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PHYS276

Lab 2

I. Introduction

In order to properly measure the vertical depth of the 4 km shaft, we will be releasing a test 1 kg mass and measure the time it takes to reach the bottom. To understand the physics behind this, this lab dives into the different factors that will affect the fall time, such as gravity, drag, and density.

II. Fall Times - Gravity and Drag

$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$$

$$g(r) = g_o\left(\frac{r}{R_{\oplus}}\right)$$

This section calculates fall times by using this second-order differential equation, as shown on the left, where t is time, y is the height, g is the gravitational acceleration, “alpha” is the drag coefficient, and “gamma” is the speed dependence of the drag. A fall time considering a constant g and no drag (“alpha” = 0) gave t ~ 28.6 seconds. Considering a variable g (still no drag) gave t ~ 28.6 seconds (but not exactly equal to the previous t, ~ a 0.001 second difference). This was calculated by replacing g with g(r), where g0 is the gravitational acceleration, r is the distance from the center of Earth, and R_E is the radius of Earth. Finally, adding in drag gives a t of ~ 83.3 seconds, which is ~ 54.7 seconds greater than the previous t. This was done by assuming “gamma” = 2, and calibrating “alpha” to be 0.0039 so the plot (Figure 2) would show a terminal velocity of ~ 50 m/s.

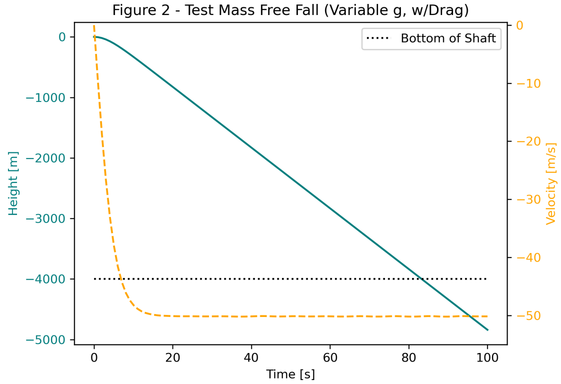

Figure 2 - Test Mass Free Fall (Variable g, w/Drag)

III. Coriolis Force

Still considering drag and a variable g, the ball will hit the shaft wall at t ~ 29.6 seconds (assuming you drop it from the center, and still with drag), which is before the ball hits the ground at t ~ 83.3 seconds. This wall bounce occurs at a depth of ____ meters. The drag did definitely play a factor since without it, the ball would’ve hit the wall at t ~ 21.9 seconds. However, the test mass should still reach the bottom, since it should be able to bounce off the wall and keep falling, possibly altering the time it hits the ground.

IV. Infinite Mines and Non-Uniform Density

By using density variation to calculate the Force in relation to the distance from the center of the Earth, we can start to see how density affects the Force of gravity. Figure 6 shows that near the center, the force is generally at its lowest. As the radius increases, the force increases (before decreasing slightly). We know from F = GMm/R^2 for planetary mass M and planet radius R that the force is

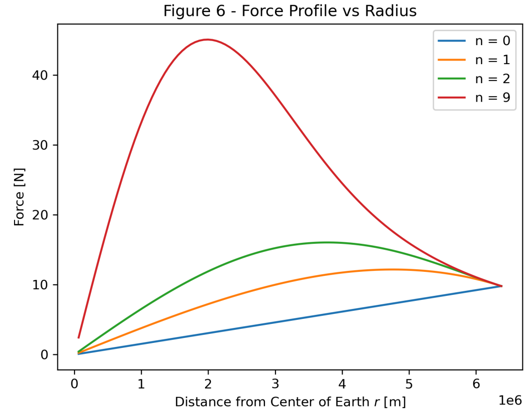

Figure 6 - Force Profile vs Radius

directly proportional to the mass of the planet (in this case: Earth). Also, consider how mass is directly proportional to density with M = pV. F = ma also shows that the force is directly proportional to the acceleration, which is inversely proportional to t^2. So, the density p must be inversely proportional to the time squared. This time can be considered as "fall-time," so rewording this, the fall-time would be inversely proportional to the square root of density. As density increases, the fall-time would decrease as the square root of density.

Without considering the density variation (homogenous Earth), the time the ball crosses the center is the same as we calculated before: t ~ 83.3 seconds. However, a non-homogenous Earth would give a crossing time of t ~ ____ seconds for n = _____.

Consider an infinitely deep mine that stretches from pole-to-pole for both the Earth and the Moon. Dropping a ball down Earth's tunnel, the ball would cross the center of Earth at t ~ 1266.5 seconds. Doing the same on the Moon, the ball would cross the center of the Moon at t ~ 1624.3 seconds.

V. Discussion and Future Work

All the values listed here were presented with a 0.1 unit precision, whereas the code has the values calculated to Python's own full precision. The constant gravitational acceleration used was an approximate value of 9.81 m/s^2. A pendulum experiment could generate a more accurate g value. Sections II and III entirely approximate that the mass is homogenously distributed, but Section IV takes care of this. The speed dependence of the drag "gamma" is assumed to be = 2, and the terminal velocity of the ball is assumed to be 50 m/s to calibrate the drag coefficient "alpha." Conducting an experiment with an actual free-fall mass over a large height, measuring the terminal velocity, and analyzing the data, could give us a more accurate average terminal velocity. With that, we could also determine a best-fit parameter for gamma that agrees with the data. For the entire lab, we've also assumed a perfectly spherical Earth. If there was some way to model the Earth's varying elevations and calculate the volume based on that new shape, that would make the mass calculated from the integral more realistic.