VAST

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```r
library(tinyVAST)
library(fmesher)
set.seed(101)
```

tinyVAST is an R package for fitting vector autoregressive spatio-temporal (VAST) models. We here explore the capacity to specify the vector-autoregressive spatio-temporal component.

Spatio-temporal autoregressive model

We first explore the ability to specify a first-order autoregressive spatio-temporal process:

```r
# Simulate settings
theta_xy = 0.4
n_x = n_y = 10
n_t = 15
rho = 0.8
spatial_sd = 0.5

# Simulate GMRFs
R_s = exp(-theta_xy * abs(outer(1:n_x, 1:n_y, FUN="-")) )
V_ss = spatial_sd^2*kronecker(R_s, R_s)
d = mvtnorm::rmvnorm(n_t, sigma=V_ss )

# Project through time and add mean
for( t in seq_len(n_t) ){
  if(t>1) d[t,] = rho*d[t-1,] + d[t,]
}
#d = d + 0.5

# Shape into longform data-frame and add error
Data = data.frame( expand.grid(time=1:n_t, x=1:n_x, y=1:n_y), "var"="logn", z=exp(as.vector(d)))
Data$n = tweedie::rtweedie( n=nrow(Data), mu=Data$z, phi=0.5, power=1.5 )
mean(Data$n==0)
#> [1] 0.046

# make mesh
mesh = fm_mesh_2d( Data[,c('x','y')] )

# fit model
mytinyVAST = fit( dsem = "logn -> logn, 1, rho",
            data = Data,
            formula = n ~ 0 + factor(time),
```

```
          spatial_graph = mesh,
          family = list( "obs"=tweedie() ),
          control = tinyVASTcontrol(quiet=TRUE, trace=0) )
mytinyVAST
#> $call
#> fit(data = Data, formula = n ~ 0 + factor(time), dsem = "logn -> logn, 1, rho",
#>     family = list(obs = tweedie()), spatial_graph = mesh, control = tinyVASTcontrol(quiet = TRUE,
#>         trace = 0))
#>
#> $opt
#> $opt$par
#>      alpha_j     alpha_j     alpha_j     alpha_j     alpha_j     alpha_j     alpha_j     alpha_j
#> -0.08323603 -0.13549103 -0.10579217 -0.14499111 -0.37823867 -0.21633304 -0.41489958 -0.67168422 -0.4
#>      alpha_j      beta_z      beta_z   log_sigma   log_sigma   log_kappa
#>   0.30040122  0.81229113  0.40988915 -0.64868475  0.04394543  0.07228543
#>
#> $opt$objective
#> [1] 1717.689
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 77
#>
#> $opt$evaluations
#> function gradient
#>      107       77
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>              Estimate Std. Error
#> alpha_j   -0.08323603 0.15196456
#> alpha_j   -0.13549103 0.18670340
#> alpha_j   -0.10579217 0.20529851
#> alpha_j   -0.14499111 0.21780791
#> alpha_j   -0.37823867 0.22691800
#> alpha_j   -0.21633304 0.23026450
#> alpha_j   -0.41489958 0.23456777
#> alpha_j   -0.67168422 0.23833635
#> alpha_j   -0.49463135 0.23869331
#> alpha_j   -0.13968722 0.23733095
#> alpha_j    0.14836185 0.23640201
#> alpha_j   -0.21516692 0.23873590
#> alpha_j   -0.20120062 0.23979214
#> alpha_j    0.16887043 0.23708655
#> alpha_j    0.30040122 0.23660035
#> beta_z     0.81229113 0.03708631
#> beta_z     0.40988915 0.03291043
```

```
#> log_sigma -0.64868475 0.05422114
#> log_sigma  0.04394543 0.07275797
#> log_kappa  0.07228543 0.10755269
#> Maximum gradient component: 0.006334975
#>
#> $run_time
#> Time difference of 21.22232 secs
```
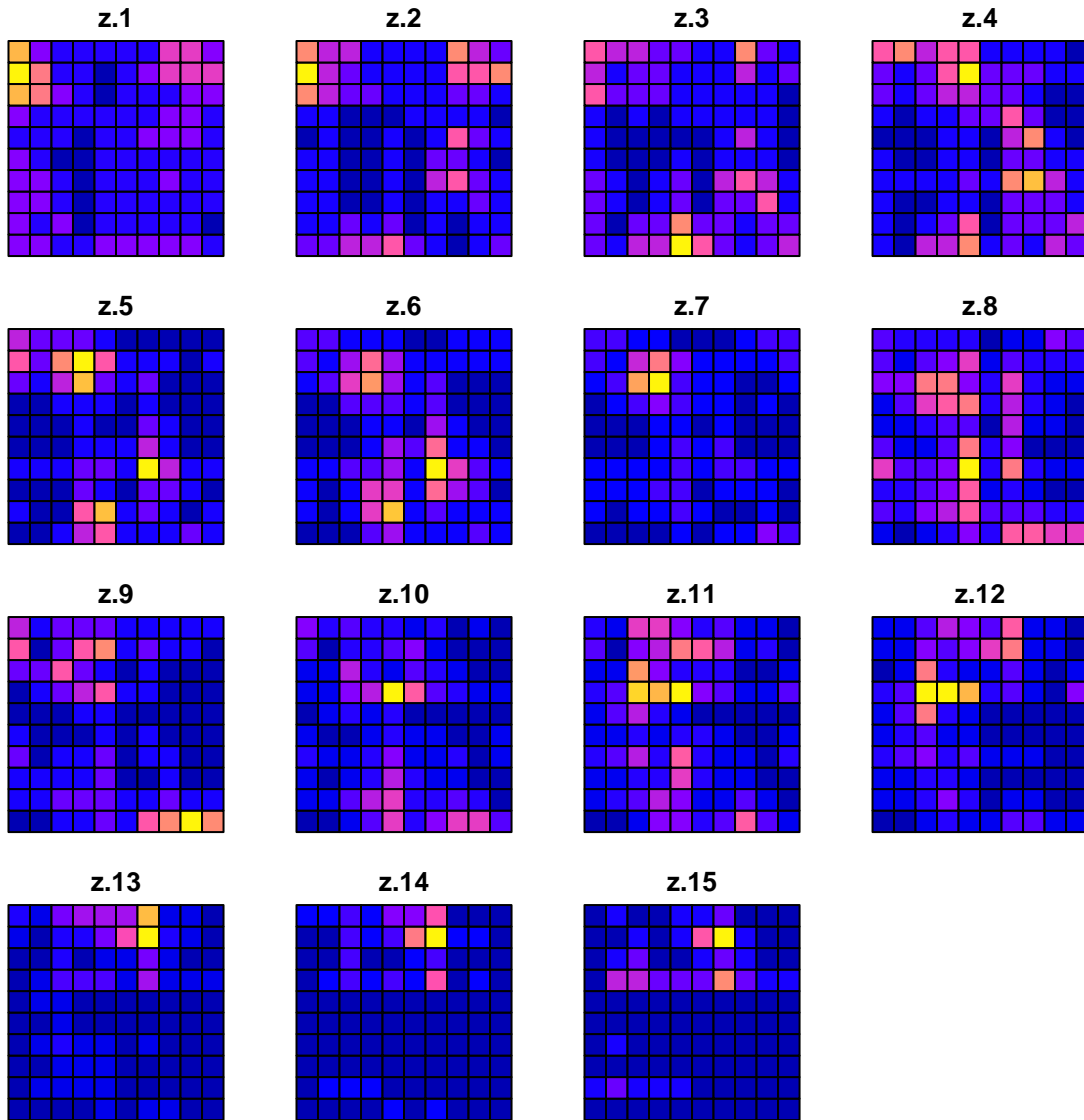
The estimated values for `beta_z` then correspond to the simulated value for `rho` and `spatial_sd`.

We can compare the true densities:

