Signals & Systems Winter 1400

Intro to Mathematica

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The very basics

Basic Operations

In[∘]:= 2 + 2

Out[∘]= 4

In[1]:= 580 / 13

Out[1]= $\frac{580}{13}$

In[2]:= N[580 / 13]

Out[2]= 44.6154

In[∘]:= 580 / 13 // N

Out[∘]= 44.6154

In[∘]:= N[580 / 13, 3]

Out[∘]= 44.6

580/13

580 / 13

$\frac{580}{13}$

Variable Assignments

In[∘]:= x = 5

Out[∘]= 5

In[∘]:= 3 * x + 2

Out[∘]= 17

In[∘]:= Clear[x]

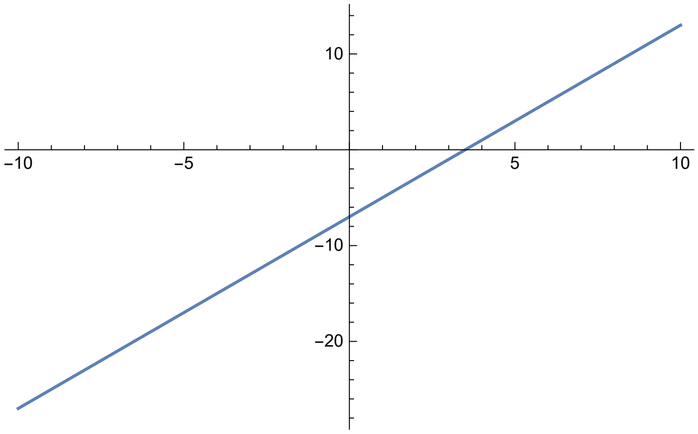
In[∘]:= x

Out[∘]= x

2D and 3D Graphics

```
Plot[2 x - 7, {x, -10, 10}]
```

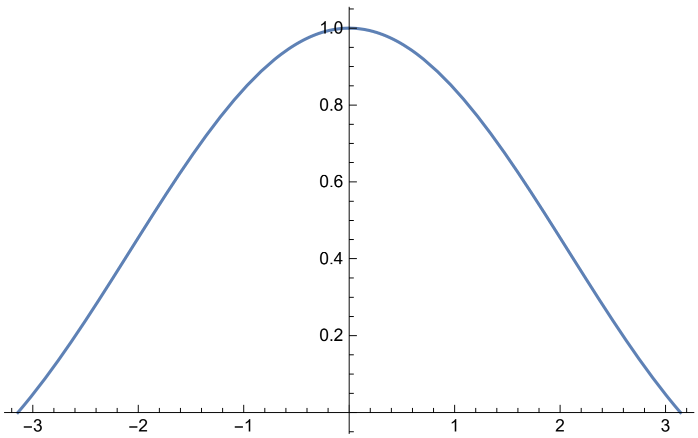
Out[∘]=



In[∘]:=

```
Plot[Sin[x] / x, {x, -Pi, Pi}]
```

Out[∘]=

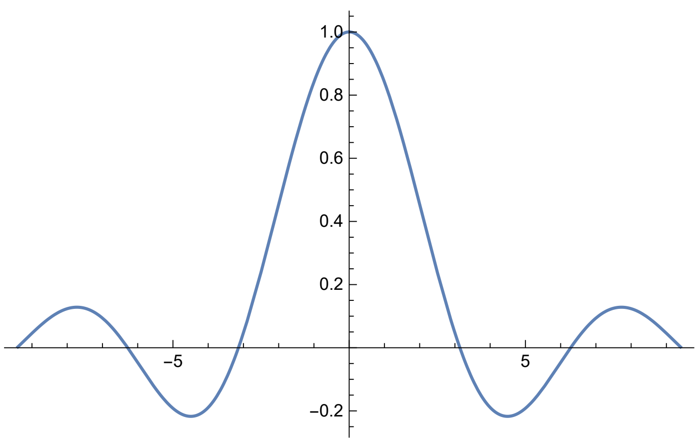


In[∘]:=

plot sin(x)/x

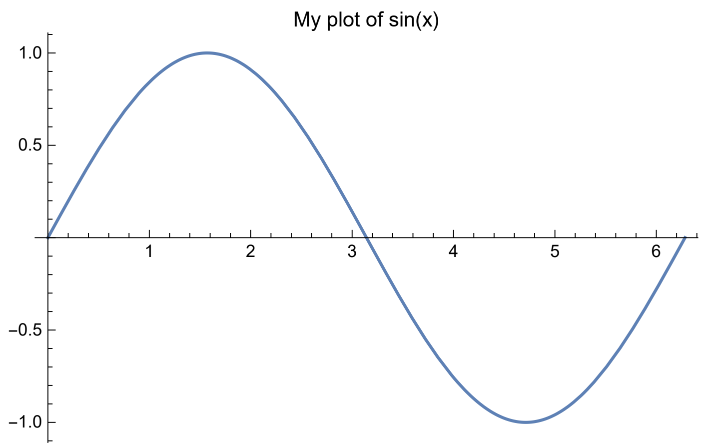
```
Plot[Sin[x] / x, {x, -9.4, 9.4}]
```

Out[∘]=



```
Plot[Sin[x], {x, 0, 2 π}, PlotLabel →  "My plot of sin(x)"]
```

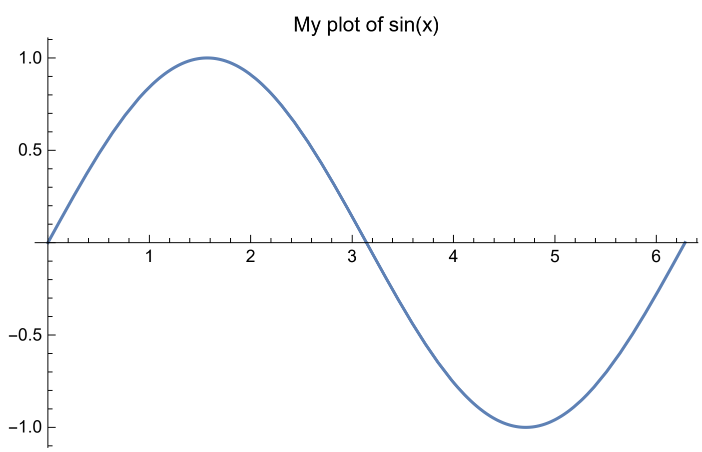
Out[ ]=



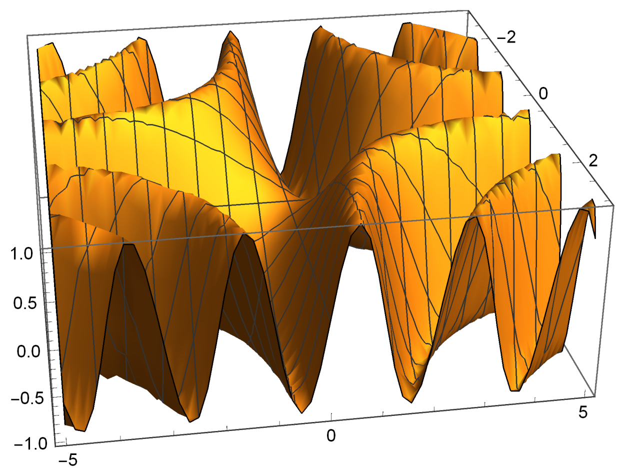
In[ ]:=

```
Plot[Sin[x], {x, 0, 2 π}, PlotLabel → "My plot of " <> "sin(x)"]
```

