VAST

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```
library(tinyVAST)
library(fmesher)
set.seed(101)
```

`tinyVAST` is an R package for fitting vector autoregressive spatio-temporal (VAST) models. We here explore the capacity to specify the vector-autoregressive spatio-temporal component.

Spatio-temporal autoregressive model

We first explore the ability to specify a first-order autoregressive spatio-temporal process:

```
# Simulate settings
theta_xy = 0.4
n_x = n_y = 10
n_t = 15
rho = 0.8
spatial_sd = 0.5

# Simulate GMRFs
R = exp(-theta_xy * abs(outer(1:n_x, 1:n_y, FUN="-")) )
d = mvtnorm::rmvnorm(n_t, sigma=spatial_sd^2*kronecker(R,R) )

# Project through time and add mean
for( t in seq_len(n_t) ){
  if(t>1) d[t,] = rho*d[t-1,] + d[t,]
}
#d = d + 0.5

# Shape into longform data-frame and add error
Data = data.frame( expand.grid(time=1:n_t, x=1:n_x, y=1:n_y), "var"="logn", z=exp(as.vector(d)))
Data$n = tweedie::rtweedie( n=nrow(Data), mu=Data$z, phi=0.5, power=1.5 )
mean(Data$n==0)
#> [1] 0.046

# make mesh
mesh = fm_mesh_2d( Data[,c('x','y')] )

# fit model
out = fit( sem = "logn -> logn, 1, rho",
           data = Data,
           formula = n ~ 0 + factor(time),
           spatial_graph = mesh,
```

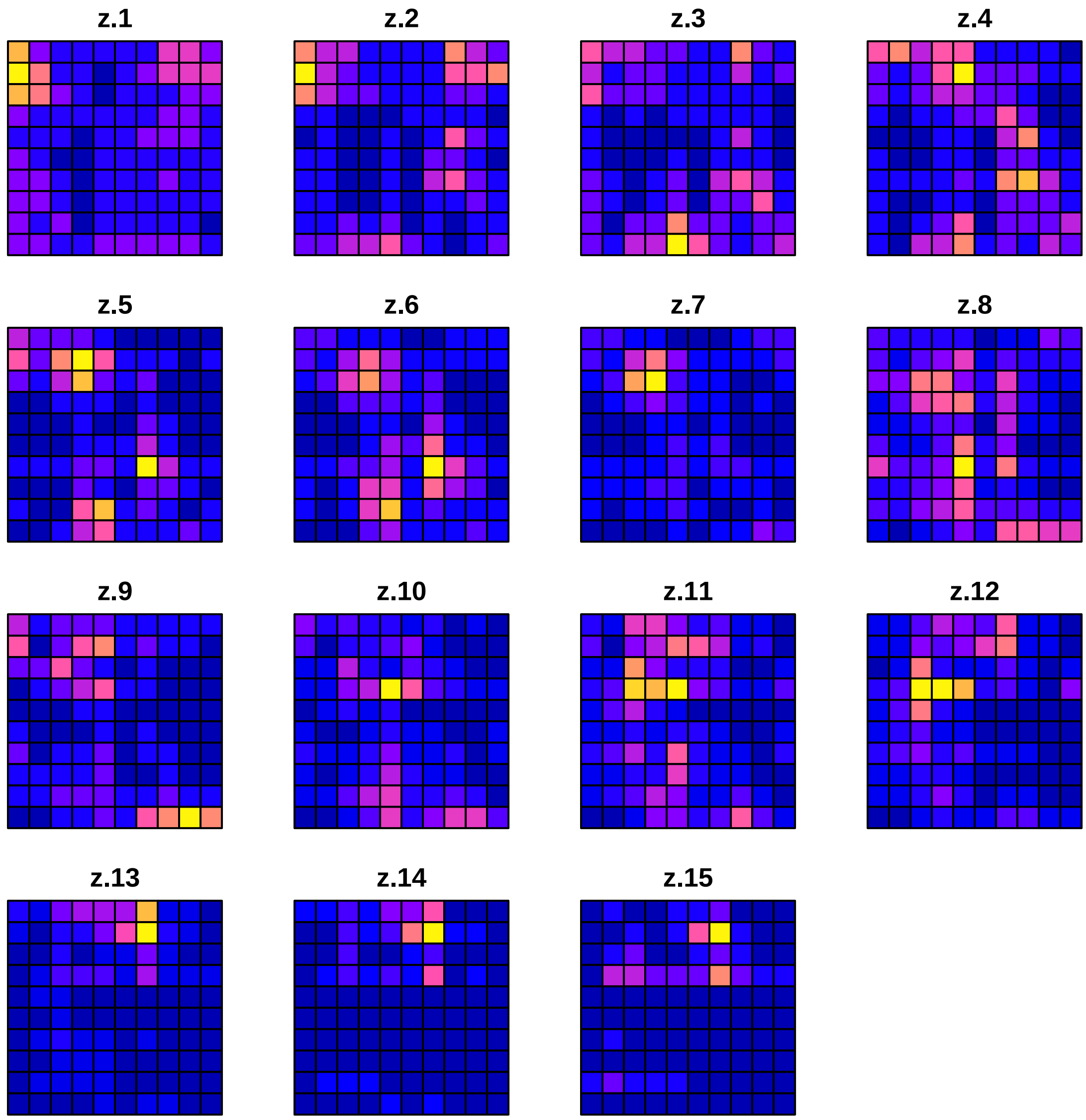
```r
          family_link = rbind("obs"=c(1,1)),
          control = tinyVASTcontrol(quiet=TRUE, trace=0) )
out
#> $call
#> fit(data = Data, formula = n ~ 0 + factor(time), sem = "logn -> logn, 1, rho",
#>     family_link = rbind(obs = c(1, 1)), spatial_graph = mesh,
#>     control = tinyVASTcontrol(quiet = TRUE, trace = 0))
#>
#> $opt
#> $opt$par
#>   log_kappa     alpha_j     alpha_j     alpha_j     alpha_j     alpha_j     alpha_j     alpha_j
#>  0.07228542 -0.08323604 -0.13549104 -0.10579217 -0.14499113 -0.37823868 -0.21633306 -0.41489958 -0.6
#>     alpha_j     alpha_j      beta_z      beta_z   log_sigma   log_sigma
#>  0.16887044  0.30040122  0.81229112  0.40988915 -0.64868475  0.04394543
#>
#> $opt$objective
#> [1] 1717.689
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 77
#>
#> $opt$evaluations
#> function gradient
#>      107       77
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>              Estimate Std. Error
#> log_kappa  0.07228542 0.10755269
#> alpha_j   -0.08323604 0.15196456
#> alpha_j   -0.13549104 0.18670340
#> alpha_j   -0.10579217 0.20529851
#> alpha_j   -0.14499113 0.21780791
#> alpha_j   -0.37823868 0.22691800
#> alpha_j   -0.21633306 0.23026450
#> alpha_j   -0.41489958 0.23456777
#> alpha_j   -0.67168423 0.23833635
#> alpha_j   -0.49463135 0.23869331
#> alpha_j   -0.13968721 0.23733095
#> alpha_j    0.14836186 0.23640201
#> alpha_j   -0.21516693 0.23873590
#> alpha_j   -0.20120060 0.23979214
#> alpha_j    0.16887044 0.23708655
#> alpha_j    0.30040122 0.23660035
#> beta_z     0.81229112 0.03708631
#> beta_z     0.40988915 0.03291043
```

