



Planning I

- getting from A to B

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Additional thanks to Jen Jen Chung for many of the slides

Today

- Motion planning
- Representing planning problems, configuration space
- Graph search methods
- Potential fields

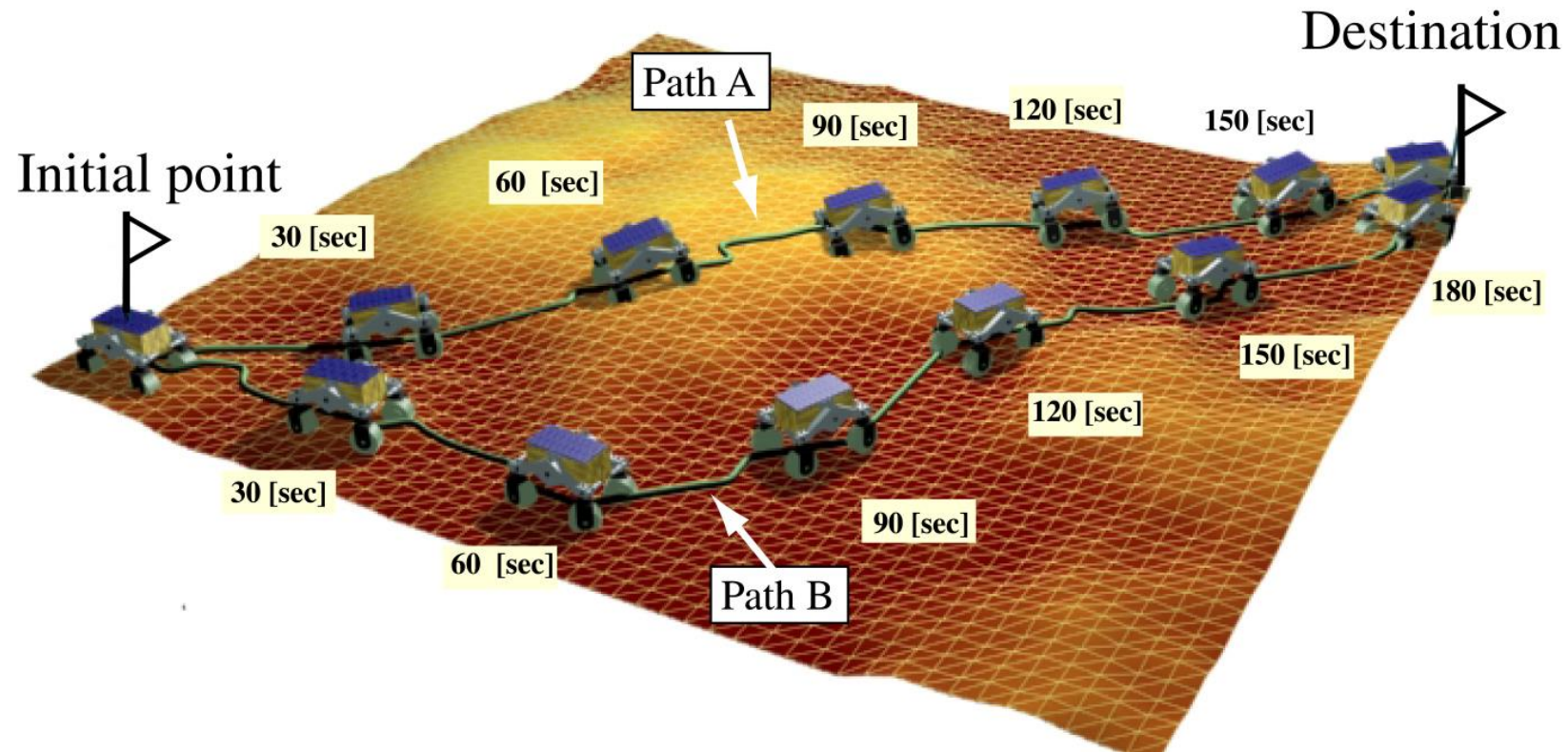
Next week – Juan Nieto

- Sampling-based methods
- Planning with uncertainty
- Recent planning research

What does planning mean?

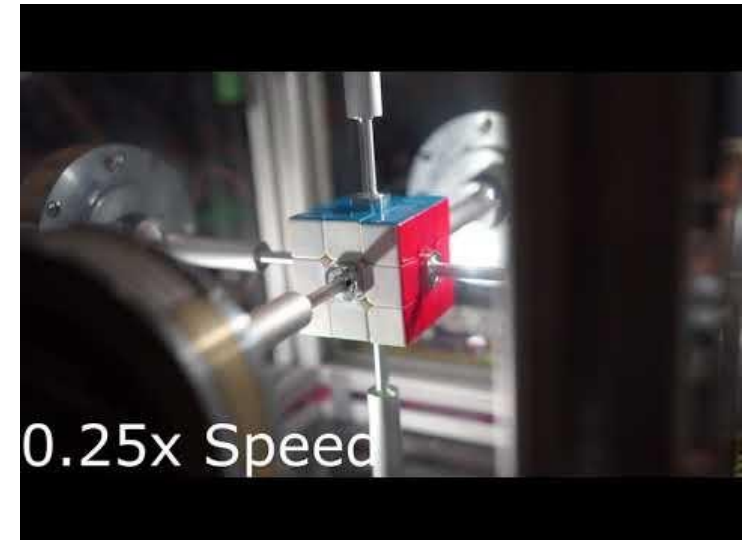
- Different things to different people
- In general, we will focus on ***motion planning*** for robotics, namely determining a set of actions to take a robot from one known state to another known state
- We are also interested in considering some of the ***constraints*** of the platform, ensuring that our generated plans are ***feasible***

What is (robot) Planning?



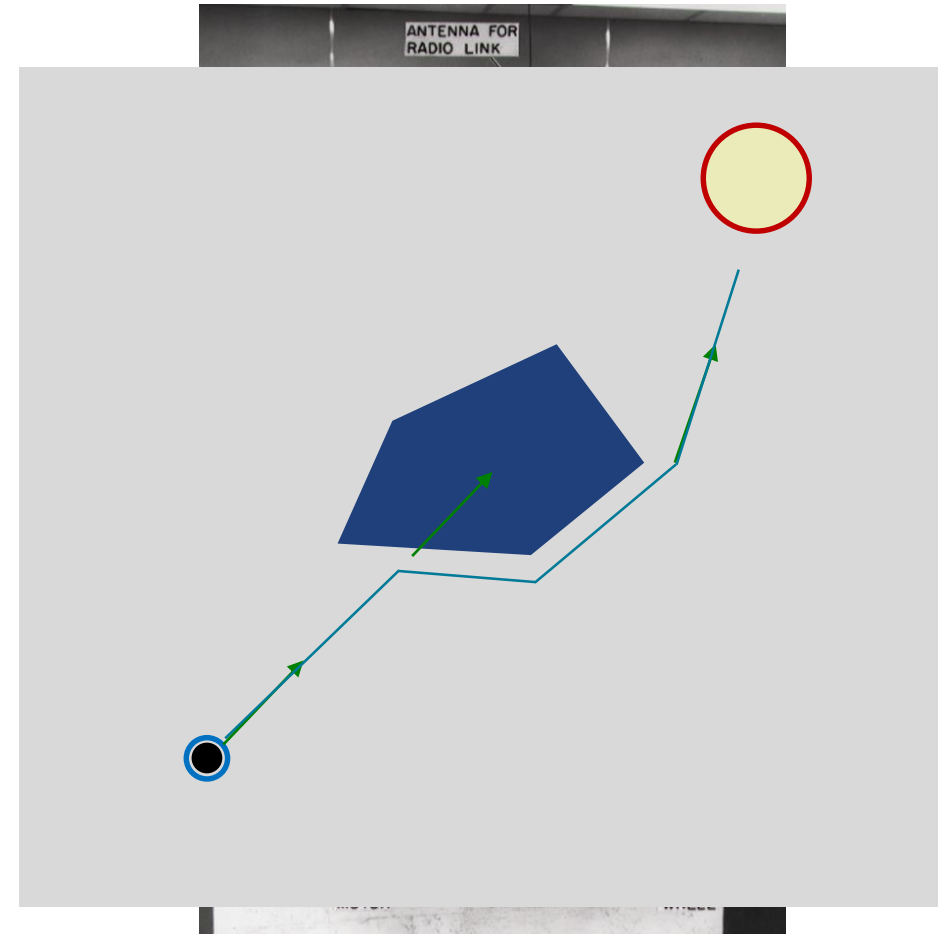
Related topics

- Control:
 - Generally concerned with reaching and maintaining a desired state in some kind of robust way
 - Often **feedback-based**
 - Success measured in terms of stability, robustness, ability to reject disturbance
- Planning in Artificial Intelligence:
 - Generally more focused on **discrete** problems
 - Classic AI 'planning' problems (spanning graphs, travelling salesman, orienteering) often appear in robotic planning approaches



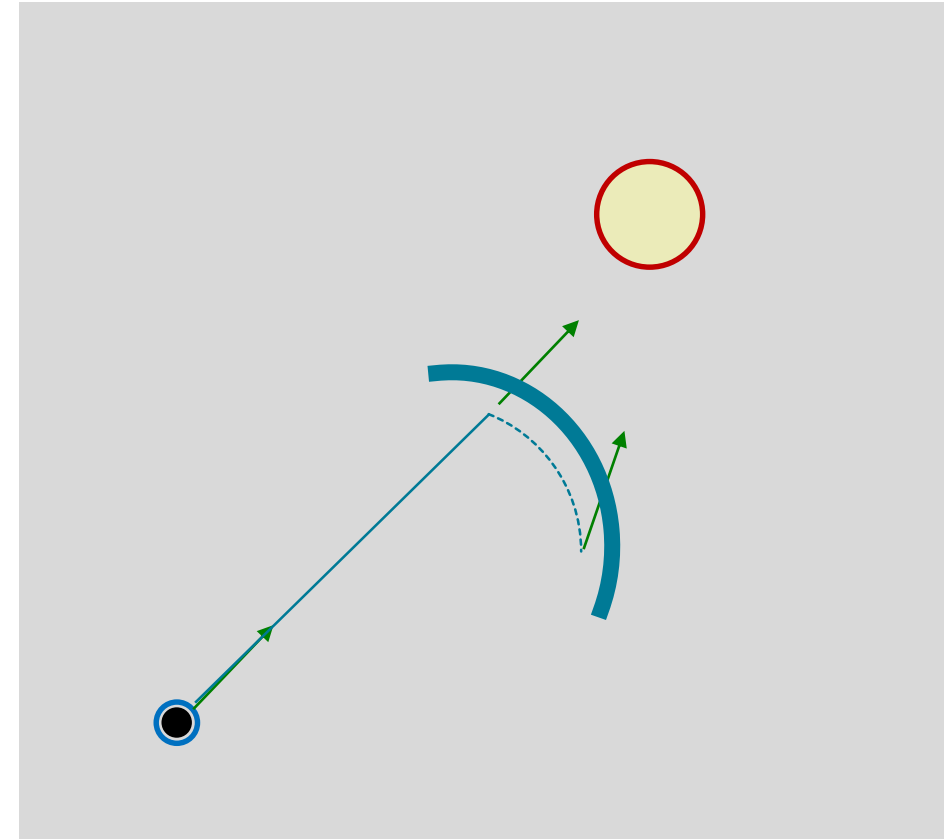
Historical robot motion planning – Adapted from Howie Choset

- Classical robotics (mid-70s)
 - Exact models, no sensing
- Reactive motion planning (mid-80s)
 - No models, rely on sensors only



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Historical robot motion planning – Adapted from Howie Choset

- Classical robotics (mid-70s)
 - Exact models, no sensing
- Reactive motion planning (mid-80s)
 - No models, rely on sensors only
- Hybrid / hierarchical (since early 90s)
 - Use models/planning at high level
 - Reactive (obstacle avoidance) at lower level
- Probabilistic (since mid 90s)
 - Incorporate uncertainty in models and sensors in all stages of planning

MOTION PLANNING

Navigation Competence

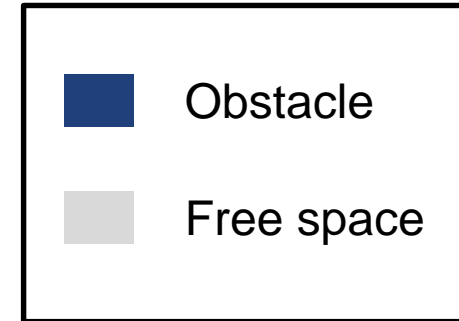
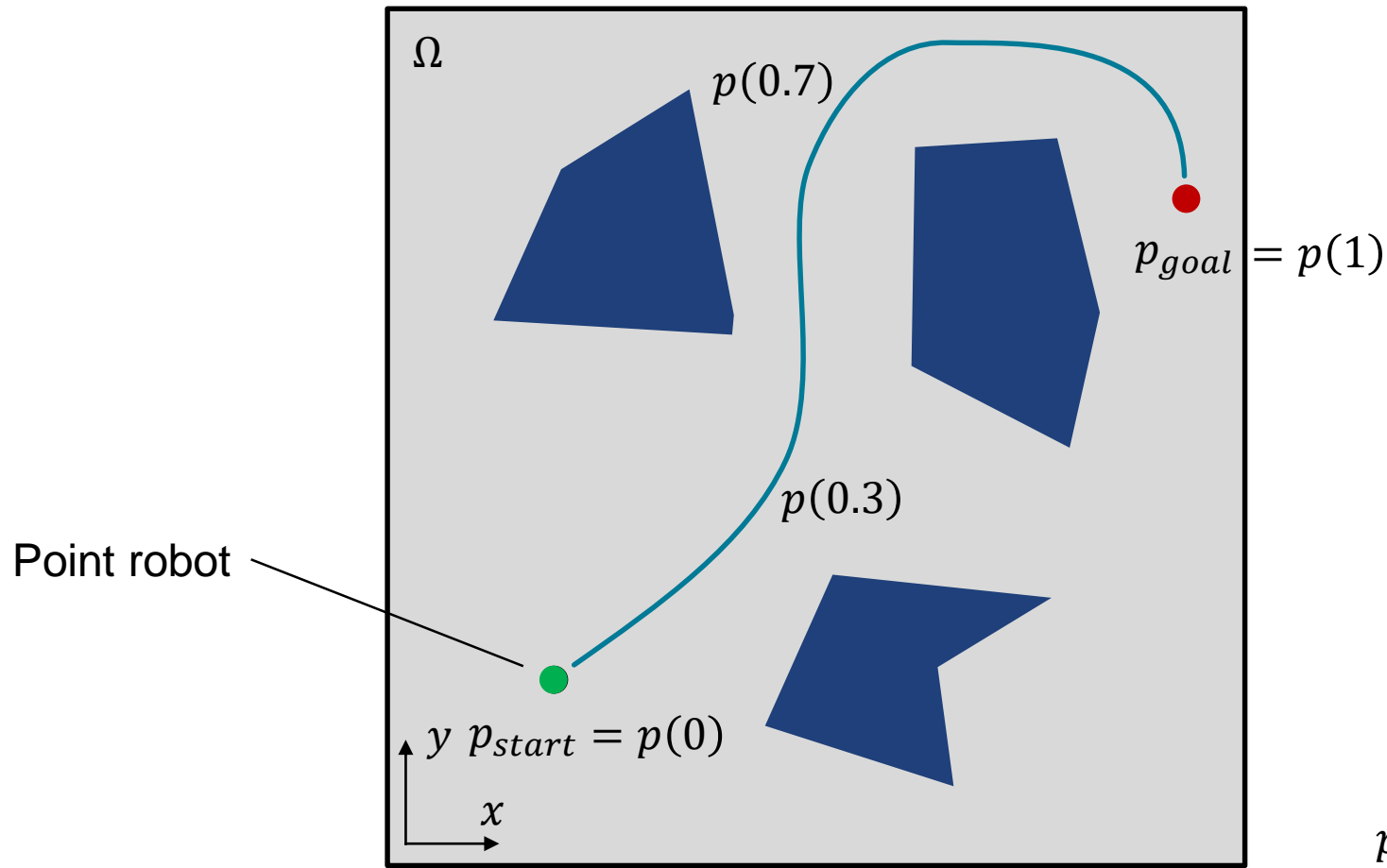
- In a nutshell, work out how the robot could *feasibly* move from one position to another
- Simplifying assumptions (for now...)
 - Our **representation** of the robot and the world is sufficiently expressive
 - We know **where we are** and **where we want to go**
 - We have a **motion model** for our robot
- Typically cast as an optimisation problem – minimise cost (time, distance, energy), within constraints

REPRESENTATION

Representation

- How the world is represented and understood by the planner (robot) is important
- Usually some degree of simplification in choosing a representation
- By choosing a suitable representation of the world, we may be able to apply existing algorithms to solve our planning problem

Representation – workspace and paths



$$\omega_{free} = \Omega - \omega_{obs}$$

$$p: [0,1] \rightarrow \omega_{free}$$

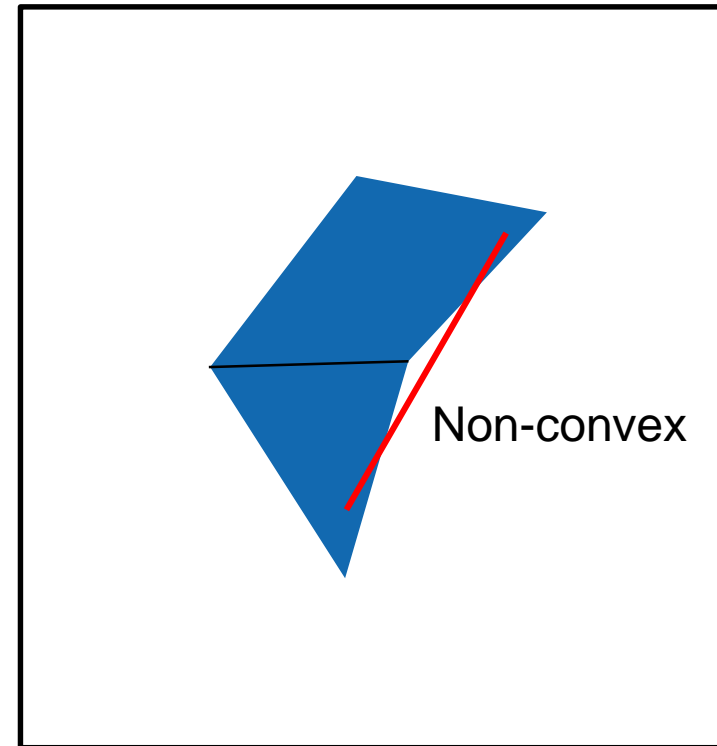
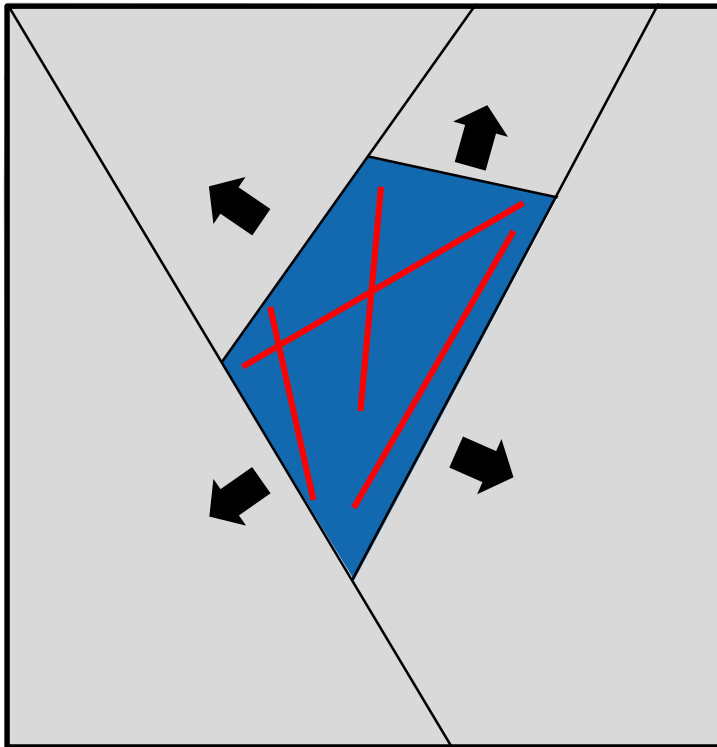
$$p(0) = p_{start}$$

