartite networks with heavy-tailed top distributions [arXiv Biology (arXiv Bio), arXiv Mathematics (arXiv Maths), and PRE], but the same for the Norwegian network. The latter has peaked distributions [Fig. 3(d)], and less four-cycles per node, with a smaller correlation slope than the others (Fig. 5).

Continuing on degree distributions, a heavy-tailed degree distribution of top nodes is expected to generate high levels of transitivity, as previously mentioned, and of degree assortativity [6], restricted, however, by the tail of the bottom

distribution. The heavier tail of the top distribution in the arXiv Bio network, for instance, induces more triangles than the top distributions of the arXiv Maths and PRE networks. At the same time, its more peaked bottom distribution induces less open triplets than the other two bottom distributions. As expected, arXiv Bio has much larger transitivity $C = 0.84$, than arXiv Maths ($C = 0.35$) and PRE ($C = 0.51$). Although the arXiv Bio top distribution results in high relative branching ($|P_{2/1}| = 24.5$), it results much more so in intermodular connectivity ($|P_{3/2}| = 72.4$). The result of $|P_{3/2}| > |P_{2/1}|$ and high C is large values of degree assortativity r_{G_s} [Eq. (6)]. As expected, assortativity in arXiv Bio is larger than that in arXiv Maths and PRE. For the Norwegian network, which displays peaked top and bottom distributions, $|P_{3/2}| = |P_{2/1}|$, thus transitivity drives r_{G_s} . Although both distributions are peaked, the top distribution is more shifted to the right, i.e., a relatively large number of triangles compared to the number of triplets. As a consequence, transitivity is high, and the projected network is degree assortative, with $r_{G_s} = 0.17$.

Let us now compare the empirical networks with the BiCM, to have a clear understanding of the role of the small-cycles motifs affecting transitivity and assortativity, apart from the effects of $P_t(d)$ and $P_b(k)$. The process of randomly