```r
library(sf)
data_wide = reshape( Data[,c('x','y','time','z')],
                     direction = "wide", idvar = c('x','y'), timevar = "time")
sf_data = st_as_sf( data_wide, coords=c("x","y"))
sf_grid = sf::st_make_grid( sf_data )
sf_plot = st_sf(sf_grid, st_drop_geometry(sf_data) )
plot(sf_plot, max.plot=n_t )
```
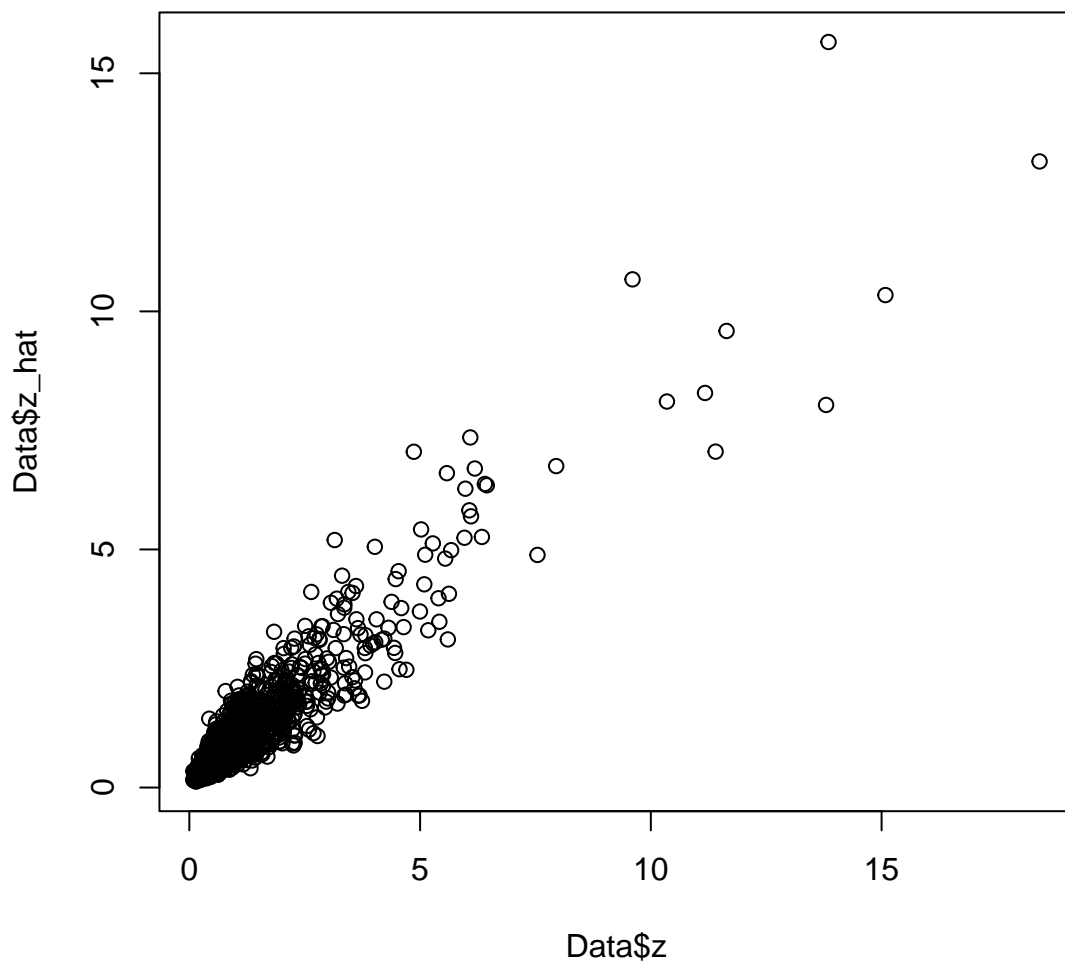
with the estimated densities:

