Apollo 11 Report

I. Introduction

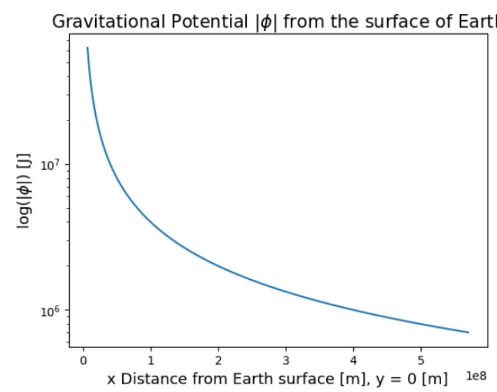
This country was put on a countdown with President Kennedy's "man, moon, decade" speech. As engineers, we now have the responsibility of achieving that goal through physics and the help of Python coding to make our calculations as accurate and efficient as possible, minimizing failure.

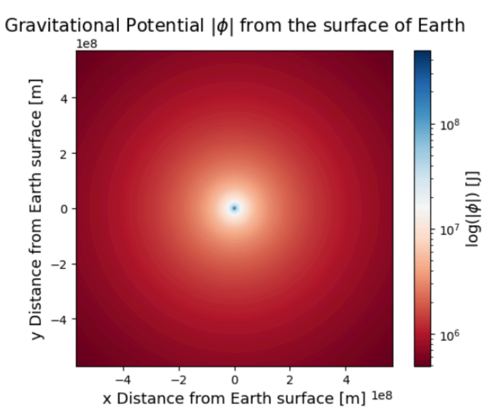
II. The gravitational potential of the Earth-Moon System

Our goal in this section is to evaluate the gravitational potential from the surface of Earth, which is the amount of potential energy an object has above the Earth. Based on mechanical physics, we recognize that the gravitational potential for a mass *M* at a distance *r* away is:

$$\Phi(r) = -\frac{GM}{r}$$

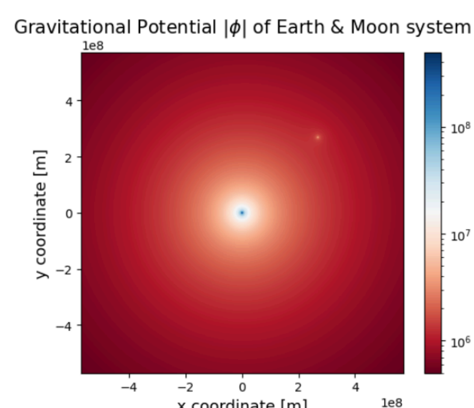
where *G* is the gravitational constant. To see how this energy can vary over a range of distances, we can make a 1D plot. First, we define a function based on the equation above, then graph its output from the surface of Earth to 1.5 times the distance from the Earth to the Moon. This results in a graph that looks like the one on the right, once we take the absolute value of gravitational potential and make the y-axis logarithmic to better see the curve, otherwise, it would drop dramatically at the start.



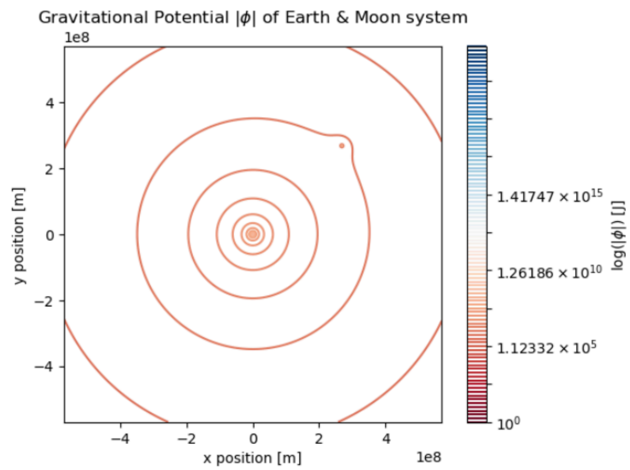


We can also visualize the gravitational potential differently with a 2D color-mesh plot. In this case, the x and y axes represent the distance from Earth's surface, while the color itself tells us the amount of gravitational potential energy. The graph is shown on the left. The color bar tells us the corresponding value of gravitational potential energy.

