بنام خدا HW#3 _ ۸۱۰۱۹۷۴۰۳ _ شایان راصف

$$out = b_3 + W_3^T x + relu\left(\tanh(b_1^T + x^T W_1) W_2^T + b_2\right) \quad , \; loss = \frac{1}{2}(out - 6)^2$$

صدگان دانشجویی

$$\boxed{\frac{\partial loss}{\partial W_{3\,(2\times1)}} = \frac{1}{2} \times 2 (out - 6) \frac{\partial out}{\partial W_3} = (out-6)\left[x + 0\right] = (out-6)\, x}$$

$$\boxed{\frac{\partial loss}{\partial b_{3\,(1\times1)}} = (out - 6) \frac{\partial out}{\partial b_3} = (out-6)}$$

$$\frac{\partial loss}{\partial W_2} = (out - 6) \frac{\partial out}{\partial W_2} = (out - 6) \frac{\partial \left[relu(\tanh(b_1^T + x^T W_1) W_2^T + b_2)\right]}{\partial W_2}$$

$$* F(x) = relu(x) \longrightarrow F' = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \xrightarrow{\text{در صفر مشتق ندارد}} F'(x) = u(x) \;,\; g(u) = \tanh(u) \to g'(u) = (1 - \tanh^2(u)) = 1 - g^2$$

$u(x)$ → step function

$$if \quad u_1 = b_1^T + x^T W_1 \;,\; u_2 = W_2 \;,\; \frac{\partial F(g(u_1))}{\partial x} = F'(g(x))\, g'(x)$$

$$\frac{\partial loss}{\partial W_{2\,(1\times3)}} = (out - 6) \frac{\partial F(g(u_1) u_2^T + b_2)}{\partial u_2} = (out - 6)\left[\frac{\partial (g(u_1) u_2^T + b_2)}{\partial u_2} \times F'(g(u_1) u_2^T + b_2)\right]$$

$$= (out - 6)\left[g(u_1) \times u(g(u_1) u_2^T + b_2)\right] = \boxed{(out-6)\left[\tanh(b_1^T + x^T W_1) \times u\left(\tanh(b_1^T + x^T W_1) W_2^T + b_2\right)\right]}$$

$$\frac{\partial loss}{\partial b_{2\,(1\times1)}} = (out-6) \frac{\partial out}{\partial b_2} = (out-6) \frac{\partial F(g(u_1) u_2^T + b_2)}{\partial b_2} = (out-6)\left[I \times F'(g(u_1) u_2^T + b_2)\right]$$

$$\boxed{= (out - 6)\left[u(\tanh(b_1^T + x^T W_1) W_2^T + b_2)\right]}$$

$$\frac{\partial loss}{\partial W_1} = (out-6) \frac{\partial F(g(u_1) u_2^T + b_2)}{\partial W_1} = (out-6) \times \frac{\partial (g(u_1) u_2^T + b_2)}{\partial W_1} \times F'(g(u_1) u_2^T + b_2)$$

$$\to g(u_1) u_2^T + b_2 = g(b_1^T + x^T W_1) u_2^T + b_2 = k(W_1) \to \frac{\partial k(W_1)}{\partial W_1} = \frac{\partial g(b_1^T + x^T W_1)}{\partial} \times g'(b_1^T + x^T W_1) u_2^T = x \times (1 - \| g(b_1^T + x^T W_1) \|^2) u_2$$

$$\boxed{\longrightarrow \frac{\partial loss}{\partial W_{1\,(2\times3)}} = \underset{(1\times1)}{(out - 6)}\; \underset{(2\times1)}{x} \underset{(1\times1)}{(1 - \| \tanh(b_1^T + x^T W_1) \|^2)} \underset{(1\times3)}{W_2} \times \underset{(1\times1)}{u(\tanh(b_1^T + x^T W_1) W_2^T + b_2)}}$$

$$\frac{\partial loss}{\partial b_{1\,(3\times1)}} = (out-6) \frac{\partial (g(u_1) u_2^T + b_2)}{\partial b_1} \times F'(g(u_1) u_2^T + b_2) \;,\; \frac{\partial k(b_1)}{\partial b_1} = \frac{\partial (b_1^T + x^T W_1)}{\partial b_1} \times g'(b_1^T + x^T W_1) W_2^T$$

