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Experimental Evaluation of Indoor Localization Algorithms

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***Abstract*—In Radio Frequency (RF)-based indoor localization scenarios, localization algorithms are needed to alleviate the impact of non-line-of-sight and multipath effects on the measurements and thereby estimate the true position precisely. Several resilient lateration algorithms have been proposed in the last couple of years which claim to minimize these effects. However, most of these algorithms were only evaluated using simulations or small static testbeds. We conducted an experiment using 25 anchor nodes and a mobile node installed on top a robotic reference system to collect ranging values. The robot has a localization error of $6.5\,\mathrm{cm}$ which is an order lower than our range measurement errors. We use this robot to collect range measurements and ground truth positions along a densely grid with approx. $10\,\mathrm{cm}$ spacing. The experiment was carried out in a hallway of our office-like building. We collected data on approx. $300\,\mathrm{m}^2$. First, we examine the influence of the anchor placement and anchor density on the ranging errors we see. Then, we evaluate and analyze the robustness of localization algorithms on our measured data to decide which one works best for a constellation of anchor placement and building. Our results show, that there are significant differences between the simulations published for lateration algorithms and actual experiments in real-world indoor localization scenarios. As we show in this paper, the distance measurement error distribution has a large influence on these algorithms.**

I. Introduction

In a world where we run multiple Global Navigation Satellite Systems (GNSSs) covering the whole planet, the problem of location awareness for all kinds of devices seems to be solved. However this is just one part of the truth, as GNSSs lack the capability to calculate a position if the target is located in an environment which restricts free visibility to the sky. To provide location information to devices which can not employ a GNSS, the research in indoor localization tries to find suitable solutions depending on the task to solve. While special cases, like room detection are already solved we still lack the availability of a general purpose system which can be deployed in any scenario and which is not dependent on maps or previously collected radio data [1], [2].

The problem of localization awareness in Wireless Sensor Networks (WSNs) is discussed since the beginning of research in that field. As the sensors or nodes of WSNs are equipped with wireless (usually radio) transceivers the WSN community tries to use these devices for localization purposes [3]. Today we can also find similar conditions in the field of consumer electronics like smartphones and devices which can be classified as being part of the Internet of Things (IOT). If a device needs to estimate the position of itself or other members of the network it must first estimate a distance or angle to a certain number of other devices with known positions. When these estimates are collected, a suitable localization algorithm is needed which takes the estimates as input and calculates a final position. If we want to measure distance estimates using radio devices we can either use the Receive Signal Strength Indication (RSSI) as a value which correlates to the distance between two devices or we may use the time of flight (TOF) of radio signals to estimate the distances [1]. Measuring the angle to another node within the network is a good alternative but usually requires special hardware which is not covered in this paper.

After we estimated the distance to an adequate number of other nodes in the network we can try to estimate a position, using a suitable localization algorithm. Initially, researchers tried to apply lateration algorithms using non-linear least squares (NLLS) or linear least squares (LLS) [4], [5] which are well known from the geodesy, especially Global Positioning System (GPS) based problems [6]. A major challenge for these algorithms is, to deal with the comparatively large amount of error and bias of our estimates which can be magnitudes higher than the errors in measurements we can get from GNSSs or other methods known from geodesy. As a consequence, several indoor localization algorithms were proposed and evaluated in simulations: Adapted Multi-Lateration (AML) [7], Iterative Clustering-based Localization Algorithm (ICLA) [8], Min-Max [9], [10], and Geo-n [11], among others. These algorithms are described in Section III.

Of the publications above, most do not report any actual experiments. Indeed, to many, simulation is the preferred method, as real world tests in buildings with physical hardware is time consuming and needs a reference system to compare the results numerically [12], [13], [14]. Sometimes, this is even impossible to researchers due to a lack of hardware. Thus, erroneous behavior of the transceiver as well as the physical behavior of radio distribution are assumed to follow a probability density function, which does not necessarily match reality.

We contribute a first exhaustive and rigorous evaluation of the aforementioned localization algorithms in an indoor scenario. Our experiment is described in Section IV. With this evaluation, we show whether algorithms actually work as advertised. Our results (see Section V) indicate, that AML and ICLA, which where specifically designed to perform well with noisy range measurements, fail to work well in our scenario, and Min-Max, which is the simplest algorithm, works surprisingly well. Geo-n performs best and is very resilient towards changes in configuration and range measurement errors.

II. Related work

We now take a look at the current state of the art of experimental evaluation in indoor localization. To decrease the amount of literature we focus on RF- and range-based localization techniques.