```
#> log_sigma -0.64868475 0.05422114
#> log_sigma  0.04394543 0.07275797
#> Maximum gradient component: 0.006323539
#>
#> $run_time
#> Time difference of 38.09037 secs
```

The estimated values for `beta_z` then correspond to the simulated value for `rho` and `spatial_sd`.

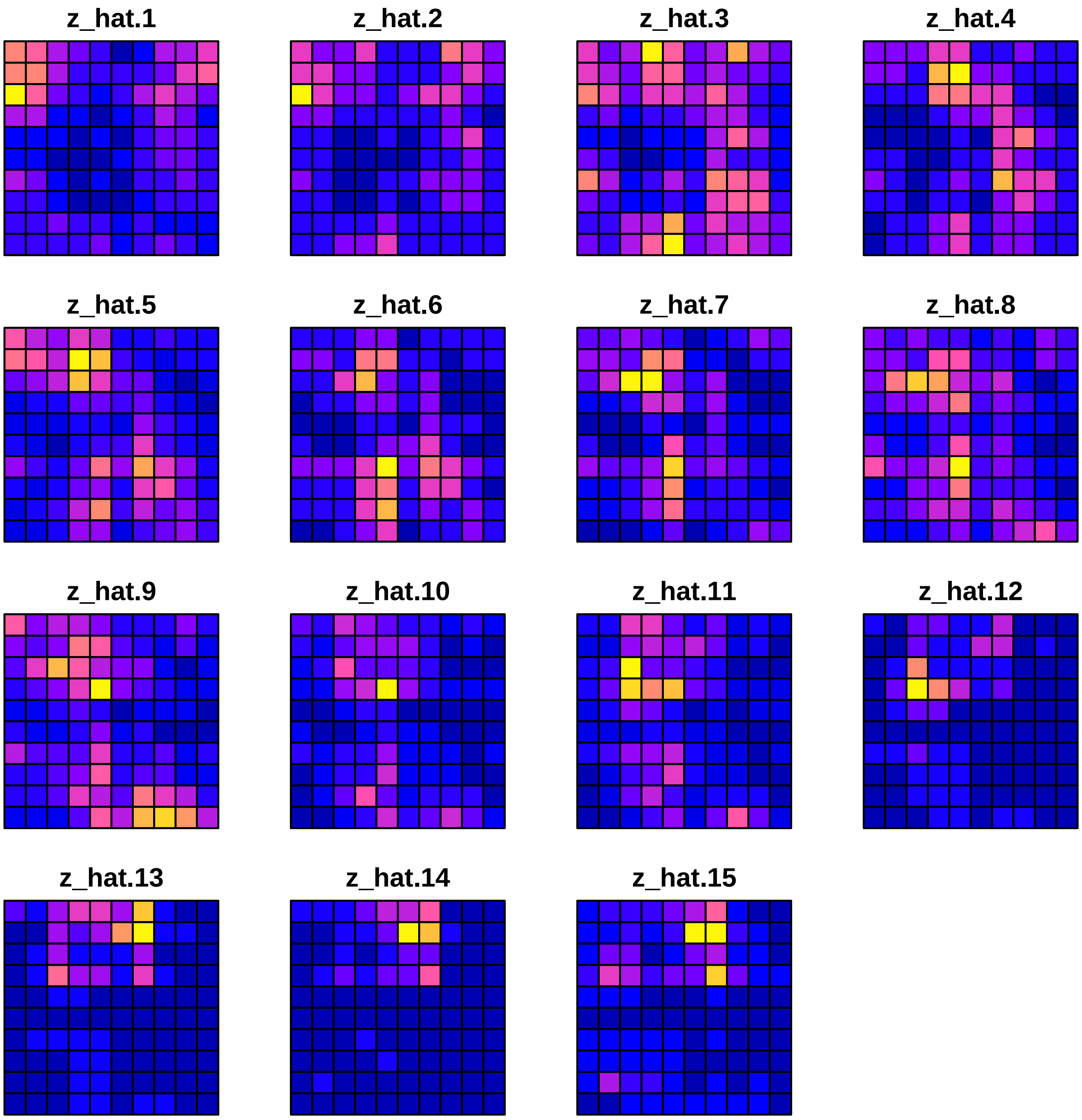
We can compare the true densities:

```
library(sf)
data_wide = reshape( Data[,c('x','y','time','z')],
                     direction = "wide", idvar = c('x','y'), timevar = "time")
sf_data = st_as_sf( data_wide, coords=c("x","y"))
sf_grid = sf::st_make_grid( sf_data )
sf_plot = st_sf(sf_grid, st_drop_geometry(sf_data) )
plot(sf_plot, max.plot=n_t )
```



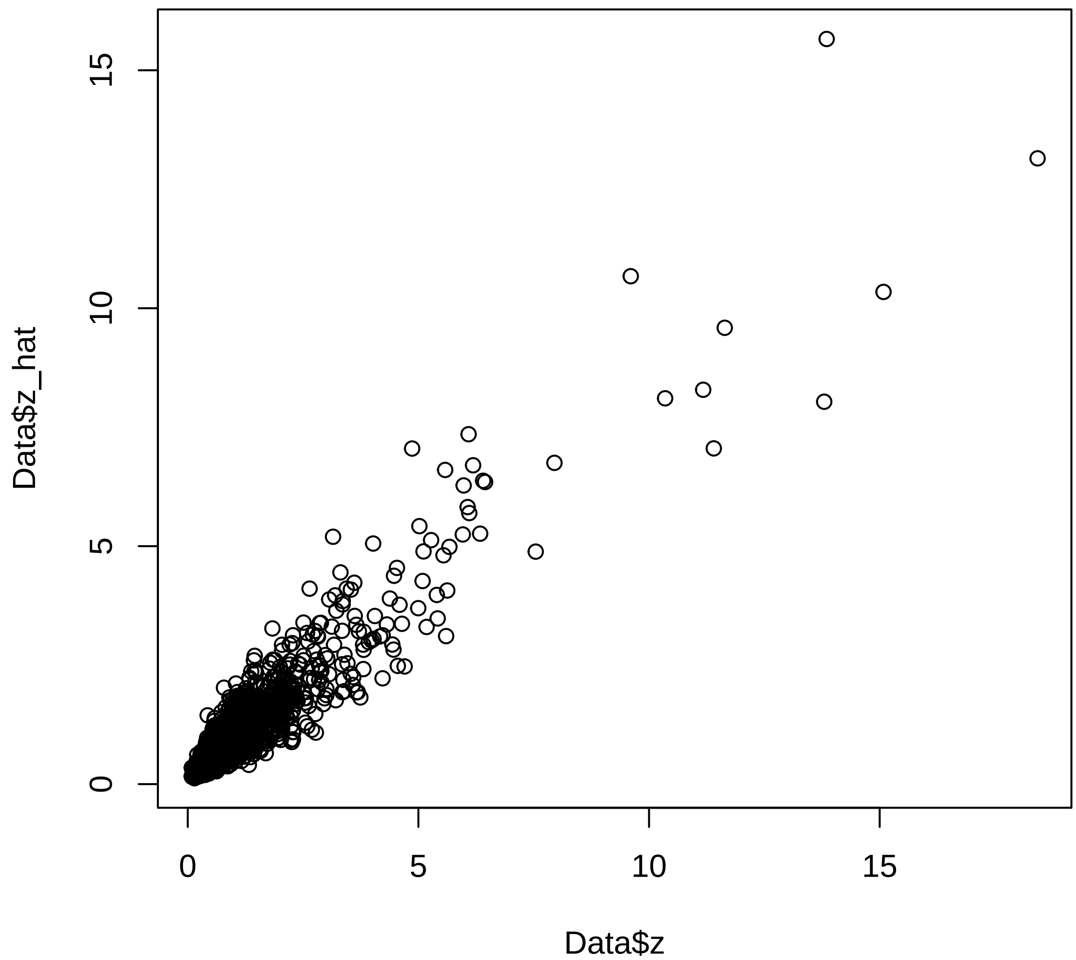
with the estimated densities:

```
Data$z_hat = predict(out)
data_wide = reshape( Data[,c('x','y','time','z_hat')],
                     direction = "wide", idvar = c('x','y'), timevar = "time")
sf_data = st_as_sf( data_wide, coords=c("x","y"))
sf_plot = st_sf(sf_grid, st_drop_geometry(sf_data) )
plot(sf_plot, max.plot=n_t )
```



where a scatterplot shows that they are highly correlated:

```
plot( x=Data$z, y=Data$z_hat )
```



We can then calculate the area-weighted total abundance and compare it with its true value:

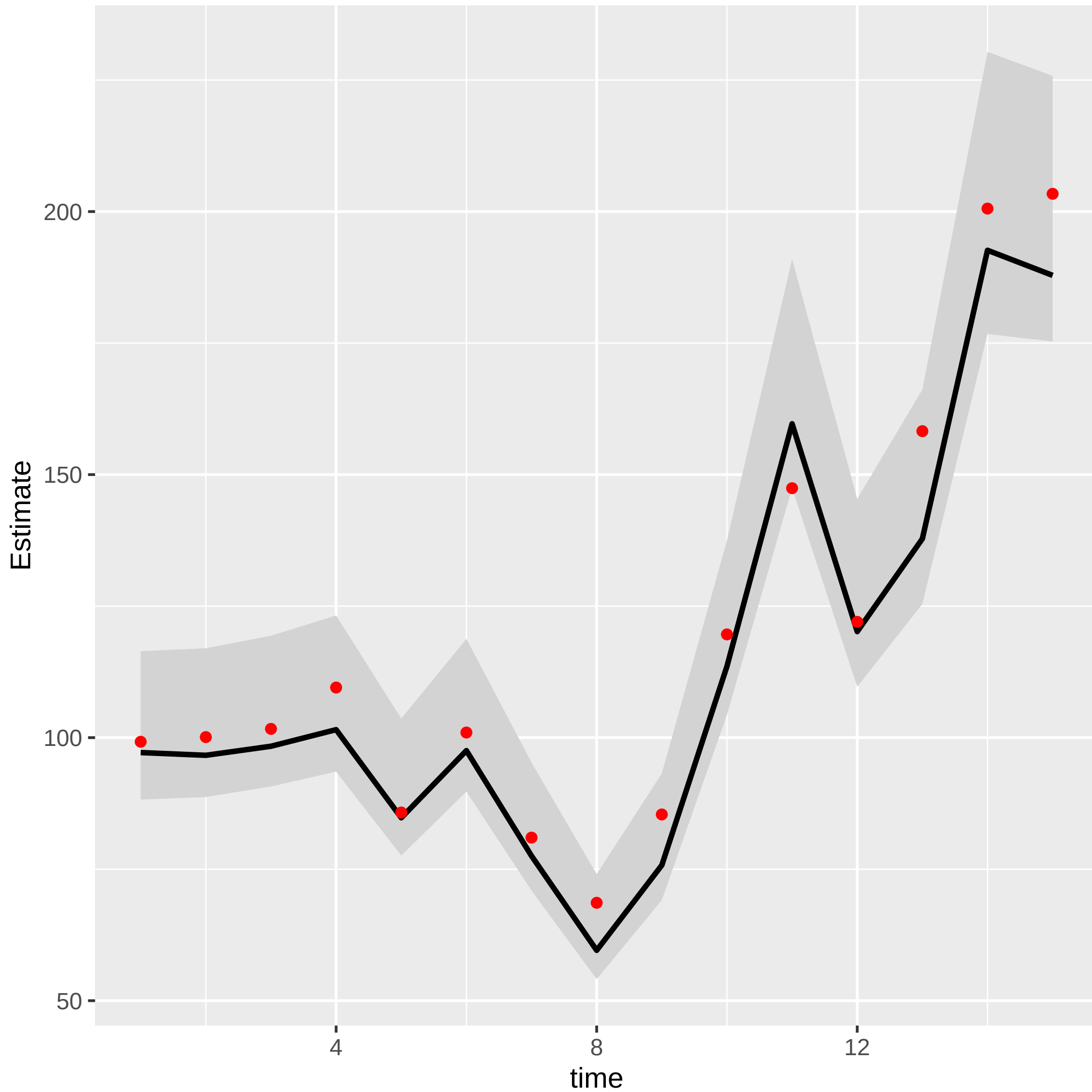
```
# Predicted sample-weighted total
(Est = sapply( seq_len(n_t), FUN=\(t) integrate_output(out, newdata=subset(Data,time==t)) ))
#>                              [,1]       [,2]       [,3]       [,4]      [,5]       [,6]      [,7]
#> Estimate                97.164903  96.643634  98.362457 101.517620 84.760587  97.538111 77.520820 59.56
#> Std. Error               7.194683   7.216494   7.309313   7.572241  6.643419   7.406226  6.207919  5.09
#> Est. (bias.correct) 102.324275 102.850496 105.043004 108.373177 90.604659 104.258111 83.102278 64.06
#> Std. (bias.correct)            NA         NA         NA         NA        NA         NA        NA
#>                            [,15]
#> Estimate               187.86973
#> Std. Error              12.88189
#> Est. (bias.correct) 200.54754
#> Std. (bias.correct)           NA