$$p(t): \forall t \in [0,1], \\ p(t) \in \omega_{free}$$

$$p(1) = p_{goal}$$

A brief aside – (polygonal) convexity

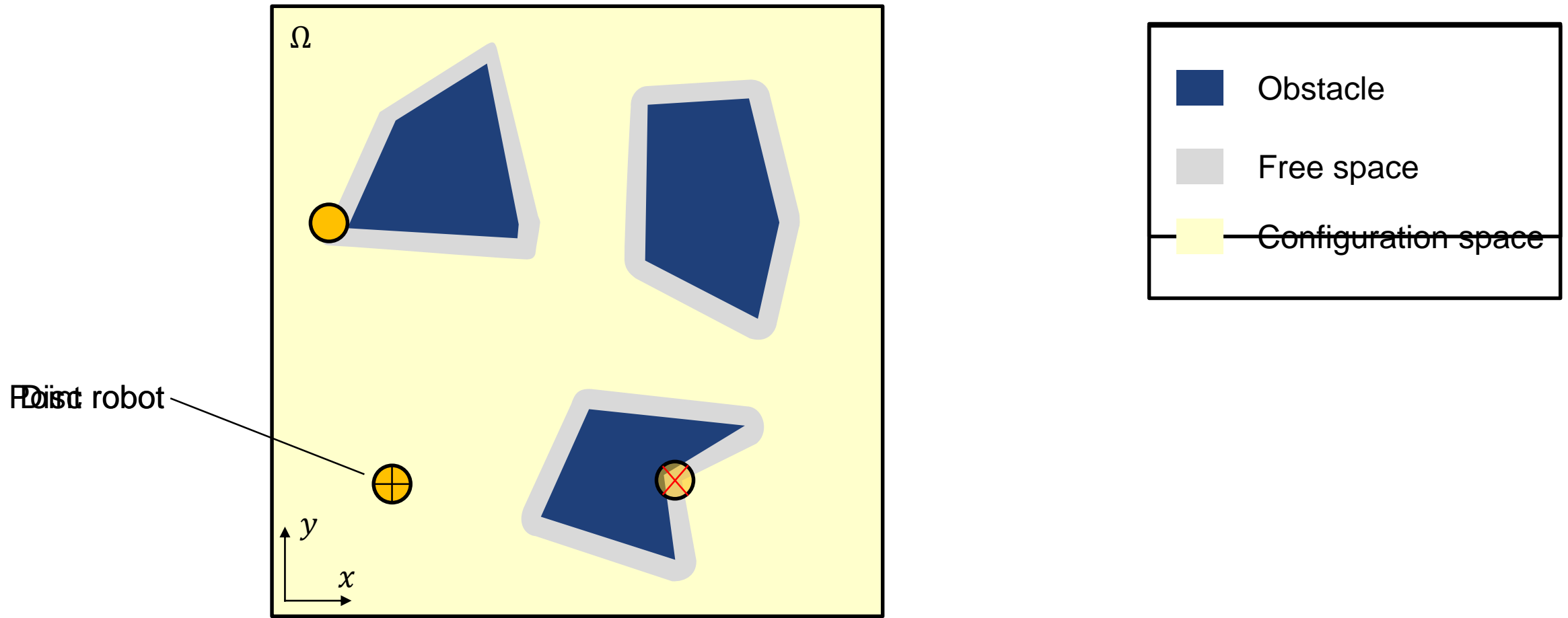
- A convex shape can be defined by a set of half-planes



Representation – Workspace and configuration space

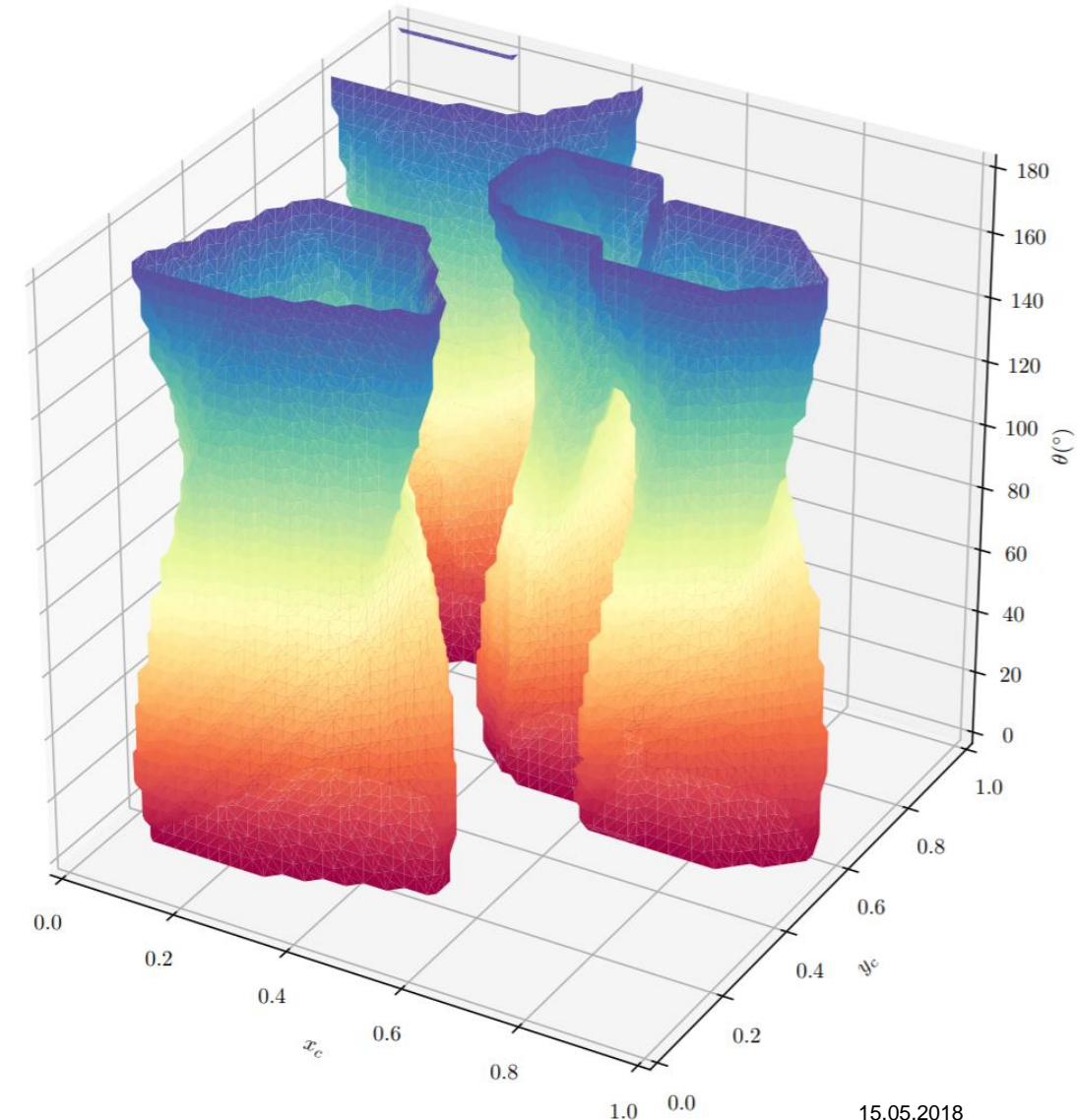
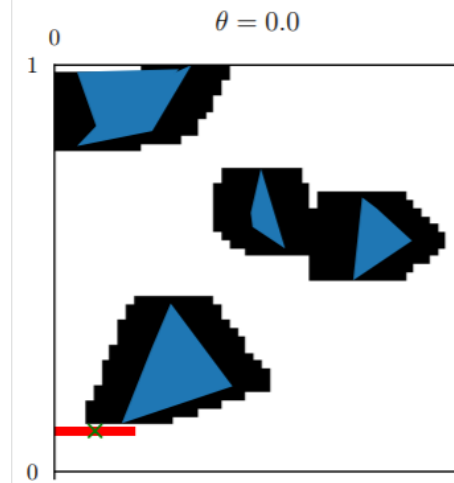
- The workspace is often the representation of the world, possibly independent of the robot itself. Often describes some notion of reachability, what space is free or occupied?
- Configuration space describes the full state of the robot in the world (actuator positions, orientation, etc.)
- Let's consider that our robot is no longer a point, but occupies an area...

Representation – configuration space



Representation – configuration space

- A robot without rotational symmetry (3DOF)



Configuration space for alternative morphologies

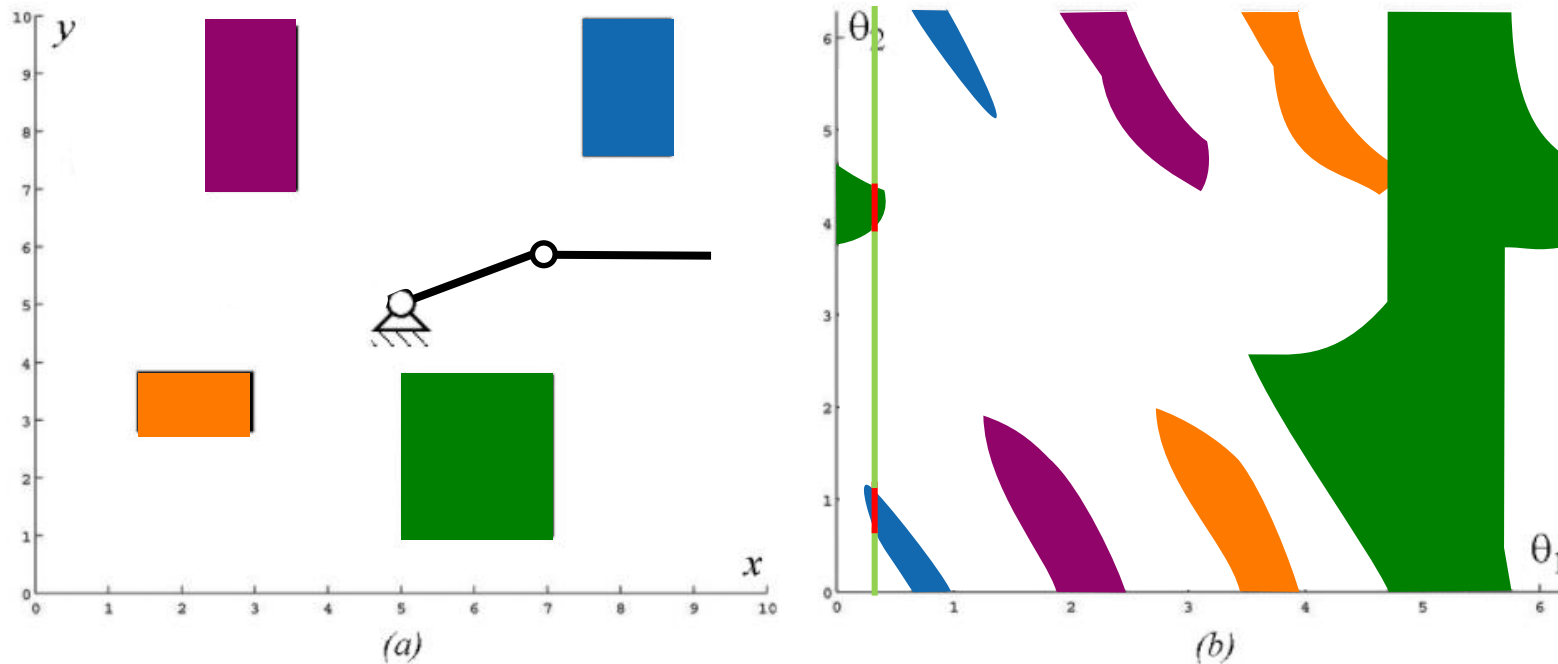


Figure 6.1

Physical space (a) and configuration space (b): (a) A two-link planar robot arm has to move from the configuration *start* to *end*. The motion is thereby constraint by the obstacles 1 to 4. (b) The corresponding configuration space shows the free space in joint coordinates (angle θ_1 and θ_2) and a path that achieves the goal.

Configuration space for alternative morphologies

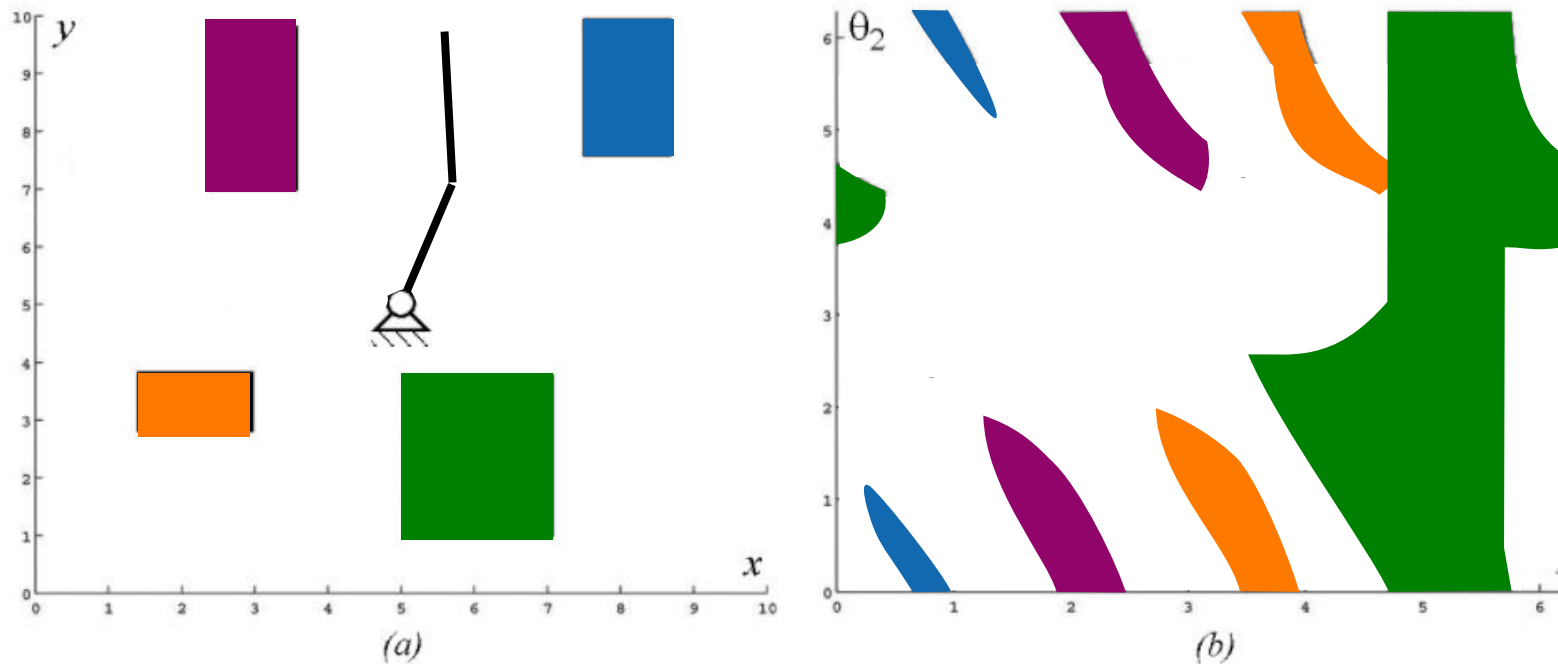


Figure 6.1

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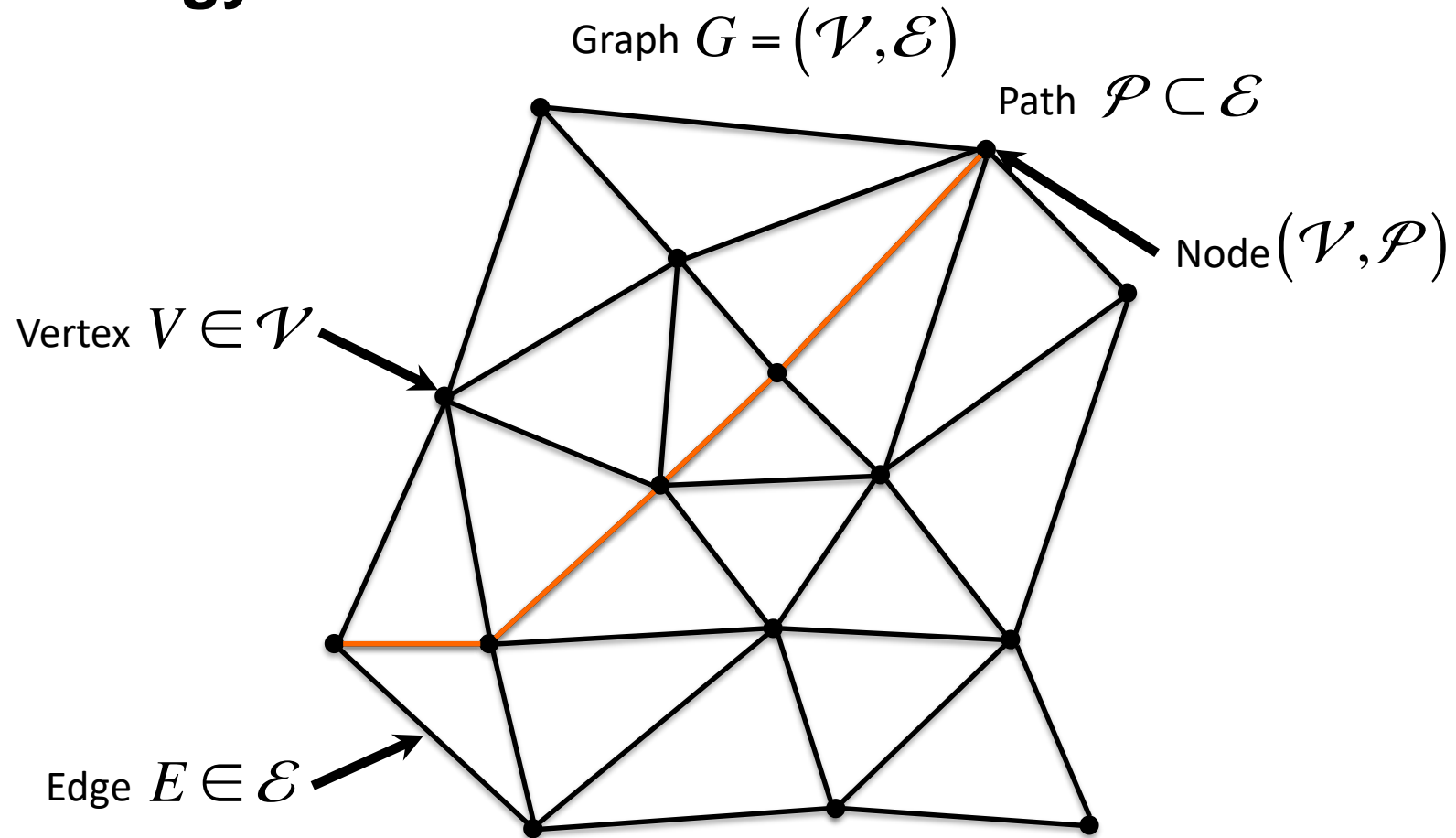
Why use configuration space?