He et al. present a testbed for evaluation the effects of non-line-of-sight (NLOS) time of arrival (TOA) based indoor localization system [15]. This paper is of special interest for us, as the authors use the same localization hardware as we do in our setup. The hardware in use is the nanoLOC device, which is a special transceiver which implements range-measurement features [16] developed by the Nanotron GmbH. He et al. describe a combined approach of simulation and field testing. The group developed a complex channel model to simulate the radio properties of the Nanotron transceiver and evaluated it by running several field tests. What makes this work outstanding is the evaluation of simulated results as well as the very realistic field testing setup. The paper also uses a good categorization of several different multipath conditions which we will also use in this paper. The authors describe four different conditions which are:

- free space — describes the perfect outdoor scenario. The receiver receives only the direct signal of the sender and no reflected signals at all
- LOS — (line of sight) sender and receiver have a direct line of sight connection but the receiver will also receive multipath parts of the senders signal
- NLOS-DP — (non line of sight - direct path) the sender has no line of sight to the receiver, but the signal travelling the direct path between sender and receiver is still detectable
- NLOS-UDP — (non line of sight - undetectable direct path) direct line of sight between sender and receiver does not exist. The part of the signal which travels the direct path is not detectable and the receiver only receives multipath parts of the signal

Besides the described paper from He et al. several different field test experiments can be found in the literature. One of the first approaches to evaluate indoor localization systems is the EVARILOS project which tries to create a standardized test environments [17]. Evarilos tries to offer standardized tests for all RF-based ranging and localization systems. An indoor localization competition was conducted by the Evarilos project in July 2014, to compare different localization algorithms. The results of this competition still need to be evaluated.

Besides the emerging testbed and standardization efforts several experiments can be found in the literature. One of the easiest approaches, as it can be realized without any modifications at the hardware or low level drivers, is to use the RSSI to calculate the distance between two sensor nodes. This technique is simple to implement but considered difficult to handle as the RSSI values are extremely sensitive to any changes in the environment [18]. A common scenario is to set up a square area and place sensor nodes around this area and then use the RSSI to localize nodes within that area. This method is described by Liu et al. [19] and in similar ways by other authors [20], [21], [22]. A drawback in this approach is that almost only free space and LOS conditions are tested which are rarely seen in real indoor localization scenarios.

Another class of field tests tries to use TOA based approaches. As the TOA measurement with radio devices can be difficult due to extremely short time periods which need to be gathered, the behavior can be imitated by using ultra-sound based ranging methods. A big advantage of this approach is that the sensor hardware needed for such an experiment is less expensive and sensors are easily available. Ultrasound based experiments were conducted by Wendeberg et al. [23] as well as by Priyantha et al. [24] and Moore et al.[25].

Bahillo et al. [26] describe one of the most realistic experiment setups we found in the literature. The authors used the results to evaluate the Robust Least-Squared Multilateration (RSLM) localization algorithm. The experiment used IEEE 802.11b devices to measure RSSI and round trip time (RTT) based ranging values. The experiment was carried out in an office floor with 8 anchors placed in NLOS positions. One mobile node was used to walk through a path of approximately 68 m. Unfortunately the paper does not disclose how the groundtruth values were gathered or how many individual positions were measured.

III. Localization algorithms

Several algorithms for distance-based location estimation have been published in the last decades. Some have been focused on low computational complexity for use in sensor networks and most recent publications have focused on resilience to measurement errors or filtering for use in indoor scenarios. These algorithms are designed to reduce the effect of distance measurement errors, e.g. ones caused by the complicated indoor multipath propagation, low Signal-to-noise ratio (SNR), reflection and link failures, and to improve the localization error [21], [27], [4], [28], [5]. Grid-scan methods [29], [30] divide the target field into several cells and are using voting based methods to select a cell as an estimated location. Refined geometry relationship [28], [25] obtains the target relative location rather than actual location, and the method is based on the range based measurements, in which measurement noise still causes estimation errors.

In our experiment we compare six algorithms. Three of them are well known algorithms and are often used for performance comparison when proposing a new localization algorithm: Multilateration using NLLS or LLS [4], [5] and Min-Max algorithm [9], [10]. The three current algorithms also use the intersection points of circles for location estimation: AML [7], ICLA [8] and Geo-n [11]. Alternative algorithms could easily be tested on our dataset and can be tested in future work. We chose this set of algorithms because we wanted to test algorithms we suppose to be representative in the field of RF-based localization.

