IN THE NAME OF GOD

Linear control system 1

Professor **Bahrami**

Project 2

Lag vs **Lead** compensation

Group : 13

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I. *Introduction*

In this project we are going to learn about lead and lag compensation , their properties and how they can affect the functionality of a system .

Lead compensator :

$$G\hat{}c(s)=K\hat{}c * \frac{s+\frac{1}{T}}{s+\frac{1}{aT}} \quad (0 < a < 1)$$

Lag compensator :

$$Gc(s)=Kc * \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad (\beta > 1)$$

II. *Part 1.2*

The system block diagram is given below:

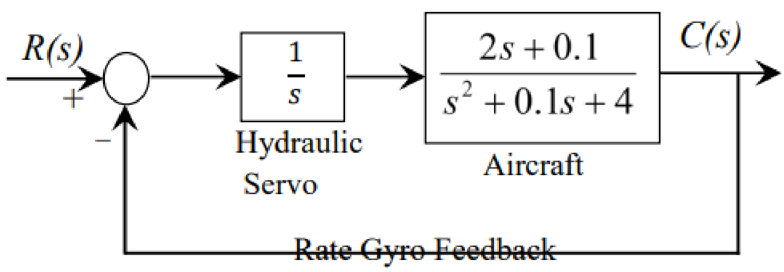

Figure 1: Block diagram of the original system

A.

- ✓ Plotting the open loop bode diagram.
- ✓ obtaining <u>Gain Margin</u>, <u>Phase Margin</u>, <u>Phase-Crossover Frequency</u> and <u>Gain-Crossover Frequency</u>.

The open loop transfer function is : Gop(s)= $\frac{2*s+0.1}{s*(s^2+0.1*s+4)}$

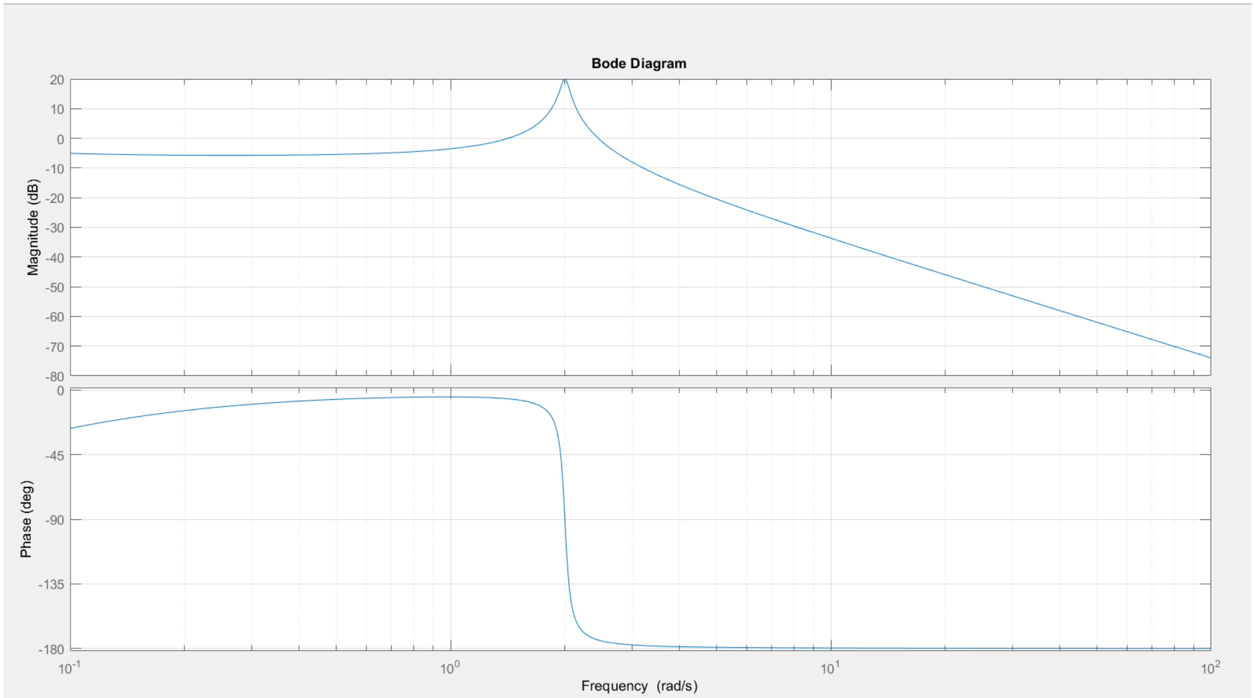

Figure 2: Bode diagram for the uncompensated system

- Phase margin:

The phase margin is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability. The gain crossover frequency is the frequency at which $|G(jw)|$,the magnitude of the openloop transfer function, is unity. The phase margin g is 180° plus the phase angle f of the open-loop transfer function at the gain crossover frequency, or

g=180°+f

- Gain margin:

The gain margin is the reciprocal of the magnitude $|G(jw)|$,at the frequency at which the phase angle is –180°. Defining the phase crossover frequency w1 to be the frequency at which the phase angle of the open-loop transfer function equals –180° gives the gain margin Kg:

$$Kg=\frac{1}{|G(jw1)|}$$

In terms of decibels:

$$Kg\ dB = 20^* \log Kg = -20 \log |G(jw)|$$

- Note that:

The gain margin of a first- or second-order system is infinite since the polar plots for such systems do not cross the negative real axis. Thus, theoretically, first- or second-order systems cannot be unstable. (Note, however, that so-called first- or second-order systems are only approximations in the sense that small time lags are neglected in deriving the system equations and are thus not truly first- or second-order systems. If these small lags are accounted for, the so-called first- or second-order systems may become unstable.)

Also as we can see from the bode plot the phase diagram will cross the frequency axis in the infinity. so the gain-margin and the phase-crossover-frequency for this system is infinite.

Finally, by analyzing the bode plot we have:

[Gm,Pm,Pcf,Gcf]={Inf, 5.8540, Inf, 2.4465}

B.

The closed-loop transfer function of the original uncompensated system is:

$$\frac{C(s)}{R(s)} = \frac{2 * s + 0.1}{s^3 + 0.1 * s^2 + 6 * s + 0.1}$$

The closed-loop poles of the uncompensated system are:

$\{-0.0417 + j * 2.4488, -0.0417 - j * 2.4488, -0.0167\}$

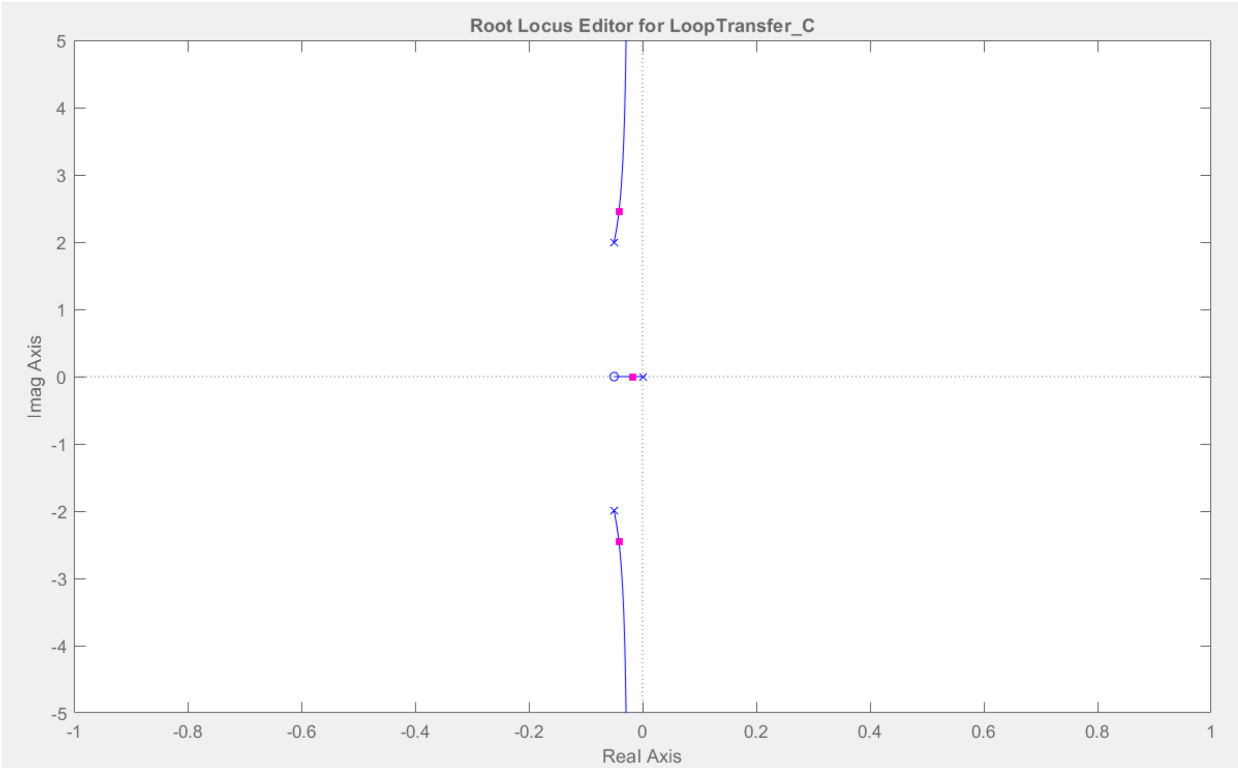

Figure 3: The closed-loop vs open-loop Root-Locus(Original system)

C.

$$Kv = \lim_{s\to 0} s * Gop(s) = \lim_{s\to 0} \frac{s*2*(s+0.05)*k}{s*(s^2+0.01*s+4)} = \frac{k}{40} \to ess = \frac{1}{kv} = 40/k$$

Since the system is stable for all the range of k, so for the infinite k we have ess=0.

D.

```
num=[2 0.1];
den=conv([1 0],[1 0.1 4]);
sys_op=tf(num,den);
sys_cl=feedback(sys_op,1);
w=logspace(-1,2,1000);
bode(sys,w)
grid on;
step(sys_cl)
```

Figure 4: coding for part 1.2

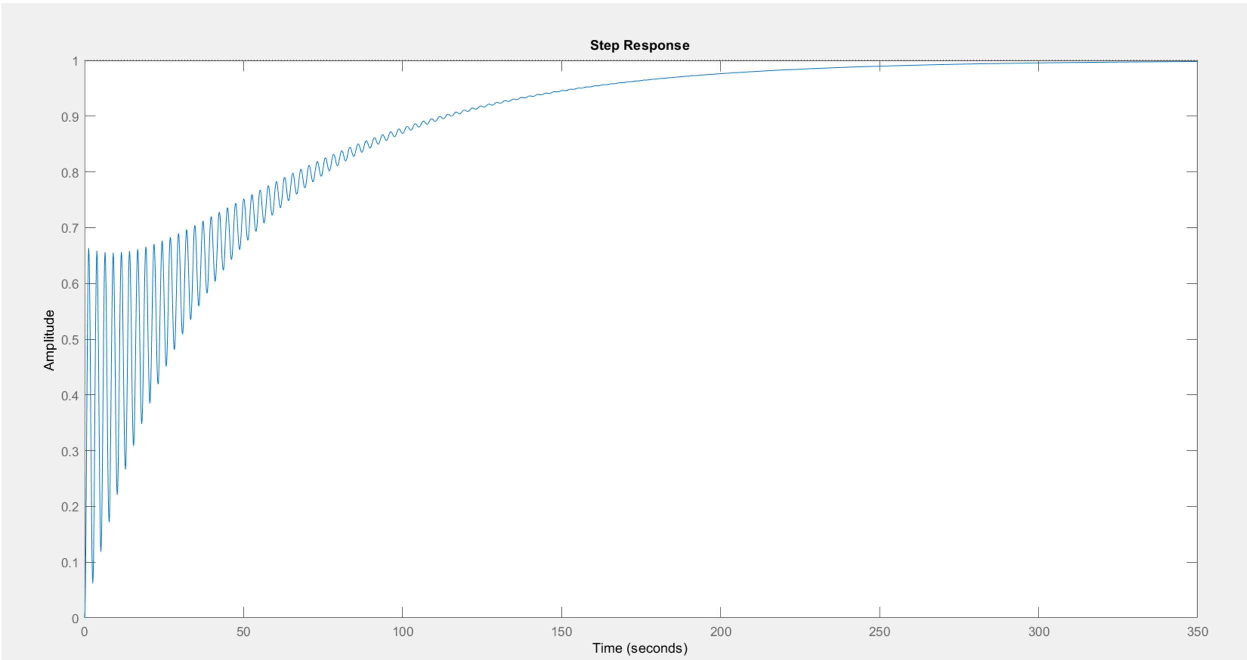

Figure 5: Step response for part 1.2(MATLAB)

By simulating the system in Simulink platform, we will have:

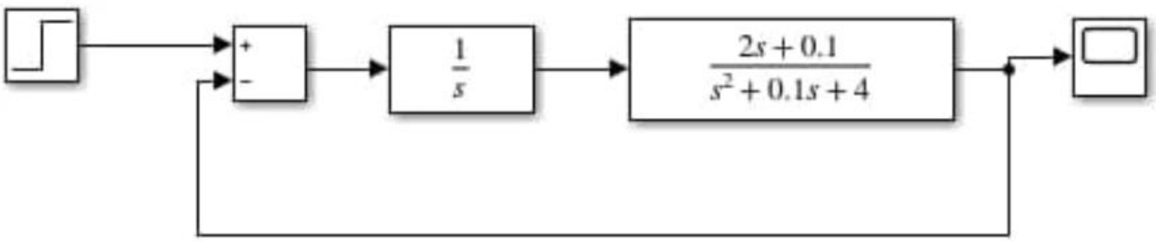

Figure 6: Block diagram of the uncompensated system in Simulink

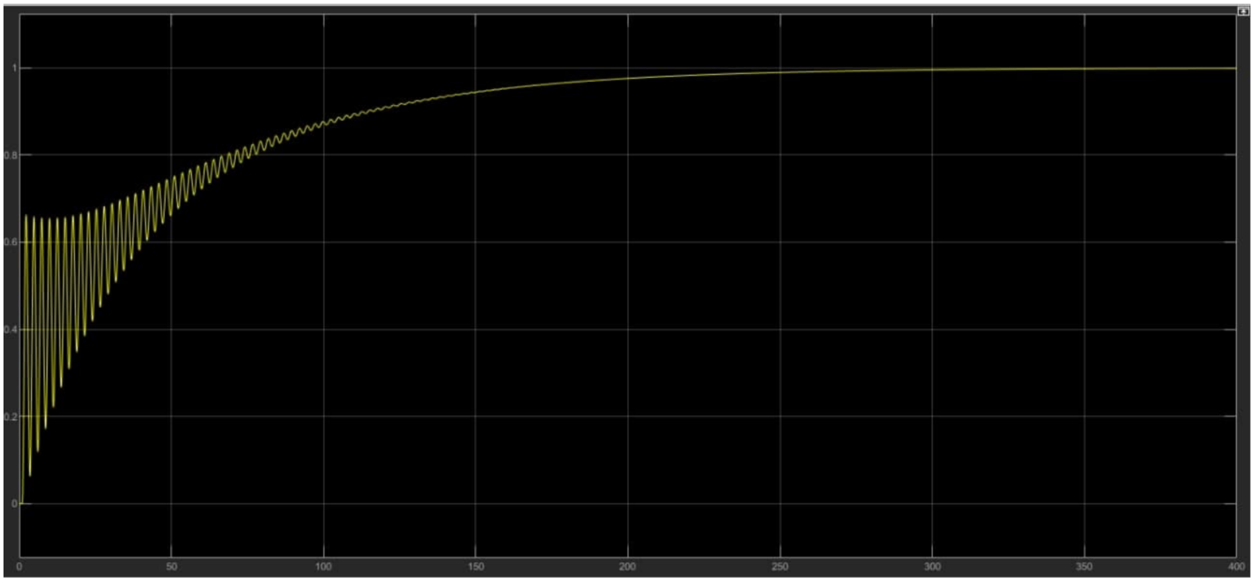

Figure 7: Step response for part 1.2(Simulink)

To design a compensator for such a system, it is desirable to cancel the zero of the plant , since it is located very close to the origin.