# True (latent) sample-weighted total
(True = tapply( Data$z, INDEX=Data$time, FUN=sum ))
```

```
#>         1         2         3         4         5         6         7         8         9        10
#>  99.21643 100.10603 101.66846 109.52622  85.76973 100.97116  80.99847  68.60738  85.39974 119.62380

#
Index = data.frame( time=seq_len(n_t), t(Est), True )
Index$low = Index[,'Est...bias.correct.'] - 1.96*Index[,'Std..Error']
Index$high = Index[,'Est...bias.correct.'] + 1.96*Index[,'Std..Error']

#
library(ggplot2)
ggplot(Index, aes(time, Estimate)) +
  geom_ribbon(aes(ymin = low,
                ymax = high),    # shadowing cnf intervals
              fill = "lightgrey") +
  geom_line( color = "black",
            linewidth = 1) +
  geom_point( aes(time, True), color = "red" )
```



Next, we compare this against the current version of VAST

```
library(VAST)
settings = make_settings( purpose="index3",
                          n_x = n_x*n_y,
                          Region = "Other",
                          bias.correct = FALSE,
                          use_anisotropy = FALSE )
settings$FieldConfig['Epsilon','Component_1'] = 0
settings$FieldConfig['Omega',] = 0
settings$RhoConfig['Epsilon2'] = 4
settings$RhoConfig['Beta1'] = 3
settings$ObsModel = c(10,2)

myVAST = fit_model( settings=settings,
                 Lat_i = Data[,'y'],
                 Lon_i = Data[,'x'],
```

```
                 t_i = Data[,'time'],
                 b_i = Data[,'n'],
                 a_i = rep(1,nrow(Data)),
                 observations_LL = cbind(Lat=Data[,'y'],Lon=Data[,'x']),
                 grid_dim_km = c(100,100),
                 newtonsteps = 0,
                 loopnum = 0,
                 control = list(eval.max=100, iter.max=100, trace=0) )
#> Warning in FishStatsUtils:::inla.barrier.fem.copy(mesh = anisotropic_mesh, : Please install the `INL
#>  which contains an implementation that runs faster!
#>              Component_1 Component_2
#> Omega                 -1          -1
#> Epsilon               -1          -2
#> Beta                  -2          -2
#> Epsilon_time          -3          -3
#> Eta1 Eta2
#>   -1   -1
#> Warning: 4 external pointers will be removed
#> Note: Library 'VAST_v14_0_1_TMBad.dll' was unloaded.
#>    Coefficient_name Number_of_coefficients   Type
#> 1          beta1_ft                      1  Fixed
#> 2          beta2_ft                     15  Fixed
#> 3    Epsilon_rho2_f                      1  Fixed
#> 4      L_epsilon2_z                      1  Fixed
#> 5         logkappa2                      1  Fixed
#> 6         logSigmaM                      1  Fixed
#> 7 Epsiloninput2_sff                   2040 Random
#>   0:     2105.7266:  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000
#>   1:     1993.4850: 0.248169 -0.00621313 -0.00940275 -0.00474146 -0.00488678 -0.0304006 -0.0100244 -
#>   2:     1815.3664: -0.625674 -0.0250910 -0.0263651 -0.0154445 -0.0135640 -0.0640075 -0.0194314 -0.0
#>   3:     1787.0493: -0.401654 0.00970346 -0.0193647 -0.00730560 -0.00492316 -0.0744611 -0.00503150 -
#>   4:     1777.8725: -0.446246 0.0170642 -0.0177169 -0.00500788 -0.00228127 -0.0780425 -0.000709483 -
#>   5:     1769.4163: -0.519117 0.0790811 0.00101357 0.0150763 0.0200457 -0.0994408 0.0309285 -0.06803
#>   6:     1760.3172: -0.511935 0.157190 0.0320130 0.0398107 0.0442341 -0.117800 0.0594775 -0.0745811
#>   7:     1750.8485: -0.566256 0.242491 0.0751094 0.0723315 0.0724627 -0.130056 0.0861515 -0.0818699
#>   8:     1747.1048: -0.570896 0.297687 0.124667 0.106758 0.0886785 -0.116014 0.0852856 -0.0872344 -0
#>   9:     1744.9699: -0.509053 0.311068 0.139401 0.117785 0.0947485 -0.112493 0.0849978 -0.0893639 -0
#>  10:     1743.1222: -0.512816 0.321511 0.166698 0.139649 0.108323 -0.104951 0.0823444 -0.0952418 -0.
#>  11:     1742.7069: -0.578092 0.317902 0.191272 0.161171 0.123579 -0.0949179 0.0755250 -0.102868 -0.
#>  12:     1741.4359: -0.553625 0.350576 0.228002 0.198994 0.151349 -0.0855427 0.0768123 -0.112226 -0.
#>  13:     1741.3951: -0.549230 0.354115 0.230379 0.201585 0.153643 -0.0848625 0.0779499 -0.112529 -0.
#>  14:     1741.3352: -0.551012 0.357402 0.232642 0.204355 0.156205 -0.0837881 0.0792783 -0.112707 -0.
#>  15:     1741.2774: -0.550206 0.360788 0.235473 0.207574 0.159120 -0.0822459 0.0806568 -0.112809 -0.
#>  16:     1741.2267: -0.551500 0.363838 0.238499 0.210823 0.161987 -0.0806066 0.0819352 -0.112867 -0.
#>  17:     1741.1721: -0.550622 0.366481 0.241688 0.214111 0.164885 -0.0786655 0.0831737 -0.112753 -0.
#>  18:     1741.1236: -0.551920 0.368996 0.244873 0.217411 0.167764 -0.0767614 0.0844078 -0.112601 -0.
#>  19:     1741.0724: -0.551195 0.371249 0.248032 0.220698 0.170664 -0.0746785 0.0856768 -0.112263 -0.
#>  20:     1741.0260: -0.552449 0.373520 0.251087 0.223967 0.173556 -0.0726804 0.0870056 -0.111886 -0.
#>  21:     1740.9777: -0.551772 0.375592 0.254042 0.227169 0.176446 -0.0705493 0.0884075 -0.111306 -0.
#>  22:     1740.9333: -0.552979 0.377691 0.256908 0.230329 0.179311 -0.0684846 0.0898717 -0.110681 -0.
#>  23:     1740.8874: -0.552354 0.379597 0.259661 0.233382 0.182143 -0.0663000 0.0914165 -0.109843 -0.
#>  24:     1740.8448: -0.553529 0.381533 0.262347 0.236393 0.184943 -0.0641761 0.0930184 -0.108966 -0.
#>  25:     1740.8009: -0.552945 0.383290 0.264911 0.239281 0.187690 -0.0619540 0.0947033 -0.107879 -0.
```