- Positions in configuration space tend to be close together for the robot
- Can be easier to solve collision checks, and join nearby poses
- Allows a level of abstraction that means solution methods can solve a wider range of problems
- Sometimes helps with wraparound conditions (rotation joints)

Continuous vs discrete state space representations

- Although continuous representations have some nice mathematical properties, they aren't always convenient
- Since computers store things digitally, there are some clever and efficient ways to create (approximate) discrete representations
- It is **very** common to convert a planning problem to some kind of (discrete) graph representation, then use one of a variety of existing search algorithms on the graph

Some Terminology



Directed graph: edges have direction

Weighted graph: edges have costs

Discrete state space representation

- Reduce continuous state space to a finite set of discrete **states**

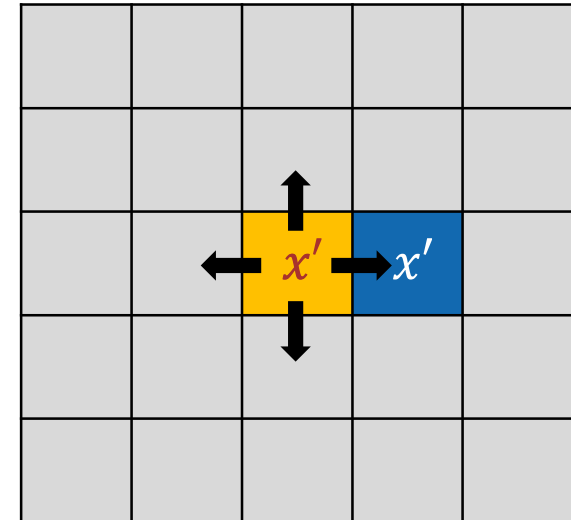
$$x \in X$$

- Also define feasible **actions** from each state

$$A(x) = \{a_0, a_1, \dots, a_n\}$$

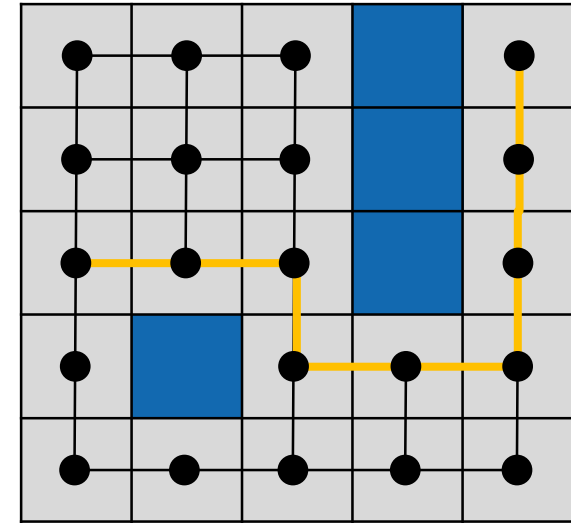
- And an associated **transition function**

$$f(x, a) = x'$$



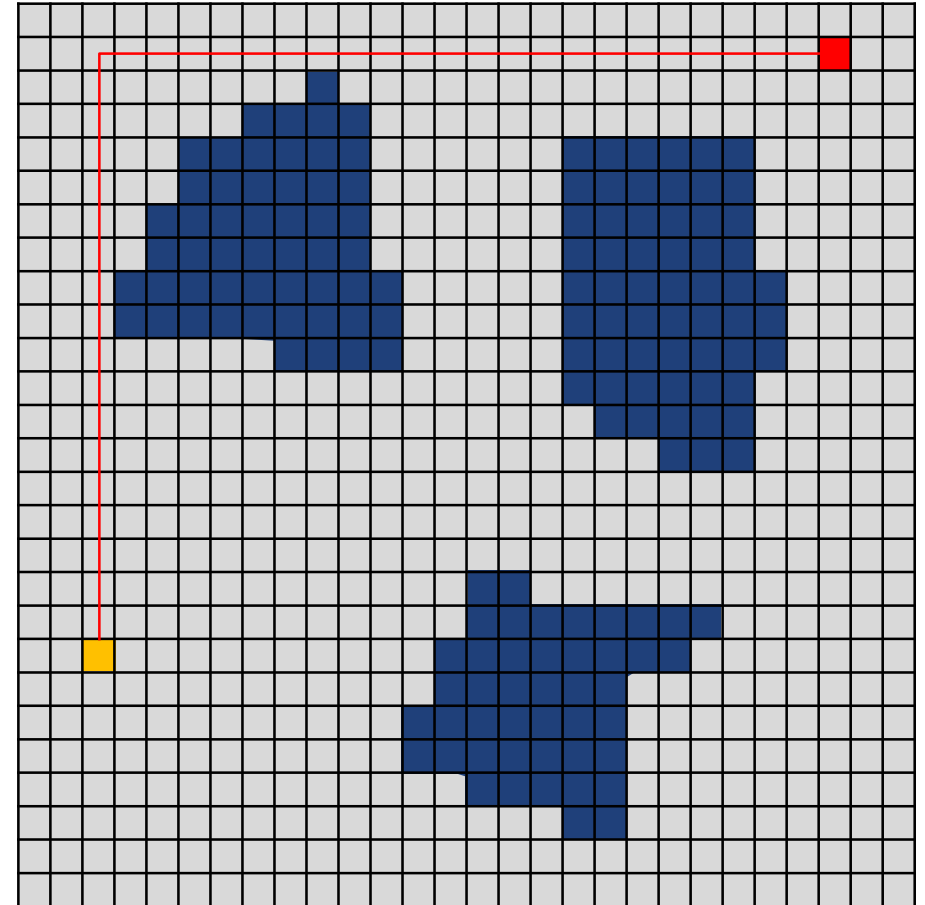
Grid \rightarrow Graph

- Consider:
 - States as vertices
 - Transitions as directed edges
- The result is a graph
- Add:
 - Start node, x_s
 - Goal node, x_g
 - Cost function $C: X \times A \rightarrow \mathbb{R}^+$
- Finding the shortest path can be treated as a graph search problem



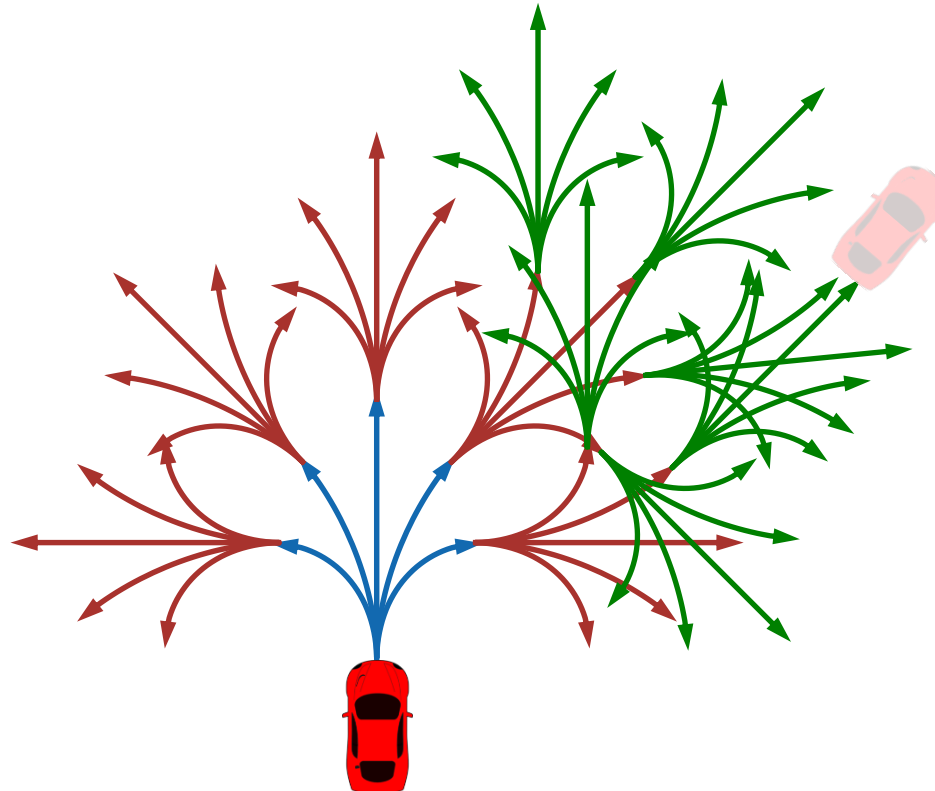
Issues with grid-based representations

- Usually suffer some loss of precision
- Selecting an appropriate grid resolution can be a challenge (multi-resolution mapping)
- Can limit the type of output path
- Suffer from poor scaling in higher dimensions



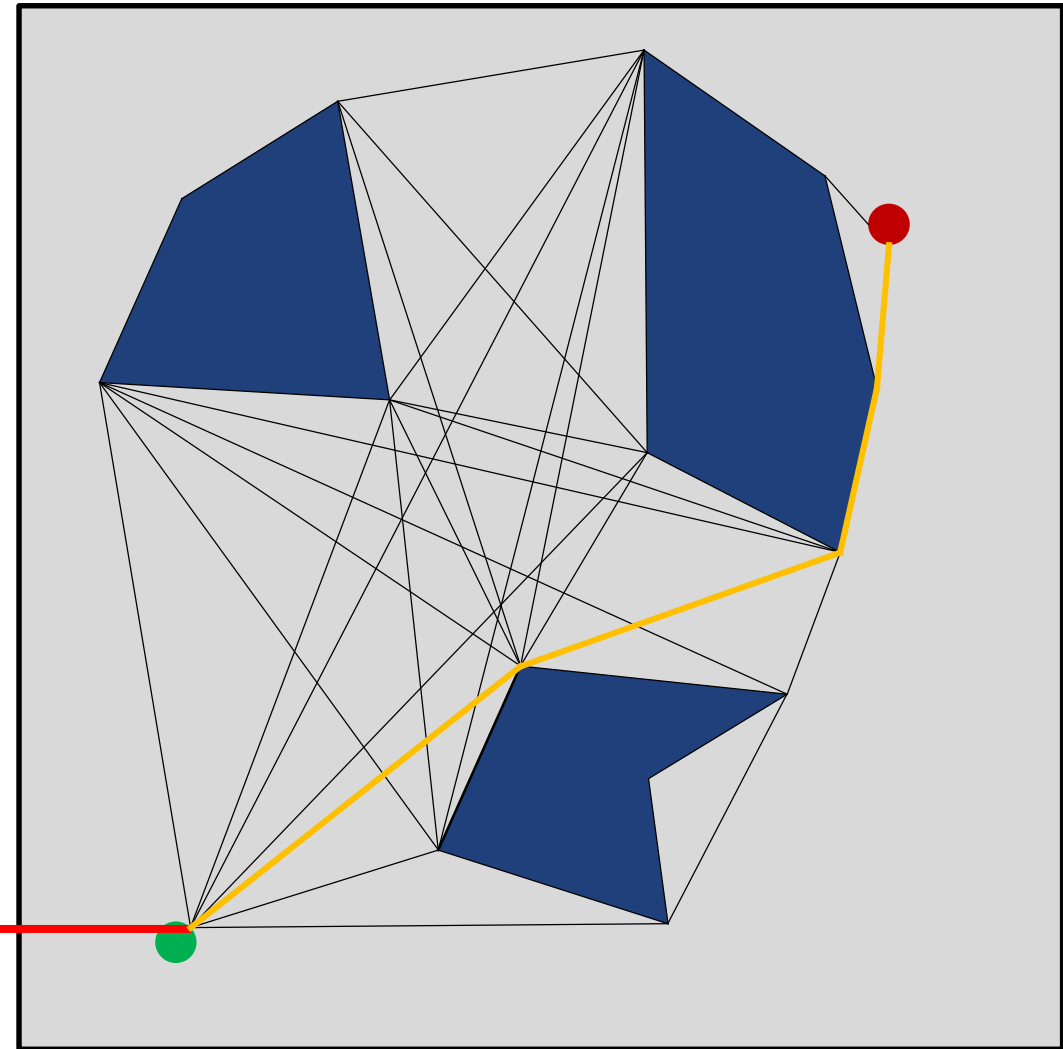
Brief aside – other graph-based representations

- Grid lattice – create a set of feasible motion primitives, and construct a tree (graph) that chains the motions into a sequence (plan)



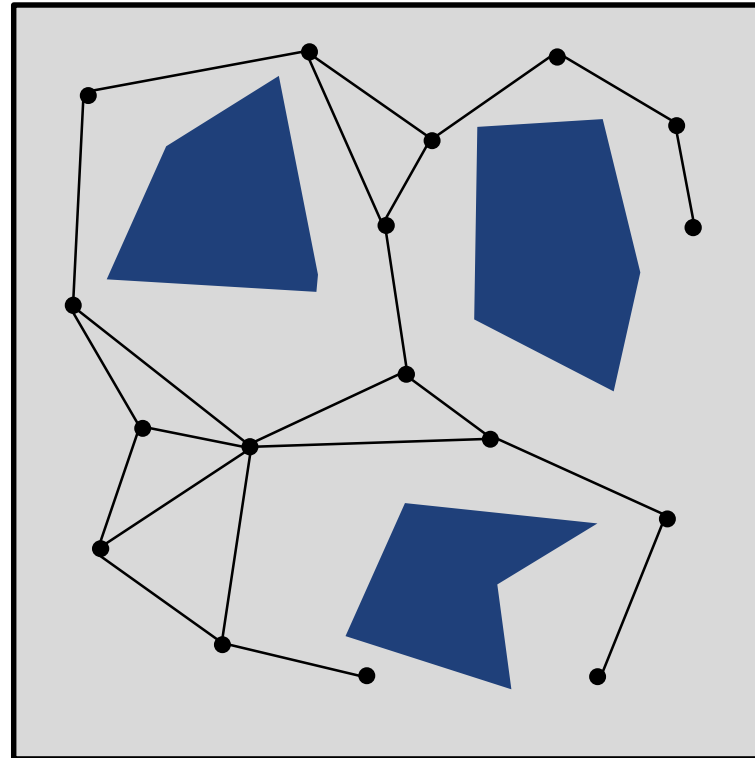
Visibility graph

- Create edges between all pairs of mutually visible vertices
- Search resulting graph
- Optimal plan!
- Limited to straight motion, 2D, polygonal obstacles



Randomly-sampled graphs

- Especially popular for sample-based methods (next lecture)
- Require careful consideration to construct graphs with guarantees



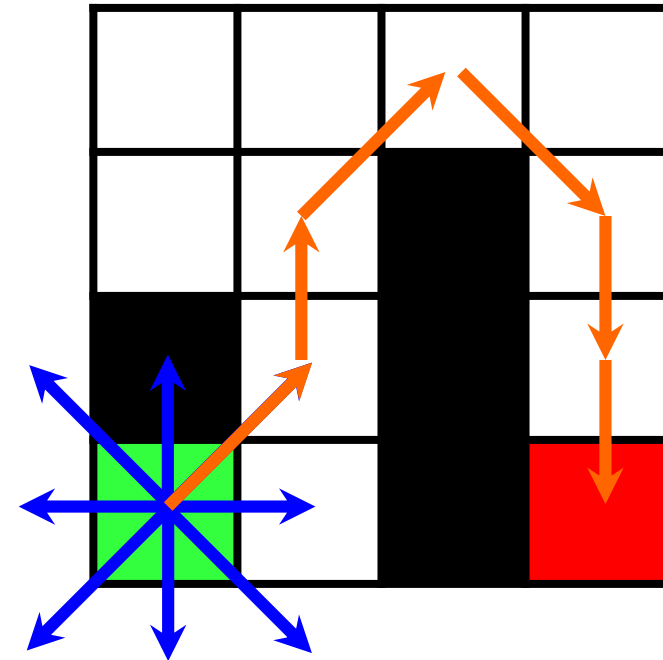
Planning with graphs

- Given a representation, a start, a goal, and a motion model, how do we actually generate a plan?
- Many planning approaches use graphs, because we know how to search graphs and computers are good at it
- Solve your planning problem in three easy steps:
 1. Convert problem to a graph
 2. Search the graph
 3. Profit!

GRAPH SEARCH METHODS

Example 1: Discretizing the World

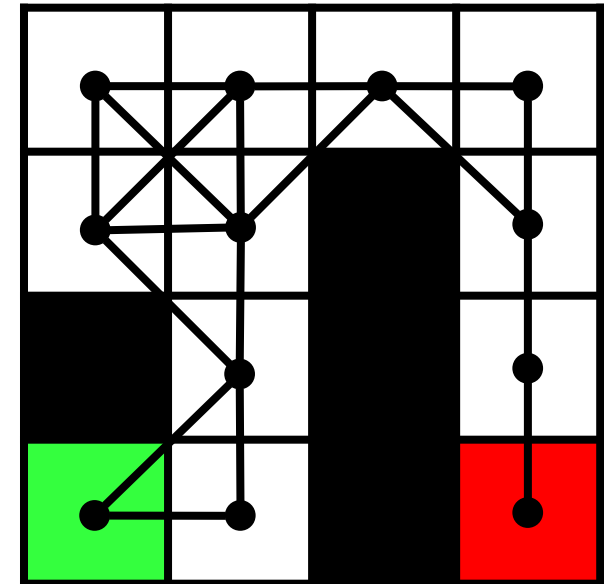
- Move from green cell to red cell
- 8-connected grid motion
- Shortest path?
- What if robot motion was restricted to 4-connected grid motion?



Assumptions about
robot motion

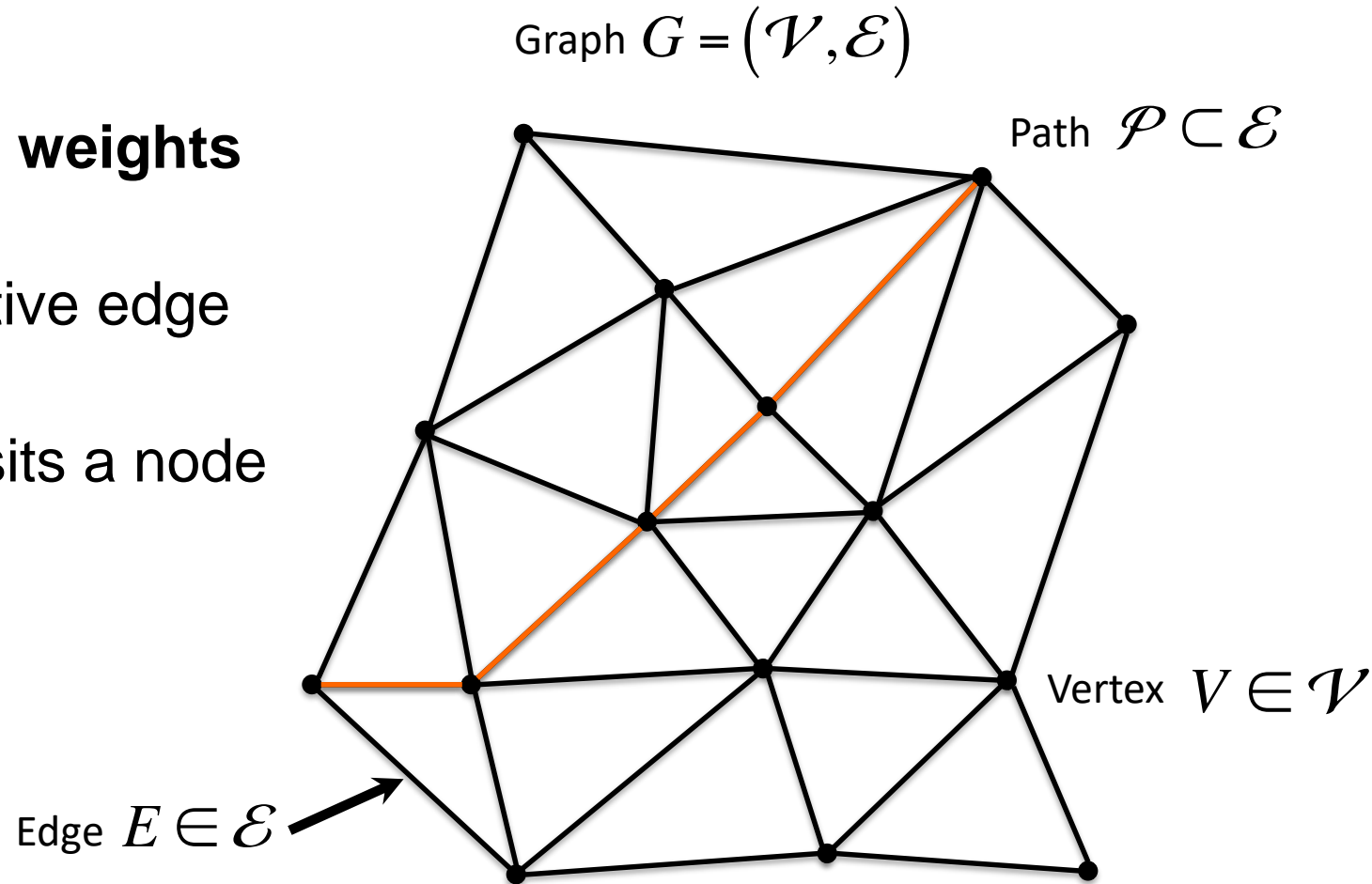
Example 1: Discretizing the World

- Search over the underlying graph
- Solve for paths from any point to any other point
- Assume all edge transitions are dynamically feasible