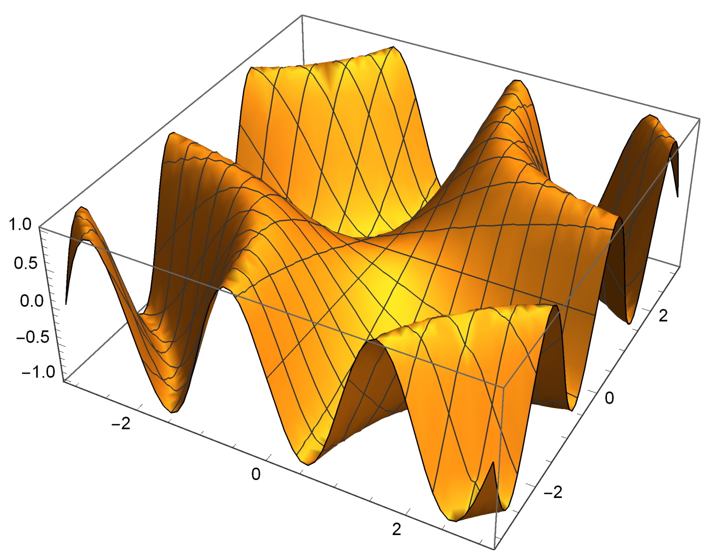
Out[ ]=



In[∘]:=
```
Plot3D[Sin[x * y], {x, -3, 3}, {y, -5, 5}]
```

Out[∘]=


In[∘]:=
plot sin(xy)
```
Plot3D[Sin[x*y], {x, -Sqrt[3*Pi], Sqrt[3*Pi]}, {y, -Sqrt[3*Pi], Sqrt[3*Pi]}]
```

Out[∘]=


In[ ]:=
```
? Plot*
```

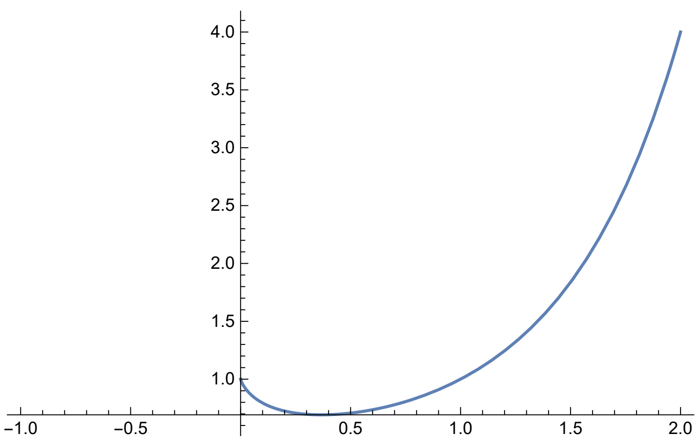
▼ System`

Plot	PlotDivision	PlotLabels	PlotMarkers	PlotRangeC-lipping	PlotRegion
Plot3D	PlotJoined	PlotLayout	PlotPoints	PlotRangeC-lipPlanes-Style	PlotStyle
Plot3Matrix	PlotLabel	PlotLegends	PlotRange	PlotRangeP-adding	PlotTheme

In[ ]:= **plot x^x**

```
Plot[x^x, {x, -1, 2}]
```

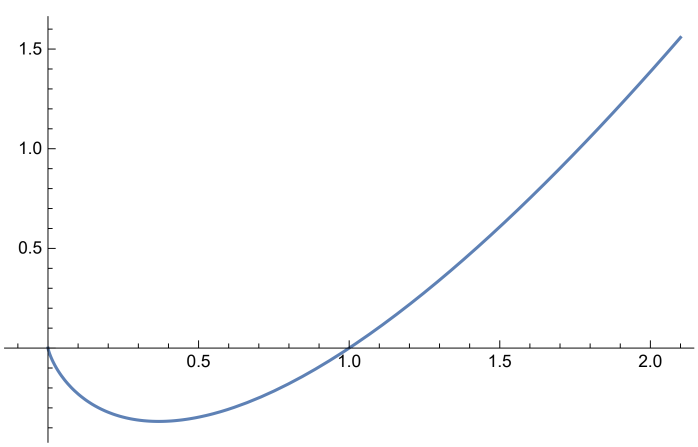
Out[ ]=



In[ ]:= **plot log x^x** »

```
Plot[Log[x^x], {x, -0.1, 2.1}]
```

Out[ ]=

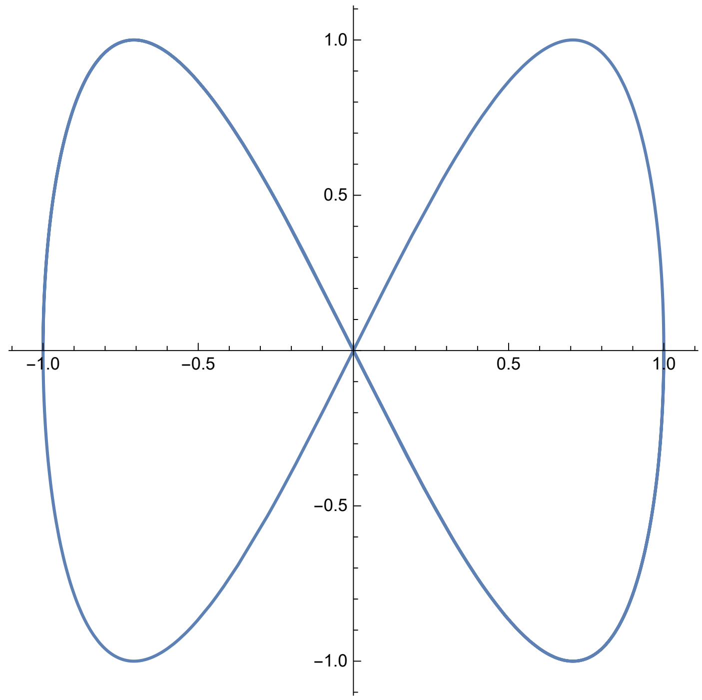


In[ ]:=

parametric plot of sin(x) , sin(2x)

```
ParametricPlot[{Sin[x], Sin[2*x]}, {x, (-3*Pi)/2, (3*Pi)/2}]
```

Out[ ]=

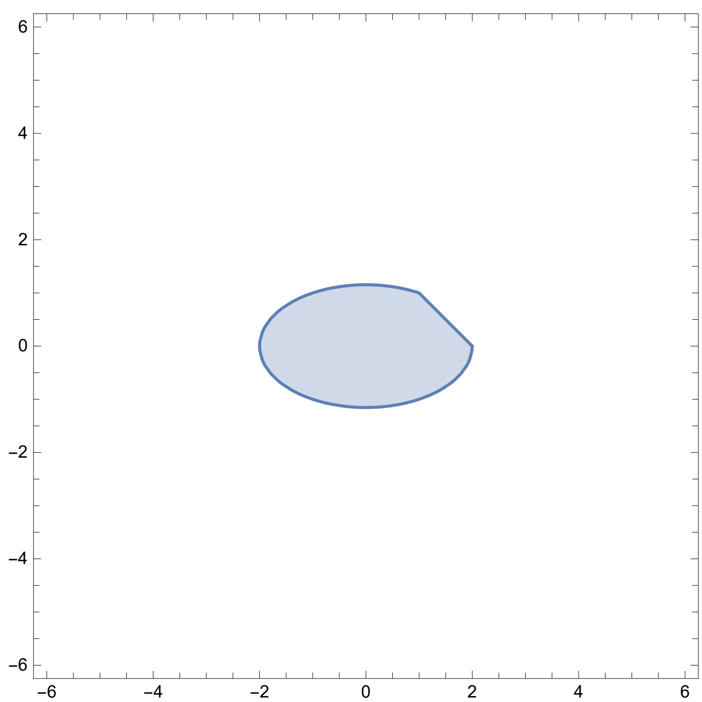


In[ ]:=

plot x^2+3*y^2 ≤ 4 and x+y≤2

```
RegionPlot[Evaluate[x^2 + 3*y^2 <= 4 && x + y <= 2], {x, -6., 6.}, {y, -6., 6.}]
```