```
Data$z_hat = predict(mytinyVAST)
data_wide = reshape( Data[,c('x','y','time','z_hat')],
                     direction = "wide", idvar = c('x','y'), timevar = "time")
sf_data = st_as_sf( data_wide, coords=c("x","y"))
sf_plot = st_sf(sf_grid, st_drop_geometry(sf_data) )
plot(sf_plot, max.plot=n_t )
```
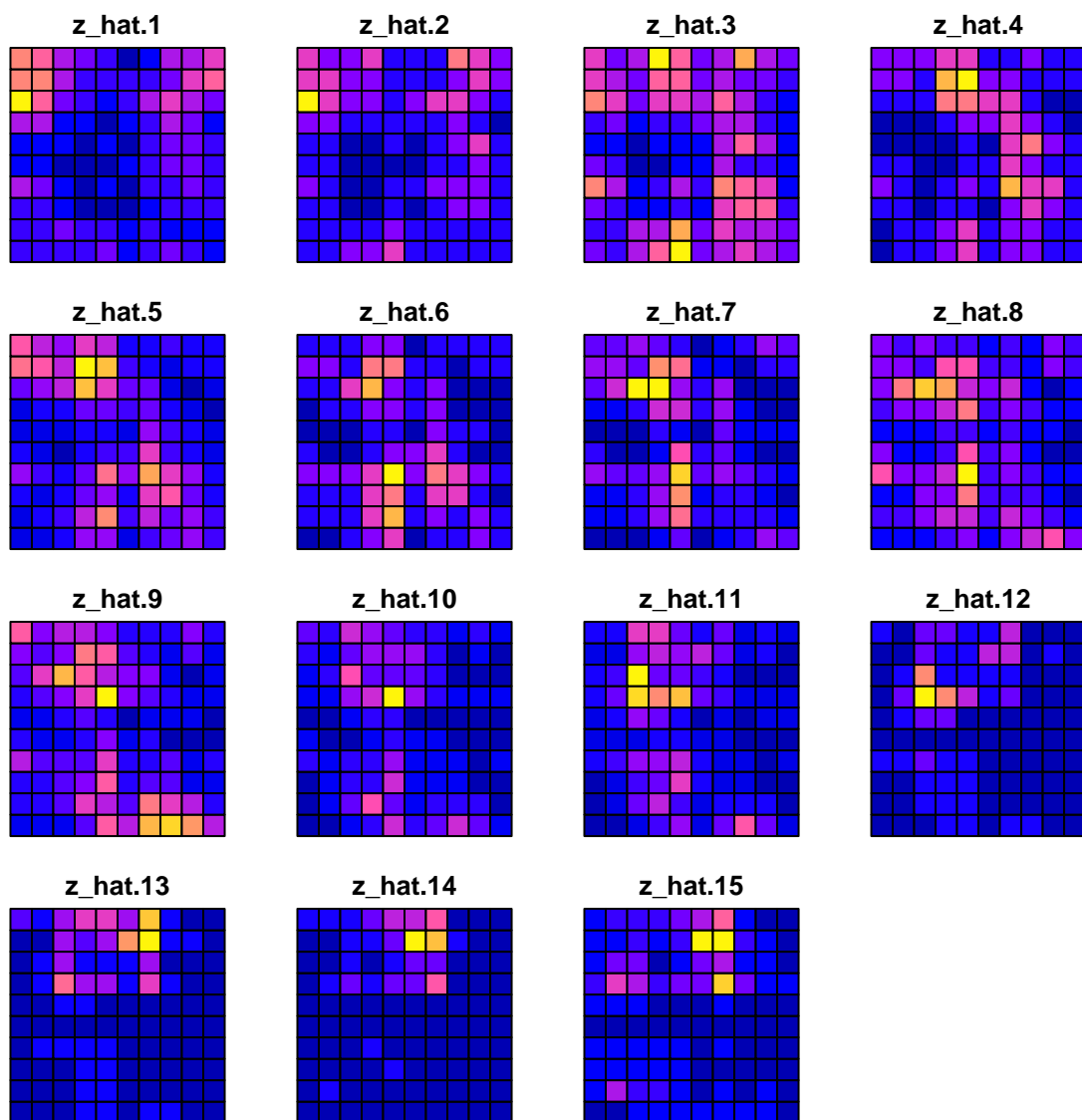
where a scatterplot shows that they are highly correlated:

```
plot( x=Data$z, y=Data$z_hat )
```

We can then calculate the area-weighted total abundance and compare it with its true value:

```
# Predicted sample-weighted total
(Est = sapply( seq_len(n_t),
   FUN=\(t) integrate_output(mytinyVAST, newdata=subset(Data,time==t)) ))
#>                        [,1]       [,2]       [,3]       [,4]      [,5]       [,6]       [,7]
#> Estimate            97.164903  96.643634  98.362457 101.517620 84.760587  97.538111 77.520820 59.56
#> Std. Error           7.194683   7.216494   7.309312   7.572241  6.643419   7.406226  6.207919  5.09
#> Est. (bias.correct) 102.324275 102.850496 105.043003 108.373177 90.604659 104.258111 83.102278 64.06
#> Std. (bias.correct)       NA         NA         NA         NA        NA         NA        NA
#>                        [,15]
#> Estimate            187.86973
#> Std. Error           12.88189
#> Est. (bias.correct) 200.54754
#> Std. (bias.correct)       NA

# True (latent) sample-weighted total
```

```
(True = tapply( Data$z, INDEX=Data$time, FUN=sum ))
#>        1          2          3          4          5          6          7          8          9         10
#>  99.21643 100.10603 101.66846 109.52622  85.76973 100.97116  80.99847  68.60738  85.39974 119.62380

#
Index = data.frame( time=seq_len(n_t), t(Est), True )
Index$low = Index[,'Est...bias.correct.'] - 1.96*Index[,'Std..Error']
Index$high = Index[,'Est...bias.correct.'] + 1.96*Index[,'Std..Error']

#
library(ggplot2)
ggplot(Index, aes(time, Estimate)) +
  geom_ribbon(aes(ymin = low,
                  ymax = high),      # shadowing cnf intervals
              fill = "lightgrey") +
  geom_line( color = "black",
             linewidth = 1) +
  geom_point( aes(time, True), color = "red" )
```
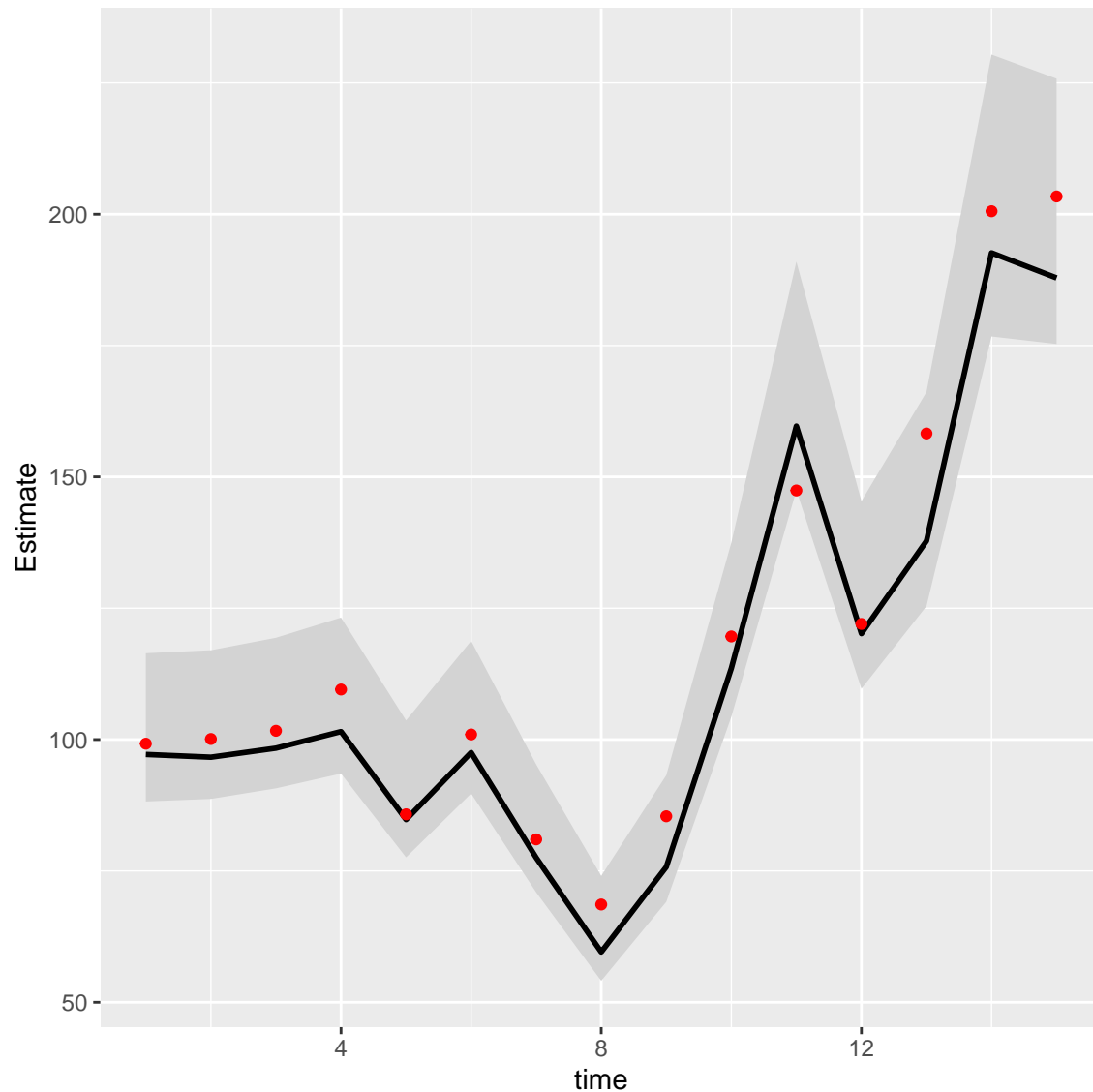
Next, we compare this against the current version of VAST