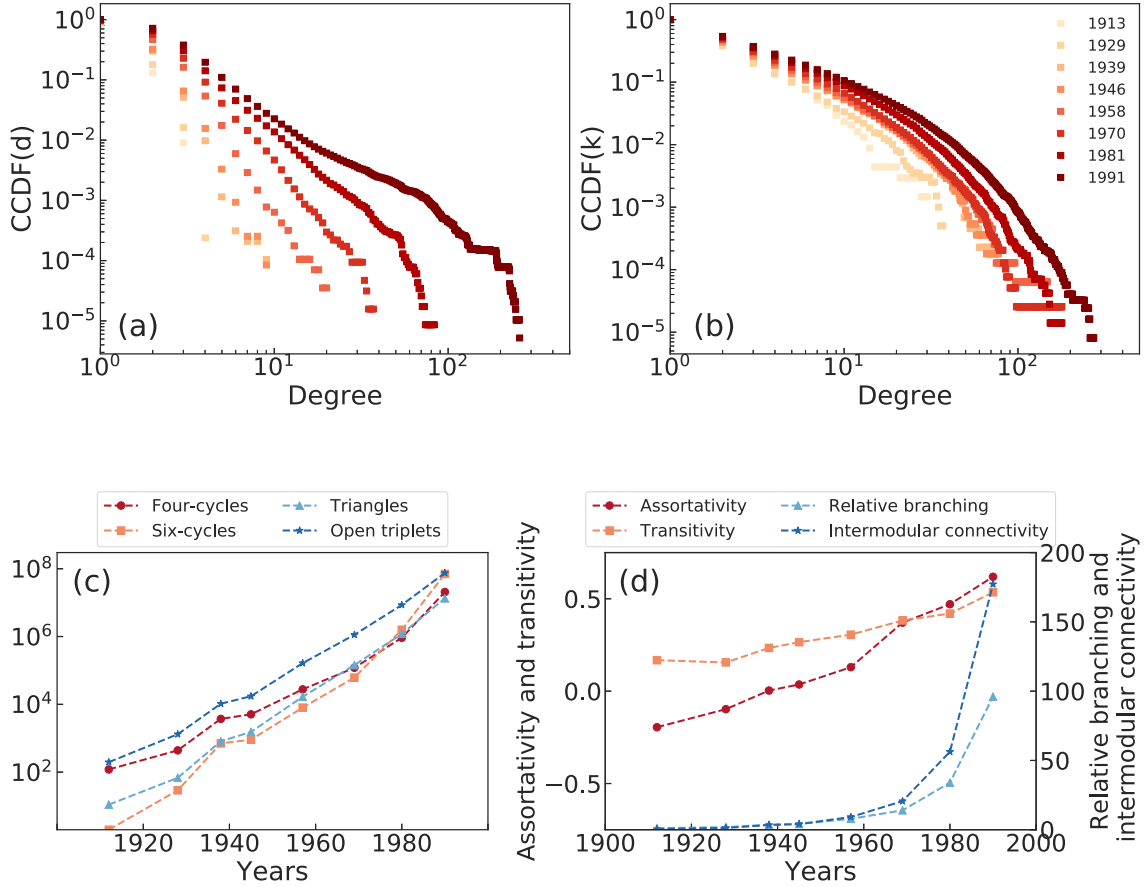


FIG. 7. Evolution of APS network topology until 1991. (a) Top and (b) bottom degree distribution; (c) four- and six-cycles, triangles, and triplets; and (d) transitivity, assortativity, relative branching, and modular connectivity. Initially the network is dissortative, due to the combination of a peaked top degree distribution, a broader bottom distribution than the top one, and the low presence of four- and six-cycles. As the network evolves, the top distribution becomes broader and the frequency of small cycles grows substantially, resulting in large transitivity and degree assortativity.

rewiring links of the BiCM breaks structural patterns found in empirical networks. At a first glance, one could expect that this would always reduce the number of triangles when projecting, as six-cycles are broken and the second term of Eq. (4) is smaller. However, not only six-cycles are broken in the BiCM, but also four-cycles and other patterns similar to those in Fig. 2. As we can see in Table II, for every case, the number of six-cycles in the BiCM diminishes significantly when compared to the empirical value, but the number of triangles in the projection actually grows. Nonetheless, the expansion of the number of open triplets in the projection, due to the BiCM rewiring, is much higher than any variation of the number of triangles. Thus, transitivity in BiCM projections is smaller than in empirical networks for every case, as the major consequence of these small-cycles motifs (concerning transitivity) is the suppression of open triplets.

Likewise, regarding assortativity, small cycles are suppressing relative branching. To explain this, let us work out an example. Extremely heavy-tailed top degree distributions, for instance, following a power-law $P_t(d) \propto k^{-\gamma}$, with $\gamma \approx 2.0$, will create some huge cliques in the projected network, increasing transitivity, intermodular connectivity, and relative branching. If the bottom distribution is peaked, intermod-

ular connectivity is higher, as the cliques in the projection are likely to be linked together, and thus the network is assortative. However, if the bottom degree distribution is extremely heavy-tailed, like the top degree distribution, projected starlike structures (i.e., a bottom node connected to several top nodes with degree $d_v = 2$ in B) are more common. Then, cliques are more distant from each other, which, in turn, can result in a dissortative projection as relative branching can dominate. The presence of four-cycles helps to prevent the appearance of starlike structures and cliques from becoming too separated, diminishing relative branching and increasing the value of r_{G_s} , from what one would be expected from the observed degree distributions $P_t(v)$ and $P_b(k)$. In Ref. [3], the authors stated that their model—based only on the degree distributions of bipartite networks (group structure)—explains only part of the observed assortativity of the case studied. They say they “conjecture that this is due to true sociological or psychological effects in the way in which acquaintanceships are formed.” Based on the results presented here, we claim that four- and six-cycles can represent, in the network, these sociological or psychological effects. While four-cycles are repeated interactions between a pair of agents (for instance, based on trust,

respect, empathy, and so on), six-cycles represent triadic closure [17,18].

In order to support these findings, let's take a particular case—the PRE network—and investigate the evolution of the network and its topology. To this end, we created eight cumulative snapshots from 1994 to 2015, with 3-yr intervals. We can notice that between 1997 and 2000, the highest-degree top node (the paper with the largest number of authors) enters the network, right-skewing the top degree distribution [Fig. 6(a)]. The consequence for the transitivity and assortativity are visible. The frequency of triangles increases, more than the number of open triplets, increasing transitivity [Figs. 6(c) and 6(d)]. Similarly, intermodular connectivity spikes, more than the relative branching, making assortativity reach an extremely high level of 0.95. As the years go by, the tail of the top distribution does not get any heavier, while the bottom distribution keeps being shifted to the right [Fig. 6(b)]. This behavior widens the gap between triplets and triangles and shortens the difference between intermodular connectivity and relative branching, slowly decreasing both transitivity and assortativity.