Out[ ]=

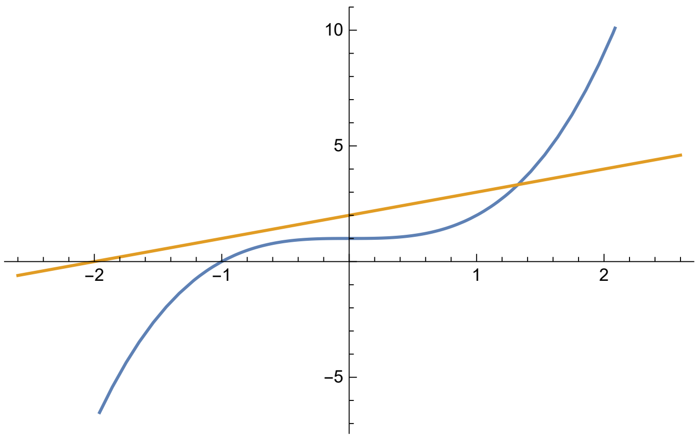


Plotting Multiple Functions Together

In[ ]:= plot x^3+1 and x+2

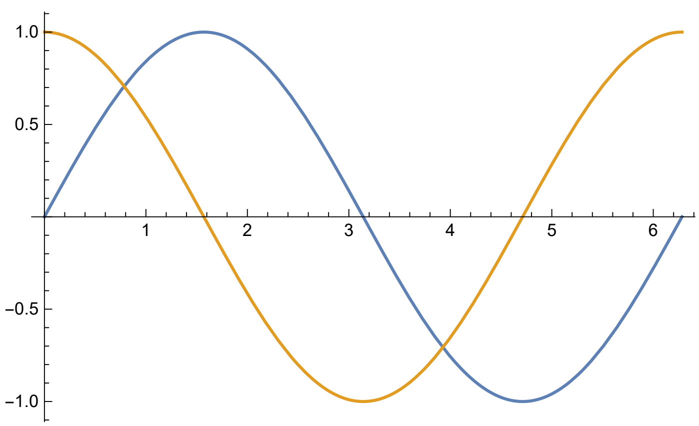
```
Plot[{x^3 + 1, x + 2}, {x, -2.6, 2.6}]
```

Out[ ]=



In[ ]:=
```
Plot[{Sin[x], Cos[x]}, {x, 0, 2 π}]
```

Out[ ]=



Defining functions

In[ ]:=
```
f[x_] := x^2
f[2]
```

Out[ ]= 4

In[ ]:=
```
f[{1, 2, 3}]
```

Out[ ]= {1, 4, 9}

In[3]:=
```
Clear[a, b]
```

In[12]:=
```
h[a_, b_] := a * b
```

```
h[10, 10]
```

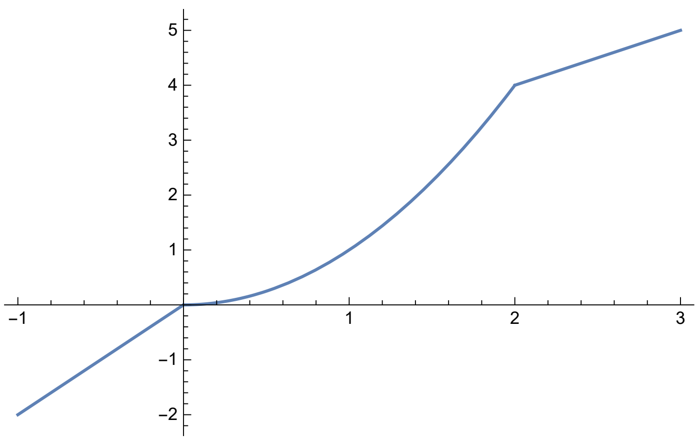
Out[13]= 100

In[14]:=
```
Clear[x, a, f, h]
```

$$h[x\_] = \begin{cases} 2\,x & x < 0 \\ x\text{^}2 & 0 \le x < 2 \\ 2 + x & x > 2 \end{cases}$$