Note that the open-loop zero at and the open-loop pole at s=0 generate a closed-loop pole between s= 0 and s= -0.05. Such a closed-loop pole becomes a dominant closed-loop pole and makes the response quite slow. Hence, it is necessary to replace this zero by a zero that is located far away from the $j\omega$ axis.

III. Part 1.3

In this part we have two important aims:

- ✓ To eliminate or decrease the fluctuation at the beginning of the step response waveform (This part is going to be fixed by the lead compensator).
- ✓ To increase the pace of the step response so that the settling time will be 2 second and also the overshoot will be less than 16 percentages (This part is going to be fixed by the lead and lag compensator).

A. As previously mentioned, we have to cancel the zero of the plant by adding a lag compensator, here is the procedure:

   The new open loop transfer function will be reformed as below:

Gop-n(s)= $2 * Kc * \frac{\left(s+\frac{1}{T}\right)*(s+0.05)}{s*\left(s+\frac{1}{\beta T}\right)*(s^2+0.1*s+4)} = K * \frac{(s+\frac{1}{T})}{s*(s^2+0.1*s+4)}$

⇨ {K=2*Kc, βT=20}

For example, by considering the zero at s=-4:

s=-4 => $\frac{1}{T}$ = 4 => T =1/4 => β =80

=> Gop-n(s)= $K * \frac{(s+4)}{s*(s^2+0.1*s+4)}$

A root-locus plot for the compensated system is shown below:

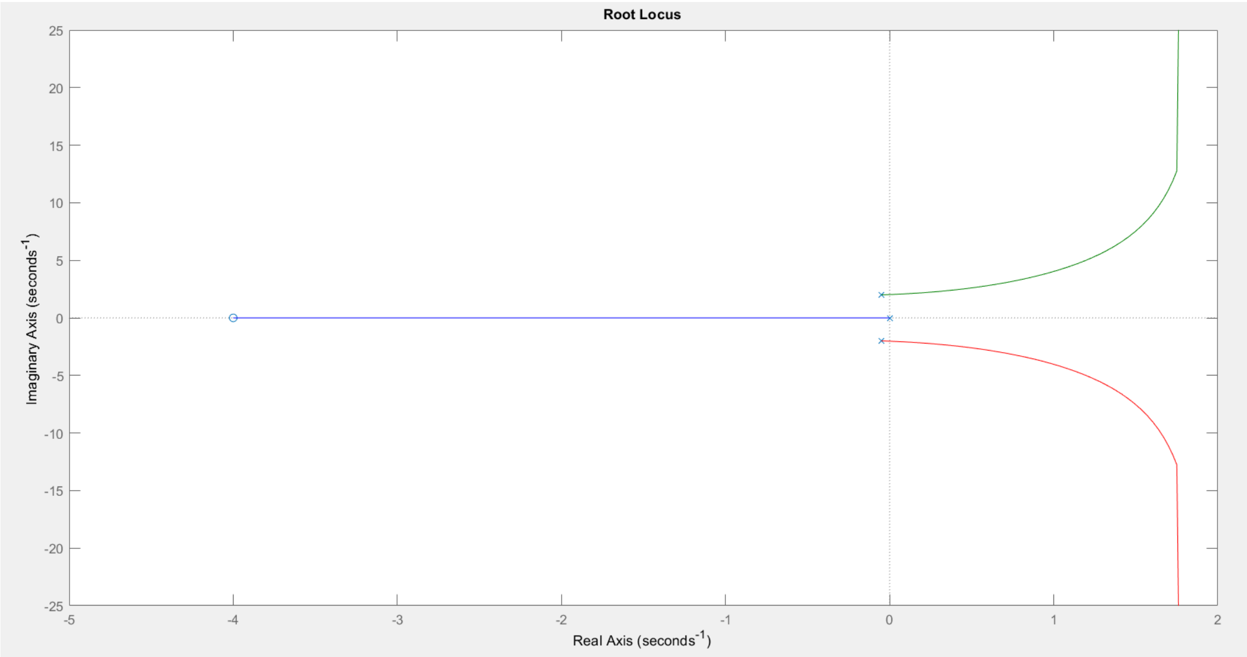

Figure 8: Root-Locus Plot of compensated system (Lag compensator).

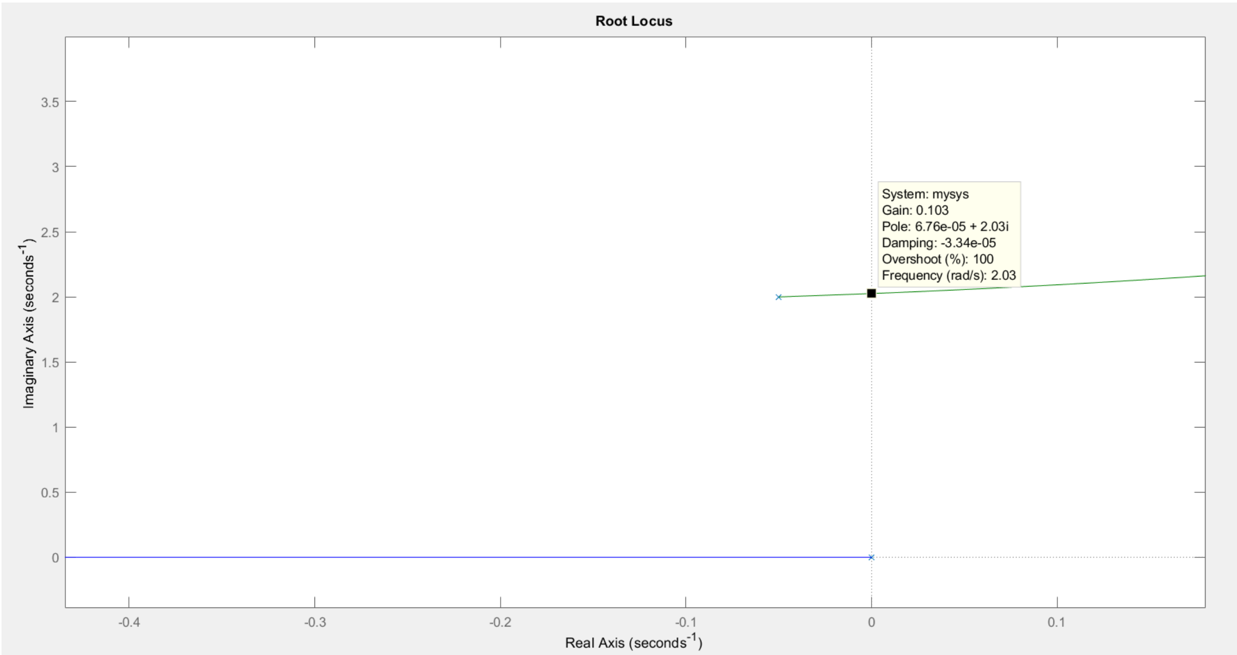

Figure 9: The boundary for the stability of the system (Lag compensator)

As we can see the maximum gain which yields the system to be unstable is 0.103 .
=> $\forall k\ s.t\ \ \exists\ k \leq 0.102\ , The\ compensated\ system\ is\ stable.$

Here are the closed-loop poles and the step response for k=0.102:

Cl-poles ={ -0.0003 + 2.0253i , -0.0003 - 2.0253i, -0.0995 + 0.0000i}

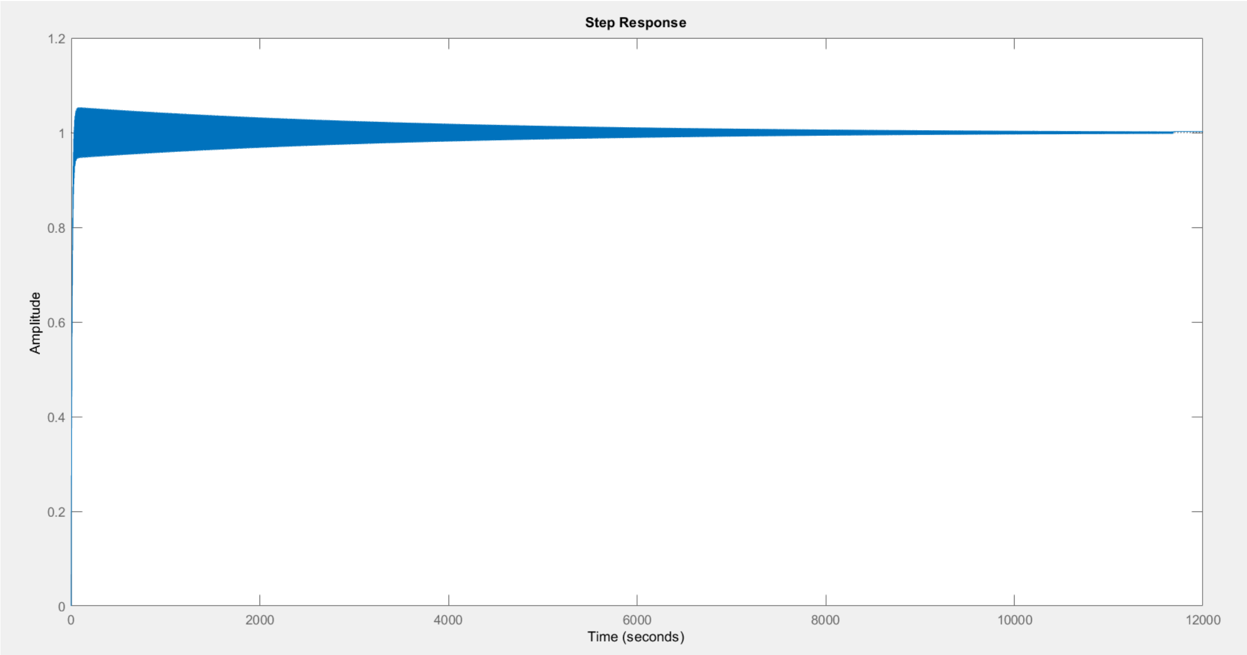

Figure 10: The step response for zero at “-4” and k= 0.102

As you can see the system is stable, and the mixed conjugate poles are the dominant poles, but still the step response is quiet slow because the real part of dominant poles is very small such that it takes 12,000 seconds to converge to the final value.

So we are going to decrease the value of gain(k) , so that we reach to an ideal boundary in which we have the best speed response.

Here are the values assigning to the gain:

a. K=0.08

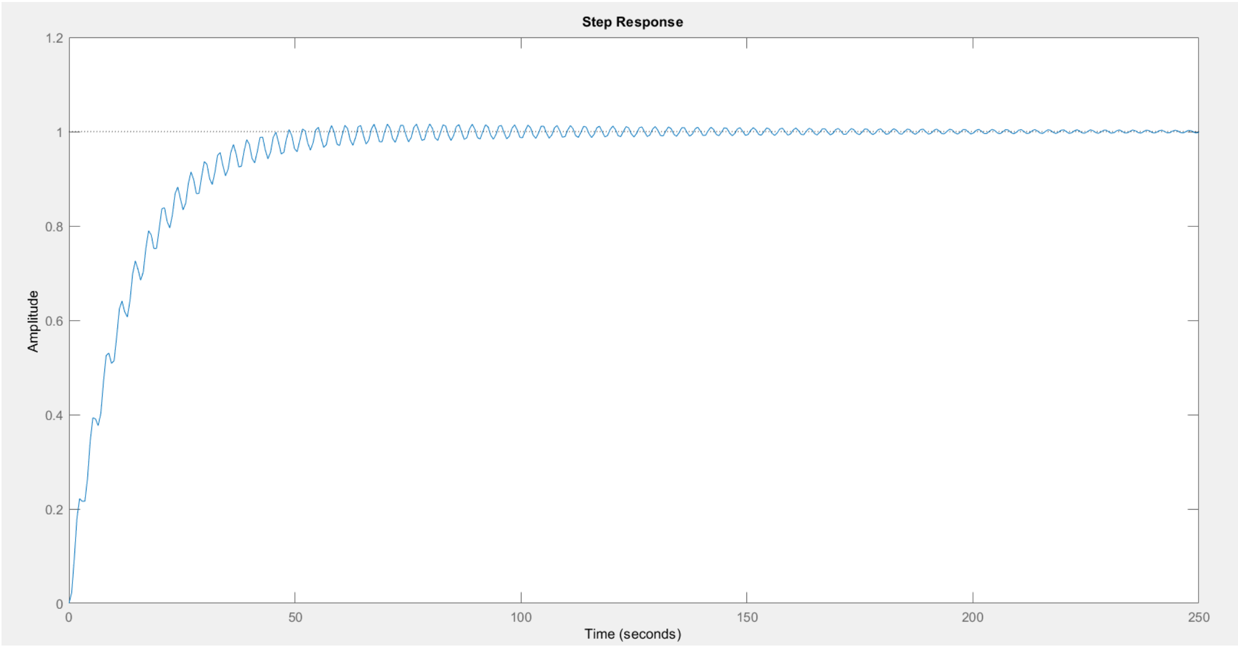

Figure 11: The step response for zero at “-4” and k= 0.08

Cl-poles={-0.0108 + 2.0195i, -0.0108 - 2.0195i ,-0.0785 + 0.0000i}

- It takes 250 seconds to converge to the final value.

b. K=0.05

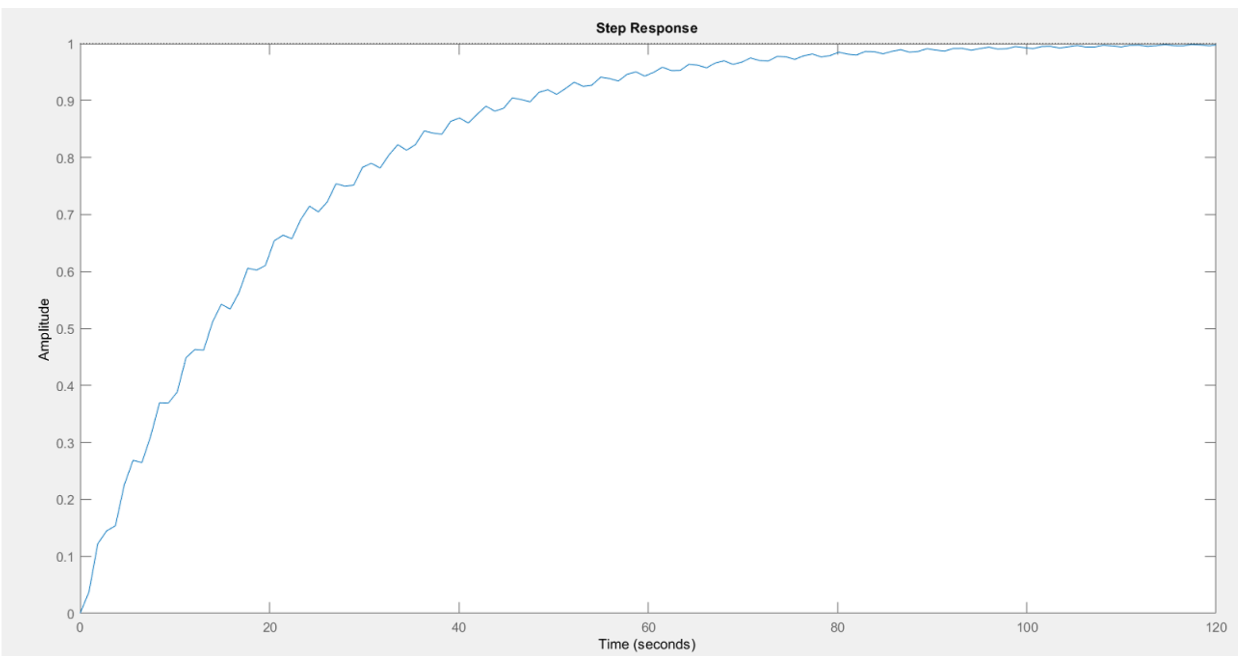

Figure 12: The step response for zero at “-4” and k= 0.05

Cl-poles={ -0.0253 + 2.0117i , -0.0253 - 2.0117i , -0.0494 + 0.0000i}

- It takes 120 seconds to converge to the final value.