A. Nonlinear Least Squares Multilateration

Given N anchor nodes with fixed locations at $b_i = (x_i, y_i)$ for $i = 1, 2, \ldots, N$ and possibly noisy range measurements r_i

from these nodes to a non-anchor node located at $u = (x, y)$, multilateration finds the most likely location of the unknown node, denoted by $\hat{u}$. From this information we formulate a system of equations:

$$\begin{aligned} (x - x_1)^2 + (y - y_1)^2 &= {r_1}^2 \\ (x - x_2)^2 + (y - y_2)^2 &= {r_2}^2 \\ &\vdots \\ (x - x_N)^2 + (y - y_N)^2 &= {r_N}^2 \end{aligned} \tag{1}$$

This problem is usually solved by minimizing the sum of the squared residuals between the observed ranges r_i and the estimated distance $||u - b_i||$:

$$\hat{u} = \underset{u}{\operatorname{argmin}} \sum_{i=1}^{N} (||u - b_i|| - r_i)^2 \tag{2}$$

This problem can be solved by any Newton type optimization algorithm [31]. These start from an initial guess at the solution and then iterate to gradually improve the estimated location until a local minimum of the objective function Eq. (2) is found. There is a non-negligible probability of finding a local minimum. Therefore, to find an estimate close to the global minimum, the algorithm should start with different starting points, which is computationally expensive. For instance, our implementation uses two starting points, the LLS solution and the centroid of all anchor coordinates.

B. Linear Least Squares Multilateration

The *nonlinear least squares* problem can be linearized by subtracting one of the equations in Eq. (1) from the remaining $N - 1$ equations. The resulting system of linear equations can be solved algorithmically to estimate a location.

C. Min-Max

The Min-Max algorithm, also known as Bounding Box algorithm, is a simple method compared to the quite expensive methods of LLS or NLLS. The main idea is to build a square (bounding box) given by $[x_i - r_i, y_i - r_i] \times [x_i + r_i, y_i + r_i]$ around each anchor location (x_i, y_i) and distance measurement r_i, and then to calculate their intersection. The location of the unlocalized node is approximated by the center of the intersection box computed by:

$$\begin{aligned} &[\max_{1 \le i \le N}(x_i - r_i), \max_{1 \le i \le N}(y_i - r_i)] \times \\ &\quad [\min_{1 \le i \le N}(x_i + r_i), \min_{1 \le i \le N}(y_i + r_i)] \end{aligned} \tag{3}$$

Variants of the Min-Max algorithm attempt to improve performance and alleviate its short-comings [32], [33]. They commonly work by assigning weights to the distances or corners of the box, thus refining the result.

D. Adapted Multi-Lateration

Similar to multilateration and Geo-n (cf. Sect. III-F), AML estimates the location using circle intersections. AML consists of three steps: intersection and elimination, first estimation and refinement. First, two intersecting circles are arbitrarily chosen. If there are two intersections, the point with the larger distance to the third anchor is eliminated. In the first estimation step, the previously computed intersection point is moved to the middle of the line connecting it with the closest point of the third anchor's circle. This is done to reduce range measurement errors and is calculated by the resemblance of triangles. Finally, the location can be further refined. The anchors not used in previous steps are added to the location estimation process with the same way used in the second step.

E. Iterative Clustering-based Localization Algorithm

In ICLA, locations are estimated by clustering intersection points. The algorithm consists of three steps. First, all intersections between every pair of circles centered at the anchor coordinate with radius equal to the estimated distance are generated. These intersections cluster around the unlocalized node. Second, the iterative clustering model (ICM) is applied to obtain the most representative intersections. Finally, the location of the unlocalized node is calculated by the centroid of all intersections of the largest group that ICM has produced. The ICM is central to the algorithm. All intersections are iteratively moved towards their moving direction and merged if they collide. The collision area is a disc with a radius equal to the size of the moving step. Points exert an attracting force proportional to their weight and influence the direction of other points. Initially, all points have identical weight. At the end, all points are classified into several different clusters according to the points left.

F. Geo-n

In response to the short-comings of the preceding algorithms, the Geo-n algorithm is developed by Will, Hillebrandt and Kyas [11]. The idea is to identify a set of close intersections of circles induced by pairwise distance measurements, clustering and filtering, and calculating a weighted centroid of the remaining intersections.

First, Geo-n calculates all pairwise intersections between the circles around an anchor with radius equal to the distance measurement. If two circles do not intersect, the point closest to both circles is calculated instead. Then, intersections that are probably beneficial for the estimation are selected. We motivate this approach using Fig. 1 (Figure taken from [11]).

Figure 1 shows the node u to localize and anchor nodes A, B, C, D, E. The black circles have a radius equal to the distance measurement. Without any measurement error, all circles would intersect at node u. The distance to anchor D is measured a bit too short while the remainder of the anchors are measured too long. Especially anchor E is measured with a large error. Two distinct circles intersect in at most two points. One intersection is likely close to the location of u. Consequently, the density of intersection points is likely highest close to u even when there is a large measurement error. These are, e.g., the six solid red points and the blue point in Fig. 1. Finding a good approximation of this cluster