Last, we would like to answer the following question: is it possible to have a one-mode degree dissortative projected network? Fisher *et al.* [4] already presented a few examples where the assortativity of networks created with group-based methods is negative. Moreover, we have shown in Ref. [16] that in its “early” days (around 40 years) the American Physical Society coauthorship network (when Physical Review was the only journal of the society) is dissortative. Figure 7 shows the evolution of the APS network topology, from 1893 to 1991 (for the complete evolution of this network, until 2015, see Ref. [16]).

We can see that, until 1929, the top distribution is highly peaked and the bottom distribution has a heavier tail. The

small cycles keep relative branching in relatively low levels until then, but that is not enough, as we still have $|P_{2/1}| > |P_{3/2}| + C$. Thus, the network is dissortative. As the network grows, the top distribution becomes more and more heavy-tailed, with a high presence of small cycles—the sociological or psychological effect of researchers repeatedly writing papers with the same collaborators, and also closing open triplets, driving the network to become degree assortative.

V. CONCLUSION

We have shown that one-mode social networks display transitivity and degree assortativity higher than those of nonsocial networks due to the structural properties of the underlying bipartite network, but not simply because they are bipartite. We claim that every social network can be expressed as a bipartite network, be it affiliations, memberships, or the simplest realization of events like accepting a friendship request on social media. The main driver of the topology of a projected one-mode network is the combination between the degree distributions of top and bottom nodes in the bipartite network, supported by small-cycles motifs, i.e., four and six-cycles. The understanding of how these important topological features—transitivity and degree assortativity—emerge can be useful to improve studies and models of spreading phenomena on social networks, especially if group-based (bipartite) structures are considered.

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- [1] M. E. J. Newman, *Phys. Rev. Lett.* **89**, 208701 (2002).
 - [2] M. E. J. Newman, *Phys. Rev. E* **67**, 026126 (2003).
 - [3] M. E. J. Newman and J. Park, *Phys. Rev. E* **68**, 036122 (2003).
 - [4] D. N. Fisher, M. J. Silk, and D. W. Franks, *Trends in Social Network Analysis* (Springer, Berlin, 2017), pp. 1–19.
 - [5] E. Estrada, *Phys. Rev. E* **84**, 047101 (2011).
 - [6] D. Vasques Filho and D. R. J. O'Neale, *Phys. Rev. E* **98**, 022307 (2018).
 - [7] D. B. Larremore, A. Clauset, and A. Z. Jacobs, *Phys. Rev. E* **90**, 012805 (2014).
 - [8] C. Seierstad and T. Opsahl, *Scand. J. Manage.* **27**, 44 (2011).
 - [9] J.-L. Guillaume and M. Latapy, *Inf. Process. Lett.* **90**, 215 (2004).
 - [10] A. Barrat, M. Barthélemy, R. Pastor-Satorras, and A. Vespignani, *Proc. Natl. Acad. Sci. USA* **101**, 3747 (2004).
 - [11] M. Coscia and L. Rossi, *Proceedings of the 2019 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining, ASONAM '19* (Association for Computing Machinery, New York, USA, 2019), pp. 286–293.
 - [12] D. Vasques Filho and D. R. J. O'Neale, [arXiv:1909.10977](https://arxiv.org/abs/1909.10977) [J. Complex Netw. (to be published)].
 - [13] M. Latapy, C. Magnien, and N. Del Vecchio, *Soc. Networks* **30**, 31 (2008).
 - [14] T. Opsahl, *Soc. Networks* **35**, 159 (2013).
 - [15] J. J. Ramasco, S. N. Dorogovtsev, and R. Pastor-Satorras, *Phys. Rev. E* **70**, 036106 (2004).
 - [16] D. Vasques Filho, Ph.D. thesis, The University of Auckland, 2018.
 - [17] G. Simmel, *Soziologie: Untersuchungen über die formen der vergesellschaftung* (BoD—Books on Demand, Berlin, 2015).
 - [18] M. S. Granovetter, *Am. J. Soc.* **78**, 1360 (1973).