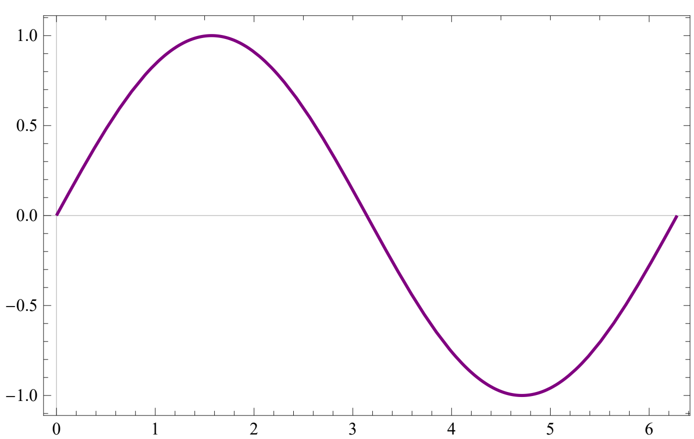
Out[15]=
$$\begin{cases} 2\,x & x < 0 \\ x^2 & 0 \le x < 2 \\ 2 + x & x > 2 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
Plot[h[x], {x, -1, 3}]
```

Out[ ]=


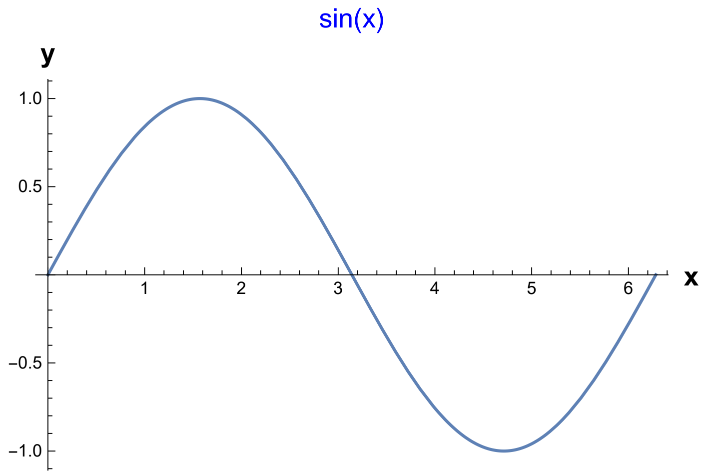
Using Options with Graphics

In[ ]:=
```
Plot[Sin[x], {x, 0, 2 π}, PlotTheme → "Scientific", PlotStyle → Purple]
```

Out[ ]=


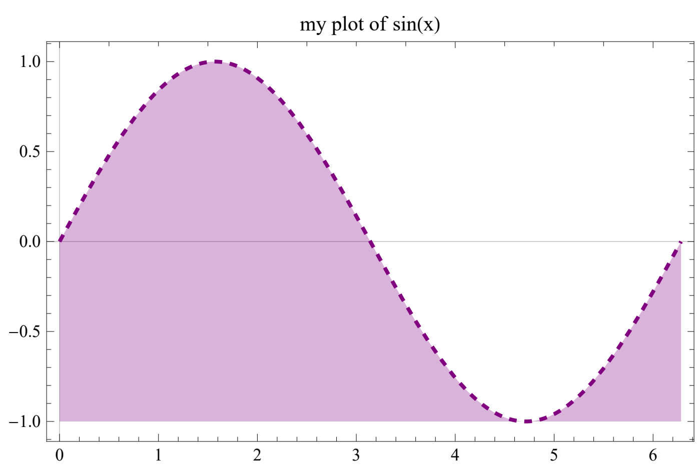
In[∘]:=
```
Plot[Sin[x], {x, 0, 2 π},
 PlotLabel → Style["sin(x)", FontSize → 14, FontFamily → "Arial", FontColor → Blue],
 AxesLabel → {Style["x", FontSize → 14, Bold], Style["y", FontSize → 14, Bold]}]
```

Out[∘]=



In[∘]:=
```
Plot[Sin[x], {x, 0, 2 π}, PlotTheme → "Scientific",
 PlotStyle → {Purple, Thick, Dashed}, Filling → Bottom, PlotLabel → "my plot of sin(x)"]
```

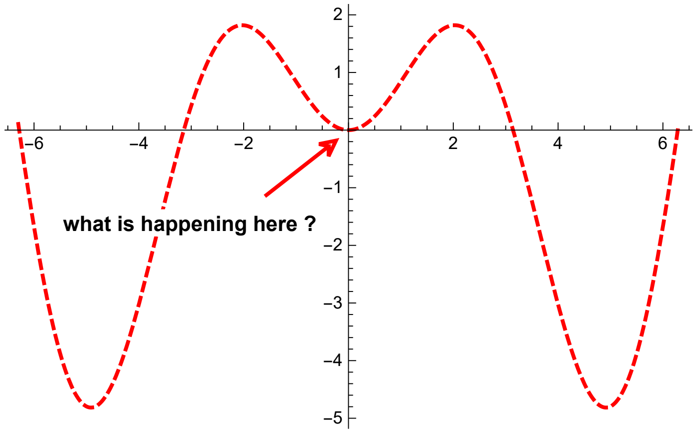
Out[∘]=



In[∘]:=

plot x*sin(x) with thick, dash, red »

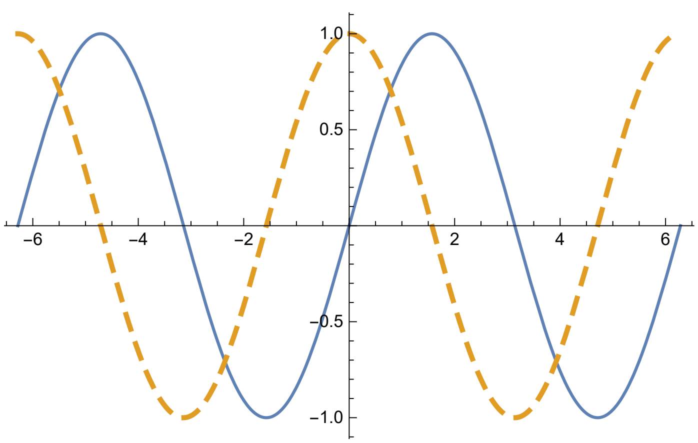
```
Plot[x*Sin[x], {x, -6.3, 6.3}, PlotStyle -> Directive[Thick, Dashed, Red]]
```



In[∘]:=

plot sin(x) and cos(x)

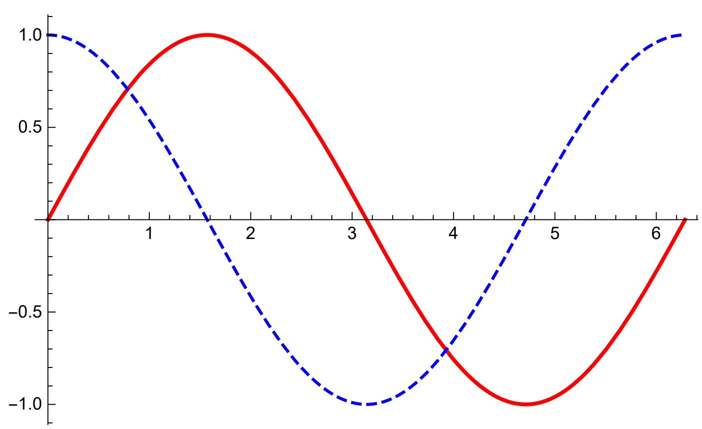
```
Plot[{Sin[x], Cos[x]}, {x, -2*Pi, 2*Pi}]
```



In[∘]:=

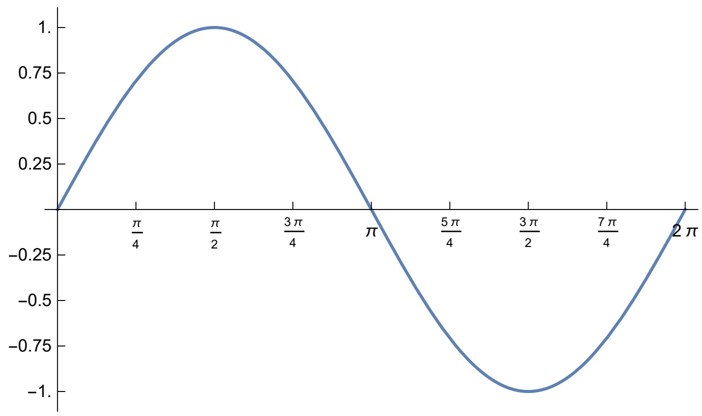
```
Plot[{Sin[x], Cos[x]}, {x, 0, 2 π}, PlotStyle → {{Red, Thick}, {Blue, Dashed}}]
```

Out[∘]=



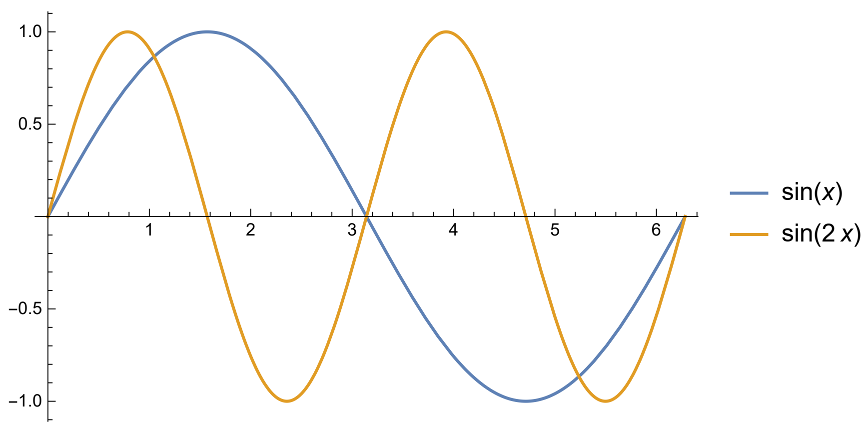
Ticks

In[ ]:=
```
Plot[Sin[x], {x, 0, 2 π}, Ticks → {Range[0, 2 π, π / 4], Range[-1, 1, 0.25]}]
```

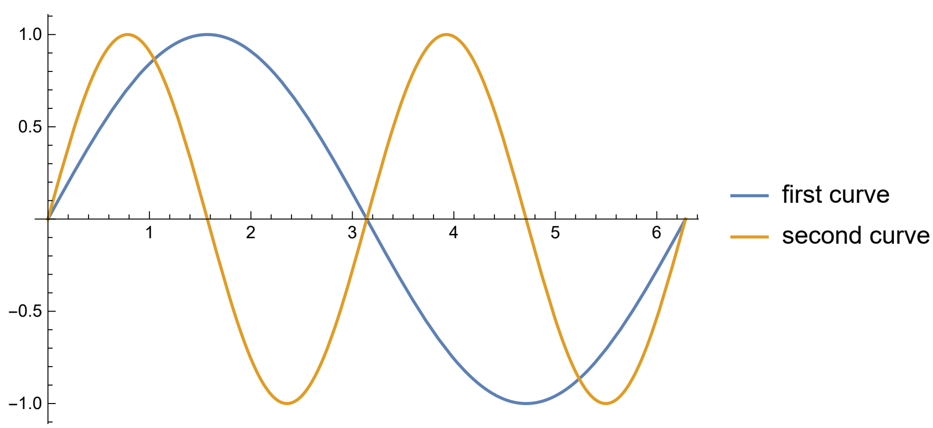
Out[ ]=


PlotLegends

In[ ]:=
```
Plot[{Sin[x], Sin[2 x]}, {x, 0, 2 π}, PlotLegends → "Expressions"]
```

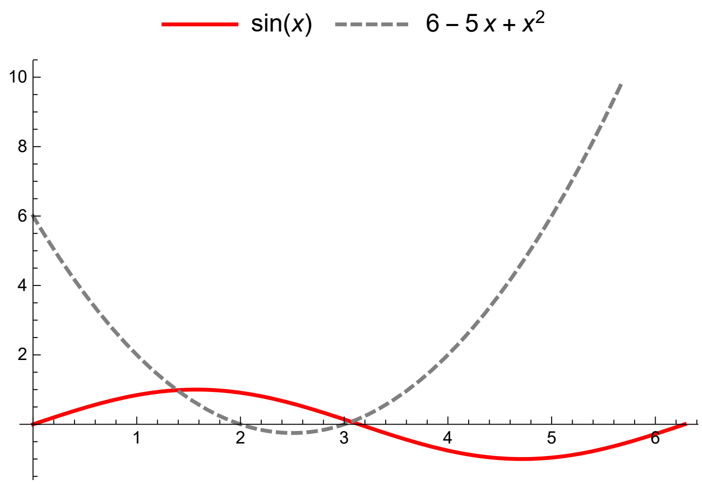
Out[ ]=


In[ ]:=
```
Plot[{Sin[x], Sin[2 x]}, {x, 0, 2 π}, PlotLegends → {"first curve", "second curve"}]
```

Out[ ]=


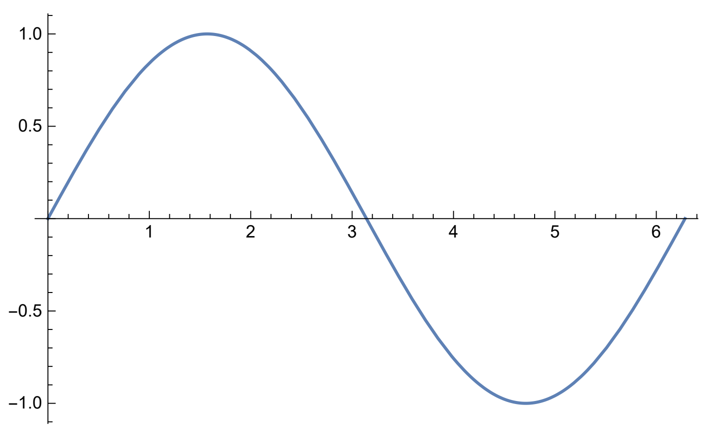
In[ ]:=
```
myPlot[eq1_, eq2_] :=
 Plot[{eq1, eq2}, {x, 0, 2 π},
  PlotStyle → {Directive[Red, Thick], Directive[Gray, Thick, Dashed]},
  PlotLegends → Placed["Expressions", Above]]
```

In[ ]:=
```
myPlot[Sin[x], x^2 - 5 * x + 6]
```

Out[ ]=


Creating Interactive Models

In[ ]:=
```
Plot[Sin[x], {x, 0, 2 π}]
```

Out[ ]=


The goal may be to compare the curve of sin(x) with the curve of sin(2 x), the curve of sin(3 x), and so on.

To begin, it is important to know that using Manipulate requires three components:

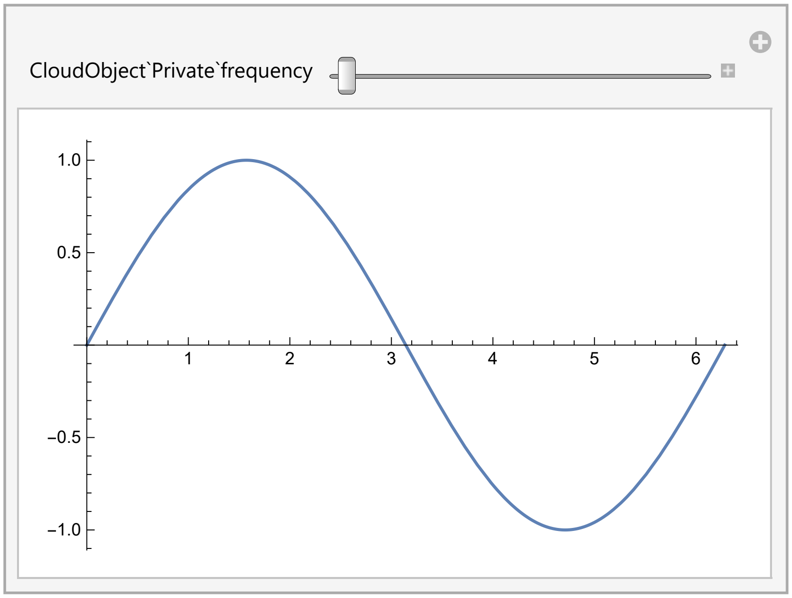
- Manipulate command
- Expression to manipulate by changing certain parameters
- Parameter specifications

Manipulate[

expression to manipulate,
parameter specifications]

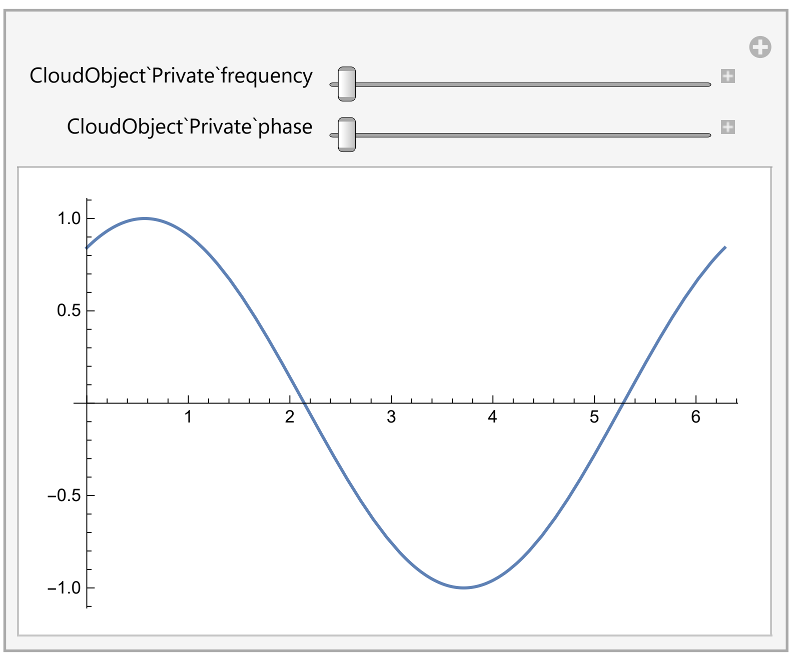
In[∘]:=
```