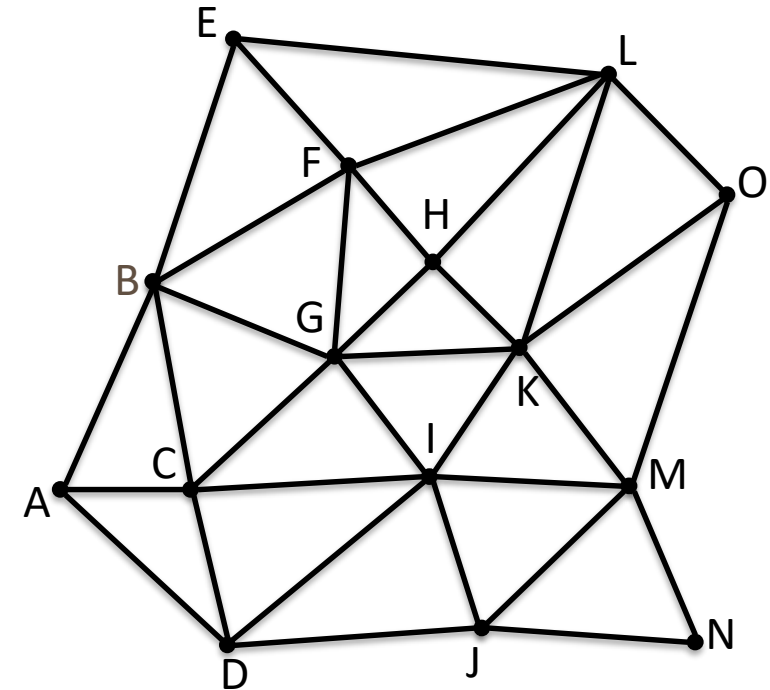
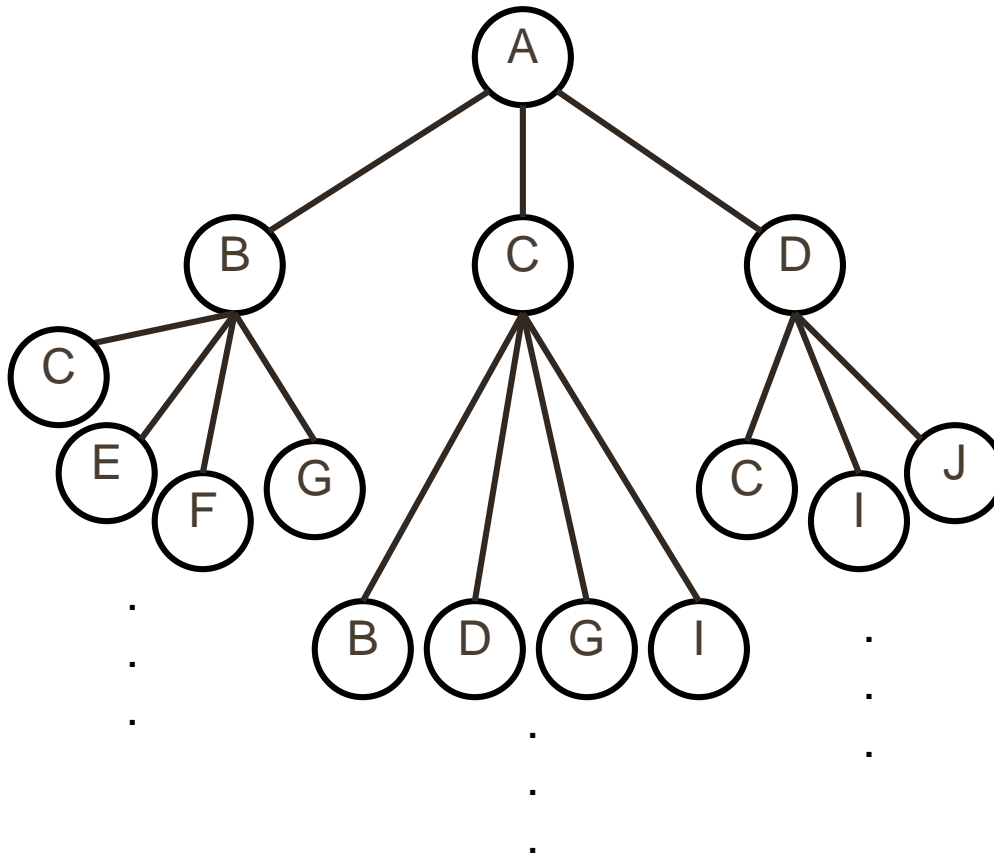
Graph Search

- Edges can also have associated **weights** (cost of traversing the edge)
- We generally only consider positive edge weights
- A minimum cost path never revisits a node
- Graph search methods
 - Breadth-first search (BFS)
 - Depth-first search (DFS)
 - Dijkstra's Algorithm
 - A*



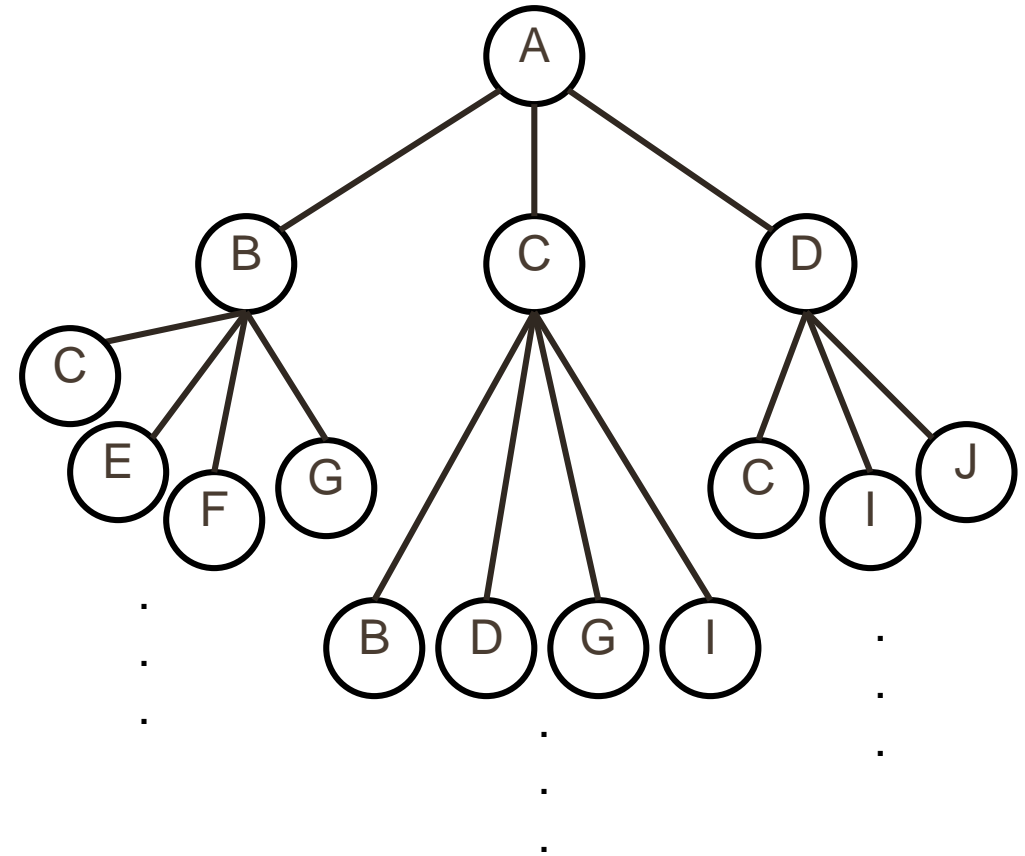
Search Trees

- We construct a “tree” through which we can search for optimal paths through the environment



Search Trees

- A search tree:
 - Start state at the root node
 - Children correspond to successors
 - A node corresponds to a unique **plan** from start to that state (follow up tree from node)
 - For most problems, we want to avoid building the whole tree
 - Use search algorithms to efficiently traverse tree



Basics of Forward Search

- Generally, start from the start, grow tree until you find a solution (path to goal)
- Expanding a node refers to adding children to the tree, pushing them onto the open set
- Try to expand as few tree nodes as possible
- **Open** set maintains a list of frontier (unexpanded) plans
 - Keeps track of what nodes to expand next
 - Often stored as a priority queue
 - For each node in the open list, we know of at least one path to it from the start
- **Closed** set keeps track of nodes that have been expanded
 - For each node in the closed list, we've already found the lowest-cost path to it from the start

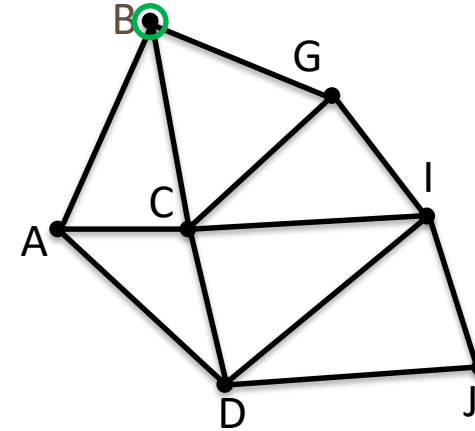
Forward Search algorithm from LaValle

```
FORWARD_SEARCH
1   $Q.Insert(x_I)$  and mark  $x_I$  as visited
2  while  $Q$  not empty do
3       $x \leftarrow Q.GetFirst()$ 
4      if  $x \in X_G$ 
5          return SUCCESS
6      forall  $u \in U(x)$ 
7           $x' \leftarrow f(x, u)$ 
8          if  $x'$  not visited
9              Mark  $x'$  as visited
10              $Q.Insert(x')$ 
11          else
12              Resolve duplicate  $x'$ 
13 return FAILURE
```

Figure 2.4: A general template for forward search.

LaValle, Steven M. *Planning algorithms*. Cambridge university press, 2006, p. 33

Breadth-First Search Example



FORWARD SEARCH

```

1  Q.Insert( $x_I$ ) and mark  $x_I$  as visited
2  while  $Q$  not empty do
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10       Q.Insert( $x'$ )
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13  return FAILURE
  
```

Open (Q):

{B}

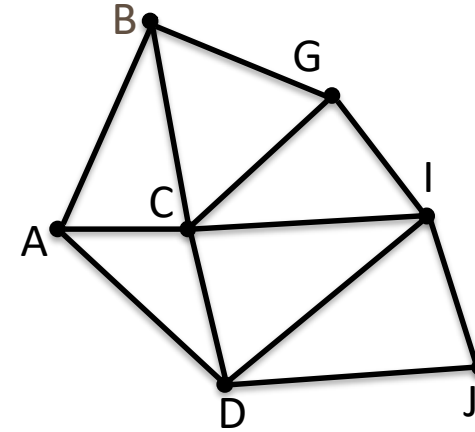
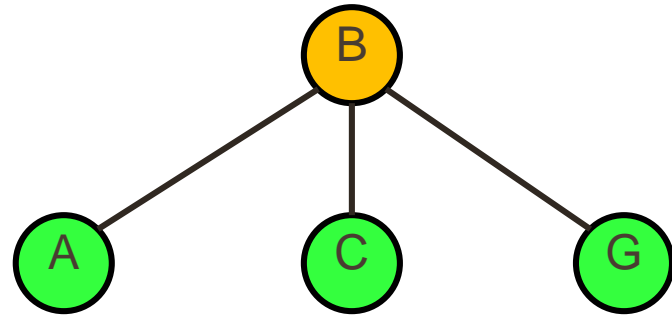
Closed:

{}

Our (BFS) queue will be FIFO:

- push (*Q.Insert*) onto the end
- pop (*Q.GetFirst*) from the front

Breadth-First Search Example



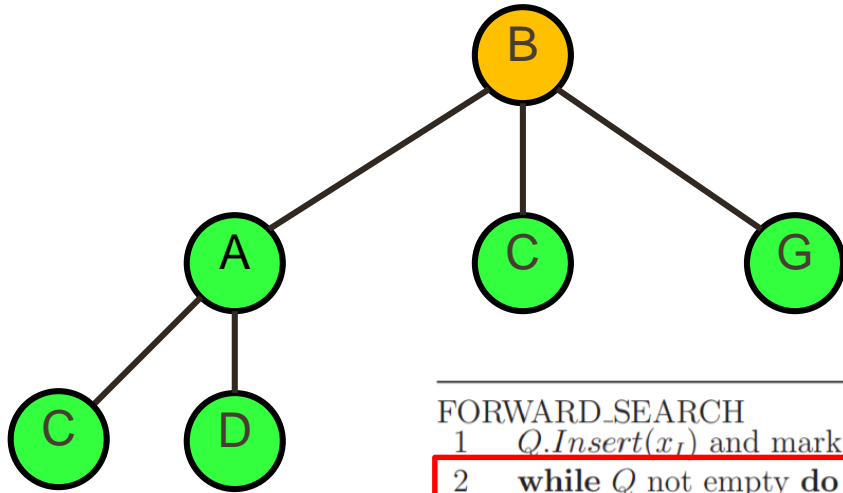
```

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2  while Q not empty do
3    x ← Q.GetFirst()
4    if x ∈ X_G
5      return SUCCESS
6    forall u ∈ U(x)
7      x' ← f(x, u)
8      if x' not visited
9        Mark x' as visited
10       Q.Insert(x')
11     else
12       Resolve duplicate x'
13  return FAILURE
  
```

Open (Q):
{A,C,G}

Closed:
{B,A,C,G}

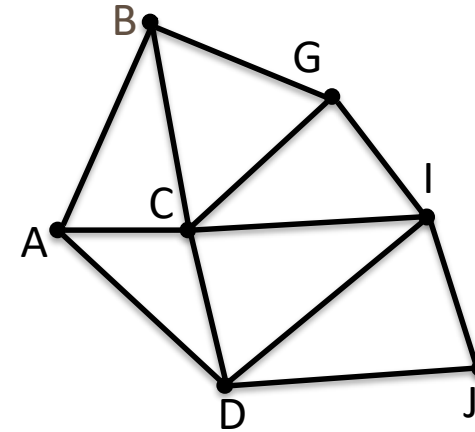
Breadth-First Search Example



FORWARD_SEARCH

```

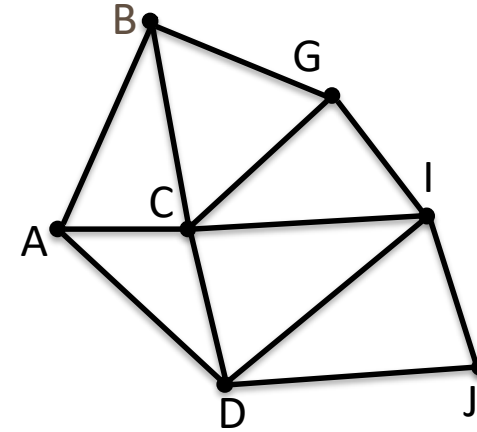
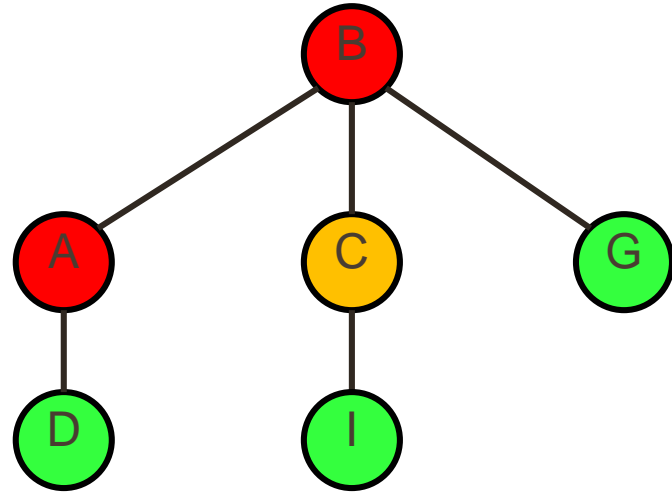
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```



Open (Q):
{C,G,D}

Closed:
{B,A,C,G,D}

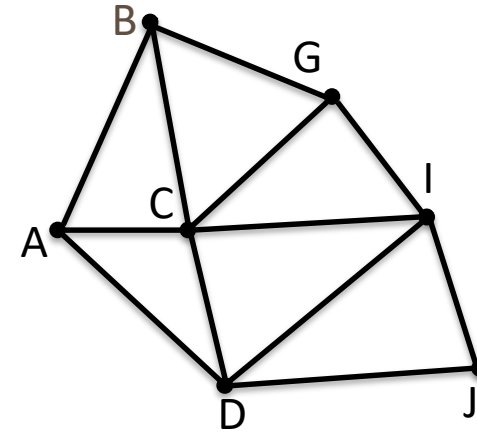
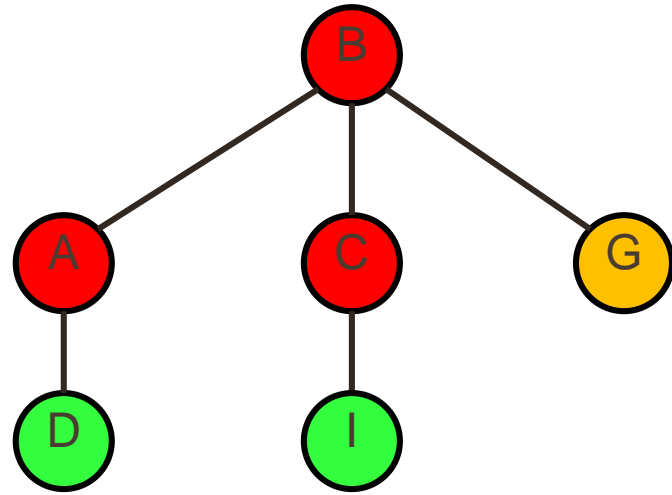
Breadth-First Search Example



Open (Q):
{G,D,I}

Closed:
{B,A,C,G,D,I}

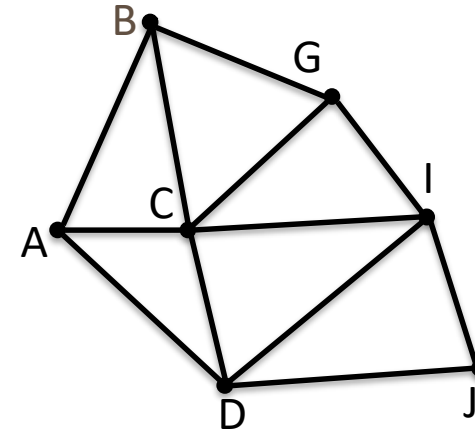
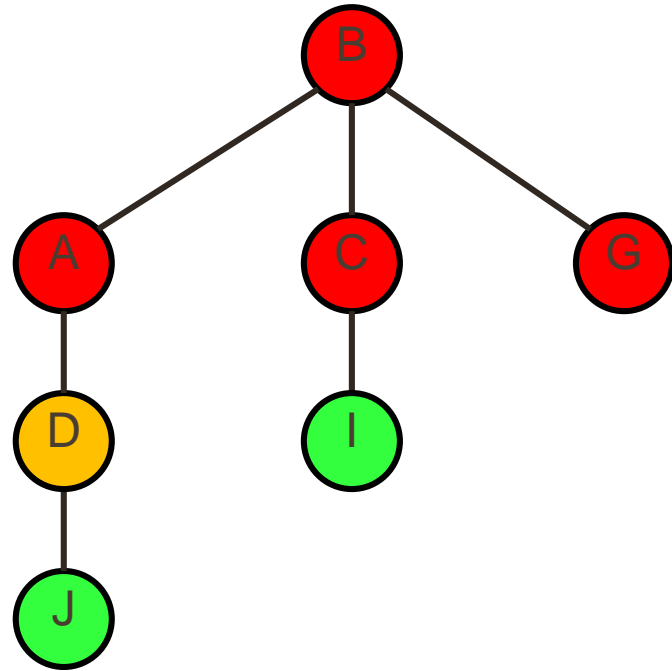
Breadth-First Search Example



Open (Q):
{D,I}

Closed:
{B,A,C,G}

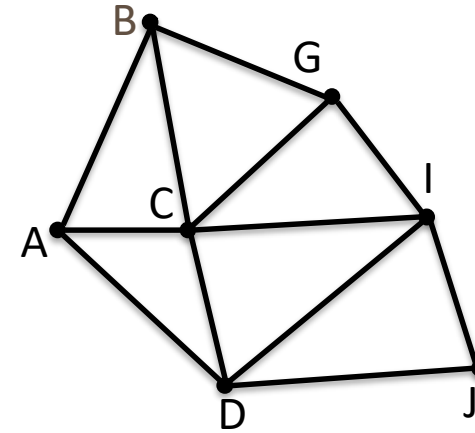
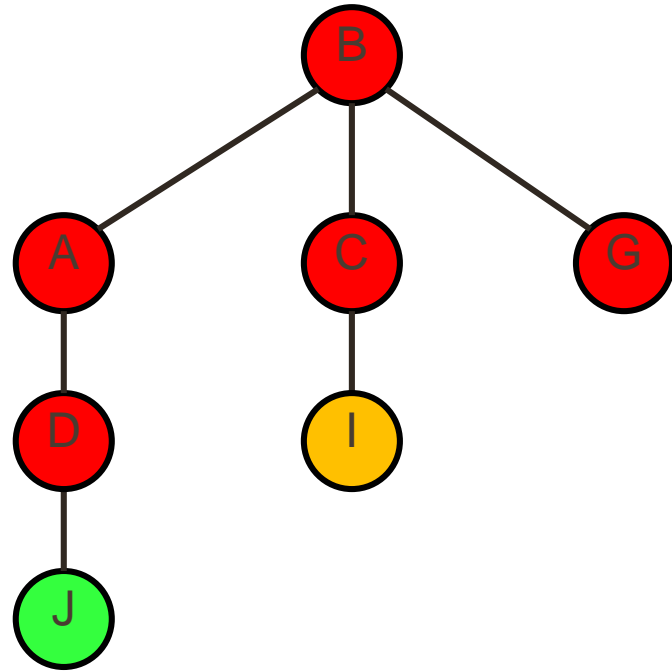
Breadth-First Search Example



Open (Q):
{I, J}

Closed:
{B, A, C, G, D, I, J}

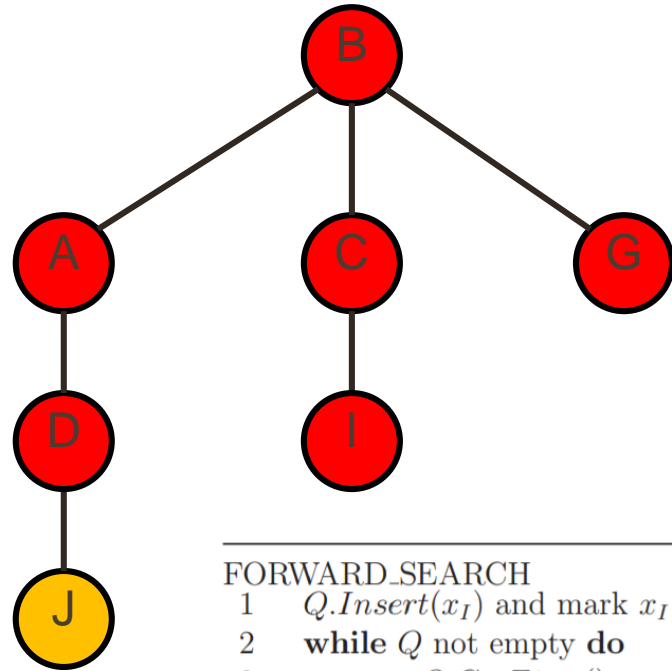
Breadth-First Search Example



Open (Q):
{J}

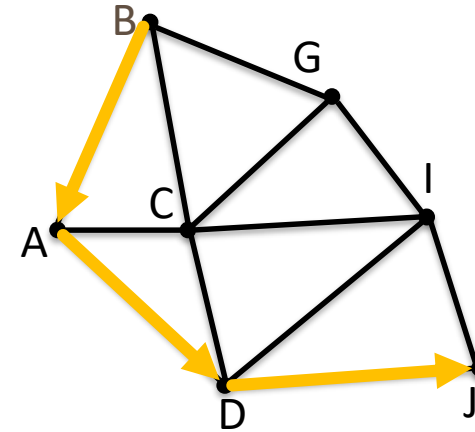
Closed:
{B,A,C,G,D,I,J}

Breadth-First Search Example