c. K=0.04

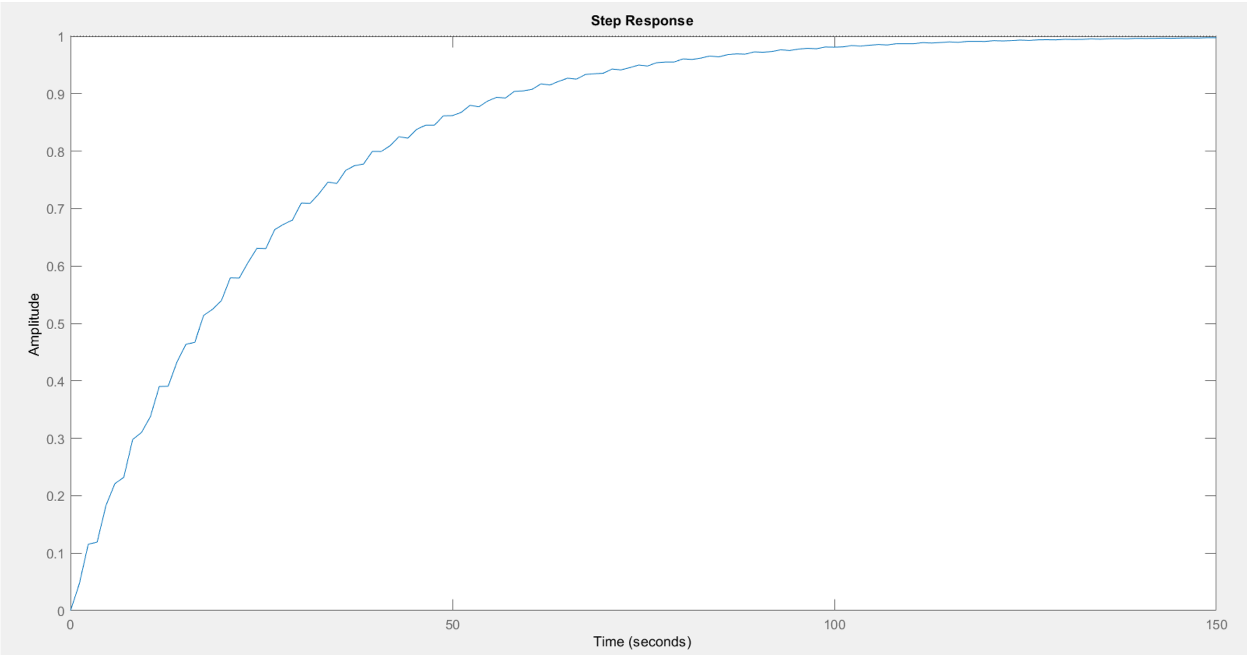

Figure 13: The step response for zero at “-4” and k= 0.04

Cl-poles ={ -0.0302 + 2.0092i , -0.0302 - 2.0092i , -0.0396 + 0.0000i}

- It takes 150 seconds to converge to the final value.

As we can see, by decreasing the value of k from 0.102 to 0.05, the pace of the step response will increase that shows less time is required to converge to the final value. Since the more the gain increases, the bigger the real part of the mixed conjugate poles will be, till we reach to the point (about k=0.04) that the real part of the third pole is closing to the real part of the dominant poles (for k=0.04 is, 0.0302 vs 0.0396) which means the third pole will be the dominant pole again and that’s why we can’t reach more speed (120 seconds for k=0.05 vs 150 seconds for k=0.04). => kc=k/2=0.025

Finally, the lag compensator is shown below:

Gc(s)= $0.025 * \frac{(s+4)}{s+0.05}$

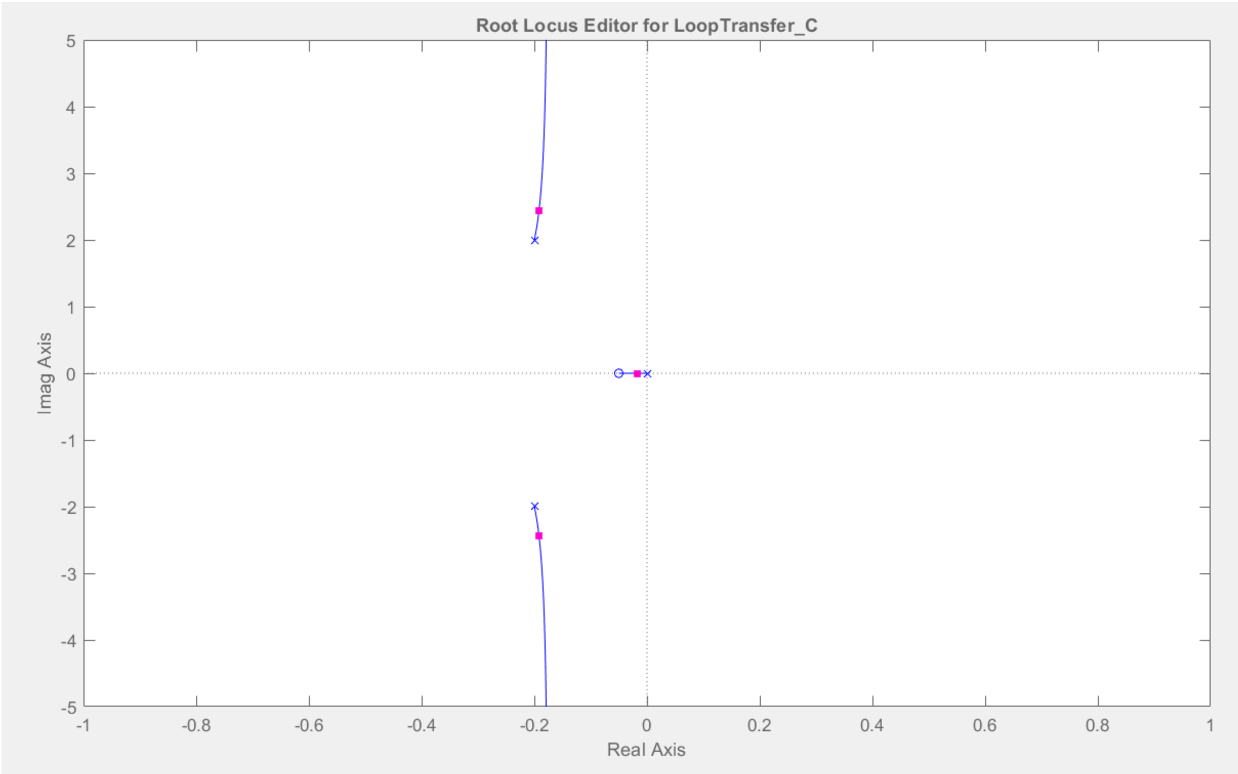

Figure 14: The closed-loop vs open-loop Root-Locus(Uncompensated)