Manipulate[Plot[Sin[frequency * x], {x, 0, 2 π}], {frequency, 1, 5}]
```

Out[∘]=



In[∘]:=
```
Manipulate[Plot[Sin[frequency * x + phase], {x, 0, 2 π}], {frequency, 1, 5}, {phase, 1, 10}]
```

Out[∘]=



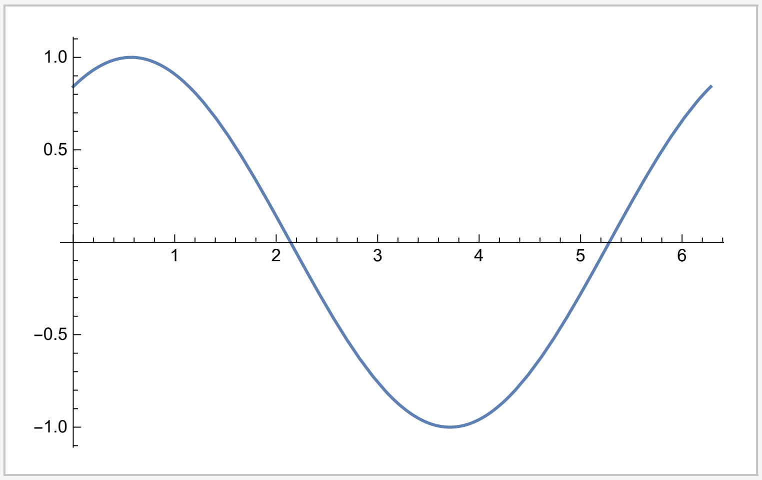
In[∘]:=
```
Manipulate[Plot[function[frequency * x + phase], {x, 0, 2 π}],
  {frequency, 1, 5}, {phase, 1, 10}, {function, {Sin, Cos, Tan, Csc, Sec, Cot}}]
```

Out[∘]=

CloudObject`Private`frequency

CloudObject`Private`phase

CloudObject`Private`function Sin



The Manipulate command is not restricted to graphical manipulation and can be used with any Mathematica expression.

In[∘]:=
```
Manipulate[Expand[(a + b) ^n], {n, 2, 10, 1}]
```

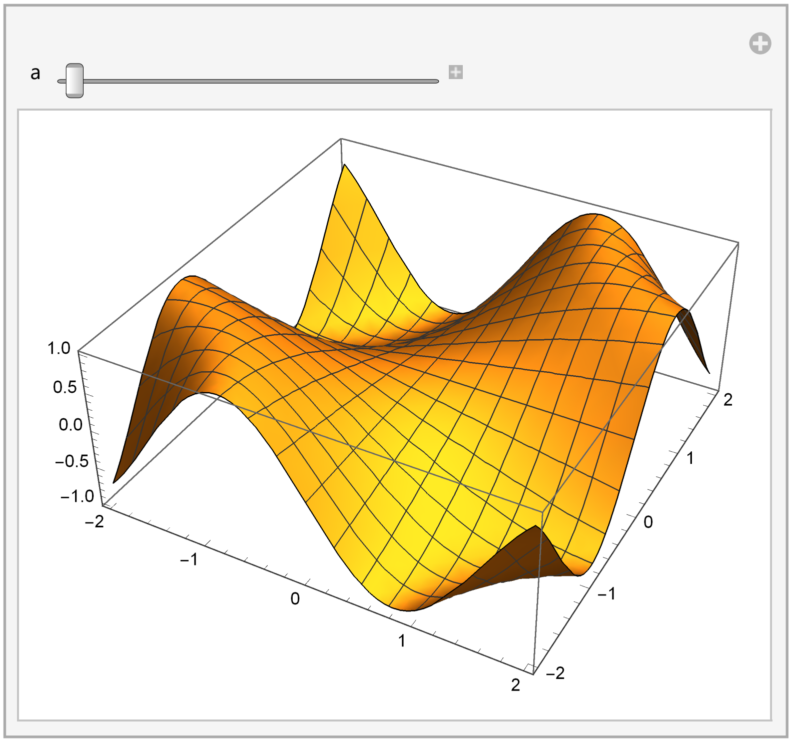
Out[∘]=

CloudObject`Private`n

$a^5 + 5 a^4 b + 10 a^3 b^2 + 10 a^2 b^3 + 5 a b^4 + b^5$

In[ ]:=
```
Manipulate[Plot3D[Sin[a x y], {x, -2, 2}, {y, -2, 2}], {a, 1, 5}]
```

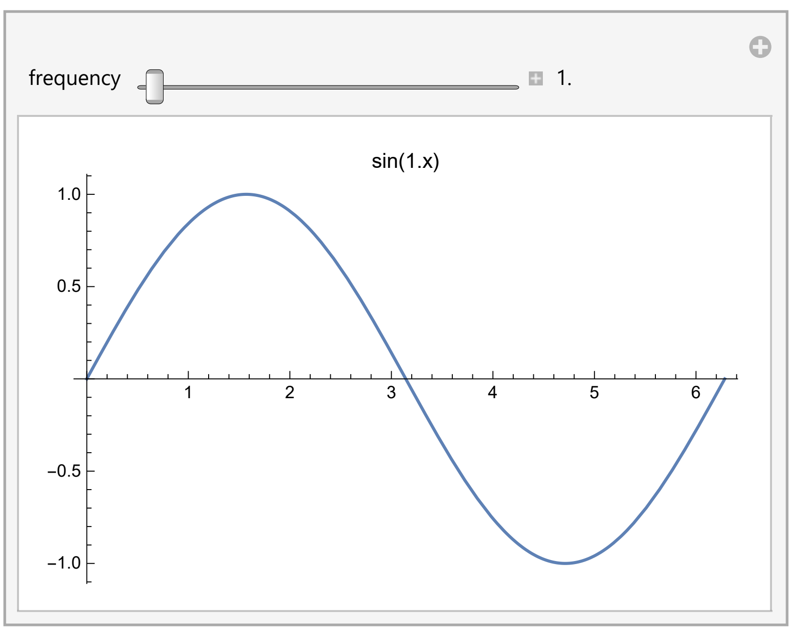
Out[ ]=



In[ ]:=

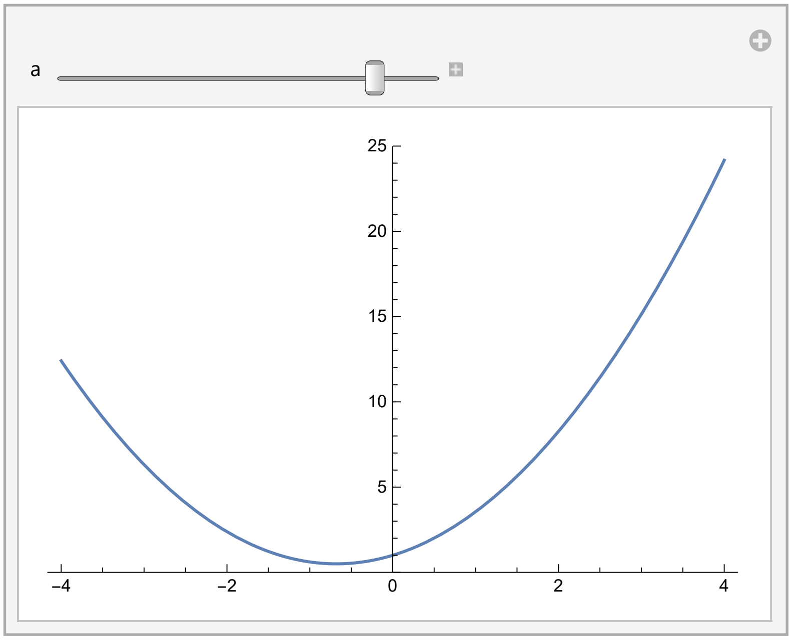
In[ ]:=
```
Manipulate[Plot[Sin[f * x], {x, 0, 2 π}, PlotLabel → "sin(" <> ToString[f] <> "x)"],
 {{f, 1, "frequency"}, 1, 5, Appearance → "Labeled"}]
```

Out[ ]=



In[16]:=
```
f[x_] := 2 * x^2 + 2 * x + 1
```

In[ ]:= `Manipulate[Plot[f[a * x], {x, -4, 4}, PlotRange → {0, 25}], {a, -1, 1}]`

Out[ ]=


Algebraic Manipulation and Equation Solving

In[ ]:= `2 a b / (b c)`

Out[ ]= $\frac{2\,a}{c}$

In[ ]:= `(a + b) (a + c) (b + c)`

Out[ ]= $(a + b)\ (a + c)\ (b + c)$

In[ ]:= `Expand[(a + b) (a + c) (b + c)]`

Out[ ]= $a^2\,b + a\,b^2 + a^2\,c + 2\,a\,b\,c + b^2\,c + a\,c^2 + b\,c^2$

In[ ]:= `Factor[a^2 b + a b^2 + a^2 c + 2 a b c + b^2 c + a c^2 + b c^2]`

Out[ ]= $(a + b)\ (a + c)\ (b + c)$

In[ ]:= `Together[1 / (x - 1) + 1 / (x + 1)]`

Out[ ]= $\frac{2\,x}{(-1 + x)\ (1 + x)}$

In[∘]:= Apart[$\frac{2x}{(x-1)(x+1)}$]

Out[∘]= $\frac{1}{-1+x}+\frac{1}{1+x}$

In[∘]:=
```
Collect[a^2 y + 2 a b y + b^2 y + 2 a x y + 2 b x y + x^2 y + c^2 x^2 y + 2 c d x^2 y + d^2 x^2 y, {x, y}]
```

Out[∘]= $(a^2+2\,a\,b+b^2)\,y+(2\,a+2\,b)\,x\,y+(1+c^2+2\,c\,d+d^2)\,x^2\,y$

In[∘]:=
```
Simplify[Sin[x]^2 + Cos[x]^2]
```

Out[∘]= 1

In[∘]:=
```
FullSimplify[(a^2 + 2 a b + b^2) y + (2 a + 2 b) x y + (1 + c^2 + 2 c d + d^2) x^2 y]
```

Out[∘]= $(1+(a^2+2\,a\,b+c^2+2\,c\,d+2\,(a+b)\,x\,y+)\,(b^2+d^2\,x^2)\,y$

In[∘]:=
```
Simplify[(x^2)^0.5]
```

Out[∘]= $(x^2)^{0.5}$

In[∘]:=
```
Simplify[(x^2)^0.5, x > 0]
```

Out[∘]= $x^{1.}$

In[∘]:=
```
TrigExpand[Sin[x^2] * Cos[2 x]]
```

Out[∘]= $\mathrm{Cos}[x]^2\,\mathrm{Sin}[x^2]-\mathrm{Sin}[x]^2\,\mathrm{Sin}[x^2]$

Basic Equation Solving