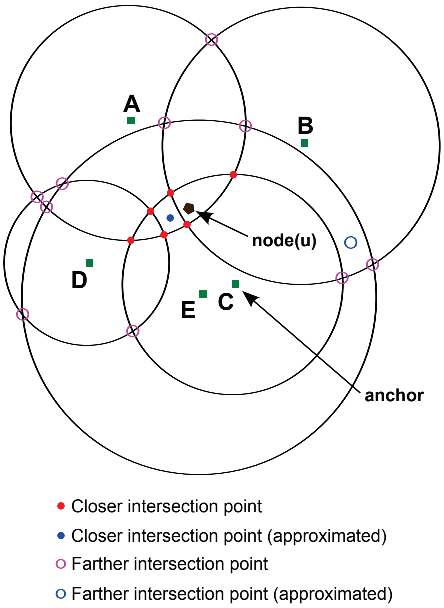


Fig. 1: Motivation of the clustering like approach

and using it for localization is the central problem solved in the Geo-n localization algorithm.

Geo-n uses two filters to remove intersections that probably don't contribute to the localization or are suspected to increase the positioning error [11]. First, those intersections contained in less than $N - 2$ closed disks of the circles are removed. Second, a median filter based on the inter-point distances removes intersections that do not improve accuracy. Note that this filter is only applied if there are at least 3 intersections left, because otherwise filtering would leave one intersection point that is probably farther from the true position than the result of all points. Finally the position of the unlocalized node is estimated as the weighted centroid of the remaining intersection points.

IV. Experimental Setup

For our experiment we use localization data gathered by an optical reference system [14], which obtains ground truth position data. We use this system to carry and track a mobile sensor node. This sensor collects range measurements to anchor nodes which were deployed on the first floor of an office building. In total, 25 anchor nodes are deployed in arbitrary rooms with unchanged office equipment and in the hallway close to the walls.

The building is a typical office building which consists of several concrete walls. The space between the concrete walls is divided into multiple office or function rooms by dry walls. During the experiment, only a few people were passing by the robot and working in the rooms.

The sensor nodes consist of a modified version of the Modular Sensor Board (MSB) A2 [34] node which is equipped with a Nanotron nanoPAN 5375 [16] transceiver. This hardware enables the sensor nodes to measure inter-node ranges using TOF in the 2.4 GHz band. We use symmetrical double-sided two-way ranging to estimate the distance of two sensor nodes.

While carrying the mobile sensor node, the reference system moves automatically through the office floor in order to traverse the free space available. The maximum speed is set to $10\,\mathrm{cm\,s^{-1}}$, but can be significantly lower while the robot performs a turn or evades obstacles. While the sensor node conducts 4 ranging measurements per second, the reference system constantly estimates its ground truth position with an average positioning error of $6.7\,\mathrm{cm}$. Both values are merged by a software running on the reference system and are then stored in a separate database back end system [35].

In Fig. 2 we show the density of the merged measurements we collected. For display purposes, the ground truth positions are snapped to a grid with a granularity of $0.3\,\mathrm{m}$ and count the number of recorded positions per cell. The average position density is 424.21 measurements per $1\,\mathrm{m}^2$. Some parts of the hallway where not accessible by our reference system and therefore appear white. Some areas, like the middle part of the hallway, were traversed multiple times by the robot and therefore have a higher density of data.

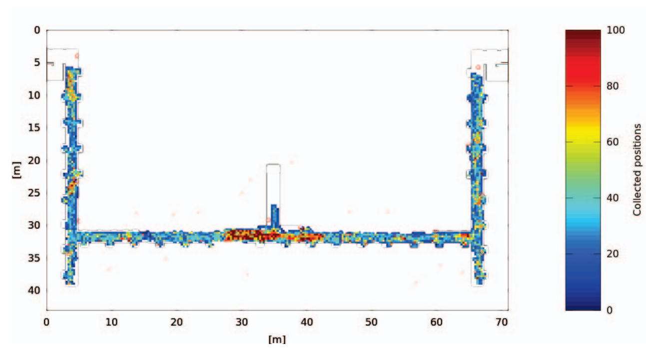


Fig. 2: Density of collected positions

Figure 3 shows the average ranging error of our sensor nodes per cell. In our setup, the nanoPAN achieves a ranging precision of around $1.95\,\mathrm{m}$ on average and the root-mean-square error (RMSE) is $3.04\,\mathrm{m}$. However, the ranging error can be as large as $50\,\mathrm{m}$. We even encountered measurement errors up to $75\,\mathrm{m}$ in rare cases. We also have a significant part of rangings which seem to be too short. Most of the known channel models do not explain the occurrence of short range measurements. This could either be a unique behavior of the Nanotron ranging system or an effect affecting other ranging systems as well which would significantly change the common channel error models we see in the literature. Especially in the middle part which was traversed multiple times, we find a lot of negative ranging errors, which has to be addressed in the future.

We use the collected data to analyze the performance of different localization algorithms and the effects of LOS versus NLOS conditions on our measurements.