```r
library(VAST)
settings = make_settings( purpose="index3",
                          n_x = n_x*n_y,
                          Region = "Other",
                          bias.correct = FALSE,
                          use_anisotropy = FALSE )
settings$FieldConfig['Epsilon','Component_1'] = 0
settings$FieldConfig['Omega',] = 0
settings$RhoConfig['Epsilon2'] = 4
settings$RhoConfig['Beta1'] = 3
settings$ObsModel = c(10,2)

myVAST = fit_model( settings=settings,
                    Lat_i = Data[,'y'],
                    Lon_i = Data[,'x'],
```

```
                t_i = Data[,'time'],
                b_i = Data[,'n'],
                a_i = rep(1,nrow(Data)),
                observations_LL = cbind(Lat=Data[,'y'],Lon=Data[,'x']),
                grid_dim_km = c(100,100),
                newtonsteps = 0,
                loopnum = 1,
                control = list(eval.max=100, iter.max=100, trace=0) )
#> Warning: The `returnclass` argument of `ne_download()` sp as of rnaturalearth 1.0.0.
#> i Please use `sf` objects with {rnaturalearth}, support for Spatial objects (sp) will be removed in
#> i The deprecated feature was likely used in the FishStatsUtils package.
#>   Please report the issue to the authors.
#> This warning is displayed once every 8 hours.
#> Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
#> Warning in FishStatsUtils:::inla.barrier.fem.copy(mesh = anisotropic_mesh, : Please install the `INL
#>  which contains an implementation that runs faster!
#>            Component_1 Component_2
#> Omega               -1          -1
#> Epsilon             -1          -2
#> Beta                -2          -2
#> Epsilon_time        -3          -3
#> Eta1 Eta2
#>   -1   -1
#>    Coefficient_name Number_of_coefficients    Type
#> 1          beta1_ft                      1   Fixed
#> 2          beta2_ft                     15   Fixed
#> 3    Epsilon_rho2_f                      1   Fixed
#> 4      L_epsilon2_z                      1   Fixed
#> 5        logkappa2                       1   Fixed
#> 6         logSigmaM                      1   Fixed
#> 7 Epsiloninput2_sff                   2040  Random
#>    0:    2105.7266:  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000
#>    1:    1993.4850: 0.248169 -0.00621313 -0.00940275 -0.00474146 -0.00488678 -0.0304006 -0.0100244 -
#>    2:    1815.3664: -0.625674 -0.0250910 -0.0263651 -0.0154445 -0.0135640 -0.0640075 -0.0194314 -0.00
#>    3:    1787.0493: -0.401654 0.00970346 -0.0193647 -0.00730560 -0.00492316 -0.0744611 -0.00503150 -0
#>    4:    1777.8725: -0.446246 0.0170642 -0.0177169 -0.00500788 -0.00228127 -0.0780425 -0.000709483 -
#>    5:    1769.4163: -0.519117 0.0790811 0.00101357 0.0150763 0.0200457 -0.0994408 0.0309285 -0.068032
#>    6:    1760.3172: -0.511935 0.157190 0.0320130 0.0398107 0.0442341 -0.117800 0.0594775 -0.0745811
#>    7:    1750.8485: -0.566256 0.242491 0.0751094 0.0723315 0.0724627 -0.130056 0.0861515 -0.0818699
#>    8:    1747.1048: -0.570896 0.297687 0.124667 0.106758 0.0886785 -0.116014 0.0852856 -0.0872344 -0
#>    9:    1744.9699: -0.509053 0.311068 0.139401 0.117785 0.0947485 -0.112493 0.0849978 -0.0893639 -0
#>   10:    1743.1222: -0.512816 0.321511 0.166698 0.139649 0.108323 -0.104951 0.0823444 -0.0952418 -0.
#>   11:    1742.7069: -0.578092 0.317902 0.191272 0.161171 0.123579 -0.0949179 0.0755250 -0.102868 -0.
#>   12:    1741.4359: -0.553625 0.350576 0.228002 0.198994 0.151349 -0.0855427 0.0768123 -0.112226 -0.
#>   13:    1741.3951: -0.549230 0.354115 0.230379 0.201585 0.153643 -0.0848625 0.0779499 -0.112529 -0.
#>   14:    1741.3352: -0.551012 0.357402 0.232642 0.204355 0.156205 -0.0837881 0.0792783 -0.112707 -0.
#>   15:    1741.2774: -0.550206 0.360788 0.235473 0.207574 0.159120 -0.0822459 0.0806568 -0.112809 -0.
#>   16:    1741.2267: -0.551500 0.363838 0.238499 0.210823 0.161987 -0.0806066 0.0819352 -0.112867 -0.
#>   17:    1741.1721: -0.550622 0.366481 0.241688 0.214111 0.164885 -0.0786655 0.0831737 -0.112753 -0.
#>   18:    1741.1236: -0.551920 0.368996 0.244873 0.217411 0.167764 -0.0767614 0.0844078 -0.112601 -0.
#>   19:    1741.0724: -0.551195 0.371249 0.248032 0.220698 0.170664 -0.0746785 0.0856768 -0.112263 -0.
#>   20:    1741.0260: -0.552449 0.373520 0.251087 0.223967 0.173556 -0.0726804 0.0870056 -0.111886 -0.
#>   21:    1740.9777: -0.551772 0.375592 0.254042 0.227169 0.176446 -0.0705493 0.0884075 -0.111306 -0.
```