```
#>  26:     1740.7599: -0.554087 0.385088 0.267422 0.242138 0.190408 -0.0597943 0.0964357 -0.106769 -0.
#>  27:     1740.7179: -0.553533 0.386717 0.269810 0.244868 0.193066 -0.0575534 0.0982451 -0.105458 -0.
#>  28:     1740.6785: -0.554646 0.388394 0.272165 0.247582 0.195703 -0.0553732 0.100088 -0.104144 -0.3
#>  29:     1740.6381: -0.554116 0.389913 0.274398 0.250169 0.198274 -0.0531233 0.101999 -0.102642 -0.3
#>  30:     1740.5999: -0.555202 0.391487 0.276618 0.252755 0.200835 -0.0509313 0.103932 -0.101154 -0.3
#>  31:     1740.5609: -0.554691 0.392909 0.278720 0.255217 0.203329 -0.0486780 0.105925 -0.0994921 -0.
#>  32:     1740.5240: -0.555753 0.394396 0.280824 0.257695 0.205823 -0.0464791 0.107931 -0.0978588 -0.
#>  33:     1740.4863: -0.555260 0.395736 0.282815 0.260052 0.208250 -0.0442245 0.109991 -0.0960631 -0.
#>  34:     1740.4504: -0.556300 0.397148 0.284822 0.262437 0.210687 -0.0420205 0.112057 -0.0943084 -0.
#>  35:     1740.4138: -0.555822 0.398418 0.286719 0.264704 0.213058 -0.0397646 0.114172 -0.0924001 -0.
#>  36:     1740.3790: -0.556842 0.399766 0.288644 0.267012 0.215447 -0.0375560 0.116289 -0.0905430 -0.
#>  37:     1740.3433: -0.556378 0.400974 0.290460 0.269203 0.217770 -0.0352983 0.118450 -0.0885390 -0.
#>  38:     1740.3094: -0.557382 0.402268 0.292316 0.271446 0.220119 -0.0330849 0.120611 -0.0865946 -0.
#>  39:     1740.2747: -0.556930 0.403424 0.294064 0.273572 0.222402 -0.0308244 0.122813 -0.0845081 -0.
#>  40:     1740.2416: -0.557918 0.404671 0.295860 0.275760 0.224718 -0.0286058 0.125012 -0.0824886 -0.
#>  41:     1740.2077: -0.557478 0.405780 0.297549 0.277831 0.226967 -0.0263417 0.127249 -0.0803302 -0.
#>  42:     1740.1754: -0.558453 0.406989 0.299295 0.279972 0.229255 -0.0241174 0.129481 -0.0782454 -0.
#>  43:     1740.1423: -0.558024 0.408057 0.300933 0.281996 0.231475 -0.0218490 0.131750 -0.0760237 -0.
#>  44:     1740.1107: -0.558986 0.409232 0.302637 0.284098 0.233741 -0.0196188 0.134013 -0.0738817 -0.
#>  45:     1739.1745: -0.567381 0.435950 0.399872 0.419534 0.401618 0.196149 0.349401 0.173937 -0.0615
#>  46:     1738.8279: -0.596524 0.471652 0.397182 0.441269 0.441486 0.225559 0.413372 0.232199 0.00229
#>  47:     1738.5935: -0.588631 0.486598 0.424664 0.445106 0.443186 0.233576 0.433439 0.257339 -0.0001
#>  48:     1738.5330: -0.588773 0.482571 0.456978 0.479450 0.446830 0.251282 0.408587 0.228845 -0.0112
#>  49:     1738.4986: -0.586549 0.503767 0.458645 0.501052 0.483951 0.237362 0.397869 0.199913 -0.0305
#>  50:     1738.4741: -0.588433 0.526758 0.465209 0.502090 0.498809 0.229918 0.397908 0.194546 -0.0755
#>  51:     1738.4317: -0.595944 0.528783 0.468489 0.500110 0.482969 0.237850 0.406101 0.192359 -0.0929
#>  52:     1738.3873: -0.593420 0.521139 0.470568 0.496121 0.466359 0.237208 0.408074 0.179406 -0.0851
#>  53:     1738.3670: -0.593640 0.520784 0.470148 0.495289 0.468669 0.236030 0.403759 0.182930 -0.0831
#>  54:     1738.3647: -0.592189 0.520681 0.470006 0.495198 0.469040 0.235710 0.402975 0.183604 -0.0827
#>  55:     1738.3536: -0.592538 0.519602 0.469976 0.494577 0.469653 0.234602 0.401502 0.185091 -0.0809
#>  56:     1738.3495: -0.591602 0.518839 0.469263 0.494263 0.469916 0.233437 0.400140 0.186819 -0.0789
#>  57:     1738.3457: -0.591405 0.518700 0.468028 0.494016 0.469236 0.232796 0.400074 0.188030 -0.0771
#>  58:     1738.3431: -0.590947 0.518128 0.467835 0.492128 0.469429 0.233065 0.400265 0.189199 -0.0761
#>  59:     1738.3414: -0.590732 0.516841 0.466226 0.493451 0.469814 0.232361 0.398789 0.191112 -0.0739
#>  60:     1738.3404: -0.590639 0.517947 0.466264 0.493252 0.469227 0.230588 0.400163 0.190894 -0.0716
#>  61:     1738.3396: -0.590399 0.518399 0.467536 0.490949 0.469168 0.231677 0.399523 0.192403 -0.0718
#>  62:     1738.3393: -0.590041 0.518003 0.467475 0.490879 0.468970 0.232448 0.398910 0.193122 -0.0722
#>  63:     1738.3384: -0.589853 0.516711 0.466863 0.491482 0.469107 0.232961 0.398825 0.192952 -0.0726
#>  64:     1738.3381: -0.589898 0.516290 0.465766 0.491697 0.470084 0.231935 0.398781 0.192302 -0.0719
#>  65:     1738.3378: -0.590047 0.517086 0.465802 0.491283 0.469494 0.231363 0.397506 0.193365 -0.0711
#>  66:     1738.3377: -0.589550 0.516628 0.465973 0.491706 0.468490 0.230889 0.397833 0.193791 -0.0706
#>  67:     1738.3376: -0.589612 0.515421 0.465693 0.491652 0.468542 0.231085 0.398274 0.193531 -0.0711
#>  68:     1738.3374: -0.589379 0.515397 0.465269 0.491285 0.468871 0.231574 0.398032 0.193246 -0.0715
#>  69:     1738.3374: -0.589387 0.516151 0.465173 0.490950 0.468610 0.231381 0.397811 0.193109 -0.0708
#>  70:     1738.3373: -0.589579 0.515992 0.465411 0.490953 0.468609 0.230659 0.397439 0.193530 -0.0708
#>  71:     1738.3373: -0.589571 0.515993 0.465395 0.490960 0.468613 0.230653 0.397427 0.193531 -0.0708
#>  72:     1738.3373: -0.589531 0.515994 0.465366 0.490973 0.468620 0.230644 0.397406 0.193533 -0.0707
#>  73:     1738.3373: -0.589531 0.515991 0.465339 0.490982 0.468627 0.230636 0.397387 0.193533 -0.0707
#>  74:     1738.3373: -0.589505 0.515976 0.465305 0.490981 0.468633 0.230638 0.397365 0.193528 -0.0707
#>  75:     1738.3373: -0.589501 0.515963 0.465270 0.490982 0.468639 0.230634 0.397341 0.193523 -0.0707
#>  76:     1738.3373: -0.589476 0.515942 0.465232 0.490970 0.468644 0.230632 0.397311 0.193511 -0.0707
#>  77:     1738.3373: -0.589470 0.515924 0.465196 0.490962 0.468644 0.230618 0.397280 0.193502 -0.0707
#>  78:     1738.3373: -0.589450 0.515903 0.465165 0.490947 0.468632 0.230592 0.397249 0.193491 -0.0707
```