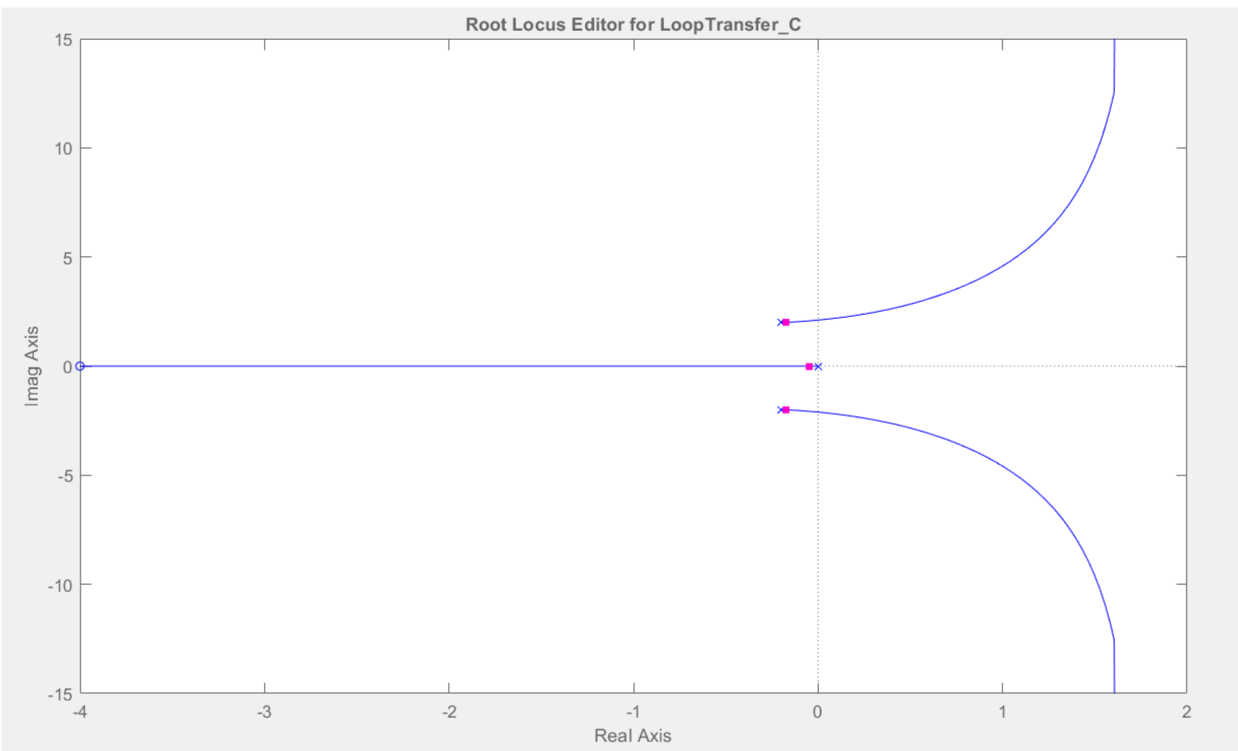

Figure 15: The closed-loop vs open-loop Root-Locus(Compensated)

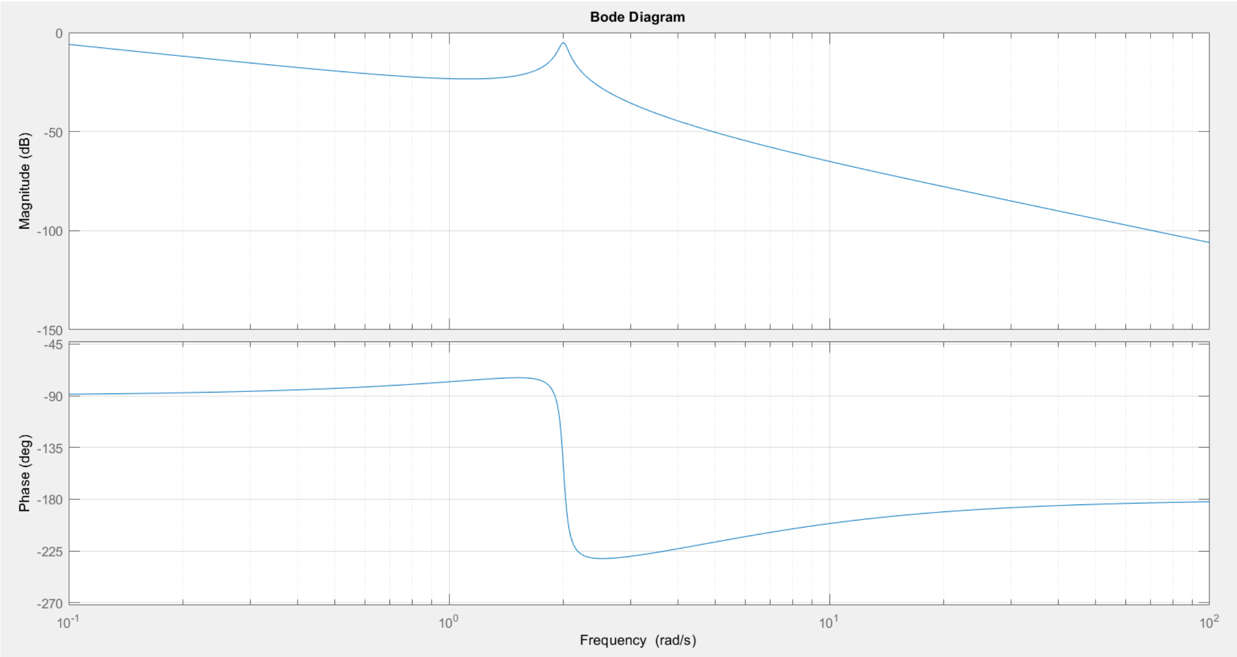

Figure 16: Bode diagram for the compensated system (Lag compensator)

This time by compensating the system by the lag compensator, the bode plot differs in compare to the uncompensated system, Since the system isn't stable for the all range of K .
As we expected , the phase diagram crosses the frequency axis in the limited frequency which illustrate that the gain-margin and the phase-crossover-frequency aren't infinite this time.
Finally after analyzing the bode plot we have :

[Gm,Pm,Pcf,Gcf]={ 6.2405 , 90.6449 , 2.0255 , 0.0500}

For the compensated system the static velocity error is:

$$Kv = \lim_{s\to 0} s * Gc(s) * G(s) = \lim_{s\to 0} 0.05 * \frac{s*(s+4)}{s*(s^2+0.1*s+4)} = 0.05 \to ess = \frac{1}{Kv} = 20$$

B.

The maximum overshoot is obtained by equation below:

percent maximum overshoot = $100 * e^{-\pi \text{ב}/\sqrt{1-\text{ב}^2}}$

The settling time If the specified percentage is 2% is:

$$ts = \frac{4}{ב * \omega}$$

Given by the problem:

$$\begin{cases} 100 * e^{-\frac{\pi ב}{\sqrt{1-ב^2}}} \leq 16.3 \rightarrow \ 0.5 \leq ב < 1 \\ \dfrac{4}{ב * \omega} \cong 2 \rightarrow \ \omega = \dfrac{2}{ב} \rightarrow \ 2 < \omega \leq 4 \end{cases}$$

For instance , we choose $\underline{\omega^2 = 14}$ and therefor ב will be 2/sqrt(14)=<u>0.5345</u>.

For the dominant poles we have:

$$s^2 + 2 * ב * \ \omega * s + \ \omega^2 = 0 \rightarrow \ \ s^2 + 4 * s + 14 = 0 \rightarrow s1{,}2 = -2 \pm 3.16228j$$

To determine by the root-locus method, we need to find the angle deficiency at the desired closed-loop pole $-2 + 3.16228j$ .The angle deficiency can be found as follows:

```
In[111]:= Solve[a^2 + 4 * a + 14 == 0, a] // N
Out[111]= {{a → -2. - 3.16228 i}, {a → -2. + 3.16228 i}}

In[110]:= Clear[a]

In[115]:= a = -2 + 3.16228 * I;
          GH = (a + 4) / (a * (a^2 + 0.1 * a + 4));
          (Arg[GH] * 180 / Pi - 180)

Out[117]= -144.509
```

Figure 17: Obtaining angle deficiency

Angle deficiency= -144.509°

Hence, the lead compensator Gˆc(s) must provide 144.509°. Since the angle deficiency is –144.509°, we need two lead compensators, each providing 72.2543°. Thus Gˆc(s) will have the following form:

Gˆc(s)= $K\hat{}c * \frac{s+\frac{1}{T}}{s+\frac{1}{aT}}^{2}$

Suppose that we choose two zeros at s= –2($\frac{1}{T} = 2$) Then the two poles of the lead compensators can be obtained from:

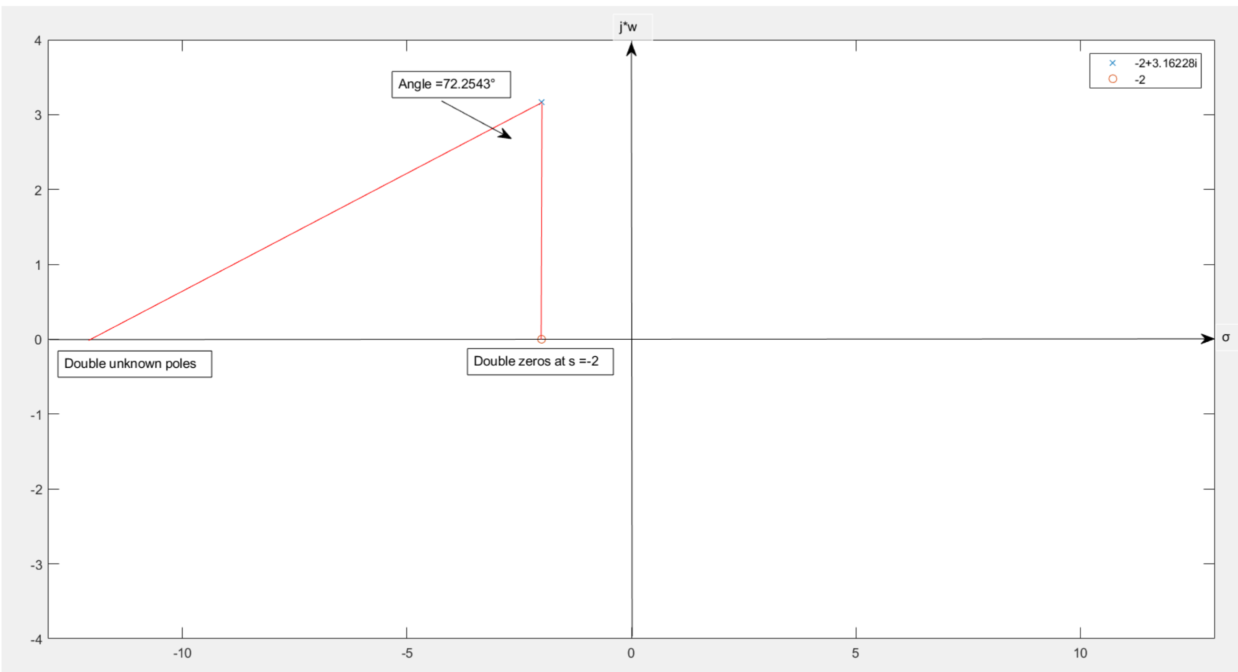

Figure 18: pole and zero of Gˆc(s)

As it is shown on the above figure, we can simply write:

$$\frac{1}{aT} = \frac{3.16228}{\tan(90° - 72.2543°)} + 2 = 11.8815 \rightarrow a = \frac{2}{11.8815} = 0.2017$$

Hence,

Gˆc(s)= $K\hat{}c * \frac{s+2}{s+11.8815}^{2}$

The entire open-loop transfer function will be:

Gf(s)= Gˆc(s)* Gc(s)*Gop(s)= $K\hat{}c * \frac{s+2}{s+11.8815}^{2} * 0.025 * \frac{s+4}{s+0.05} * \frac{2*s+0.1}{s*(s^2+0.1*s+4)} \rightarrow$

$$\text{Gf(s)} = \frac{K\hat{}c * 0.05 * (s+2)^2 * (s+4)}{s * (s+11.8815)^2 * (s^2+0.1*s+4)}$$

the magnitude condition becomes:

$$\left|\frac{K\hat{}c * 0.05 * (s+2)^2 * (s+4)}{s * (s+11.8815)^2 * (s^2+0.1*s+4)}\right| = 1\,, for\ (s = -2 + 3.16228j) \rightarrow$$

$$K\hat{}c = \left|\frac{s * (s+11.8815)^2 * (s^2+0.1*s+4)}{0.05 * (s+2)^2 * (s+4)}\right|\,, for\ (s = -2 + 3.16228j) \rightarrow$$

$K\hat{}c$ =2697

⇨ $\text{Gf(s)} = \frac{134.85*(s+2)^2*(s+4)}{s*(s+11.8815)^2*(s^2+0.1*s+4)}$

Now that the compensator has been designed, we shall examine the transient-response characteristics using MATLAB. The closed-loop transfer function is given by:

$$\frac{\text{C(s)}}{\text{R(s)}} = \frac{134.85 * (s+2)^2 * (s+4)}{s * (s+11.8815)^2 * (s^2+0.1*s+4) + 88.0227 * (s+2)^2 * (s+4)}$$

As a brief summary we can say that; Lead compensation basically speeds up the response and increases the stability of the system. Lag compensation improves the steady-state accuracy of the system, but reduces the speed of the response. If improvements in both transient response and steady-state response are desired, then both a lead compensator and a lag compensator may be used simultaneously.

Here are the poles for the final compensated system:

Final-poles=

{ -9.4834 + 9.0560i, -9.4834 - 9.0560i, -2.0000 + 3.1623i,

-2.0000 -3.1623i, -0.8963 + 0.0000i}

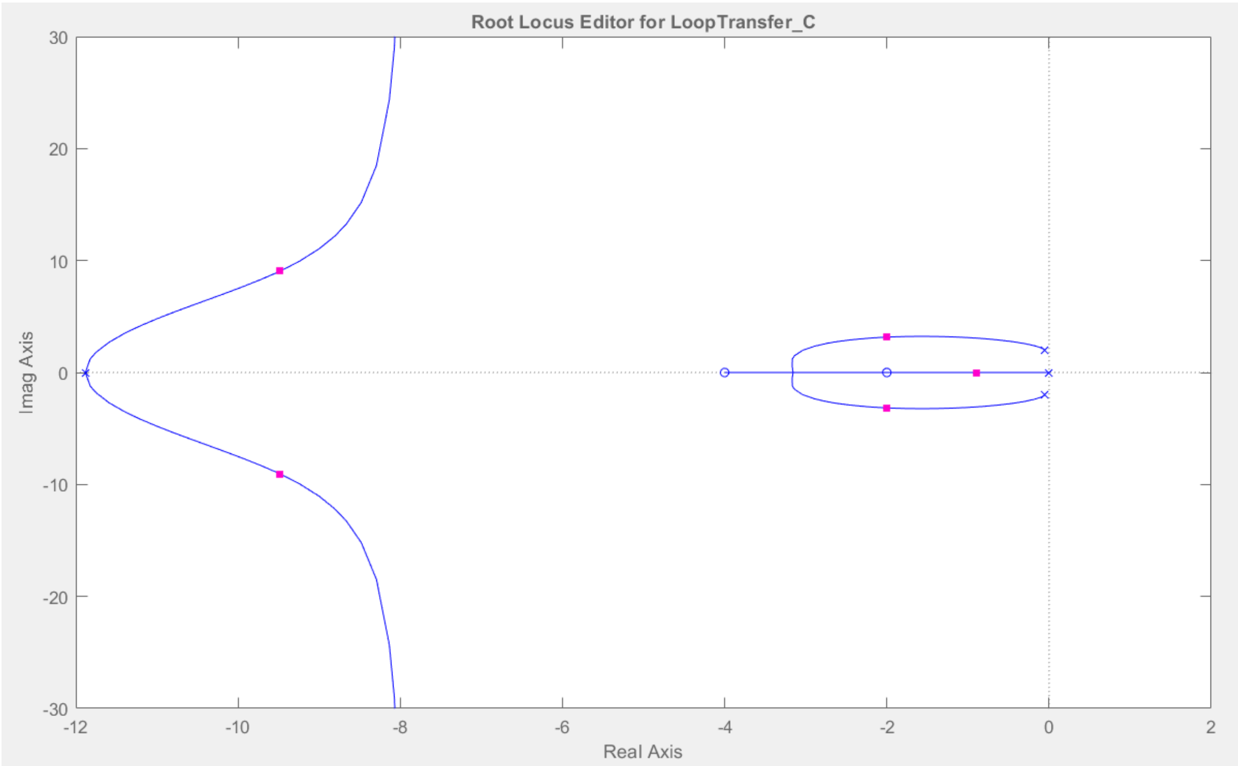

Figure 19: The closed-loop vs open-loop Root-Locus(Final-Compensation)