```

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1   $Q.Insert(x_I)$  and mark  $x_I$  as visited
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12    Resolve duplicate  $x'$ 
13  return FAILURE
  
```



Open (Q):
 $\{\}$

Closed:
 $\{B, A, C, G, D, I, J\}$

Final path solution: $B \rightarrow A \rightarrow D \rightarrow J$

Other solutions may exist but have the same number or more transitions

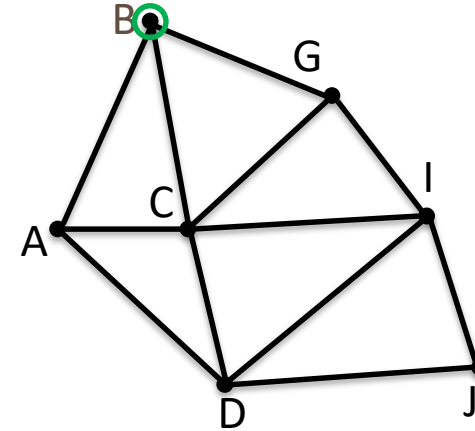
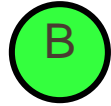
Breadth-First Search

- Complete (will find the solution if it exists)
- Guaranteed to find the shortest (number of edges) path
 - First solution found is the optimal path
- What about non-uniform edge weights? (... Dijkstra)
- Time complexity $O(|V|+|E|)$
- Consider another approach: Depth-first search

Depth-first search

- Instead of searching across levels of the tree, DFS starts at the root node and explores as far as possible along each branch before backtracking
- Similar implementation to BFS, but with a stack (last-in first-out) queue

Depth-First Search Example



FORWARD SEARCH

```

1  Q.Insert( $x_I$ ) and mark  $x_I$  as visited
2  while  $Q$  not empty do
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{B}

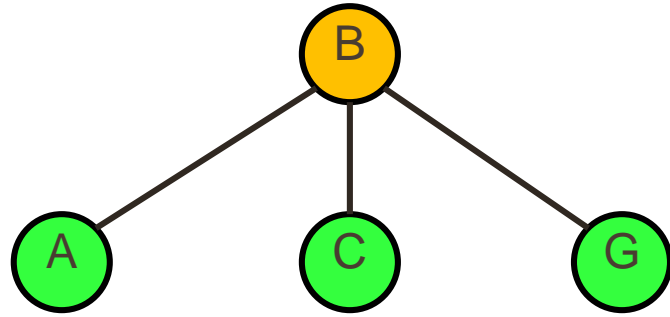
Closed:

{}

Our (DFS) queue will be LIFO:

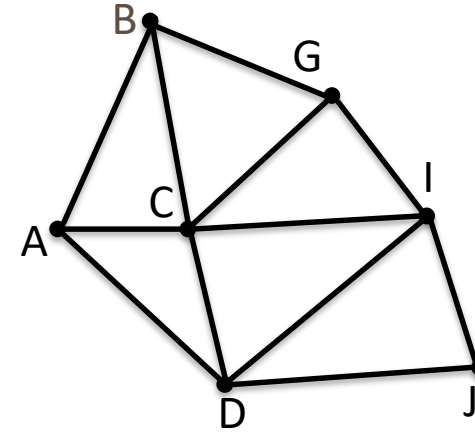
- push (*Q.Insert*) onto the front
- pop (*Q.GetFirst*) from the front

Depth-First Search Example



```

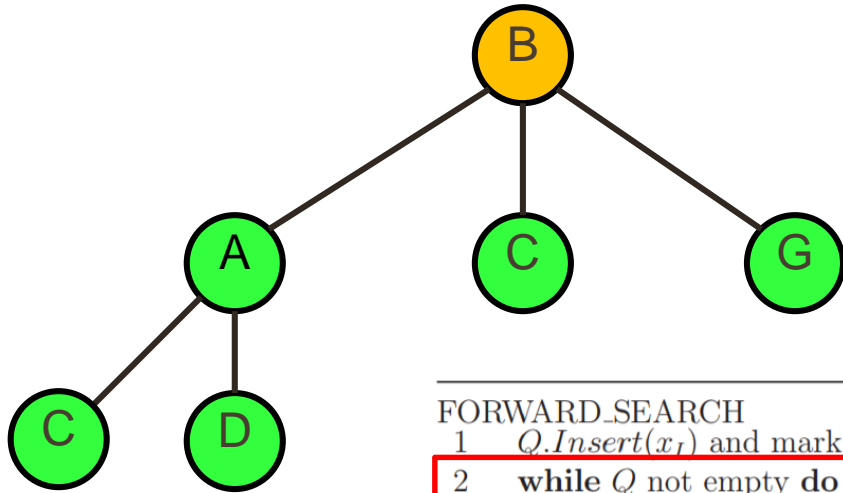
FORWARD_SEARCH
1  Q.Insert( $x_I$ ) and mark  $x_I$  as visited
2  while Q not empty do
3     $x \leftarrow Q.GetFirst()$ 
4    if  $x \in X_G$ 
5      return SUCCESS
6    forall  $u \in U(x)$ 
7       $x' \leftarrow f(x, u)$ 
8      if  $x'$  not visited
9        Mark  $x'$  as visited
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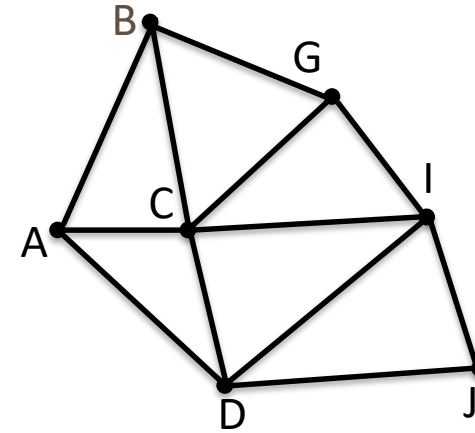
Depth-First Search Example



FORWARD_SEARCH

```

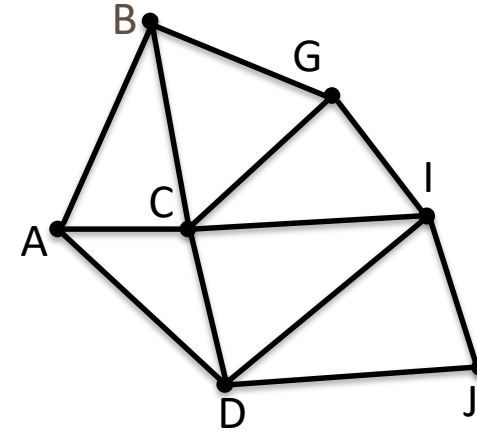
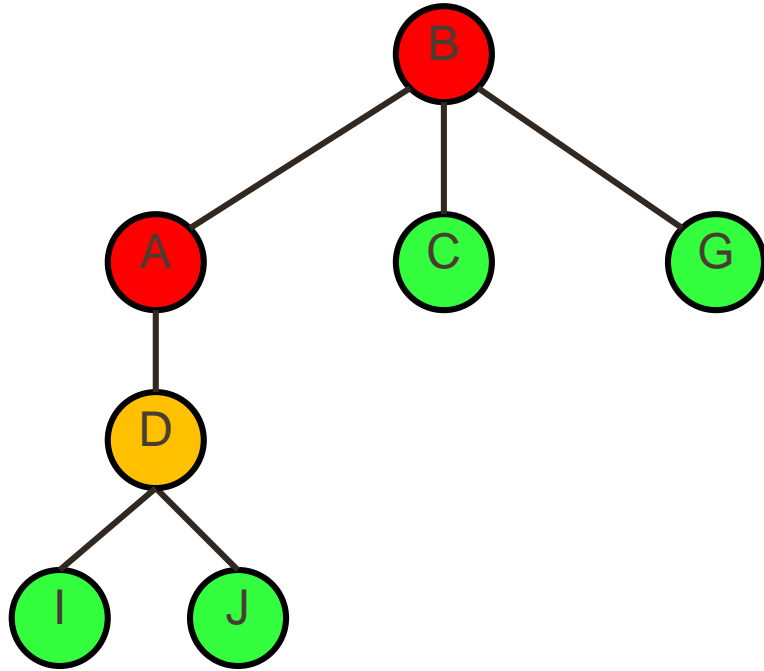
1   $Q.Insert(x_I)$  and mark  $x_I$  as visited
2  while  $Q$  not empty do
3     $x \leftarrow Q.GetFirst()$ 
4    if  $x \in X_G$ 
5      return SUCCESS
6    forall  $u \in U(x)$ 
7       $x' \leftarrow f(x, u)$ 
8      if  $x'$  not visited
9        Mark  $x'$  as visited
10        $Q.Insert(x')$ 
11    else
12      Resolve duplicate  $x'$ 
13  return FAILURE
  
```



Open (Q):
{D,C,G}

Closed:
{B,A,C,G,D}

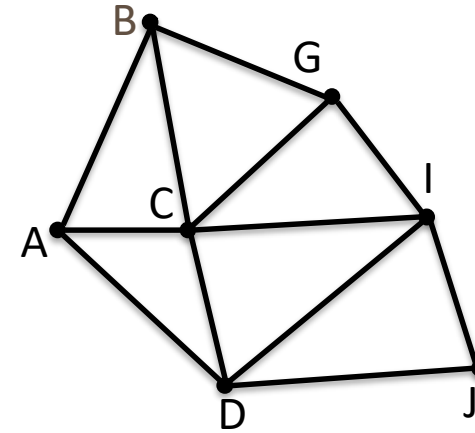
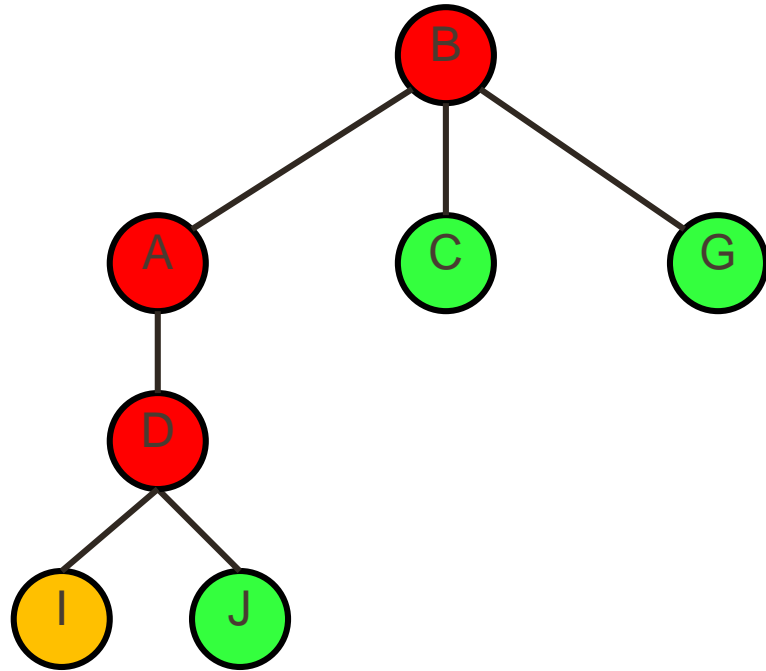
Depth-First Search Example



Open (Q):
{I, J, C, G}

Closed:
{B, A, C, G, D, I, J}

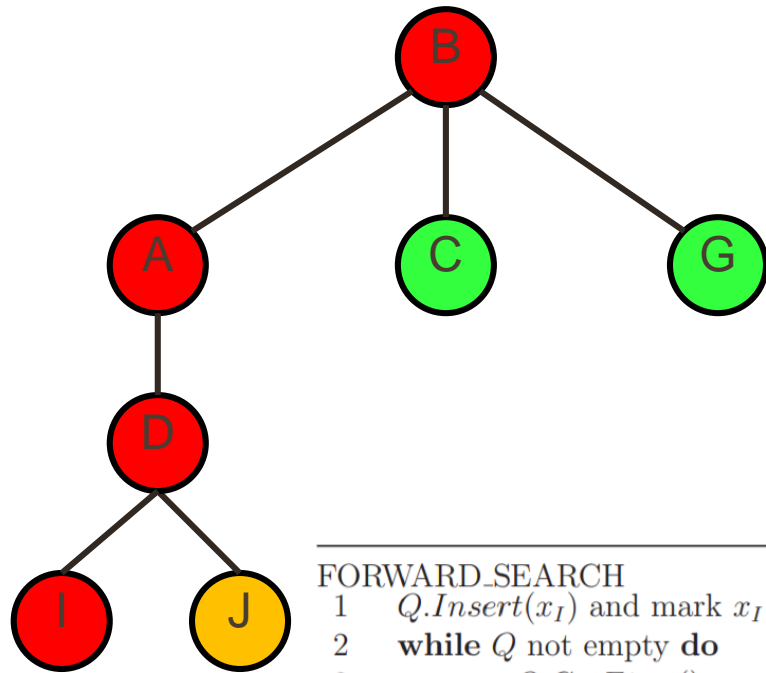
Depth-First Search Example



Open (Q):
{I,C,G}

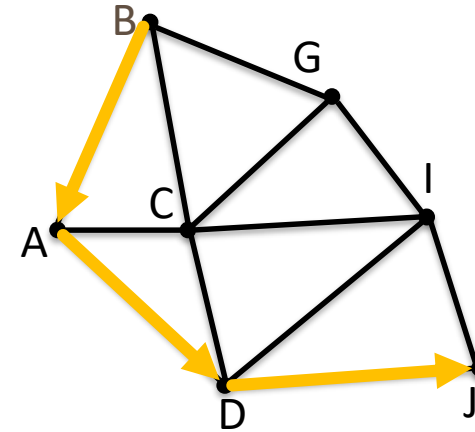
Closed:
{B,A,C,G,D,I,J}

Breadth-First Search Example