```
#>  79:     1738.3373: -0.589448 0.515887 0.465135 0.490939 0.468618 0.230562 0.397221 0.193484 -0.0707
#>  80:     1738.3373: -0.589432 0.515873 0.465106 0.490918 0.468596 0.230532 0.397195 0.193472 -0.0707
#>  81:     1738.3373: -0.589431 0.515857 0.465076 0.490910 0.468581 0.230503 0.397167 0.193468 -0.0707
#>  82:     1738.3373: -0.589416 0.515833 0.465055 0.490892 0.468558 0.230471 0.397140 0.193461 -0.0707
#>  83:     1738.3373: -0.589414 0.515832 0.465019 0.490888 0.468554 0.230450 0.397115 0.193455 -0.0706
#>  84:     1738.3373: -0.589401 0.515828 0.464989 0.490864 0.468537 0.230431 0.397087 0.193438 -0.0706
#>  85:     1738.3373: -0.589401 0.515803 0.464977 0.490865 0.468524 0.230402 0.397068 0.193441 -0.0706
#>  86:     1738.3373: -0.589387 0.515777 0.464969 0.490856 0.468503 0.230363 0.397056 0.193445 -0.0706
#>  87:     1738.3373: -0.589385 0.515781 0.464934 0.490839 0.468490 0.230348 0.397029 0.193431 -0.0706
#>  88:     1738.3373: -0.589374 0.515769 0.464908 0.490825 0.468485 0.230335 0.396999 0.193416 -0.0706
#>  89:     1738.3373: -0.589373 0.515738 0.464904 0.490800 0.468466 0.230302 0.396980 0.193406 -0.0706
#>  90:     1738.3373: -0.589346 0.515737 0.464863 0.490812 0.468459 0.230282 0.396958 0.193412 -0.0706
#>  91:     1738.3373: -0.589355 0.515745 0.464814 0.490819 0.468427 0.230274 0.396935 0.193411 -0.0706
#>  92:     1738.3373: -0.589353 0.515721 0.464819 0.490766 0.468436 0.230252 0.396916 0.193382 -0.0706
#>  93:     1738.3373: -0.589346 0.515700 0.464810 0.490745 0.468409 0.230219 0.396887 0.193371 -0.0706
#>  94:     1738.3373: -0.589337 0.515689 0.464780 0.490748 0.468364 0.230185 0.396866 0.193376 -0.0706
#>  95:     1738.3373: -0.589338 0.515677 0.464768 0.490731 0.468346 0.230141 0.396879 0.193382 -0.0706
#>  96:     1738.3373: -0.589341 0.515672 0.464734 0.490714 0.468326 0.230152 0.396840 0.193351 -0.0706
#>  97:     1738.3373: -0.589333 0.515663 0.464710 0.490699 0.468307 0.230136 0.396799 0.193330 -0.0706
#>  98:     1738.3373: -0.589319 0.515645 0.464699 0.490673 0.468292 0.230092 0.396783 0.193323 -0.0706
#>  99:     1738.3373: -0.589319 0.515623 0.464680 0.490645 0.468260 0.230067 0.396786 0.193314 -0.0706
#> 100:     1738.3373: -0.589335 0.515616 0.464667 0.490640 0.468249 0.230043 0.396745 0.193293 -0.0706
#> 101:     1738.3373: -0.589315 0.515603 0.464638 0.490619 0.468221 0.230020 0.396728 0.193274 -0.0707
#> 102:     1738.3373: -0.589305 0.515592 0.464606 0.490574 0.468212 0.230019 0.396733 0.193271 -0.0706
#> 103:     1738.3373: -0.589304 0.515589 0.464616 0.490574 0.468193 0.229976 0.396692 0.193245 -0.0706
#> 104:     1738.3373: -0.589304 0.515559 0.464589 0.490557 0.468168 0.229960 0.396668 0.193243 -0.0707
#> 105:     1738.3373: -0.589302 0.515566 0.464581 0.490553 0.468152 0.229949 0.396668 0.193226 -0.0707
#> 106:     1738.3373: -0.589302 0.515566 0.464581 0.490553 0.468152 0.229949 0.396668 0.193226 -0.0707
myVAST
#> fit_model(.) result
#> $par
#>       beta1_ft       beta2_ft       beta2_ft       beta2_ft       beta2_ft       beta2_ft       beta
#>    -0.58930072     0.51556561     0.46458135     0.49054919     0.46815470     0.22995167     0.3966
#>       beta2_ft       beta2_ft       beta2_ft       beta2_ft       beta2_ft   L_epsilon2_z      logka
#>     0.77135919     0.40831811     0.42839446     0.79998250     0.91170569     0.49266122    -4.3006
#>
#> $objective
#> [1] 1738.337
#>
#> $iterations
#> [1] 6
#>
#> $evaluations
#> function gradient
#>       12        7
#>
#> $time_for_MLE
#> Time difference of 2.056543 secs
#>
#> $max_gradient
#> [1] 0.00056998
#>
#> $Convergence_check
```