V. Results

To evaluate the localization results we calculate the estimated positions using one of the six algorithms at a time. We then compare these estimates to the ground truth value of the reference system and plot the localization errors on a floor plan of the office building. To examine the impact of the

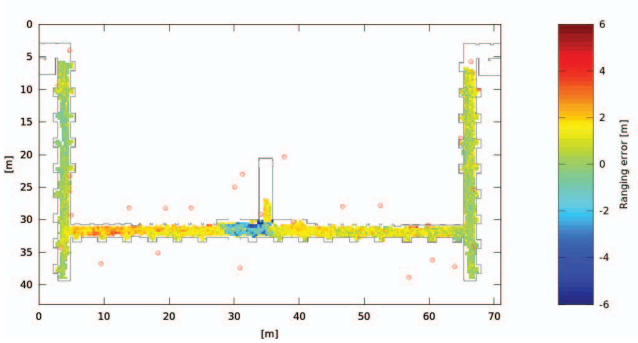


Fig. 3: Ranging error of the nanoPAN transceiver

anchor geometry on the localization performance we evaluate two different settings. In V-A we use all available anchors which are distributed on both sides of the floor. As a second scenario V-B we disable some anchors on the outside of the floors so that the reference system is outside of the convex hull of the anchors most of the time.

Our expectation on the results predicts different effects. The first and somewhat trivial assumption is, that we will see a strong correlation of the spatial distribution of the ranging error as shown in Fig. 3 with the position error of the algorithm. Depending on the algorithms resilience towards NLOS effects we also expect an error amplification or attenuation in the areas of strongly error prone measurements.

A second expectation is to see differences in the performance with different spatial distributions of the anchors. Hillebrandt, Will and Kyas [36] ran several simulations on the algorithms we use anchors which suggest such effects. Finally we also expect to see that some algorithms which are not optimal in the average might perform better in special situations such as unfortunate anchor geometry settings or NLOS situations.

To compare both scenarios we depicted the spatial localization performance of each algorithm in Fig. 4 and Fig. 5, and calculated the overall performance of each algorithm in Tab. I.

A. *Inside the convex hull*

If we take a look at the pictures in Fig. 4 we see significant differences of the spatial localization error for all algorithms. The LLS algorithm shown in Fig. 4a is an exception because its overall performance is so poor that we can not compare the spatial distribution of the error. The reason for this behaviour might be, that the LLS approach is very sensitive to a bias in the measurements and as we know our system has such a bias. For example the likelihood that the estimates contain a positive error is much higher than the likelihood of a negative error.

The spatial distribution of the error shows the same trend for all remaining five algorithms. Positions with a high error are alike for all algorithms as well as positions with a relatively low error and seems to be strongly correlated to the measurement error which is depicted in Fig. 3. Upon closer inspection we also see some small but remarkable differences between the algorithms. Both AML and ICLA seem to amplify the measurement error in the upper half of the vertical hallways to the left and right of our building. ICLA also seems to fail in areas with a relatively low distance error like the area around the (50 m, 32 m) position. The Min-Max algorithm shows the effects which already were suggested by Hillebrandt, Will and Kyas [36] which result in poor performance as soon as we leave the convex hull of the anchors. This is clearly visible at the (3 m, 38 m) position. Another bad spot can be found at (48 m, 32 m).

We assumed beforehand that it might make sense for a real world application to be able to dynamically switch between different indoor localization algorithms depending on the current situation. As we can see in Fig. 4 this might not be necessary because we see that Min-Max (Fig. 4e) and Geo-n (Fig. 4f) clearly outperform the other algorithms for every possible scenario in that experiment including unfortunate anchor constellations as well as NLOS conditions.