```
Solve[x^2 + 2 x - 1 == 0, x]
```

Out[∘]= $\{\{x\to -1-\sqrt{2}\},\{x\to -1+\sqrt{2}\}\}$

In[∘]:=
```
Solve[{2 x + y == 12, x + 4 y == 34}, {x, y}]
```

Out[∘]= $\{\{x\to 2, y\to 8\}\}$

In[∘]:=
```
ReplaceAll[{x, x + 1, x + 2}, x → 2]
```

Out[∘]= {2, 3, 4}

In[∘]:=
```
x /. x → y^2 (* replace all*)
```

Out[∘]= y^2

In[∘]:=
```
Solve[x^2 + 2 x - 1 == 0, x]
```

Out[∘]= $\{\{x\to -1-\sqrt{2}\},\{x\to -1+\sqrt{2}\}\}$

In[]:=
```
(x + 3) /. Solve[x^2 + 2 x - 1 == 0, x]
```
Out[]= $\{2 - \sqrt{2}, 2 + \sqrt{2}\}$

Other Commands for Solving Equations

In[]:=
```
Solve[x^2 - y^3 == 1, {x, y}]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[]= $\{\{x \to -\sqrt{1 + y^3}\}, \{x \to \sqrt{1 + y^3}\}\}$

In[]:=
```
Reduce[x^2 - y^3 == 1, {x, y}]
```
Out[]= $y == (-1 + x^2)^{1/3} \,||\, y == -(-1)^{1/3} (-1 + x^2)^{1/3} \,||\, y == (-1)^{2/3} (-1 + x^2)^{1/3}$

```
FindRoot[Sin[x^2] - Cos[x] == 0, {x, π}]
```
Out[]= $\{x \to 3.29304\}$