```
#> [1] "The model is likely not converged"
#>
#> $number_of_coefficients
#>  Total  Fixed Random
#>   2060     20   2040
#>
#> $AIC
#> [1] 3516.675
#>
#> $diagnostics
#>             Param starting_value     Lower         MLE     Upper final_gradient
#> 1        beta1_ft    -0.58930212      -Inf -0.58930072       Inf  -2.488046e-04
#> 2        beta2_ft     0.51556568      -Inf  0.51556561       Inf   6.611767e-06
#> 3        beta2_ft     0.46458080      -Inf  0.46458135       Inf  -1.701449e-05
#> 4        beta2_ft     0.49055252      -Inf  0.49054919       Inf   2.270791e-04
#> 5        beta2_ft     0.46815187      -Inf  0.46815470       Inf  -2.159211e-04
#> 6        beta2_ft     0.22994935      -Inf  0.22995167       Inf  -1.972091e-04
#> 7        beta2_ft     0.39666757      -Inf  0.39666650       Inf   7.372993e-05
#> 8        beta2_ft     0.19322635      -Inf  0.19322490       Inf   1.272622e-04
#> 9        beta2_ft    -0.07072354      -Inf -0.07072449       Inf   8.157662e-05
#> 10       beta2_ft     0.11680234      -Inf  0.11680344       Inf  -9.136430e-05
#> 11       beta2_ft     0.47100949      -Inf  0.47101096       Inf  -1.192637e-04
#> 12       beta2_ft     0.77135982      -Inf  0.77135919       Inf   3.421843e-05
#> 13       beta2_ft     0.40831862      -Inf  0.40831811       Inf   3.709032e-05
#> 14       beta2_ft     0.42839433      -Inf  0.42839446       Inf  -2.773916e-05
#> 15       beta2_ft     0.79997985      -Inf  0.79998250       Inf  -1.693684e-04
#> 16       beta2_ft     0.91170841      -Inf  0.91170569       Inf   1.992522e-04
#> 17   L_epsilon2_z     0.49266061      -Inf  0.49266122       Inf  -3.431190e-04
#> 18      logkappa2    -4.30062175 -6.214608 -4.30062279 -3.565449   1.082641e-04
#> 19 Epsilon_rho2_f     0.85035619 -0.990000  0.85035928  0.990000  -5.699800e-04
#> 20      logSigmaM     0.10422369      -Inf  0.10422305 10.000000   8.291044e-05
#>
#> $SD
#> sdreport(.) result
#>                   Estimate Std. Error
#> beta1_ft       -0.58930072 0.05080467
#> beta2_ft        0.51556561 0.14414517
#> beta2_ft        0.46458135 0.17371753
#> beta2_ft        0.49054919 0.19200615
#> beta2_ft        0.46815470 0.20433941
#> beta2_ft        0.22995167 0.21476798
#> beta2_ft        0.39666650 0.21934844
#> beta2_ft        0.19322490 0.22481460
#> beta2_ft       -0.07072449 0.22973074
#> beta2_ft        0.11680344 0.23059098
#> beta2_ft        0.47101096 0.22964742
#> beta2_ft        0.77135919 0.22895294
#> beta2_ft        0.40831811 0.23199191
#> beta2_ft        0.42839446 0.23305156
#> beta2_ft        0.79998250 0.23091833
#> beta2_ft        0.91170569 0.23072494
#> L_epsilon2_z    0.49266122 0.04727208
#> logkappa2      -4.30062279 0.13652887
```

```
#> Epsilon_rho2_f  0.85035928 0.03527714
#> logSigmaM       0.10422305 0.07108160
#> Maximum gradient component: 0.00056998
#>
#> $time_for_sdreport
#> Time difference of 8.762033 secs
#>
#> $time_for_run
#> Time difference of 39.7857 secs
```

Or with sdmTMB

```
#library(INLA)
library(sdmTMB)
mesh = make_mesh(Data, c("x","y"), n_knots=n_x*n_y )

start_time = Sys.time()
mysdmTMB = sdmTMB(
  formula = n ~ 0 + factor(time),
  data = Data,
  mesh = mesh,
  spatial = "off",
  spatiotemporal = "ar1",
  time = "time",
  family = tweedie()
)
Sys.time() - start_time
#> Time difference of 37.73523 secs
```

Bivariate spatio-temporal autoregressive model

We next highlight how to specify a bivariate spatio-temporal model with a cross-laggged (vector autoregressive) interaction.

```
# Simulate settings
theta_xy = 0.2
n_x = n_y = 10
n_t = 20
B = rbind( c( 0.5, -0.25),
           c(-0.1,  0.50) )

# Simulate GMRFs
R = exp(-theta_xy * abs(outer(1:n_x, 1:n_y, FUN="-")) )
d1 = mvtnorm::rmvnorm(n_t, sigma=0.2*kronecker(R,R) )
d2 = mvtnorm::rmvnorm(n_t, sigma=0.2*kronecker(R,R) )
d = abind::abind( d1, d2, along=3 )

# Project through time and add mean
for( t in seq_len(n_t) ){
  if(t>1) d[t,,] = t(B%*%t(d[t-1,,])) + d[t,,]
}
```

```r
# Shape into longform data-frame and add error
Data = data.frame( expand.grid(time=1:n_t, x=1:n_x, y=1:n_y, "var"=c("d1","d2")), z=exp(as.vector(d)))
Data$n = tweedie::rtweedie( n=nrow(Data), mu=Data$z, phi=0.5, power=1.5 )

# make mesh
mesh = fm_mesh_2d( Data[,c('x','y')] )

# Define sem
sem = "
  d1 -> d1, 1, b11
  d2 -> d2, 1, b22
  d2 -> d1, 1, b21
  d1 -> d2, 1, b12
  d1 <-> d1, 0, var1
  d2 <-> d2, 0, var1
"