```

FORWARD_SEARCH
1   $Q.Insert(x_I)$  and mark  $x_I$  as visited
2  while  $Q$  not empty do
3     $x \leftarrow Q.GetFirst()$ 
4    if  $x \in X_G$ 
5      return SUCCESS
6  forall  $u \in U(x)$ 
7     $x' \leftarrow f(x, u)$ 
8    if  $x'$  not visited
9      Mark  $x'$  as visited
10      $Q.Insert(x')$ 
11  else
12    Resolve duplicate  $x'$ 
13  return FAILURE
  
```

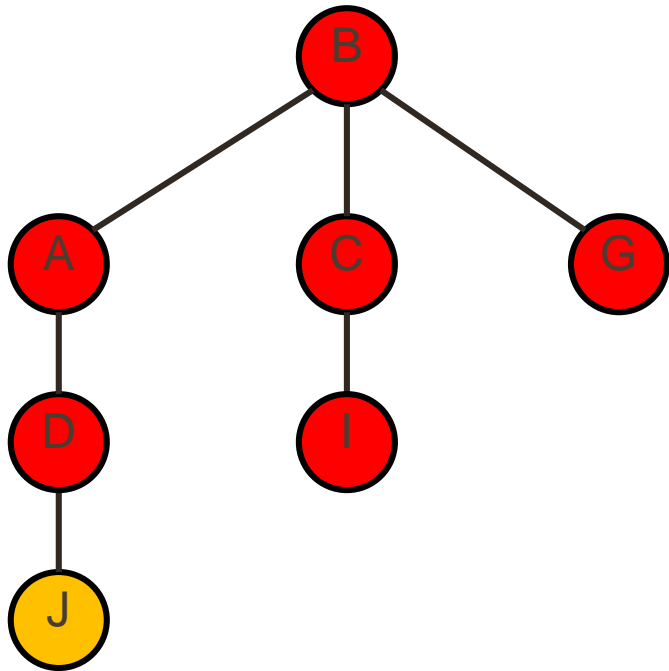


Open (Q):
 {}

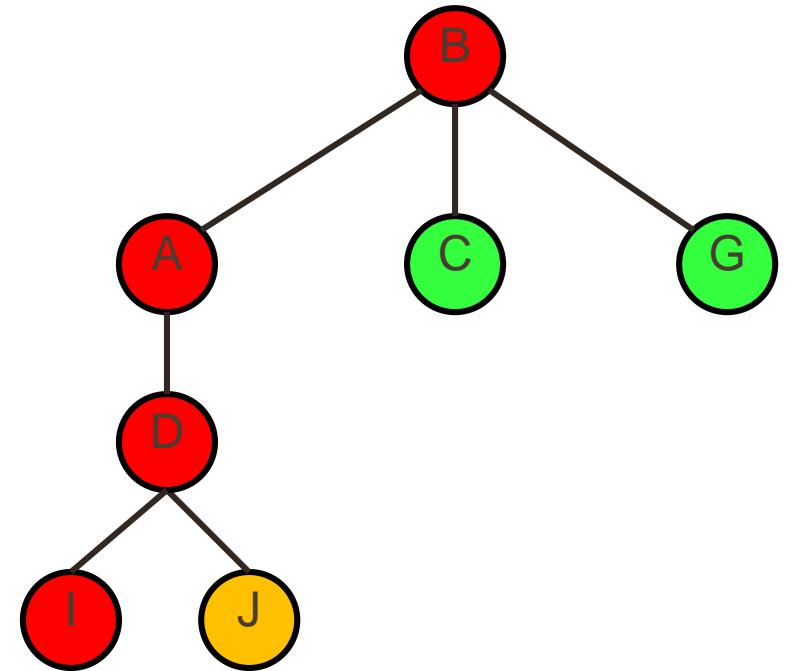
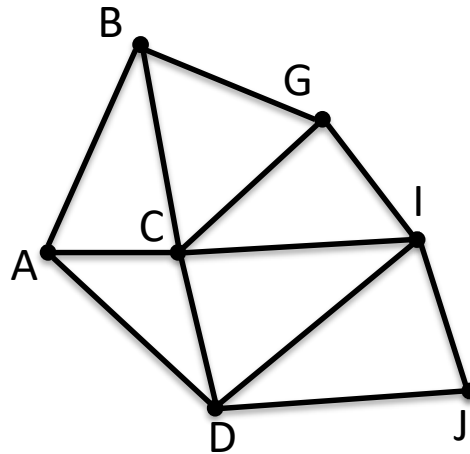
Closed:
 {B,A,C,G,D,I,J}

Final path solution: $B \rightarrow A \rightarrow D \rightarrow J$

Search tree comparison



BFS



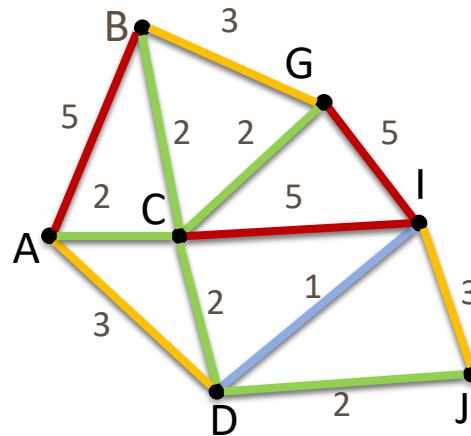
DFS

Depth-First Search

- Lower memory footprint than BFS with high-branching
- Not often used for path search, sometimes used to completely explore a graph
- Both BFS and DFS are simple to implement, but might be inefficient. More complex algorithms are faster, but generally more difficult to implement
- Seems like we want a compromise, search promising paths while we can, then go back up if they aren't working out
- DFS not complete for infinite trees (may explore an incorrect branch infinitely deep, never come back up, BFS *is* complete)

Costs on Actions

- What about non-uniform edge weights (costs)?



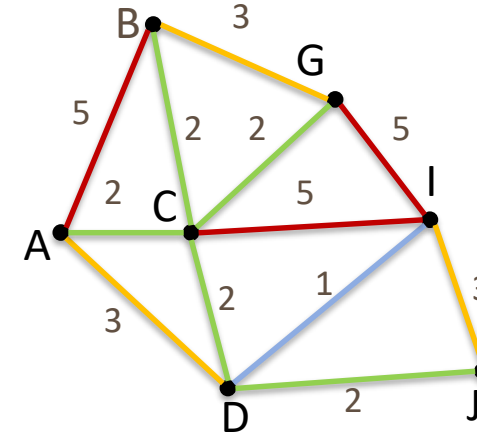
- Dijkstra's Algorithm and A* search

Dijkstra's Algorithm

- Published by Edsger Dijkstra in 1959
- Basic idea of expanding in order of closest to start (BFS with edge costs)
- One of the most commonly used routing algorithms in graph traversal problems
- Asymptotically the fastest known single-source shortest path algorithm for arbitrary directed graphs
- **Open** queue is ordered according to currently known best cost to arrive

Dijkstra's Algorithm Example

B(0)



FORWARD SEARCH

```

1  Q.Insert( $x_I$ ) and mark  $x_I$  as visited
2  while  $Q$  not empty do
3       $x \leftarrow Q.GetFirst()$ 
4      if  $x \in X_G$ 
5          return SUCCESS
6      forall  $u \in U(x)$ 
7           $x' \leftarrow f(x, u)$ 
8          if  $x'$  not visited
9              Mark  $x'$  as visited
10             Q.Insert( $x'$ )
11      else
12          Resolve duplicate  $x'$ 
13  return FAILURE
  
```

Open (Q):

{B(0)}

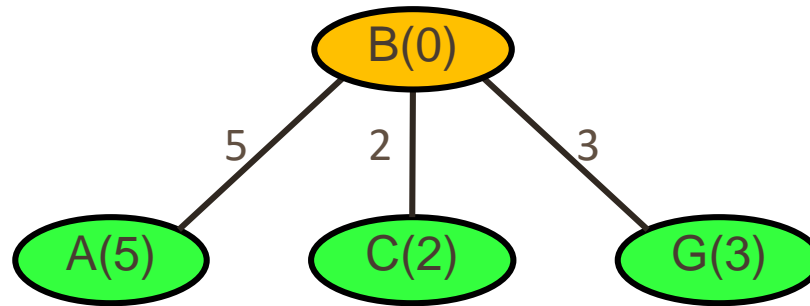
Closed:

{B(0)}

Our Dijkstra queue will be ordered by cost to arrive:

- push (*Q.Insert*) by cost
- pop (*Q.GetFirst*) from the front, and add it to the closed list

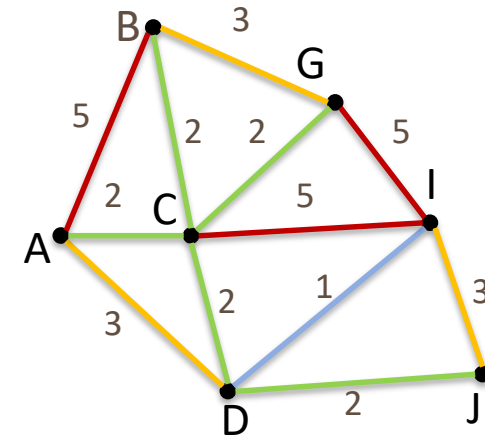
Dijkstra's Algorithm Example



FORWARD_SEARCH

```

1   $Q.Insert(x_I)$  and mark  $x_I$  as visited
2  while  $Q$  not empty do
3     $x \leftarrow Q.GetFirst()$ 
4    if  $x \in X_G$ 
5      return SUCCESS
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7       $x' \leftarrow f(x, u)$ 
8      if  $x'$  not visited
9        Mark  $x'$  as visited
10        $Q.Insert(x')$ 
11    else
12      Resolve duplicate  $x'$ 
13  return FAILURE
  
```



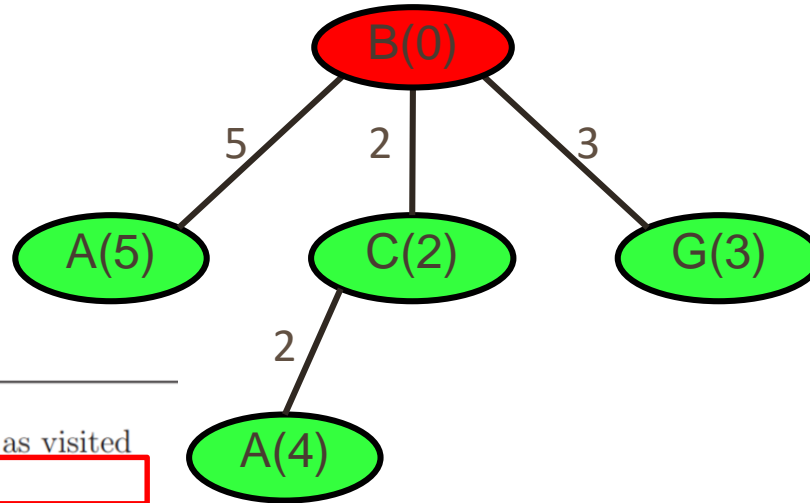
Open (Q):

{ C (2),
G (3),
A (5) }

Closed:

{ B (0) }

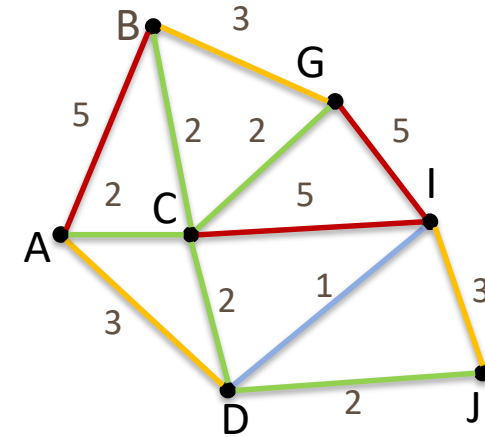
Dijkstra's Algorithm Example



FORWARD_SEARCH

```

1  Q.Insert( $x_I$ ) and mark  $x_I$  as visited
2  while Q not empty do
3       $x \leftarrow Q.GetFirst()$ 
4      if  $x \in X_G$ 
5          return SUCCESS
6      forall  $u \in U(x)$ 
7           $x' \leftarrow f(x, u)$ 
8          if  $x'$  not visited
9              Mark  $x'$  as visited
10             Q.Insert( $x'$ )
11          else
12              Resolve duplicate  $x'$ 
13  return FAILURE
  
```



Open (Q):

{ G (3),
A (4) }

Closed:

{ B (0),
C (2) }

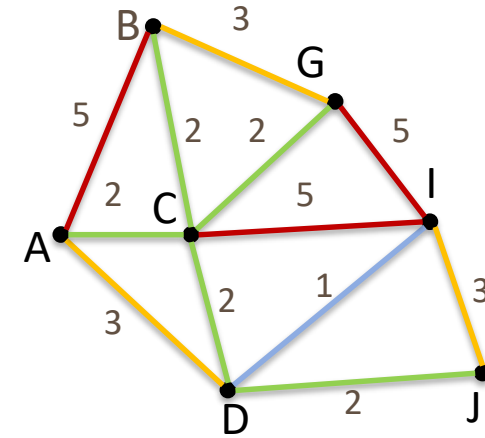
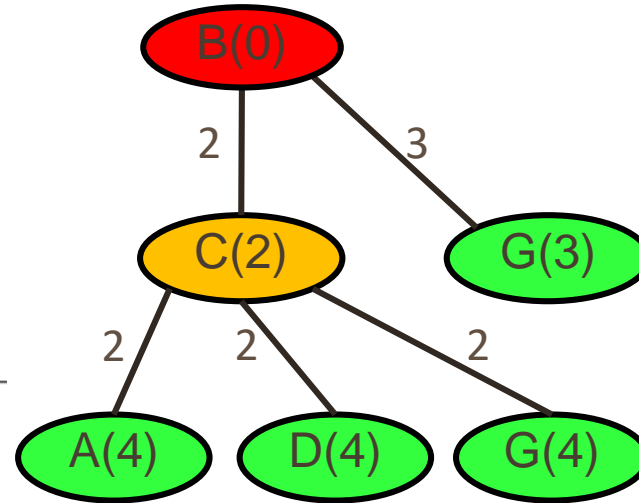
Dijkstra's Algorithm Example

FORWARD_SEARCH

```

1   $Q.Insert(x_I)$  and mark  $x_I$  as visited
2  while  $Q$  not empty do
3     $x \leftarrow Q.GetFirst()$ 
4    if  $x \in X_G$ 
5      return SUCCESS
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7       $x' \leftarrow f(x, u)$ 
8      if  $x'$  not visited
9        Mark  $x'$  as visited
10        $Q.Insert(x')$ 
11    else
12      Resolve duplicate  $x'$ 
13  return FAILURE

```



Open (Q):

{ G (3),
A (4),
D (4) }

Closed:

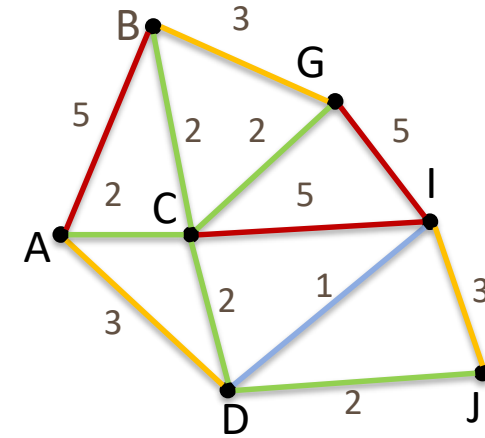
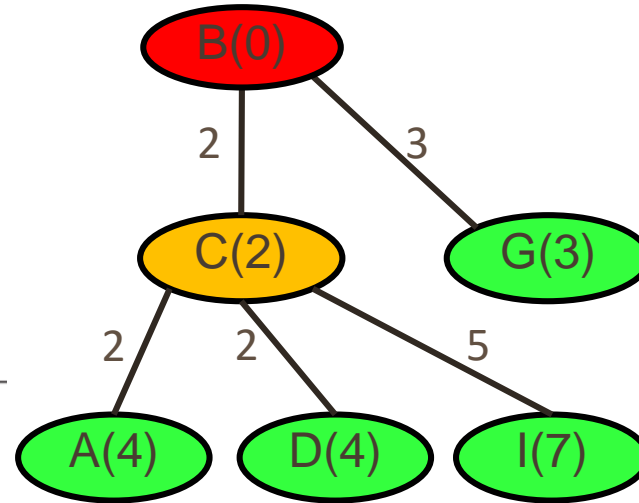
{ B (0),
C (2) }

Dijkstra's Algorithm Example

FORWARD_SEARCH