In[]:=
```
Reduce[Sin[x^2] - Cos[x] == 0, x]
```
Out[]= $$C[1] \in \mathbb{Z} \,\&\&\, \left(x == \frac{1}{2}\left(-1 - \sqrt{1 + 2\pi - 16\pi C[1]}\right) \,||\, x == \frac{1}{2}\left(-1 + \sqrt{1 + 2\pi - 16\pi C[1]}\right) \,||\, x == \frac{1}{2}\left(-1 - \sqrt{1 + 2\pi - 8(\pi + 2\pi C[1])}\right) \,||\, x == \frac{1}{2}\left(-1 + \sqrt{1 + 2\pi - 8(\pi + 2\pi C[1])}\right) \,||\, x == \frac{1}{2}\left(1 - \sqrt{1 + 2\pi - 16\pi C[1]}\right) \,||\, x == \frac{1}{2}\left(1 + \sqrt{1 + 2\pi - 16\pi C[1]}\right) \,||\, x == \frac{1}{2}\left(1 - \sqrt{1 + 2\pi - 8(\pi + 2\pi C[1])}\right) \,||\, x == \frac{1}{2}\left(1 + \sqrt{1 + 2\pi - 8(\pi + 2\pi C[1])}\right)\right)$$

Calculus

Differentiation

In[]:=
```
D[x^2 Sin[x], x]
```
Out[]= $x^2 \,\text{Cos}[x] + 2 x \,\text{Sin}[x]$

In[]:=
```
D[x^2 Sin[x], {x, 3}]
```
Out[]= $6 \,\text{Cos}[x] - x^2 \,\text{Cos}[x] - 6 x \,\text{Sin}[x]$

In[]:=
```
Sin'[x]
```
Out[]= Cos[x]

In[∘]:=
```
f[x_] := x^3 - 2 x^2 - 5 x + 6
```

In[∘]:=
```
f'[x]
```

Out[∘]= $-5 - 4 x + 3 x^2$

```
{D[f[x], x], f'[x]} (*equivalant*)
```

Out[∘]= $\{-5 - 4 x + 3 x^2, -5 - 4 x + 3 x^2\}$

In[∘]:=
```
Table[{x, f[x], f'[x], f''[x]}, {x, 1, 10}] // TableForm
```

Out[∘]//TableForm=

1	0	-6	2
2	-4	-1	8
3	0	10	14
4	18	27	20
5	56	50	26
6	120	79	32
7	216	114	38
8	350	155	44
9	528	202	50
10	756	255	56

In[∘]:=
```
D[x^2 Cos[x y] + y^2 Sin[x y], x, y]
```

Out[∘]= $-x^3 y \,\text{Cos}[x y] + 3 y^2 \,\text{Cos}[x y] - 3 x^2 \,\text{Sin}[x y] - x y^3 \,\text{Sin}[x y]$

In[∘]:=
```
D[Sin[x]^10, {x, 4}]
```

Out[∘]= $5040 \,\text{Cos}[x]^4 \,\text{Sin}[x]^6 - 4680 \,\text{Cos}[x]^2 \,\text{Sin}[x]^8 + 280 \,\text{Sin}[x]^{10}$

In[∘]:=
```
Simplify[%]
```

Out[∘]= $10 \left(141 + 238 \,\text{Cos}[2 x] + 125 \,\text{Cos}[4 x]\right) \text{Sin}[x]^6$

In[∘]:=
```
D[x g[x], x]
```

Out[∘]= $g[x] + x g'[x]$

In[∘]:=
```
D[x g[x], {x, 2}]
```

Out[∘]= $2 g'[x] + x g''[x]$

In[∘]:=
```
D[g[x^2] g''[x], x] /. g → Sin
```

Out[∘]= $-2 x \,\text{Cos}[x^2] \,\text{Sin}[x] - \text{Cos}[x] \,\text{Sin}[x^2]$

Limits

In[∘]:=
```
Limit[1/x, x → 1]
```

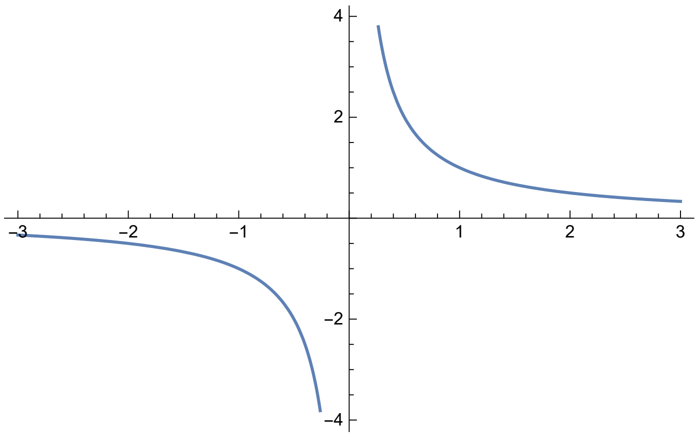
Out[∘]= 1

In[∘]:=
```
Limit[1/x, x → Infinity]
```

Out[∘]= 0

Direction

```
In[•]:= Plot[1/x, {x, -3, 3}]
```

Out[•]=



```
In[•]:= Limit[1/x, x → 0, Direction → +1] (*left*)
Out[•]= -∞

In[•]:= Limit[1/x, x → 0, Direction → -1] (*right*)
Out[•]= ∞

In[•]:= Limit[1/x, x → 0] (*default direction*)
Out[•]= Indeterminate

In[•]:= Limit[x^a, a → ∞]
Out[•]= ConditionalExpression[∞, Log[x] > 0]

In[•]:= Limit[x^a, a → ∞, Assumptions → x > 1]
Out[•]= ∞

In[•]:= Limit[x^a, a → ∞, Assumptions → x == 1]
Out[•]= 1
```