Since our mission involves going to the Moon, we should also consider the gravitational potential energy of the Moon. In other words, we're considering the situation as an Earth-Moon system. We can add the gravitational potential energy



of the Moon to our color-mesh plot by using the same function, just applying it to the Moon. Finally, we add the Earth's and Moon's energies together, and plot that instead. This result is the graph on the right. We can see a hint of the Moon's presence towards the upper-right corner. This makes sense since the Moon has less mass than the Earth, it will have less gravitational potential energy.



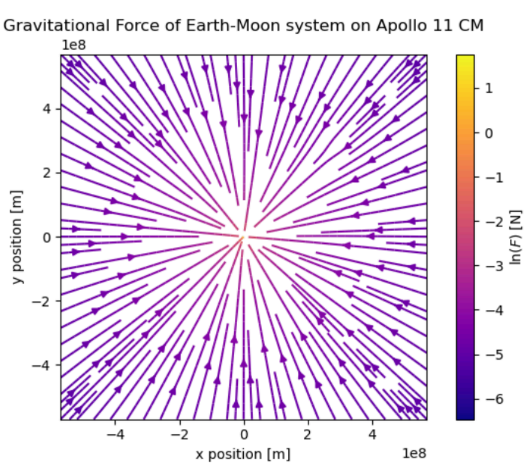
Another way we can visualize the Earth-Moon system's gravitational potential energy is through a 2D contour plot, located on the left. While similar to the color-mesh plot, the radial aspect is represented through level curves, a constant value of gravitational potential energy located along that line. Again, the colors still represent the magnitude of gravitational potential energy.

III. The gravitational force of the Earth-Moon system

Our goal in this section is to find the gravitational force of the Earth-Moon system exerted onto the Apollo Command Module (CM). As the CM tries to navigate this space, it will go under gravitational forces from both the Earth and the Moon. From mechanical physics, the gravitational force F that a mass $M1$ exerts on mass $m2$ is:

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

where $r21$ is the displacement vector from $M1$ to $m2$.



Like before, we can create a function based off of this equation. However, to plot the force on the CM based on these two bodies, we will need to calculate the forces individually. Meaning, we calculate the force from the Earth on the CM, and the force from the Moon on the CM, to ultimately add them together for a total force. Only then can we plot this force, specifically using a 2D streamplot. This type of plot can tell us the direction of this force with the arrows, as well as its magnitude with the color.

IV. Projected performance of the Saturn V Stage 1

A huge concern that can determine the success or failure of the mission is how we can manage to get the Saturn V rocket off the ground in the first place. Since our rocket is multi-stage, we'll first look at Stage 1 to see how high it can get off the ground based on the fuel exhaust propelling the rocket forwards. To start, we find the burn time T for the amount of fuel we have, given by:

$$T = \frac{m_0 - m_f}{\dot{m}}$$

where m0 is the initial mass that considers fuel, rocket parts, and payload; mf is the final mass after all fuel was burned; and mdot is the fuel burn rate, assuming that it's constant. After calculating this, I found a total burn time of ~ 158 seconds.
Next, we consider the rocket's change in velocity, which is:

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

where ve is the fuel exhaust velocity, g is the gravitational acceleration of the Earth, and t is time. m(t) is the amount of mass at a given time, noted by:

$$m(t) = m_0 - \dot{m}t$$

If we create a function for change in velocity, then integrate it from the launch time to the burn time, we can find the altitude *h* that Stage 1 gives us. Using scipy.integrate.quad() to integrate, I found a peak altitude of 74.1 +/- 6e-11 km for the rocket at the end of the burn.

V. Discussion and Future Work

We found a peak altitude of 74.1 +/- 6e-11 km for the rocket at the end of the burn time of ~ 158 seconds. However, the initial test resulted in an altitude of about 70 km with a burn time of 160 seconds. Like I mentioned earlier, we assumed that the burn rate was constant. But in reality, it most likely isn't, which can contribute to this underapproximation of burn time. There are also other factors such as air drag that could slow down the rocket, resulting in a lower altitude like the test shows.