```

1  Q.Insert( $x_I$ ) and mark  $x_I$  as visited
2  while Q not empty do
3       $x \leftarrow Q.GetFirst()$ 
4      if  $x \in X_G$ 
5          return SUCCESS
6      forall  $u \in U(x)$ 
7           $x' \leftarrow f(x, u)$ 
8          if  $x'$  not visited
9              Mark  $x'$  as visited
10             Q.Insert( $x'$ )
11          else
12              Resolve duplicate  $x'$ 
13  return FAILURE
  
```



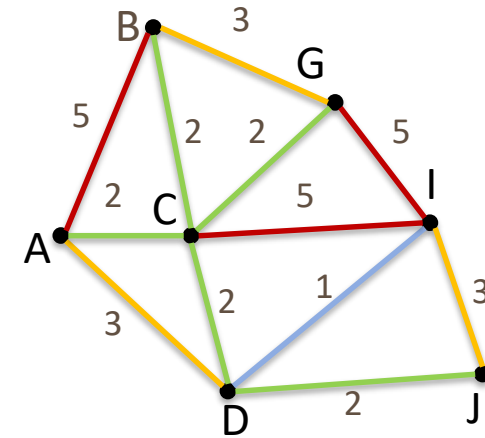
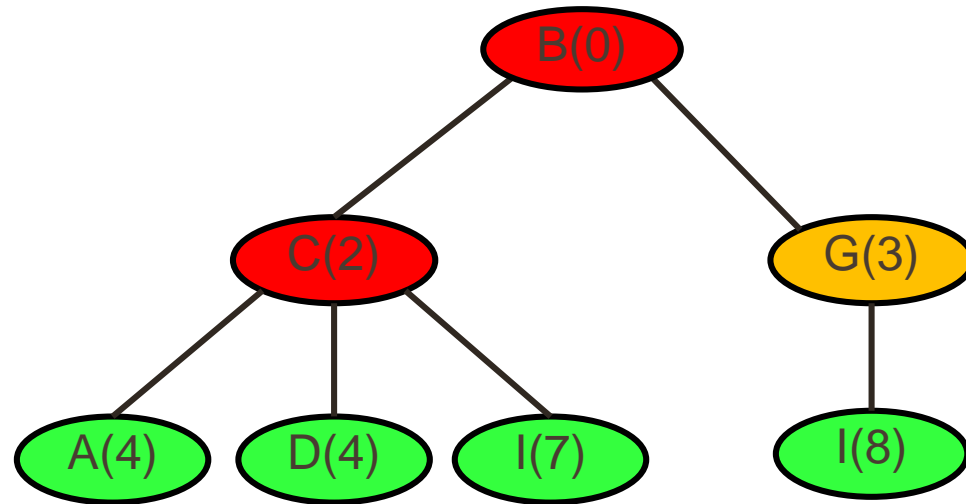
Open (Q):

{ G (3),
A (4),
D (4),
I (7) }

Closed:

{ B (0),
C (2) }

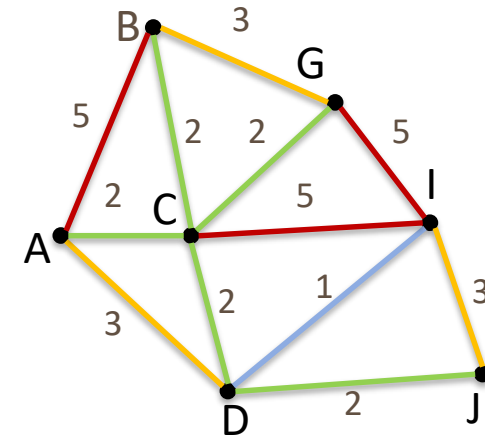
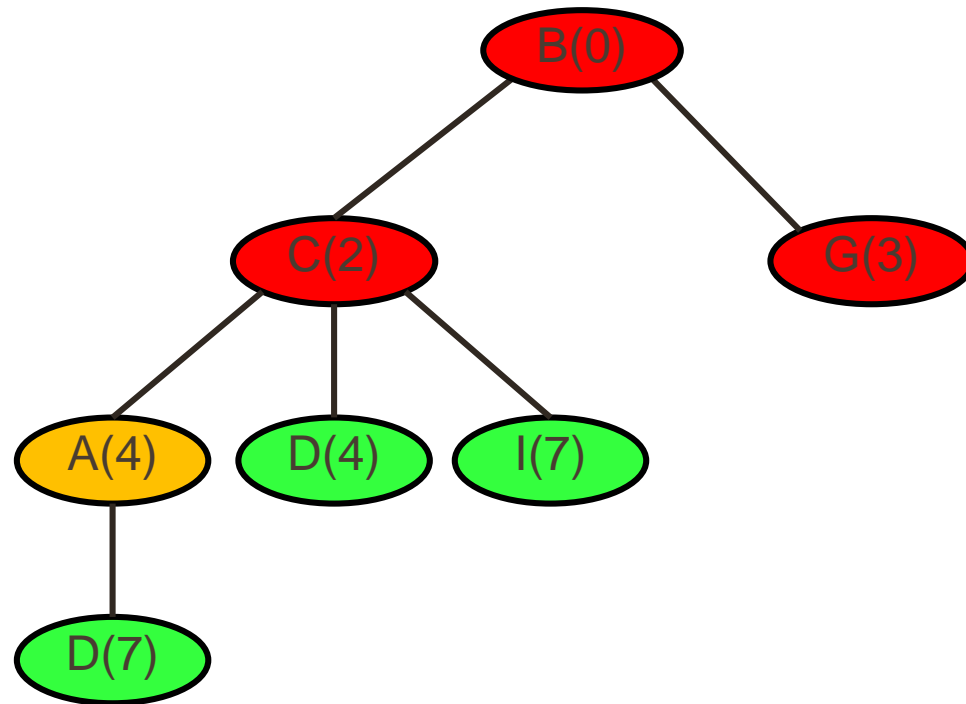
Dijkstra's Algorithm Example



Open (Q):
 { A (4),
 D (4),
 I (7) }

Closed:
 { B (0),
 C (2),
 G (3) }

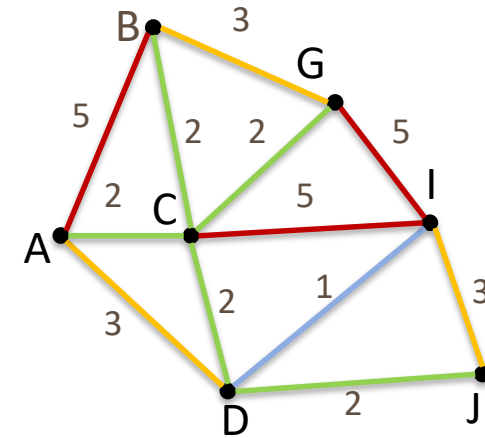
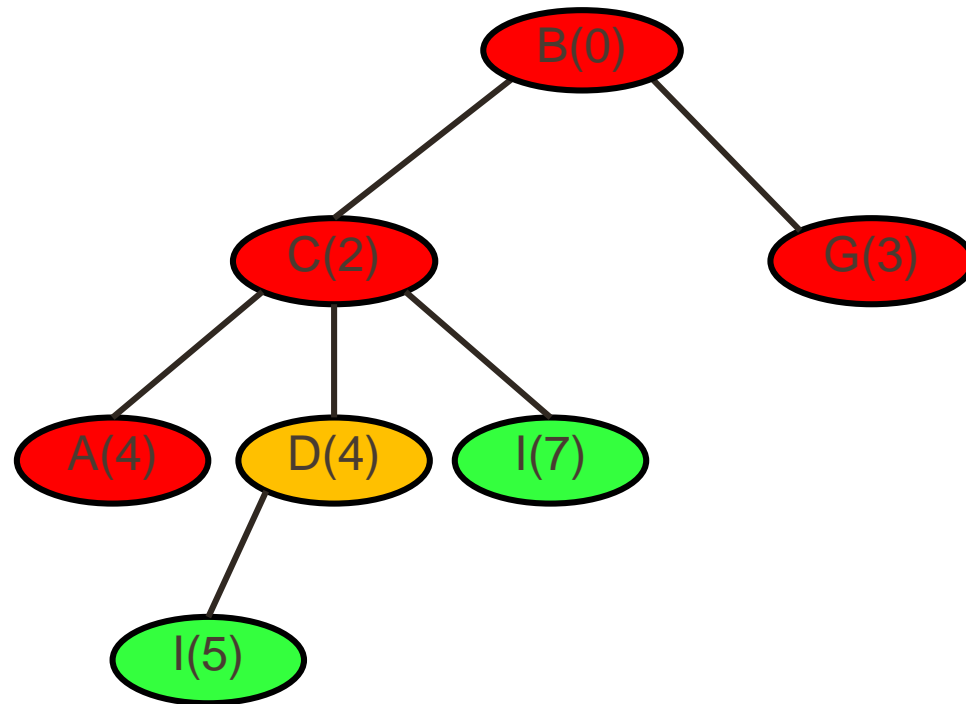
Dijkstra's Algorithm Example



Open (Q):
 { D (4),
 I (7) }

Closed:
 { B (0),
 C (2),
 G (3),
 A (4) }

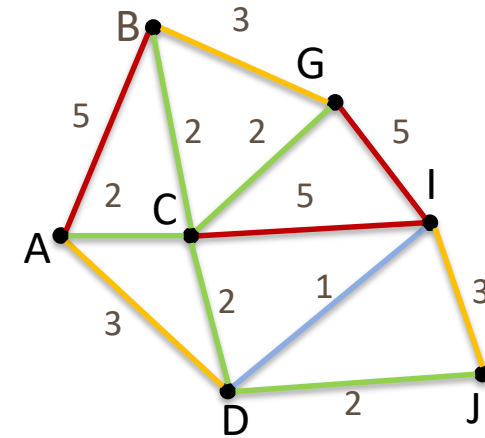
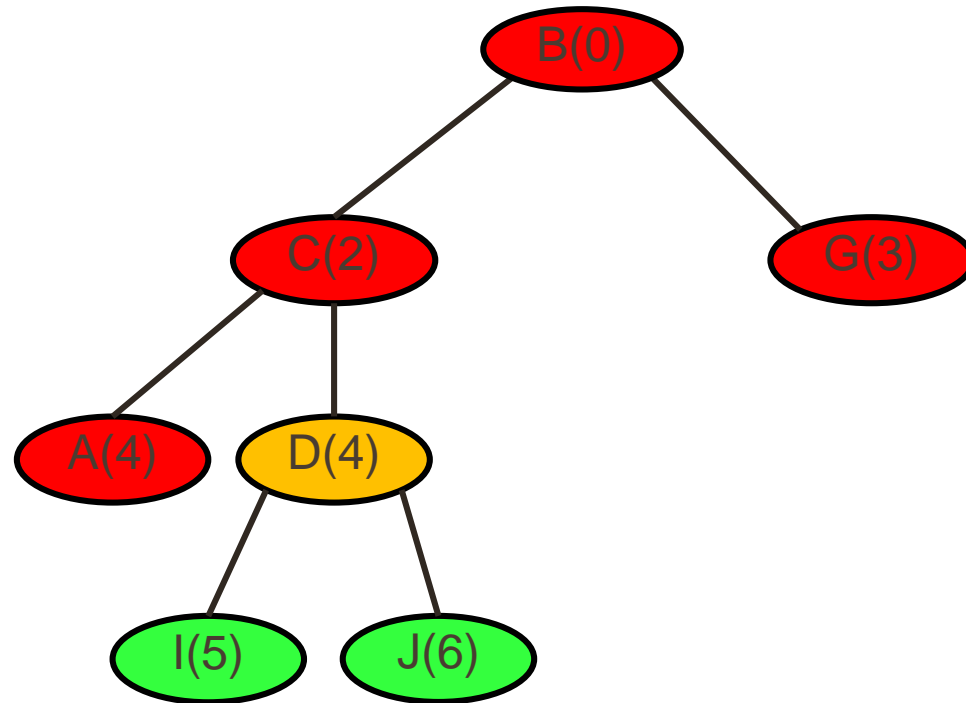
Dijkstra's Algorithm Example



Open (Q):
{ I (5) }

Closed:
{ B (0),
C (2),
G (3),
A (4),
D (4) }

Dijkstra's Algorithm Example



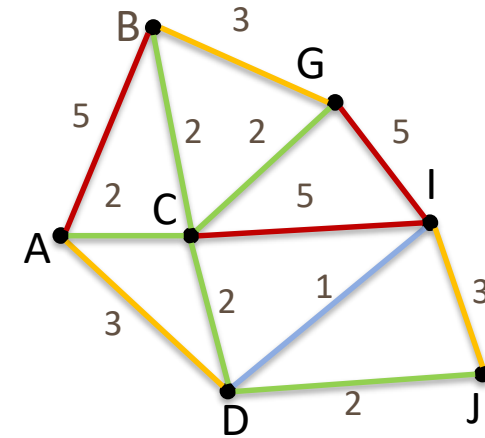
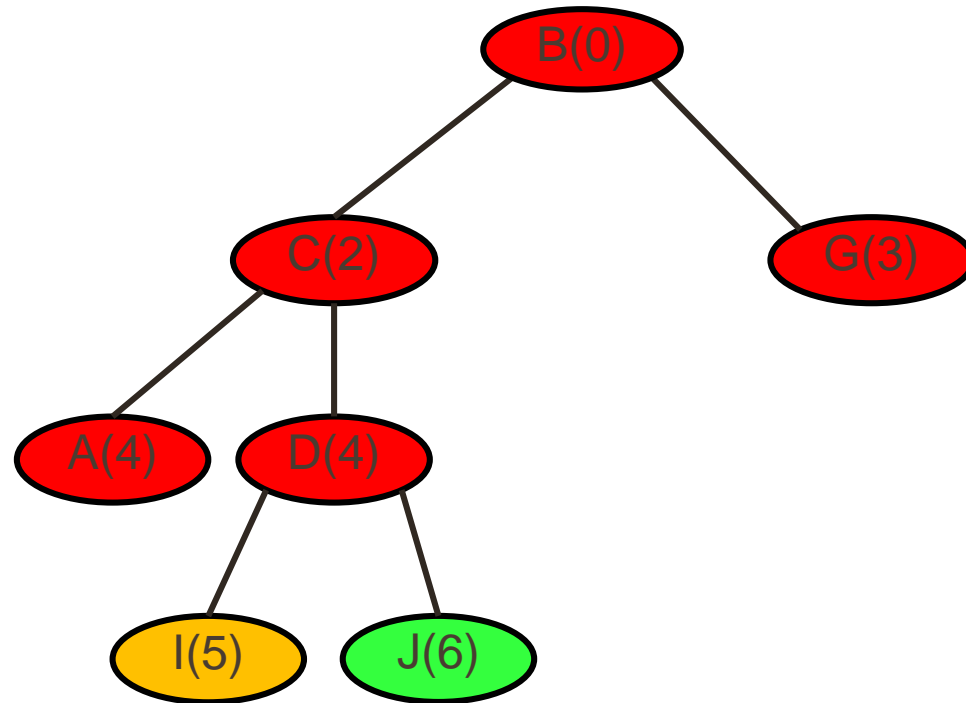
Open (Q):

{ I (5),
J (6) }

Closed:

{ B (0),
C (2),
G (3),
A (4),
D (4) }

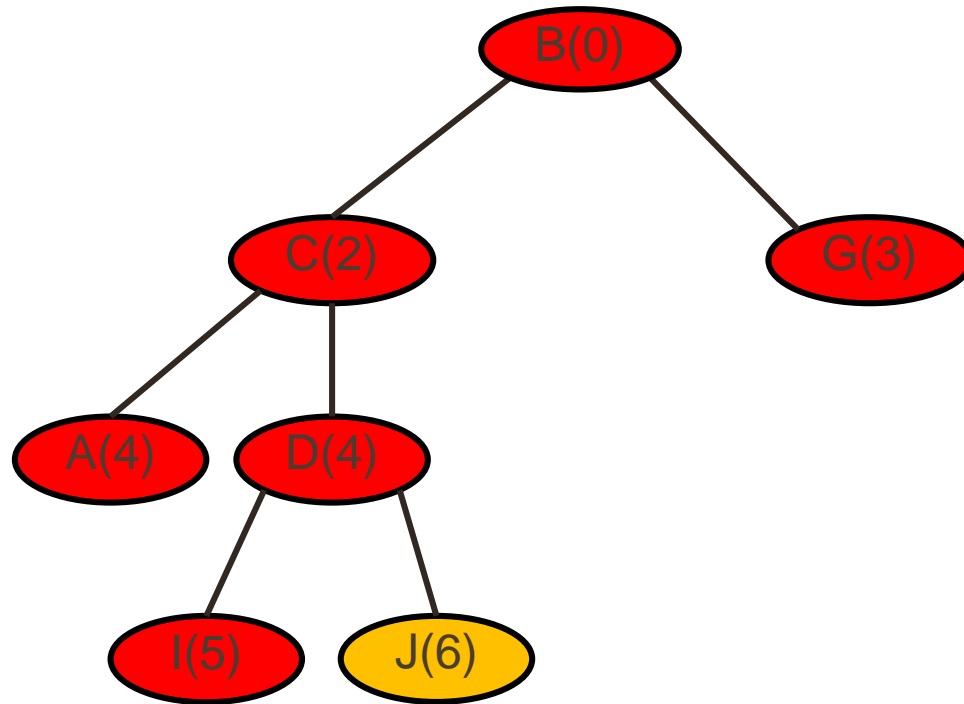
Dijkstra's Algorithm Example



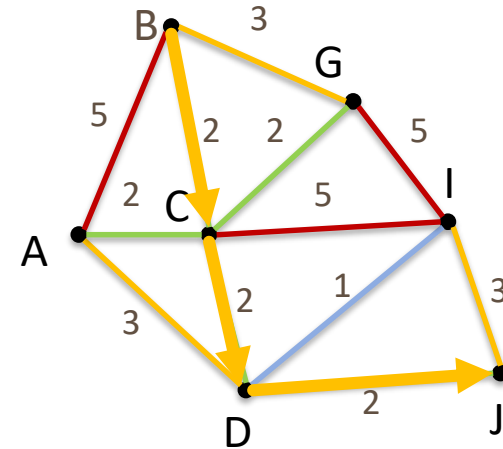
Open (Q):
 { J (6) }

Closed:
 { B (0),
 C (2),
 G (3),
 A (4),
 D (4),
 I (5) }

Dijkstra's Algorithm Example



Final path solution: $B \rightarrow C \rightarrow D \rightarrow J$
with path cost 6



Open (Q):
{ }

Closed:
{ B (0),
C (2),
G (3),
A (4),
D (4),
I (5),
J (6) }

Dijkstra's Algorithm

- At the end, we can recover the lowest-cost route from the start to any node (or any node with cost $<$ goal if we terminate at a goal)
- Quite easy to implement, but requires a little bit of careful management with the priority queue
- Doesn't really know the goal exists until it reaches it
 - Could we guide the search to expand nodes that are closer to the goal earlier?
