

ECE/CS 559 Neural Networks, Fall 2023 - Homework #4
Due: 10/03/2023, 11pm.

- **Q1 (100 pts)** In this computer project, we will use a neural network for curve fitting.

1. Draw $n = 300$ real numbers uniformly at random on $[0, 1]$, call them x_1, \dots, x_n .
2. Draw n real numbers uniformly at random on $[-\frac{1}{10}, \frac{1}{10}]$, call them ν_1, \dots, ν_n .
3. Let $d_i = \sin(20x_i) + 3x_i + \nu_i$, $i = 1, \dots, n$. Plot the points (x_i, d_i) , $i = 1, \dots, n$.

We will consider a $1 \times N \times 1$ neural network with one input, $N = 24$ hidden neurons, and 1 output neuron. The network will thus have $3N + 1$ weights including biases. Let \mathbf{w} denote the vector of all these $3N + 1$ weights. The output neuron will use the activation function $\phi(v) = v$; all other neurons will use the activation function $\phi(v) = \tanh v$. Given input x , we use the notation $f(x, \mathbf{w})$ to represent the network output.

4. Use the backpropagation algorithm with online learning to find the optimal weights/network that minimize the mean-squared error (MSE) $\frac{1}{n} \sum_{i=1}^n (d_i - f(x_i, \mathbf{w}))^2$. Use some η of your choice. Plot the number of epochs vs the MSE in the backpropagation algorithm. Hint: Since this is a very simple network, you can just manually derive the derivatives if you like without using the BP algorithm.

Hint: As discussed in class, for a given fixed η , the algorithm may not always result in a monotonically decreasing MSE (the descent may overshoot the locally optimal point). You may have to modify the gradient descent algorithm in such a way that you decrease η (e.g. via $\eta \leftarrow 0.9\eta$) whenever you detect that the MSE has increased. Also, beginning with a very large η may result in an immediate divergence of the weights.

5. Let us call the weights resulting from the backpropagation algorithm (when it converges) as \mathbf{w}_0 . The curve $(x, f(x, \mathbf{w}_0))$, $x \in [0, 1]$ will then be a fit to the points (x_i, d_i) , $i = 1, \dots, n$. Plot the curve $f(x, \mathbf{w}_0)$ as x ranges from 0 to 1 on top of the plot of points in (c). The fit should be a “good” fit. If you are not getting good fit, try different hyperparameters until you do.
6. Your report should include a pseudocode of your training algorithm including all gradient descent update equations written out explicitly. The pseudocode should be written in such a way that anyone would be able to implement your algorithm without knowing anything about neural networks. As usual, upload a copy of your code to box with the filename 04-IDNumber-LastName.py.