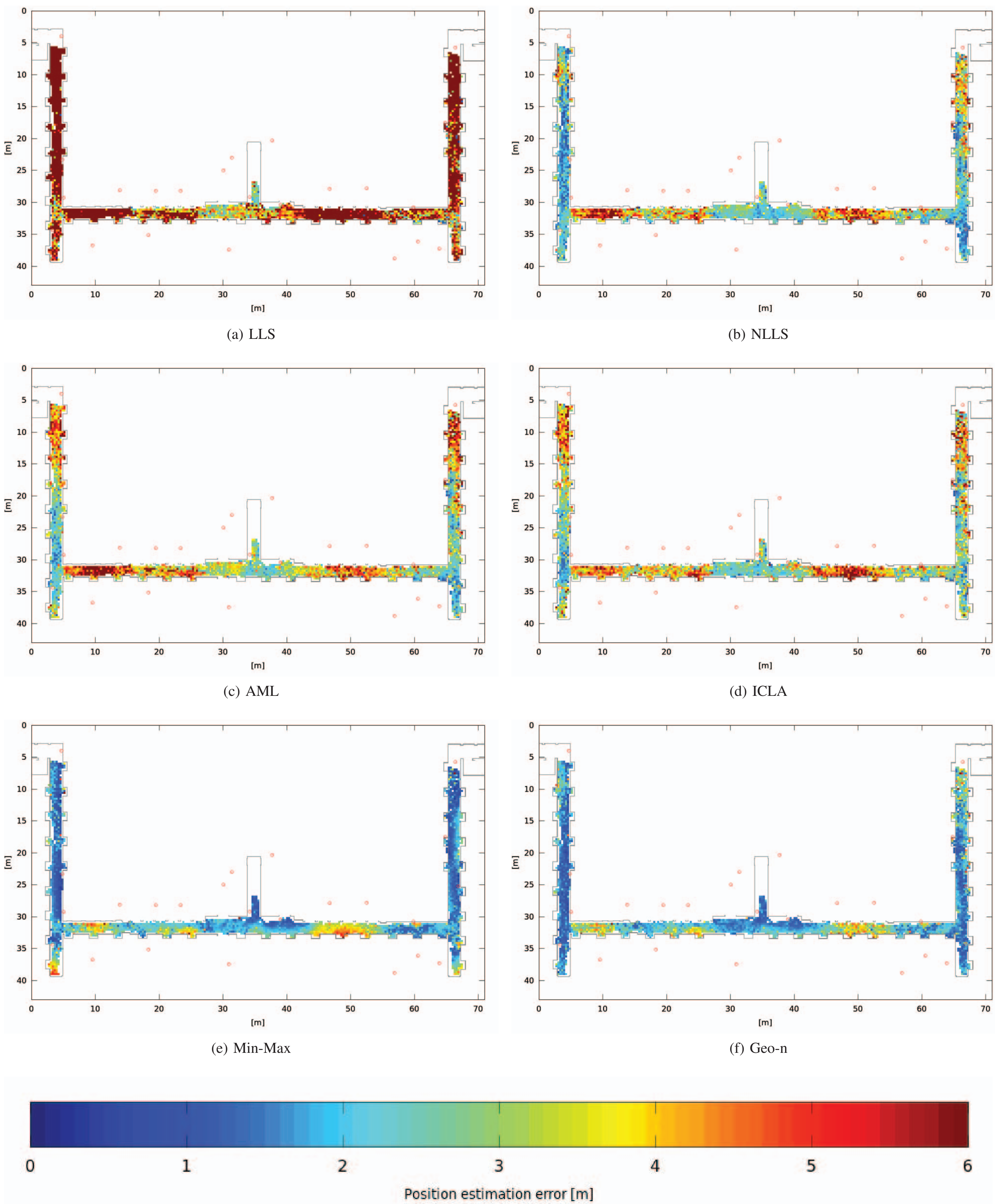
In contrast to Kuruoglu et al. [7], we also see that AML performs much worse than the bounding box approach implemented by the Min-Max algorithm. AML might not be as robust as suggested when it comes to noisy measurements in a real world application. ICLA also performs worse than estimated by simulation [8]. The authors show that Min-Max performs much worse than ICLA in simulation and state that ICLA is accurate and error tolerant, which can not be confirmed by our experiment. The reason for this might be that the error model used for simulation approaches might not be sufficient for use in indoor environments. We see that the Min-Max localization algorithm clearly outperforms all other algorithms we compare, except Geo-n.

B. *Outside of the convex hull*

A common assumption for experiments indoor localization is that the target is somehow surrounded by anchor nodes and therefore within the convex hull of anchors. While this assumption might hold for fully instrumented buildings there are a lot of scenarios where staying in the convex hull is impossible for example in rescue scenarios we might only be able to position anchor nodes at one side of the disaster area.

To better understand the impact of anchor placement on the performance of these algorithms, we now take a look on the results when some anchors are removed from the data after the experiment. Due to the design of the ranging system, this does not influence the actual measurements. We removed 8 anchors so that the horizontal floor and both areas below that floor lie outside of the convex hull of all remaining 17 anchors.

As shown on Fig. 5, most of the algorithms we compare perform worse when applied in a scenario where more than half of the traversed space lies outside of the convex hull. But in case of Geo-n (Fig. 5f), the results look better than expected. In Tab. I we compare the mean average error (MAE) and the RMSE of both scenarios. The relative difference of the position estimation error of Geo-n in both scenarios is only 10.7 % (MAE) and 10.1 % (RMSE). In case of NLLS the difference is with 4.1 % (MAE) and 9.8 % (RMSE) even smaller. But Geo-n outperforms all other algorithm in both scenarios. As expected, Min-Max can not compete anymore due to its bounding box approach.