```
#>  22:     1740.9333: -0.552979 0.377691 0.256908 0.230329 0.179311 -0.0684846 0.0898717 -0.110681 -0..
#>  23:     1740.8874: -0.552354 0.379597 0.259661 0.233382 0.182143 -0.0663000 0.0914165 -0.109843 -0..
#>  24:     1740.8448: -0.553529 0.381533 0.262347 0.236393 0.184943 -0.0641761 0.0930184 -0.108966 -0..
#>  25:     1740.8009: -0.552945 0.383290 0.264911 0.239281 0.187690 -0.0619540 0.0947033 -0.107879 -0..
#>  26:     1740.7599: -0.554087 0.385088 0.267422 0.242138 0.190408 -0.0597943 0.0964357 -0.106769 -0..
#>  27:     1740.7179: -0.553533 0.386717 0.269810 0.244868 0.193066 -0.0575534 0.0982451 -0.105458 -0..
#>  28:     1740.6785: -0.554646 0.388394 0.272165 0.247582 0.195703 -0.0553732 0.100088 -0.104144 -0.3.
#>  29:     1740.6381: -0.554116 0.389913 0.274398 0.250169 0.198274 -0.0531233 0.101999 -0.102642 -0.3.
#>  30:     1740.5999: -0.555202 0.391487 0.276618 0.252755 0.200835 -0.0509313 0.103932 -0.101154 -0.3.
#>  31:     1740.5609: -0.554691 0.392909 0.278720 0.255217 0.203329 -0.0486780 0.105925 -0.0994921 -0..
#>  32:     1740.5240: -0.555753 0.394396 0.280824 0.257695 0.205823 -0.0464791 0.107931 -0.0978588 -0..
#>  33:     1740.4863: -0.555260 0.395736 0.282815 0.260052 0.208250 -0.0442245 0.109991 -0.0960631 -0..
#>  34:     1740.4504: -0.556300 0.397148 0.284822 0.262437 0.210687 -0.0420205 0.112057 -0.0943084 -0..
#>  35:     1740.4138: -0.555822 0.398418 0.286719 0.264704 0.213058 -0.0397646 0.114172 -0.0924001 -0..
#>  36:     1740.3790: -0.556842 0.399766 0.288644 0.267012 0.215447 -0.0375560 0.116289 -0.0905430 -0..
#>  37:     1740.3433: -0.556378 0.400974 0.290460 0.269203 0.217770 -0.0352983 0.118450 -0.0885390 -0..
#>  38:     1740.3094: -0.557382 0.402268 0.292316 0.271446 0.220119 -0.0330849 0.120611 -0.0865946 -0..
#>  39:     1740.2747: -0.556930 0.403424 0.294064 0.273572 0.222402 -0.0308244 0.122813 -0.0845081 -0..
#>  40:     1740.2416: -0.557918 0.404671 0.295860 0.275760 0.224718 -0.0286058 0.125012 -0.0824886 -0..
#>  41:     1740.2077: -0.557478 0.405780 0.297549 0.277831 0.226967 -0.0263417 0.127249 -0.0803302 -0..
#>  42:     1740.1754: -0.558453 0.406989 0.299295 0.279972 0.229255 -0.0241174 0.129481 -0.0782454 -0..
#>  43:     1740.1423: -0.558024 0.408057 0.300933 0.281996 0.231475 -0.0218490 0.131750 -0.0760237 -0..
#>  44:     1740.1107: -0.558986 0.409232 0.302637 0.284098 0.233741 -0.0196188 0.134013 -0.0738817 -0..
#>  45:     1739.1745: -0.567381 0.435950 0.399872 0.419534 0.401618 0.196149 0.349401 0.173937 -0.0615.
#>  46:     1738.8279: -0.596524 0.471652 0.397182 0.441269 0.441486 0.225559 0.413372 0.232199 0.00229.
#>  47:     1738.5935: -0.588631 0.486598 0.424664 0.445106 0.443186 0.233576 0.433439 0.257339 -0.0001.
#>  48:     1738.5330: -0.588773 0.482571 0.456978 0.479450 0.446830 0.251282 0.408587 0.228845 -0.0112.
#>  49:     1738.4986: -0.586549 0.503767 0.458645 0.501052 0.483951 0.237362 0.397869 0.199913 -0.0305.
#>  50:     1738.4741: -0.588433 0.526758 0.465209 0.502090 0.498809 0.229918 0.397908 0.194546 -0.0755.
#>  51:     1738.4317: -0.595944 0.528783 0.468489 0.500110 0.482969 0.237850 0.406101 0.192359 -0.0929.
#>  52:     1738.3873: -0.593420 0.521139 0.470568 0.496121 0.466359 0.237208 0.408074 0.179406 -0.0851.
#>  53:     1738.3670: -0.593640 0.520784 0.470148 0.495289 0.468669 0.236030 0.403759 0.182930 -0.0831.
#>  54:     1738.3647: -0.592189 0.520681 0.470006 0.495198 0.469040 0.235710 0.402975 0.183604 -0.0827.
#>  55:     1738.3536: -0.592538 0.519602 0.469976 0.494577 0.469653 0.234602 0.401502 0.185091 -0.0809.
#>  56:     1738.3495: -0.591602 0.518839 0.469263 0.494263 0.469916 0.233437 0.400140 0.186819 -0.0789.
#>  57:     1738.3457: -0.591405 0.518700 0.468028 0.494016 0.469236 0.232796 0.400074 0.188030 -0.0771.
#>  58:     1738.3431: -0.590947 0.518128 0.467835 0.492128 0.469429 0.233065 0.400265 0.189199 -0.0761.
#>  59:     1738.3414: -0.590732 0.516841 0.466226 0.493451 0.469814 0.232361 0.398789 0.191112 -0.0739.
#>  60:     1738.3404: -0.590639 0.517947 0.466264 0.493252 0.469227 0.230588 0.400163 0.190894 -0.0716.
#>  61:     1738.3396: -0.590399 0.518399 0.467536 0.490949 0.469168 0.231677 0.399523 0.192403 -0.0718.
#>  62:     1738.3393: -0.590041 0.518003 0.467475 0.490879 0.468970 0.232448 0.398910 0.193122 -0.0722.
#>  63:     1738.3384: -0.589853 0.516711 0.466863 0.491482 0.469107 0.232961 0.398825 0.192952 -0.0726.
#>  64:     1738.3381: -0.589898 0.516290 0.465766 0.491697 0.470084 0.231935 0.398781 0.192302 -0.0719.
#>  65:     1738.3378: -0.590047 0.517086 0.465802 0.491283 0.469494 0.231363 0.397506 0.193365 -0.0711.
#>  66:     1738.3377: -0.589550 0.516628 0.465973 0.491706 0.468490 0.230889 0.397833 0.193791 -0.0706.
#>  67:     1738.3376: -0.589612 0.515421 0.465693 0.491652 0.468542 0.231085 0.398274 0.193531 -0.0711.
#>  68:     1738.3374: -0.589379 0.515397 0.465269 0.491285 0.468871 0.231574 0.398032 0.193246 -0.0715.
#>  69:     1738.3374: -0.589387 0.516151 0.465173 0.490950 0.468610 0.231381 0.397811 0.193109 -0.0708.
#>  70:     1738.3373: -0.589579 0.515992 0.465411 0.490953 0.468609 0.230659 0.397439 0.193530 -0.0708.
#>  71:     1738.3373: -0.589571 0.515993 0.465395 0.490960 0.468613 0.230653 0.397427 0.193531 -0.0708.
#>  72:     1738.3373: -0.589531 0.515994 0.465366 0.490973 0.468620 0.230644 0.397406 0.193533 -0.0707.
#>  73:     1738.3373: -0.589531 0.515991 0.465339 0.490982 0.468627 0.230636 0.397387 0.193533 -0.0707.
#>  74:     1738.3373: -0.589505 0.515976 0.465305 0.490981 0.468633 0.230638 0.397365 0.193528 -0.0707.
```