Integration

Indefinite Integration

```
In[•]:= Integrate[x^2 + 2 x + 1, x]
```

Out[•]= $x + x^2 + \frac{x^3}{3}$

In[∘]:= $\int (x\text{^}2 + 2\,x + 1)\, dx$

Out[∘]= $x + x^2 + \frac{x^3}{3}$

In[∘]:=

```
integrate of sin(x)*cos(x)
Integrate[Sin[x] * Cos[x], x]
```

Out[∘]= $-\frac{1}{2}\,\mathrm{Cos}[x]^2$

Definite Integration

In[∘]:=

```
Integrate[x^2 Exp[x], {x, 0, 1}]
```

Out[∘]= $-2 + e$

In[∘]:=

```
N[%]
```

Out[∘]= 0.718282

In[∘]:=

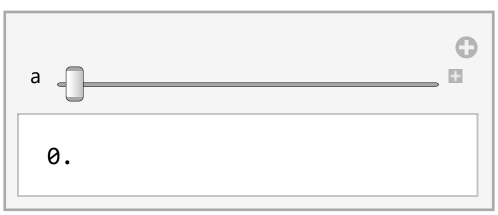
```
Integrate[x^2 Exp[x], {x, a, b}]
```

Out[∘]= $-\left(2 + (-2 + a)\,a\right)\,e^{a} + \left(2 + (-2 + b)\,b\right)\,e^{b}$

In[∘]:=

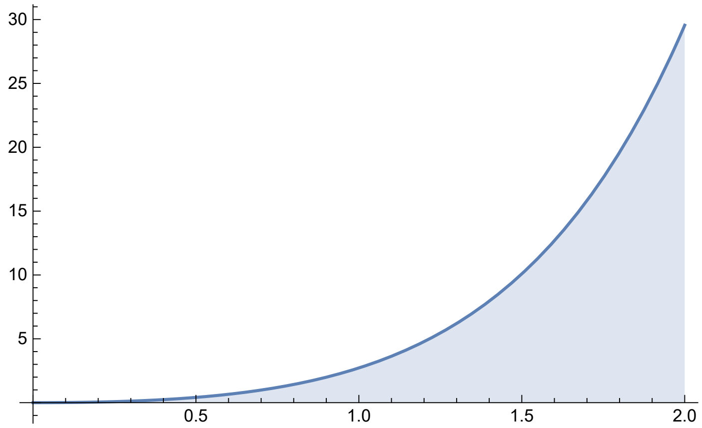
```
Manipulate[
Integrate[x^2 Exp[x], {x, 0, a}],
 {a, 0, 8}]
```

Out[∘]=



In[]:=
```
Plot[x^2 Exp[x], {x, 0, 2}, Filling → Axis]
```

Out[]=



In[]:=
```
Integrate[x^3 Sin[y] + y^2 Cos[x^2], {x, -1, 1}, {y, -2, x}]
```

Out[]= $\frac{8}{3}\sqrt{2\pi}\ \text{FresnelC}\left[\sqrt{\frac{2}{\pi}}\right]$

Differential Equations

Solving Symbolically with DSolve

In[]:=
```
DSolve[y'[x] == x^2 Sin[x], y[x], x]
```

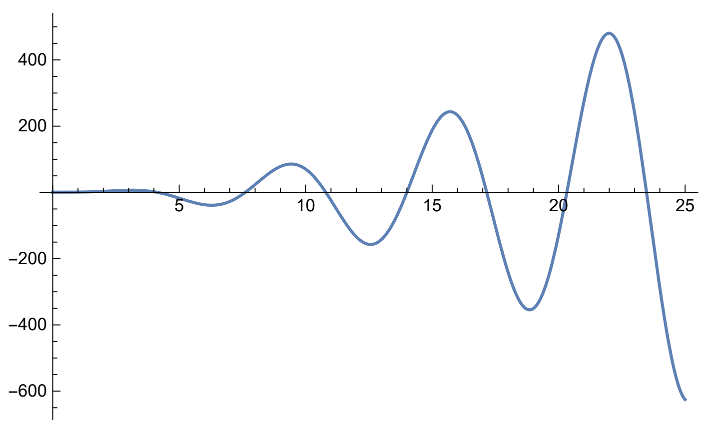
Out[]= $\{\{y[x] \to C[1] - (-2 + x^2)\ \text{Cos}[x] + 2\ x\ \text{Sin}[x]\}\}$

In[]:=
```
DSolve[{y'[x] == x^2 Sin[x], y[1] == 1}, y[x], x]
```

Out[]= $\{\{y[x] \to 1 - \text{Cos}[1] + 2\ \text{Cos}[x] - x^2\ \text{Cos}[x] - 2\ \text{Sin}[1] + 2\ x\ \text{Sin}[x]\}\}$

In[]:=
```
soln = DSolveValue[{y'[x] == x^2 Sin[x], y[1] == 1}, y[x], x];
Plot[soln, {x, 0, 25}]
```

Out[]=



In[∘]:=
```
soln = DSolveValue[{y''[x] + y'[x] + y[x] == 0}, y[x], x]
```

Out[∘]= $e^{-x/2}\, C[2]\, \mathrm{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + e^{-x/2}\, C[1]\, \mathrm{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]$

```
solnTable = Table[soln /. {C[1] → i, C[2] → j}, {i, -1, 1, 1}, {j, -1, 1, 1}] (*TableForm*)
```

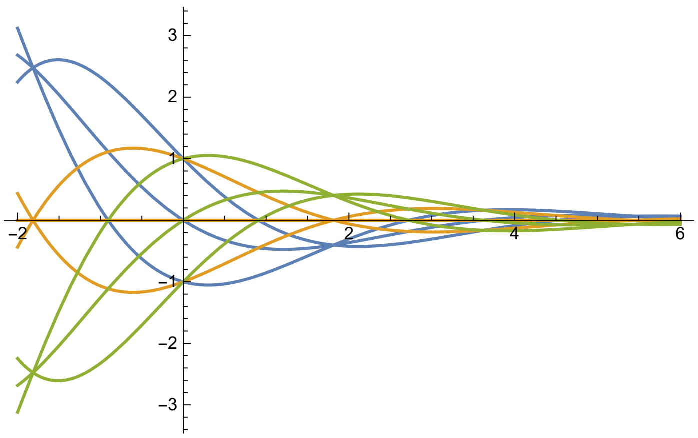
Out[∘]= $\left\{\left\{-e^{-x/2}\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] - e^{-x/2}\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right],\ -e^{-x/2}\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right],\ e^{-x/2}\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] - e^{-x/2}\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right\},\right.$

$\left\{-e^{-x/2}\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right],\ 0,\ e^{-x/2}\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right]\right\},$

$\left.\left\{-e^{-x/2}\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + e^{-x/2}\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right],\ e^{-x/2}\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right],\ e^{-x/2}\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + e^{-x/2}\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right\}\right\}$

In[∘]:=
```
Plot[solnTable, {x, -2, 6}, PlotRange → All]
```

Out[∘]=



Solving Numerically with NDSolve

In[∘]:=
```
NDSolveValue[{y'[x] == x^2 Sin[x], y[1] == 1}, y, {x, 0, 10}]
```

Out[∘]= InterpolatingFunction[Domain: {{0., 10.}} Output: scalar]

In[∘]:=
```
res = NDSolveValue[{y'[x] == x^2 Sin[x], y[1] == 1}, y, {x, 0, 10}]
```

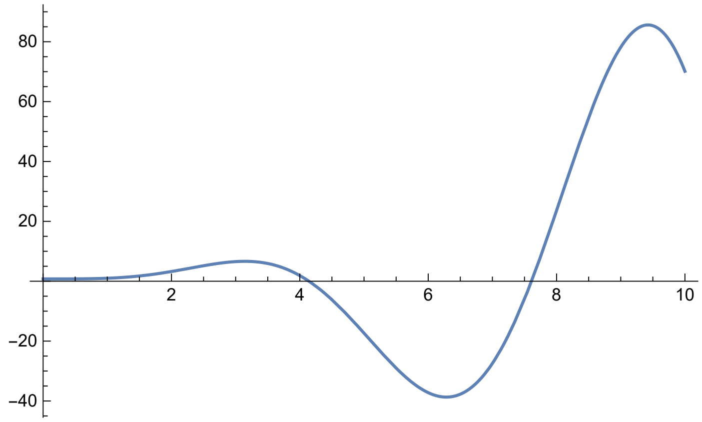
Out[∘]= InterpolatingFunction[Domain: {{0., 10.}} Output: scalar]

In[∘]:=
```
res[4]
```

Out[∘]= 1.87333

In[]:=
```
Plot[res[x], {x, 0, 10}]
```

Out[]=



The END.