(a) LLS

(b) NLLS

(c) AML

(d) ICLA

(e) Min-Max

(f) Geo-n

Fig. 4: Spatial localization error of different algorithms

TABLE I: Performance parameters of different algorithms

Algorithm	25 anchors		17 anchors	
	MAE	RMSE	MAE	RMSE
Geo-n	1.97 m	2.47 m	2.18 m	2.72 m
Min-Max	2.05 m	2.56 m	3.07 m	3.72 m
NLLS	2.90 m	3.78 m	3.02 m	4.15 m
ICLA	3.30 m	4.41 m	3.80 m	5.44 m
AML	3.46 m	4.46 m	4.14 m	5.53 m
LLS	12.65 m	82.68 m	42.96 m	585.97 m

Another observation is that in contrast to the results in Section V-A and the overall performance shown in Tab. I we see that Min-Max (Fig. 5e) is able to perform better in the flanking floors than Geo-n (Fig. 5f). The area arround ((45 m - 55 m, 32 m) is also quite interesting in this case. If we compare the performance of AML, ICLA, NLLS and Geo-n in that area we can see that the algorithms perform better with a reduced anchor set. This fact show that it might be useful to rule out anchors which suffer from NLOS effects.

VI. Conclusion

By using an automated reference system and a large network setup we conducted the first large scale experiment on radio range based indoor localization in the context of WSNs. We used one mobile node and 25 anchor nodes which were distributed over a whole office floor of a concrete and drywall building.

The results show the tremendous shortcomings of pure simulation in the field of indoor localization. Two algorithms that promise to address problems with outliers and ranging measurement errors, namely AML and ICLA, perform worse than NLLS, the algorithm on which they try to improve. One of the most interesting observations for us was the impact of the spatial anchor constellation on the performance of the algorithms. This observation needs to be taken into account for further experiments. We have to accept that we generate a significant experimental bias if we only evaluate an algorithm within- or out of the convex hull of anchors. It would be useful to take this into for future setups.

Although our measurements only reflect the parameters of the building we used, it is very likely that the system would perform in a related manner in any building of the same type. Nevertheless, our results are only valid for the office building we used. We will repeat the experiment in another kind of building to corroborate the results. Still, our results clearly show a lack of suitable simulation methods and models in the field of indoor localization.

As future work, we want to repeat the experiment in different locations and with different ranging hardware to get more representative results. The results would be useful to develop a generic model of the error behavior of indoor localization systems and create a standardized benchmark for indoor localization algorithms.

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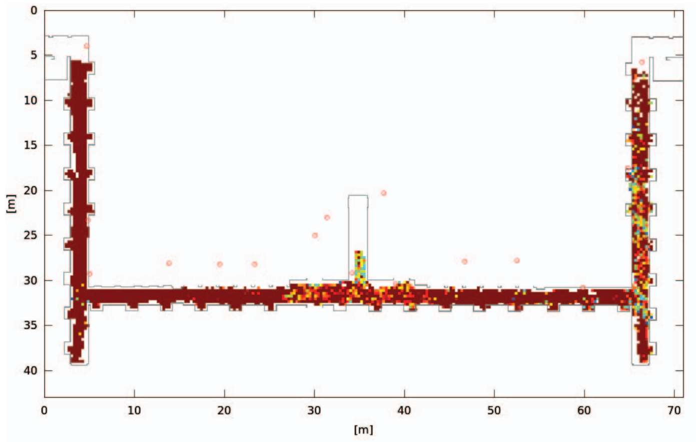
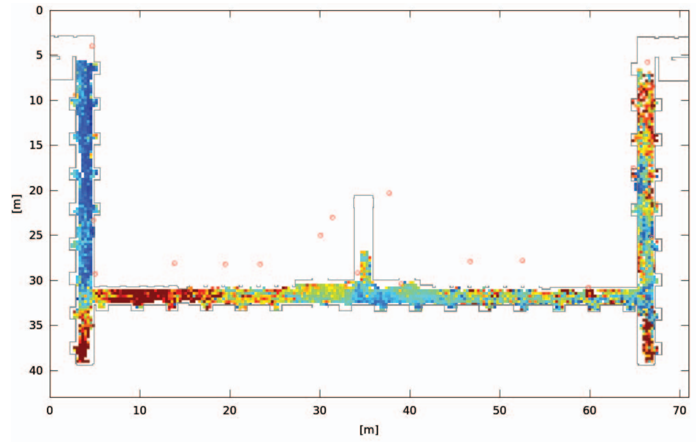
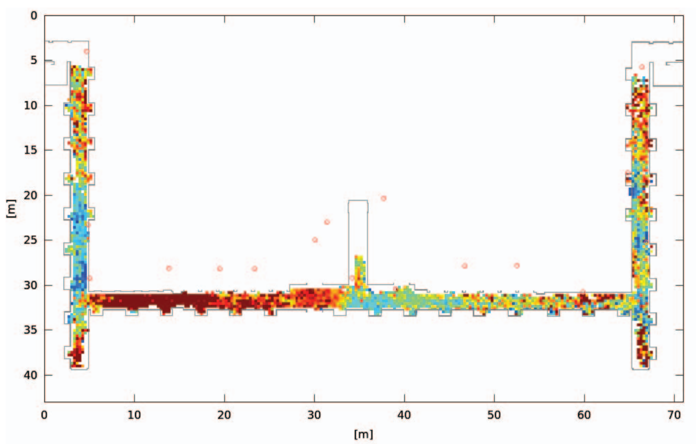