```
#>   75:     1738.3373: -0.589501 0.515963 0.465270 0.490982 0.468639 0.230634 0.397341 0.193523 -0.0707
#>   76:     1738.3373: -0.589476 0.515942 0.465232 0.490970 0.468644 0.230632 0.397311 0.193511 -0.0707
#>   77:     1738.3373: -0.589470 0.515924 0.465196 0.490962 0.468644 0.230618 0.397280 0.193502 -0.0707
#>   78:     1738.3373: -0.589450 0.515903 0.465165 0.490947 0.468632 0.230592 0.397249 0.193491 -0.0707
#>   79:     1738.3373: -0.589448 0.515887 0.465135 0.490939 0.468618 0.230562 0.397221 0.193484 -0.0707
#>   80:     1738.3373: -0.589432 0.515873 0.465106 0.490918 0.468596 0.230532 0.397195 0.193472 -0.0707
#>   81:     1738.3373: -0.589431 0.515857 0.465076 0.490910 0.468581 0.230503 0.397167 0.193468 -0.0707
#>   82:     1738.3373: -0.589416 0.515833 0.465055 0.490892 0.468558 0.230471 0.397140 0.193461 -0.0707
#>   83:     1738.3373: -0.589414 0.515832 0.465019 0.490888 0.468554 0.230450 0.397115 0.193455 -0.0706
#>   84:     1738.3373: -0.589401 0.515828 0.464989 0.490864 0.468537 0.230431 0.397087 0.193438 -0.0706
#>   85:     1738.3373: -0.589401 0.515803 0.464977 0.490865 0.468524 0.230402 0.397068 0.193441 -0.0706
#>   86:     1738.3373: -0.589387 0.515777 0.464969 0.490856 0.468503 0.230363 0.397056 0.193445 -0.0706
#>   87:     1738.3373: -0.589385 0.515781 0.464934 0.490839 0.468490 0.230348 0.397029 0.193431 -0.0706
#>   88:     1738.3373: -0.589374 0.515769 0.464908 0.490825 0.468485 0.230335 0.396999 0.193416 -0.0706
#>   89:     1738.3373: -0.589373 0.515738 0.464904 0.490800 0.468466 0.230302 0.396980 0.193406 -0.0706
#>   90:     1738.3373: -0.589346 0.515737 0.464863 0.490812 0.468459 0.230282 0.396958 0.193412 -0.0706
#>   91:     1738.3373: -0.589355 0.515745 0.464814 0.490819 0.468427 0.230274 0.396935 0.193411 -0.0706
#>   92:     1738.3373: -0.589353 0.515721 0.464819 0.490766 0.468436 0.230252 0.396916 0.193382 -0.0706
#>   93:     1738.3373: -0.589346 0.515700 0.464810 0.490745 0.468409 0.230219 0.396887 0.193371 -0.0706
#>   94:     1738.3373: -0.589337 0.515689 0.464780 0.490748 0.468364 0.230185 0.396866 0.193376 -0.0706
#>   95:     1738.3373: -0.589338 0.515677 0.464768 0.490731 0.468346 0.230141 0.396879 0.193382 -0.0706
#>   96:     1738.3373: -0.589341 0.515672 0.464734 0.490714 0.468326 0.230152 0.396840 0.193351 -0.0706
#>   97:     1738.3373: -0.589333 0.515663 0.464710 0.490699 0.468307 0.230136 0.396799 0.193330 -0.0706
#>   98:     1738.3373: -0.589319 0.515645 0.464699 0.490673 0.468292 0.230092 0.396783 0.193323 -0.0706
#>   99:     1738.3373: -0.589319 0.515623 0.464680 0.490645 0.468260 0.230067 0.396786 0.193314 -0.0706
#>  100:     1738.3373: -0.589335 0.515616 0.464667 0.490640 0.468249 0.230043 0.396745 0.193293 -0.0706
#>  101:     1738.3373: -0.589315 0.515603 0.464638 0.490619 0.468221 0.230020 0.396728 0.193274 -0.0707
#>  102:     1738.3373: -0.589305 0.515592 0.464606 0.490574 0.468212 0.230019 0.396733 0.193271 -0.0706
#>  103:     1738.3373: -0.589304 0.515589 0.464616 0.490574 0.468193 0.229976 0.396692 0.193245 -0.0706
#>  104:     1738.3373: -0.589304 0.515559 0.464589 0.490557 0.468168 0.229960 0.396668 0.193243 -0.0707
#>  105:     1738.3373: -0.589302 0.515566 0.464581 0.490553 0.468152 0.229949 0.396668 0.193226 -0.0707
#>  106:     1738.3373: -0.589302 0.515566 0.464581 0.490553 0.468152 0.229949 0.396668 0.193226 -0.0707
myVAST
#> fit_model(.) result
#> $par
#>       beta1_ft        beta2_ft        beta2_ft        beta2_ft        beta2_ft        beta2_ft        beta
#>     -0.5893007       0.5155656       0.4645814       0.4905492       0.4681547       0.2299517       0.396
#>       beta2_ft        beta2_ft        beta2_ft        beta2_ft        beta2_ft     L_epsilon2_z        logka
#>      0.7713592       0.4083181       0.4283945       0.7999825       0.9117057       0.4926612       -4.300
#>
#> $objective
#> [1] 1738.337
#>
#> $iterations
#> [1] 6
#>
#> $evaluations
#> function gradient
#>       12        7
#>
#> $time_for_MLE
#> Time difference of 1.119554 secs
#>
```