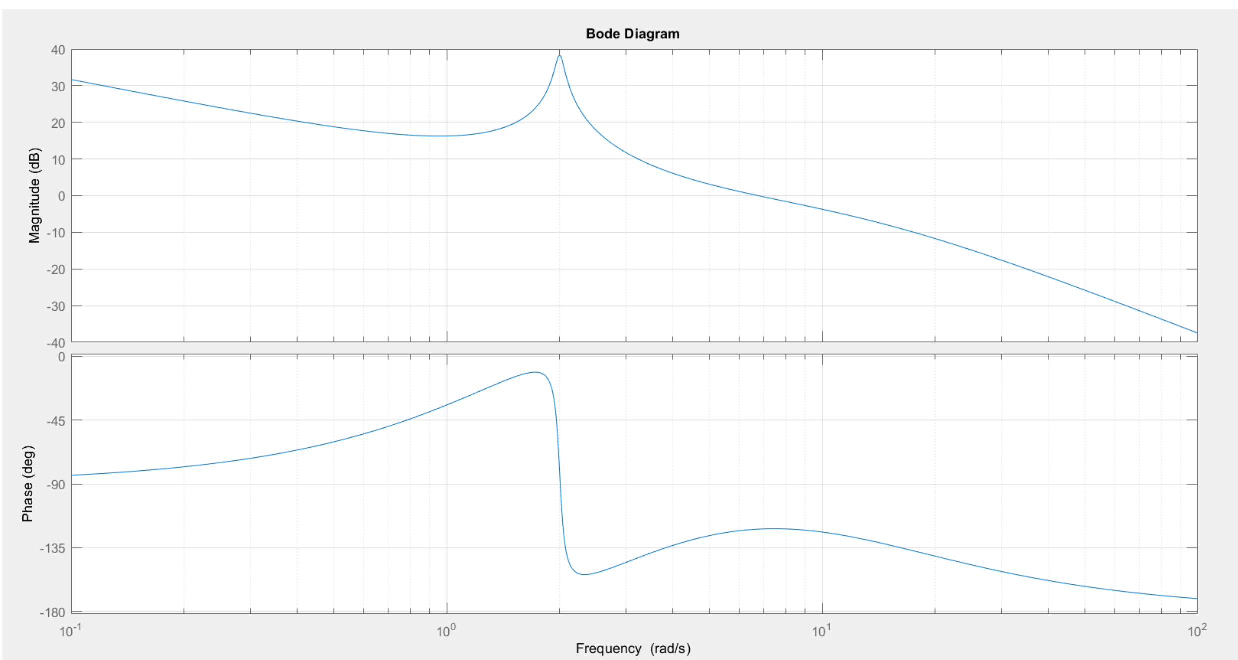

Figure 20: Bode diagram for the final-compensated system
(Lag & Lead compensators)

The root-locus for the final-compensated system shows the stability for all range of gain k, so we can say that the bode diagram of the last part and the first part are literally the same, since they don't cross over the frequency axis, but they are in conflict with the bode diagram of part 'a', because it has a boundary for being stable.

Finally, by analyzing the bode plot we have:

[Gm,Pm,Pcf,Gcf]={ Inf, 58.0735, Inf , 6.7558}

The static velocity error is:

$$Kv = \lim_{s\to 0} s * Gf(s) = \lim_{s\to 0} \frac{s*134.85*(s+2)^2*(s+4)}{s*(s+11.8815)^2*(s^2+0.1*s+4)} = 3.8209 \rightarrow$$

$$ess = \frac{1}{Kv} = 0.2617$$

To increase the static velocity , we can increases β(a coefficient of the lag compensator) that yields the step response to be slower than before.

Here is the final step response by the Lead-Lag compensators:

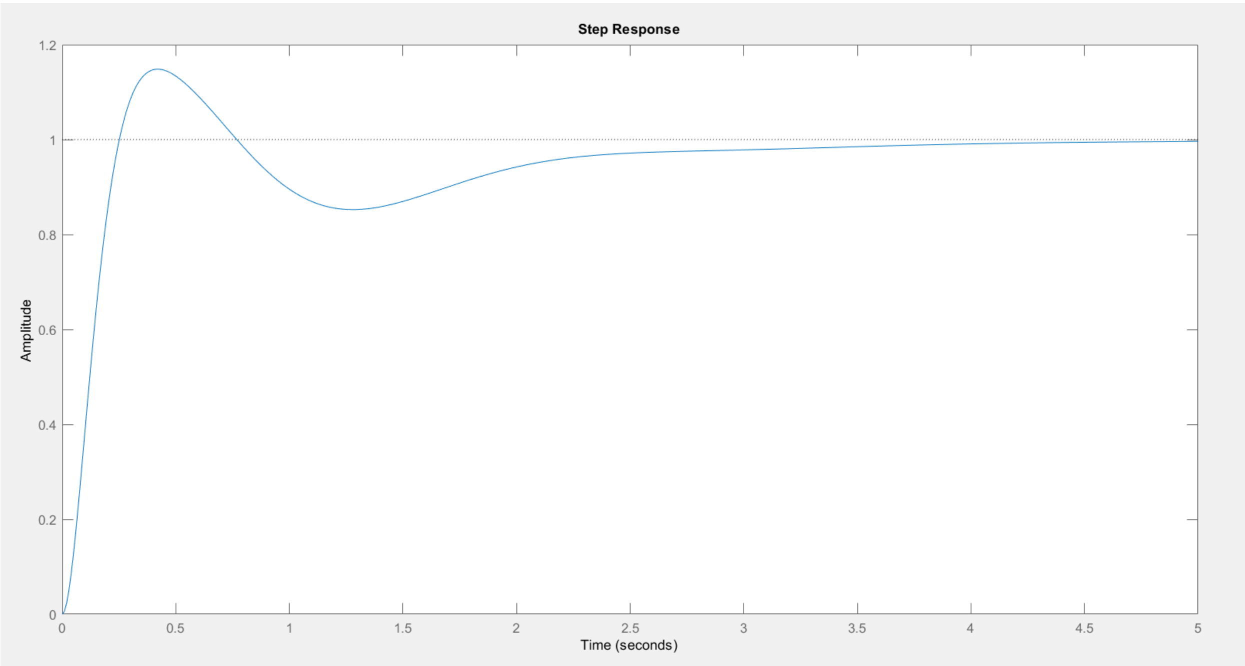

Figure 21: Final step response for the compensated system

Let's check the overshoot and the settling time:

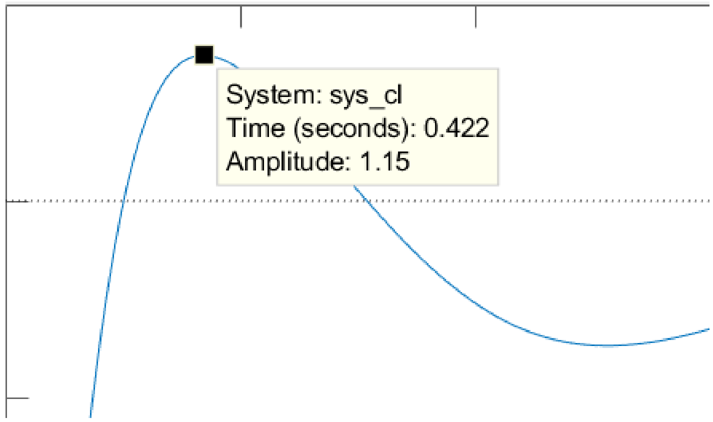

Figure 22: final overshoot

- What we expected (for ב=<u>0.5345):</u>

percent maximum overshoot = $100 * e^{-\pi \text{ב}/\sqrt{1-\text{ב}^2}}$=13.7133%
From the plot : overshoot =(1.15-1)/1*100=15% <16.3% (Expected 13.7133%)

- For overshoot=15% => $100 * e^{-\pi \text{ב}/\sqrt{1-\text{ב}^2}}$= 15% => $\text{ב} \cong 0.517$
  => $ts = \frac{4}{sqrt(14)*\text{ב}} = 2.0678$ (Expected 2)

  Hence, the answer is acceptable …

THE END…