$$\to \frac{\partial k(b_1)}{\partial b_1} = I_{3\times3} \times \underset{(1\times1)}{(1 - \| g(b_1^T + x^T W_1) \|^2)} W_2^T \to \boxed{\frac{\partial loss}{\partial b_1} = (out - 6) \times \underset{(1\times1)}{(1 - \| \tanh(b_1^T + x^T W_1) \|^2)} \underset{(3\times1)}{W_2^T} \underset{(1\times1)}{u(\tanh(b_1^T + x^T W_1) W_2^T + b_2)}}$$

I ماتریس همانی

$a = 3 ,\; b = 0$

۳- آپدیت وزن‌ها: ابتدا مقادیر وزن‌ها را بصورت زیر جایگزین می‌کنیم:

$$W_1' = \begin{pmatrix} 1.3 & -0.73 & 0 \\ 0.2 & -1.3 & 0.03 \end{pmatrix} ,\; b_1' = \begin{pmatrix} 0 \\ 0.01 \\ 1.3 \end{pmatrix} ,\; W_2' = \begin{pmatrix} 1.135 & -0.5 & 0.03 \end{pmatrix} ,\; b_2 = 0.1$$

$$W_3' = \begin{pmatrix} -0.3 \\ 1 \end{pmatrix} ,\; b_3' = 0.1 ,\; x' = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\rightarrow out' = 0.1 + (-0.3 \;\; 1)\begin{pmatrix} 2 \\ 3 \end{pmatrix} + relu\left[\tanh\left((0 \;\; 0.01 \;\; 1.3) + (2 \;\; 3)\begin{pmatrix} 1.3 & -0.73 & 0 \\ 0.2 & -1.3 & 0.03 \end{pmatrix}\right)\begin{pmatrix} 1.135 \\ -0.5 \\ 0.03 \end{pmatrix} + 0.1\right] \simeq 4.147$$

$$W_3^2 = W_3' - \alpha \frac{\partial loss}{\partial W_3'} = \begin{pmatrix} -0.3 \\ 1 \end{pmatrix} - 0.1 \times \left[(4.147-6)\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right] = \begin{pmatrix} 0.006 \\ 1.459 \end{pmatrix}$$

$$b_3^2 = b_3' - \alpha \frac{\partial loss}{\partial b_3'} = 0.1 - 0.1 \times \left[(4.147-6)\right] = 0.253$$

$$W_2^2 = W_2' - \alpha \frac{\partial loss}{\partial W_2'} = (1.135 \;\; -0.5 \;\; 0.03) - 0.1\left[(4.147-6)\underbrace{\left(\tanh\left((0 \;\; 0.01 \;\; 1.3) + (2 \;\; 3)\begin{pmatrix} 1.3 & -0.73 & 0 \\ 0.2 & -1.3 & 0.03 \end{pmatrix}\right)\right)}_{(0.997 \;\; -0.999 \;\; 0.883)}\right.$$

$$\left. \times \underbrace{u}_{\text{unit step}}\underbrace{\left((-0.997 \;\; -0.999 \;\; 0.883)\begin{pmatrix} 1.135 \\ -0.5 \\ 0.03 \end{pmatrix} + 0.1\right)}_{1.977>0}\right] = (1.503 \;\; -0.653 \;\; 0.165)$$

$$b_2^2 = b_2' - \alpha \frac{\partial loss}{\partial b_2'} = 0.1 - 0.1 \times \left[(4.147-6) \times \underbrace{u(1.97)}_{1}\right] = 0.253$$

$$W_1^2 = W_1' - \alpha \frac{\partial loss}{\partial W_1'} = \begin{pmatrix} 1.3 & -0.73 & 0 \\ 0.2 & -1.3 & 0.03 \end{pmatrix} - 0.1 \times \left[(4.147-6)\begin{pmatrix} 2 \\ 3 \end{pmatrix}\left(1 - \underbrace{\|(0.997 \;\; -0.999 \;\; 0.883)\|^2}_{2.772}\right)(1.135 \;\; -0.5 \;\; 0.03)\right]$$