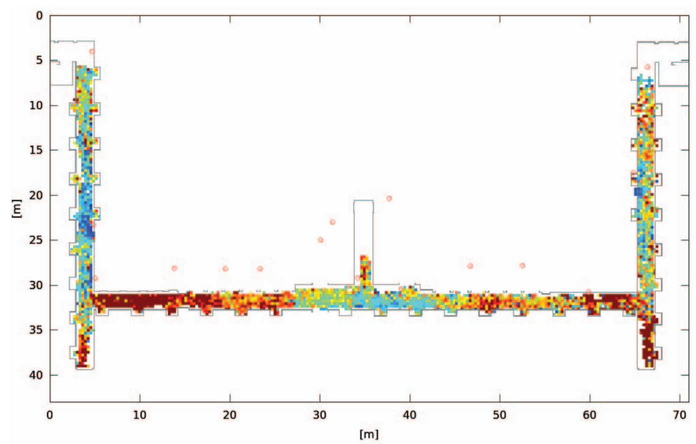
(a) LLS



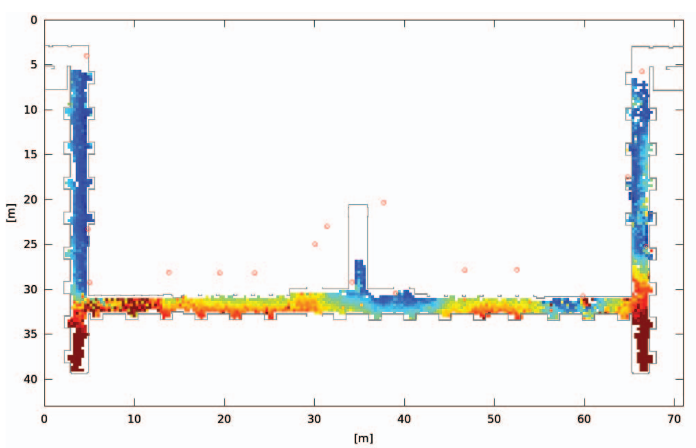
(b) NLLS



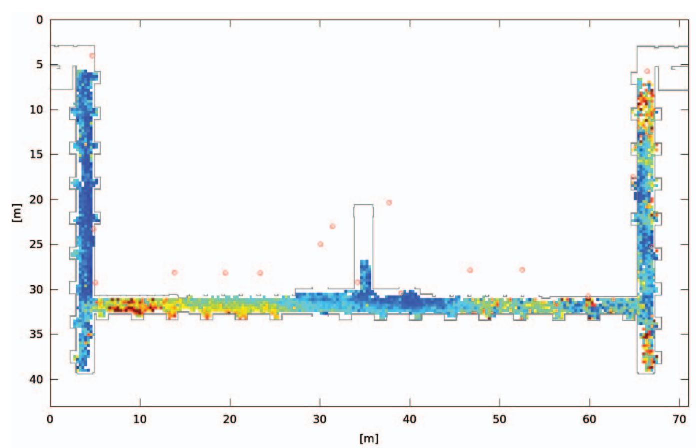
(c) AML



(d) ICLA



(e) Min-Max



(f) Geo-n

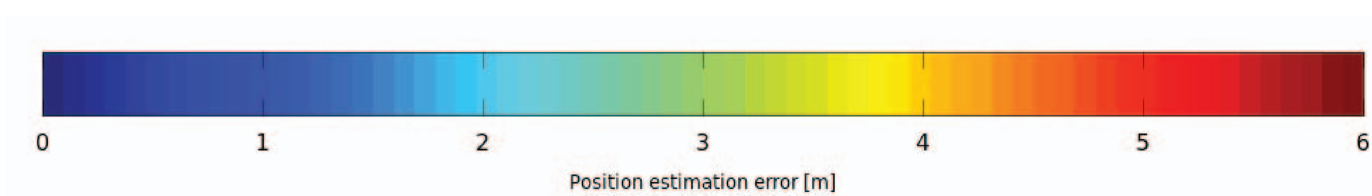


Fig. 5: Spatial localization error of different algorithms with fewer anchors

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