# fit model
out = fit( sem = sem,
           data = Data,
           formula = n ~ 0 + var,
           spatial_graph = mesh,
           family_link = rbind("obs"=c(1,1)),
           control = tinyVASTcontrol(quiet=TRUE, trace=0) )
out
#> $call
#> fit(data = Data, formula = n ~ 0 + var, sem = sem, family_link = rbind(obs = c(1,
#>     1)), spatial_graph = mesh, control = tinyVASTcontrol(quiet = TRUE,
#>     trace = 0))
#>
#> $opt
#> $opt$par
#>    log_kappa      alpha_j      alpha_j       beta_z       beta_z       beta_z       beta_z       bet
#> -0.669057076 -0.090128409 -0.002000367  0.509529377  0.529236398 -0.200418888 -0.117205371  0.294319
#>
#> $opt$objective
#> [1] 4365.006
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 52
#>
#> $opt$evaluations
#> function gradient
#>       66       53
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
```

```
#> sdreport(.) result
#>               Estimate Std. Error
#> log_kappa -0.669057076 0.09746707
#> alpha_j   -0.090128409 0.09771149
#> alpha_j   -0.002000367 0.09611298
#> beta_z     0.509529377 0.07886506
#> beta_z     0.529236398 0.07336727
#> beta_z    -0.200418888 0.08304602
#> beta_z    -0.117205371 0.07264403
#> beta_z     0.294319087 0.01800602
#> log_sigma -0.646266097 0.02660900
#> log_sigma  0.012846242 0.04964136
#> Maximum gradient component: 0.004519086
#>
#> $run_time
#> Time difference of 3.720452 mins
```

The values for `beta_z` again correspond to the specified value for interaction-matrix `B`

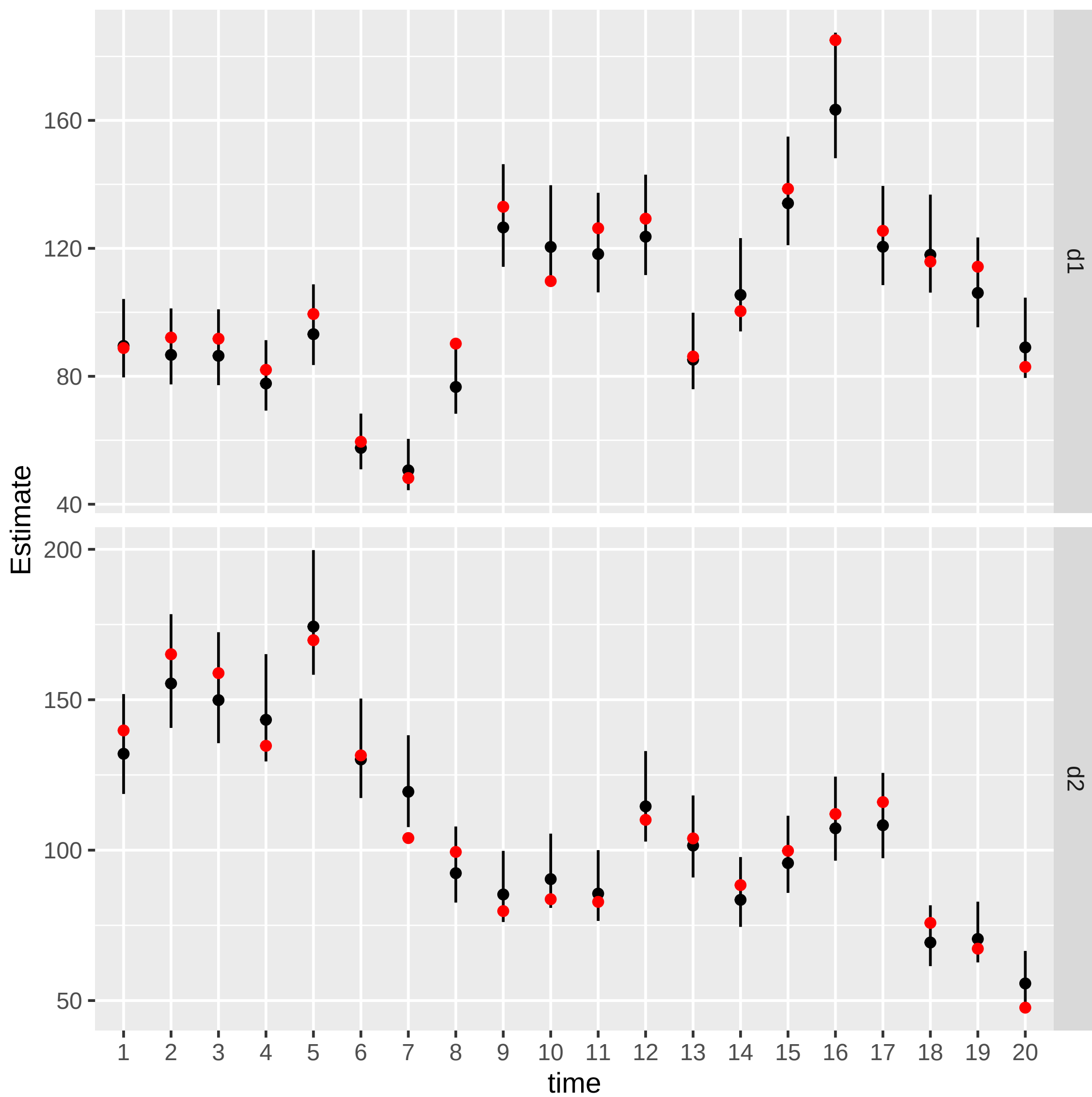
We can again calculate the area-weighted total abundance and compare it with its true value:

```
# Predicted sample-weighted total
Est1 = sapply( seq_len(n_t), FUN=\(t) integrate_output(out, newdata=subset(Data,time==t & var=="d1")) )
Est2 = sapply( seq_len(n_t), FUN=\(t) integrate_output(out, newdata=subset(Data,time==t & var=="d2")) )

# True (latent) sample-weighted total
True = tapply( Data$z, INDEX=list("time"=Data$time,"var"=Data$var), FUN=sum )

#
Index = data.frame( expand.grid(dimnames(True)), "True"=as.vector(True) )
Index = data.frame( Index, rbind(t(Est1), t(Est2)) )
Index$low = Index[,'Est...bias.correct.'] - 1.96*Index[,'Std..Error']
Index$high = Index[,'Est...bias.correct.'] + 1.96*Index[,'Std..Error']

#
library(ggplot2)
ggplot(Index, aes( time, Estimate )) +
  facet_grid( rows=vars(var), scales="free" ) +
  geom_segment(aes(y = low,
                   yend = high,
                   x = time,
                   xend = time) ) +
  geom_point( aes(x=time, y=Estimate), color = "black") +
  geom_point( aes(x=time, y=True), color = "red" )
```


Estimate
160
120
80
40
200
150
100
50
d1
d2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
time