```
#> $max_gradient
#> [1] 0.0005689832
#>
#> $Convergence_check
#> [1] "The model is likely not converged"
#>
#> $number_of_coefficients
#>   Total  Fixed Random
#>    2060     20   2040
#>
#> $AIC
#> [1] 3516.675
#>
#> $diagnostics
#>            Param starting_value    Lower        MLE    Upper final_gradient
#> 1        beta1_ft    -0.58930212     -Inf -0.5893007      Inf  -2.600747e-04
#> 2        beta2_ft     0.51556570     -Inf  0.5155656      Inf   6.548257e-06
#> 3        beta2_ft     0.46458081     -Inf  0.4645814      Inf  -1.605389e-05
#> 4        beta2_ft     0.49055252     -Inf  0.4905492      Inf   2.232692e-04
#> 5        beta2_ft     0.46815188     -Inf  0.4681547      Inf  -2.132041e-04
#> 6        beta2_ft     0.22994936     -Inf  0.2299517      Inf  -1.950281e-04
#> 7        beta2_ft     0.39666757     -Inf  0.3966665      Inf   7.238240e-05
#> 8        beta2_ft     0.19322635     -Inf  0.1932249      Inf   1.259944e-04
#> 9        beta2_ft    -0.07072353     -Inf -0.0707245      Inf   8.084947e-05
#> 10       beta2_ft     0.11680234     -Inf  0.1168035      Inf  -9.055590e-05
#> 11       beta2_ft     0.47100948     -Inf  0.4710110      Inf  -1.182167e-04
#> 12       beta2_ft     0.77135983     -Inf  0.7713592      Inf   3.340028e-05
#> 13       beta2_ft     0.40831863     -Inf  0.4083181      Inf   3.659812e-05
#> 14       beta2_ft     0.42839434     -Inf  0.4283945      Inf  -2.734301e-05
#> 15       beta2_ft     0.79997986     -Inf  0.7999825      Inf  -1.664807e-04
#> 16       beta2_ft     0.91170841     -Inf  0.9117057      Inf   1.962860e-04
#> 17   L_epsilon2_z     0.49266061     -Inf  0.4926612      Inf  -3.363051e-04
#> 18       logkappa2    -4.30062174 -6.214608 -4.3006228 -3.565449   1.055535e-04
#> 19 Epsilon_rho2_f     0.85035619 -0.990000  0.8503593  0.990000  -5.689832e-04
#> 20       logSigmaM     0.10422369     -Inf  0.1042231 10.000000   9.101179e-05
#>
#> $SD
#> sdreport(.) result
#>                 Estimate Std. Error
#> beta1_ft       -0.5893007 0.05080467
#> beta2_ft        0.5155656 0.14414517
#> beta2_ft        0.4645814 0.17371754
#> beta2_ft        0.4905492 0.19200615
#> beta2_ft        0.4681547 0.20433942
#> beta2_ft        0.2299517 0.21476799
#> beta2_ft        0.3966665 0.21934845
#> beta2_ft        0.1932249 0.22481460
#> beta2_ft       -0.0707245 0.22973075
#> beta2_ft        0.1168035 0.23059099
#> beta2_ft        0.4710110 0.22964743
#> beta2_ft        0.7713592 0.22895295
#> beta2_ft        0.4083181 0.23199191
#> beta2_ft        0.4283945 0.23305157
```

```
#> beta2_ft         0.7999825 0.23091834
#> beta2_ft         0.9117057 0.23072494
#> L_epsilon2_z     0.4926612 0.04727208
#> logkappa2       -4.3006228 0.13652887
#> Epsilon_rho2_f   0.8503593 0.03527714
#> logSigmaM        0.1042231 0.07108160
#> Maximum gradient component: 0.0005689832
#>
#> $time_for_sdreport
#> Time difference of 4.718721 secs
#>
#> $time_for_run
#> Time difference of 21.76643 secs
```

Or with sdmTMB

```
library(sdmTMB)
mesh = make_mesh(Data, c("x","y"), n_knots=n_x*n_y )

start_time = Sys.time()
mysdmTMB = sdmTMB(
  formula = n ~ 0 + factor(time),
  data = Data,
  mesh = mesh,
  spatial = "off",
  spatiotemporal = "ar1",
  time = "time",
  family = tweedie()
)
sdmTMBtime = Sys.time() - start_time
```

The models all have similar runtimes

```
Times = c( "tinyVAST" = mytinyVAST$run_time,
           "VAST" = myVAST$total_time,
           "sdmTMB" = sdmTMBtime )
knitr::kable( cbind("run times (sec.)"=Times), digits=1)
```

	run times (sec.)
tinyVAST	21.2
VAST	23.6
sdmTMB	20.5

Delta models

We can also fit these data using a delta model