$$= \begin{pmatrix} 0.568 & 0.7471 & -0.0763 \\ -0.898 & -0.893 & 0.0056 \end{pmatrix}$$

$$b_1^2 = b_1' - \alpha \frac{\partial loss}{\partial b_1'} = \begin{pmatrix} 0 \\ 0.01 \\ 1.3 \end{pmatrix} - 0.1 \times \left[(4.147-6) \times (1 - 2.772)\begin{pmatrix} 1.135 \\ -0.5 \\ 0.03 \end{pmatrix} \times 1\right] = \begin{pmatrix} -0.366 \\ 0.7456 \\ 1.2919 \end{pmatrix}$$

$$\rightarrow out^2 = 0.253 + (0.006 \;\; 1.459)\begin{pmatrix} 2 \\ 3 \end{pmatrix} + relu\left[\tanh\left((-0.366 \;\; 0.7456 \;\; 1.2919) + (2 \;\; 3)\begin{pmatrix} 0.568 & 0.7471 & -0.0763 \\ -0.8981 & -0.893 & 0.0056 \end{pmatrix}\right)\right.$$

$$\left. \times \begin{pmatrix} 1.503 \\ -0.653 \\ 0.165 \end{pmatrix} + 0.253\right] \boxed{\simeq 4.1642}$$

$$W_3^3 = W_3^2 - \alpha \frac{\partial loss}{\partial W_3^2} = \begin{pmatrix} 0.006 \\ 1.459 \end{pmatrix} - 0.1\left[(4.1642-6)\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right] = \begin{pmatrix} 0.278 \\ 1.866 \end{pmatrix}$$

$$b_3^3 = b_3^2 - \alpha \frac{\partial loss}{\partial b_3^2} = 0.253 - 0.1\left[(4.1642-6)\right] = 0.389$$

$$W_2^3 = W_2^2 - \alpha \frac{\partial loss}{\partial W_2^2} = (1.503 \;\; -0.653 \;\; 0.165) - 0.1\left[(4.1642-6)\left(\tanh\left((-0.366 \;\; 0.7456 \;\; 1.2919) + (2 \;\; 3) \times \begin{pmatrix} 0.568 & 0.7471 & -0.0763 \\ -0.898 & -0.893 & 0.0056 \end{pmatrix}\right)\right)\right.$$

$$\left. \times u\underbrace{\left((-0.958 \;\; -0.978 \;\; 0.855)\begin{pmatrix} 1.503 \\ -0.653 \\ 0.165 \end{pmatrix} + 0.253\right)}_{<0}\right] = (1.503 \;\; -0.653 \;\; 0.165)$$

$$b_2^3 = b_2^2 - \alpha \frac{\partial loss}{\partial b_2^2} = 0/253 - 0/1 \times 0 = 0/253$$

چون $u(\tanh(b_1^{2T} + x^T w_1^2) w_2^{2T} + b_2^2) = 0$ ، همه b_1^3 ، w_1^3 ، b_2^3 ، w_2^3 دارای این ترم هستند پس مقادیر این در نما در $epch = 2$ تغییر نمی کند.

$$w_1^3 = \begin{pmatrix} 0/568 & 0/7471 & -0/0763 \\ -0/898 & -0/893 & 0/056 \end{pmatrix} , \quad b_1^3 = \begin{pmatrix} -0/366 \\ 0/7456 \\ 1/2979 \end{pmatrix}$$

$$\longrightarrow out^3 = 0/389 + (0/278 \quad 1/866)\begin{pmatrix} 2 \\ 3 \end{pmatrix} + relu\Big[\tanh\Big((-0/366 \quad 0/7456 \quad 1/2979) + (2 \; 3)\begin{pmatrix} 0/568 & 0/7471 & -0/0763 \\ -0/898 & -0/893 & 0/056 \end{pmatrix}\Big)$$

$$\times \begin{pmatrix} 1/5.3 \\ -0/653 \\ 0/765 \end{pmatrix} + 0/253\Big] \simeq \boxed{6/54}$$