```
# fit model
mydelta = fit( data = Data,
```

```
                formula = n ~ 1,
                delta_formula = ~ 0 + factor(time),
                delta_dsem = "logn -> logn, 1, rho",
                family = list( "obs"=independent_delta() ),
                spatial_graph = mesh,
                control = tinyVASTcontrol(quiet=TRUE, trace=0) )
mydelta
#> $call
#> fit(data = Data, formula = n ~ 1, delta_formula = ~0 + factor(time),
#>     delta_dsem = "logn -> logn, 1, rho", family = list(obs = independent_delta()),
#>     spatial_graph = mesh, control = tinyVASTcontrol(quiet = TRUE,
#>         trace = 0))
#>
#> $opt
#> $opt$par
#>        alpha_j       alpha2_j       alpha2_j       alpha2_j       alpha2_j       alpha2_j       alpha2_j
#>   3.032022e+00   4.467177e-01   4.113254e-01   5.053690e-01   3.694708e-01   7.254509e-02   3.336953e-01   1
#>       alpha2_j       alpha2_j       alpha2_j       alpha2_j        beta2_z        beta2_z      log_sigma
#>   5.325019e-01   2.865528e-01   7.913769e-01   9.836626e-01   1.012746e+00   8.306752e-07   2.565052e-01
#>
#> $opt$objective
#> [1] 2069.593
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 88
#>
#> $opt$evaluations
#> function gradient
#>      137       89
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#> Warning in sqrt(diag(object$cov.fixed)): NaNs produced
#>            Estimate Std. Error
#> alpha_j    3.032022e+00 0.12325402
#> alpha2_j   4.467177e-01 0.13555038
#> alpha2_j   4.113254e-01 0.13488713
#> alpha2_j   5.053690e-01 0.13488702
#> alpha2_j   3.694708e-01 0.13423421
#> alpha2_j   7.254509e-02 0.13423422
#> alpha2_j   3.336953e-01 0.13488712
#> alpha2_j   1.252914e-01 0.13622434
#> alpha2_j  -1.360709e-01 0.13555039
#> alpha2_j   2.348312e-01 0.14050758
#> alpha2_j   4.421359e-01 0.13690899
#> alpha2_j   7.414033e-01 0.13555050
```

```
#> alpha2_j    5.325019e-01 0.13690900
#> alpha2_j    2.865528e-01 0.13903204
#> alpha2_j    7.913769e-01 0.13488708
#> alpha2_j    9.836626e-01 0.13555026
#> beta2_z     1.012746e+00        NaN
#> beta2_z     8.306752e-07        NaN
#> log_sigma   2.565052e-01 0.01869238
#> Warning:
#> Hessian of fixed effects was not positive definite.
#> Maximum gradient component: 0.006397981
#>
#> $run_time
#> Time difference of 0.127759 secs
```

Bivariate spatio-temporal autoregressive model

We next highlight how to specify a bivariate spatio-temporal model with a cross-laggged (vector autoregressive) interaction.

```
# Simulate settings
theta_xy = 0.2
n_x = n_y = 10
n_t = 20
B = rbind( c( 0.5, -0.25),
           c(-0.1,  0.50) )

# Simulate GMRFs
R = exp(-theta_xy * abs(outer(1:n_x, 1:n_y, FUN="-")) )
d1 = mvtnorm::rmvnorm(n_t, sigma=0.2*kronecker(R,R) )
d2 = mvtnorm::rmvnorm(n_t, sigma=0.2*kronecker(R,R) )
d = abind::abind( d1, d2, along=3 )

# Project through time and add mean
for( t in seq_len(n_t) ){
  if(t>1) d[t,,] = t(B%*%t(d[t-1,,])) + d[t,,]
}

# Shape into longform data-frame and add error
Data = data.frame( expand.grid(time=1:n_t, x=1:n_x, y=1:n_y, "var"=c("d1","d2")), z=exp(as.vector(d)))
Data$n = tweedie::rtweedie( n=nrow(Data), mu=Data$z, phi=0.5, power=1.5 )

# make mesh
mesh = fm_mesh_2d( Data[,c('x','y')] )

# Define DSEM
dsem = "
  d1 -> d1, 1, b11
  d2 -> d2, 1, b22
  d2 -> d1, 1, b21
  d1 -> d2, 1, b12
  d1 <-> d1, 0, var1
  d2 <-> d2, 0, var1
```

15

```
"

# fit model
out = fit( dsem = dsem,
           data = Data,
           formula = n ~ 0 + var,
           spatial_graph = mesh,
           family = list( "obs"=tweedie() ),
           control = tinyVASTcontrol(quiet=TRUE, trace=0) )
out
#> $call
#> fit(data = Data, formula = n ~ 0 + var, dsem = dsem, family = list(obs = tweedie()),
#>     spatial_graph = mesh, control = tinyVASTcontrol(quiet = TRUE,
#>         trace = 0))
#>
#> $opt
#> $opt$par
#>      alpha_j      alpha_j       beta_z       beta_z       beta_z       beta_z       beta_z       log_si
#> -0.090128406 -0.002000401  0.509529381  0.529236409 -0.200418877 -0.117205373  0.294319079 -0.646266
#>
#> $opt$objective
#> [1] 4365.006
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 52
#>
#> $opt$evaluations
#> function gradient
#>       66       53
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>            Estimate Std. Error
#> alpha_j   -0.090128406 0.09771149
#> alpha_j   -0.002000401 0.09611298
#> beta_z     0.509529381 0.07886506
#> beta_z     0.529236409 0.07336727
#> beta_z    -0.200418877 0.08304602
#> beta_z    -0.117205373 0.07264403
#> beta_z     0.294319079 0.01800602
#> log_sigma -0.646266085 0.02660900
#> log_sigma  0.012846253 0.04964136
#> log_kappa -0.669057063 0.09746707
#> Maximum gradient component: 0.004501045
#>
#> $run_time
```

```
#> Time difference of 2.188533 mins
```

The values for `beta_z` again correspond to the specified value for interaction-matrix B

We can again calculate the area-weighted total abundance and compare it with its true value:

```r
# Predicted sample-weighted total
Est1 = sapply( seq_len(n_t), FUN=\(t) integrate_output(out, newdata=subset(Data,time==t & var=="d1")) )
Est2 = sapply( seq_len(n_t), FUN=\(t) integrate_output(out, newdata=subset(Data,time==t & var=="d2")) )

# True (latent) sample-weighted total
True = tapply( Data$z, INDEX=list("time"=Data$time,"var"=Data$var), FUN=sum )

#
Index = data.frame( expand.grid(dimnames(True)), "True"=as.vector(True) )
Index = data.frame( Index, rbind(t(Est1), t(Est2)) )
Index$low = Index[,'Est...bias.correct.'] - 1.96*Index[,'Std..Error']
Index$high = Index[,'Est...bias.correct.'] + 1.96*Index[,'Std..Error']

#
library(ggplot2)
ggplot(Index, aes( time, Estimate )) +
  facet_grid( rows=vars(var), scales="free" ) +
  geom_segment(aes(y = low,
                   yend = high,
                   x = time,
                   xend = time) ) +
  geom_point( aes(x=time, y=Estimate), color = "black") +
  geom_point( aes(x=time, y=True), color = "red" )
```