 - Can we do it without breaking the condition that a node is only accepted with its lowest cost of arrival?

A* Heuristic Search

- Heuristic:
 - Any *optimistic estimate* of how close a state is to a goal
 - Designed for a particular search problem
 - Examples: Manhattan distance, Euclidean distance

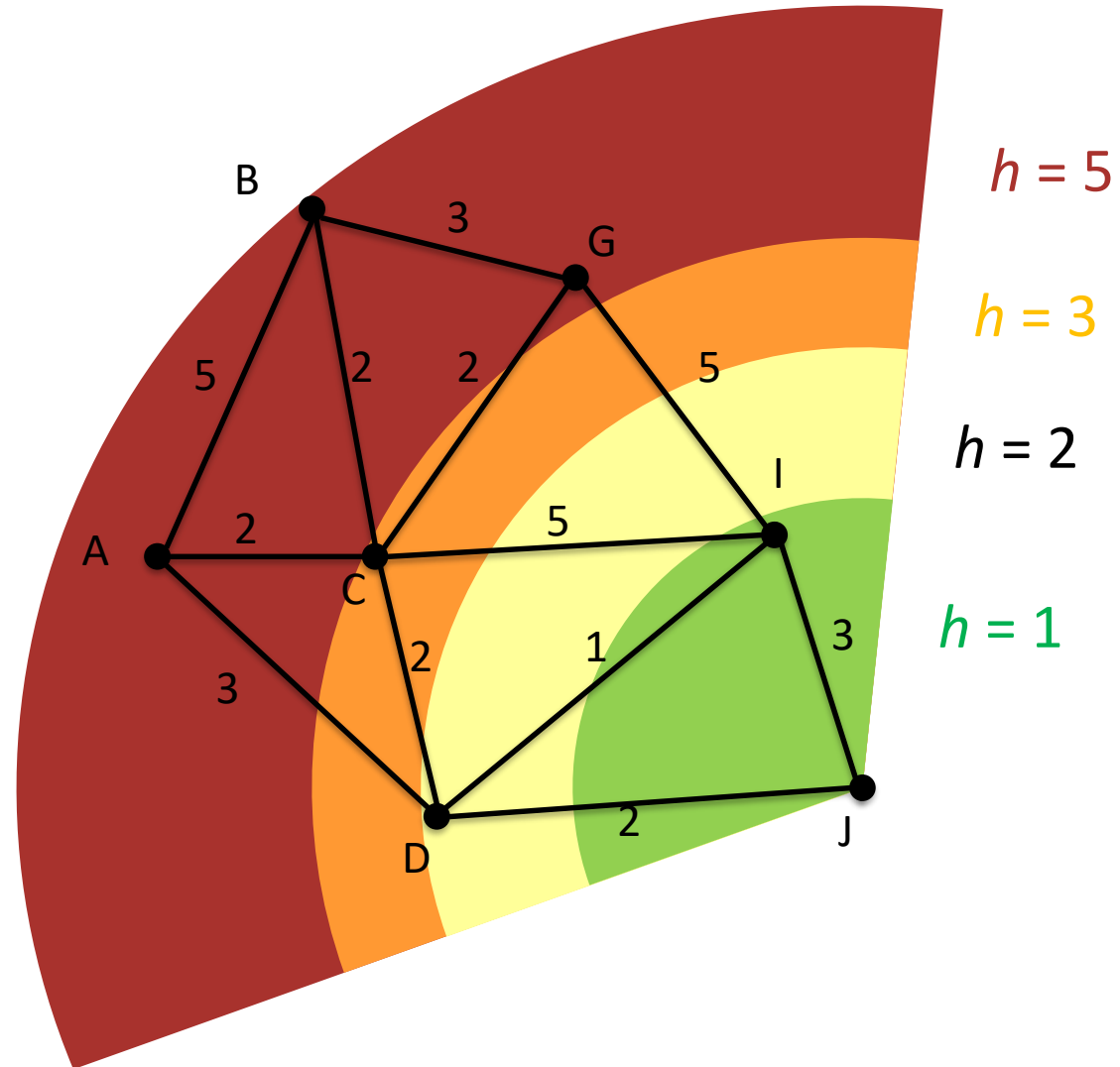
- A* Priority:

$$f(n) = g(n) + h(n)$$

Cost to arrive

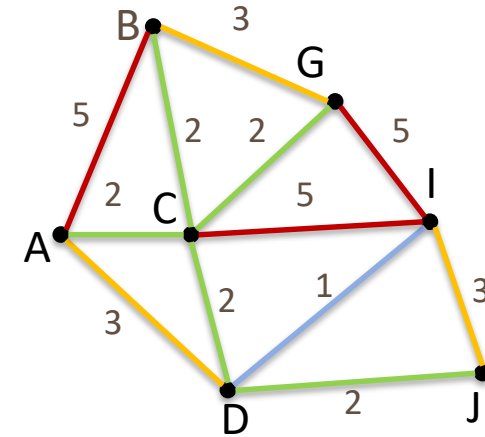
Heuristic cost to goal

A* Heuristic



A* Algorithm Example

B(0)



Open (Q):

{B(0)}

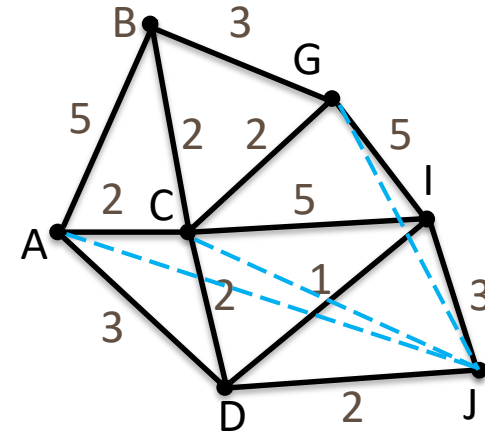
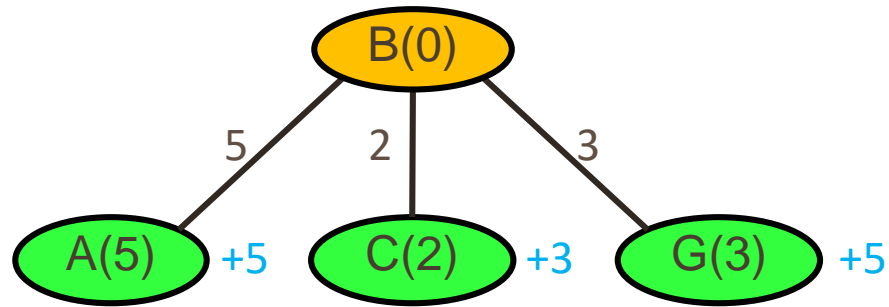
Closed:

{B(0)}

Our A* queue will be ordered by cost to arrive + heuristic:

- push ($Q.Insert$) by A* priority, $f(n)$
- pop ($Q.GetFirst$) from the front, and add it to the closed list

A* Algorithm Example



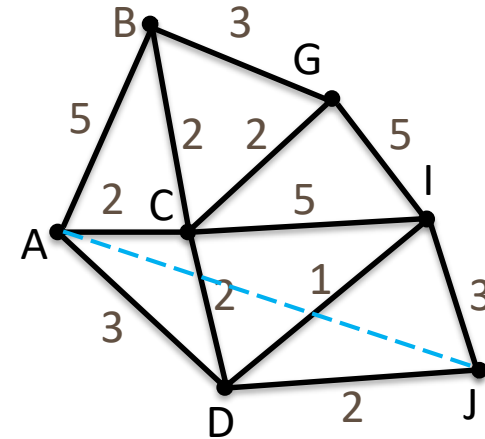
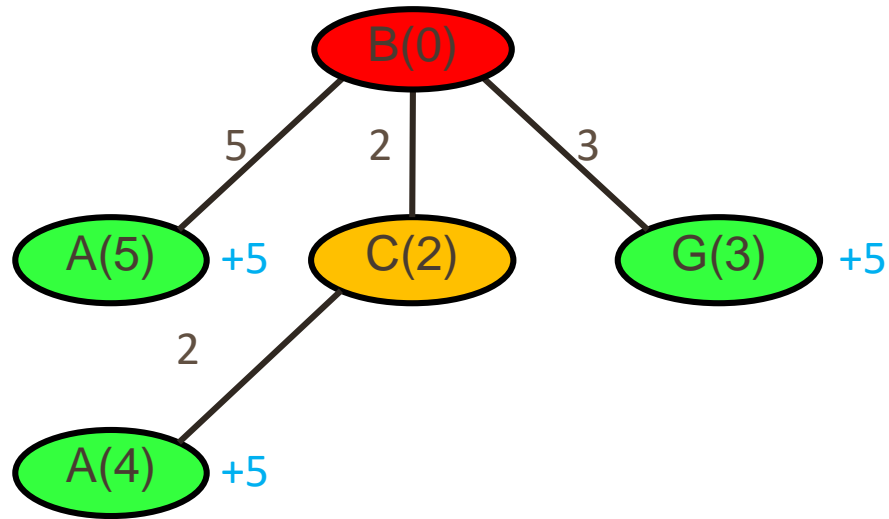
Open (Q):

{ C (2+3),
G (3+5),
A (5+5) }

Closed:

{ B (0) }

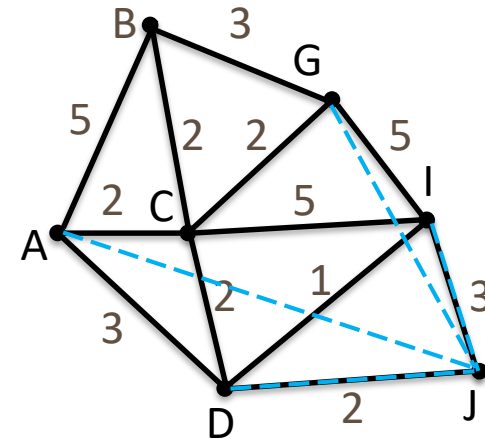
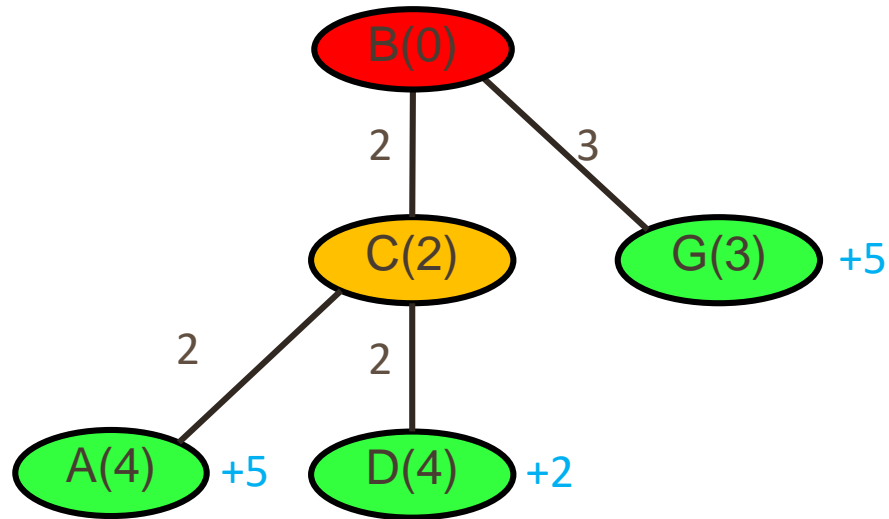
A* Algorithm Example



Open (Q):
 { G (3+5),
 A (4+5) }

Closed:
 { B (0),
 C (2) }

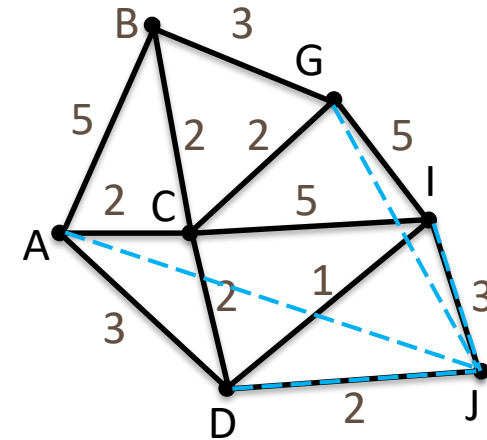
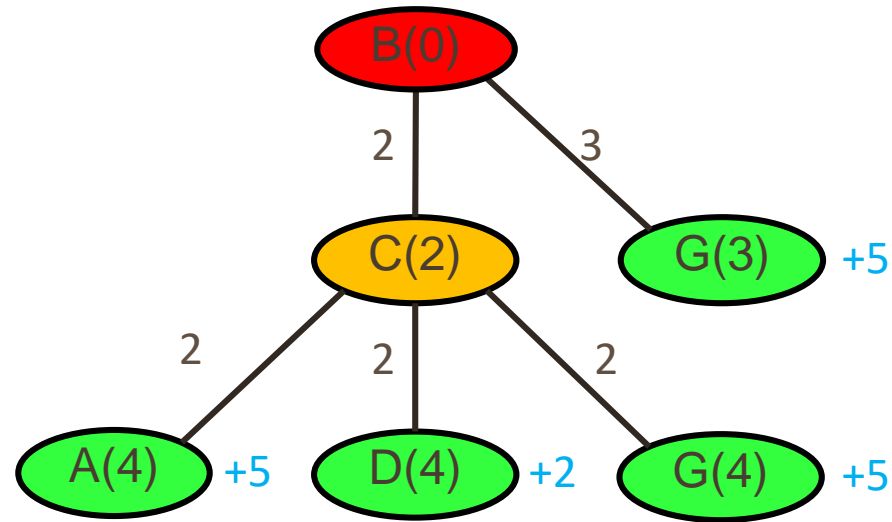
A* Algorithm Example



Open (Q):
 { D (4+2),
 G (3+5),
 A (4+5) }

Closed:
 { B (0),
 C (2) }

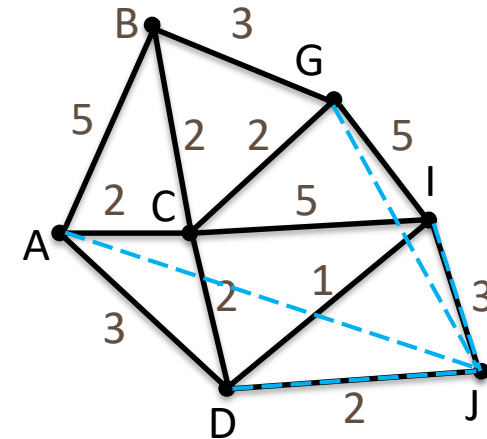
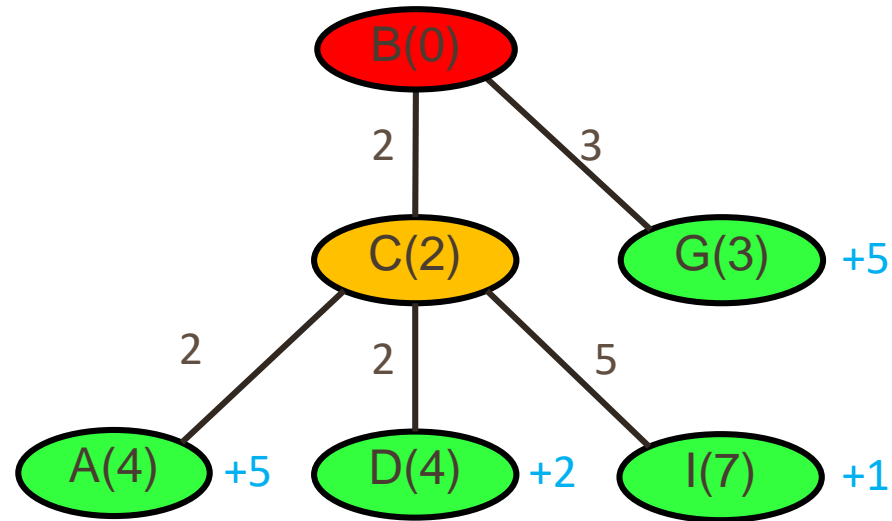
A* Algorithm Example



Open (Q):
 { D (4+2),
 G (3+5),
 A (4+5) }

Closed:
 { B (0),
 C (2) }

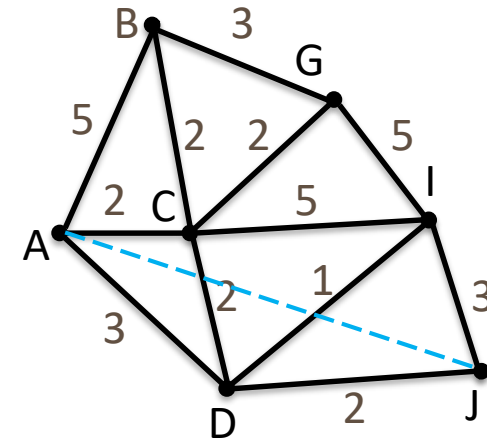
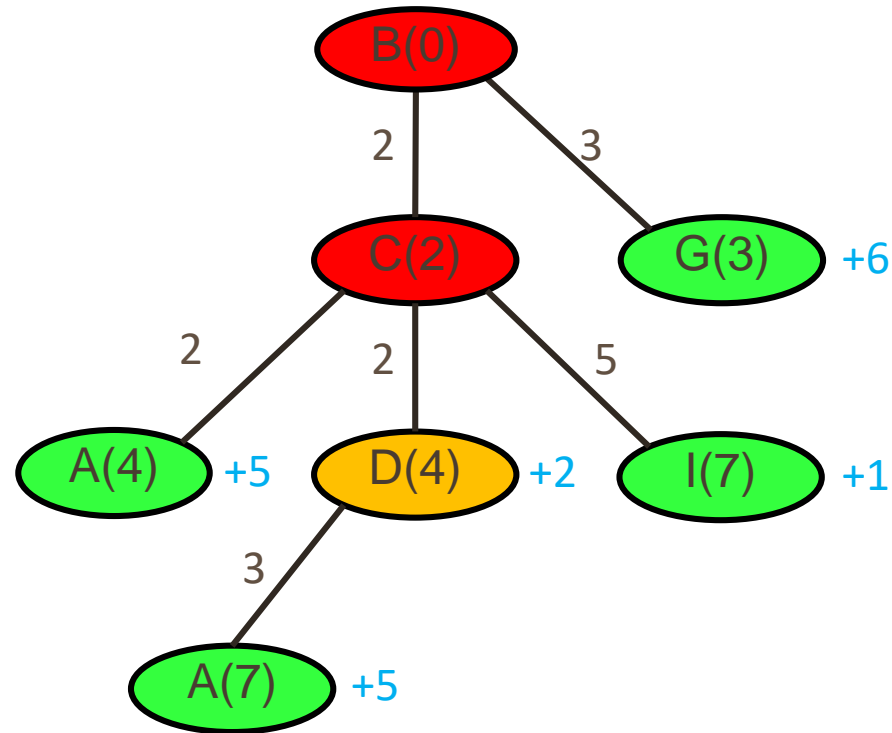
A* Algorithm Example



Open (Q):
 { D (4+2),
 I (7+1),
 G (3+5),
 A (4+5) }

Closed:
 { B (0),
 C (2) }

A* Algorithm Example



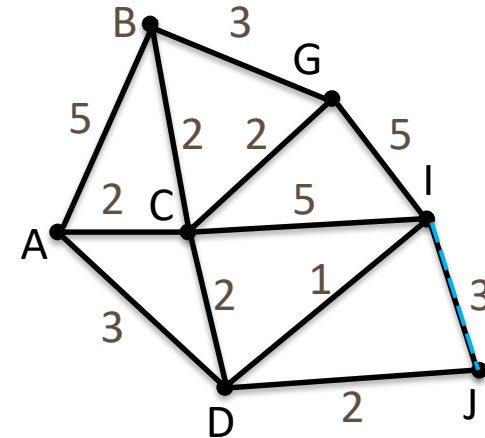
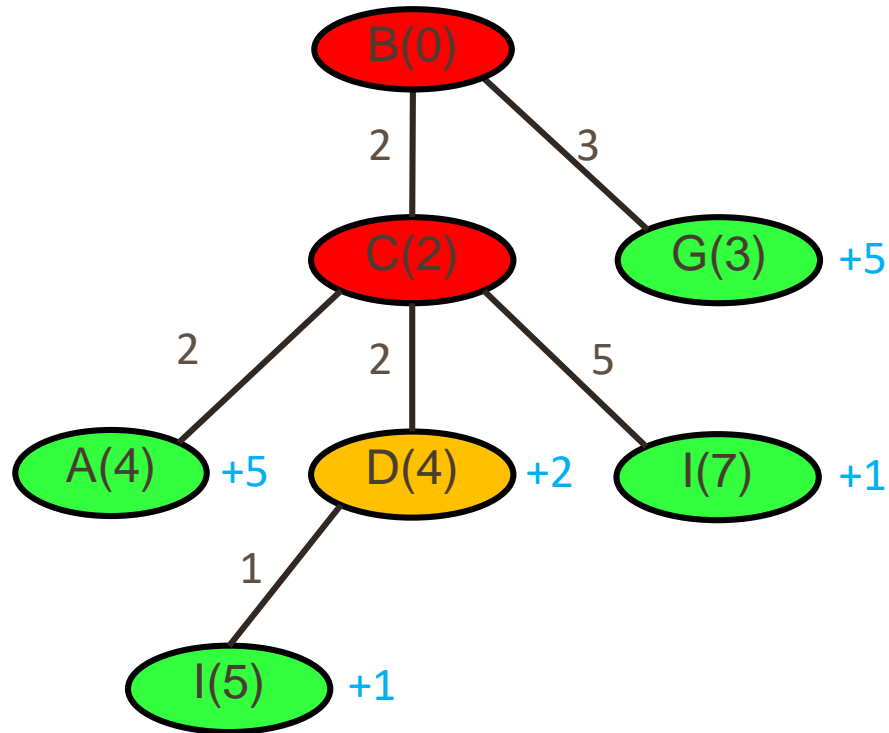
Open (Q):

{ I (5+1),
G (3+5),
A (4+5) }

Closed:

{ B (0),
C (2),
D (4) }

A* Algorithm Example



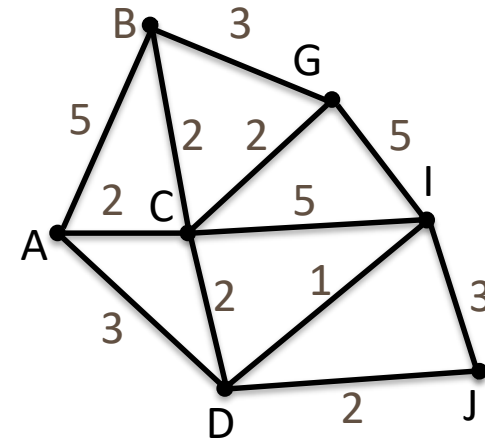
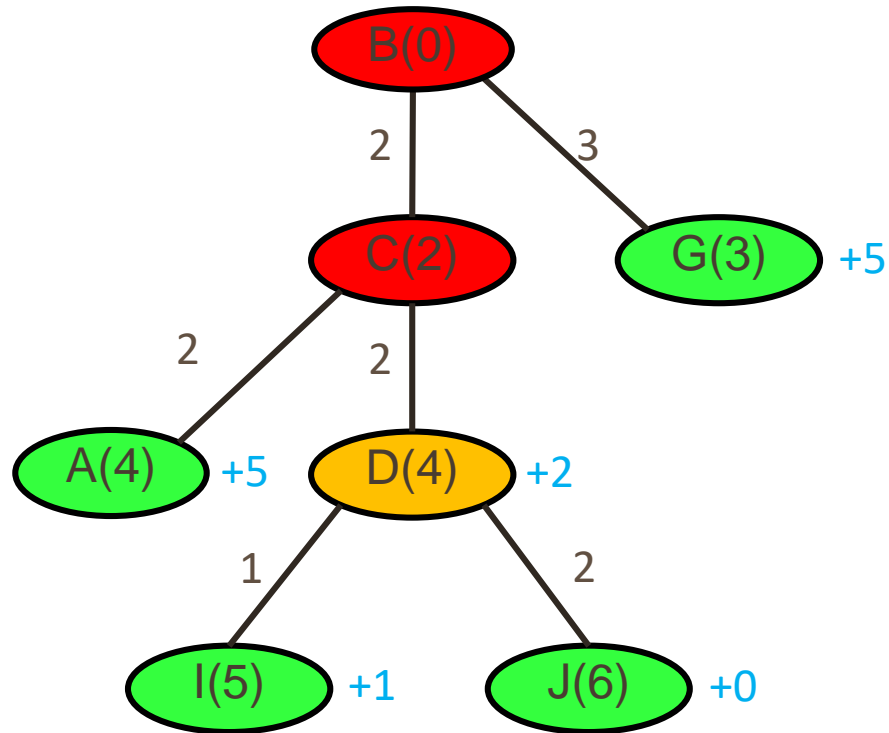
Open (Q):

{ I (5+1),
G (3+5),
A (4+5) }

Closed:

{ B (0),
C (2),
D (4) }

A* Algorithm Example



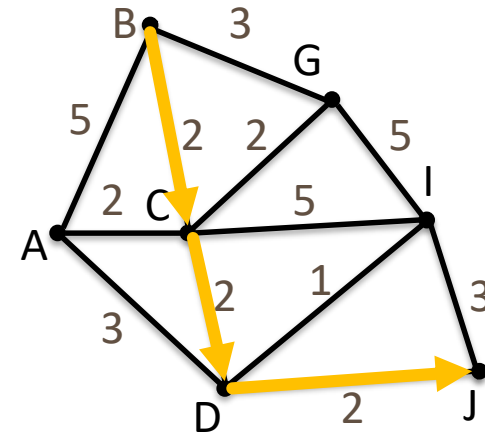
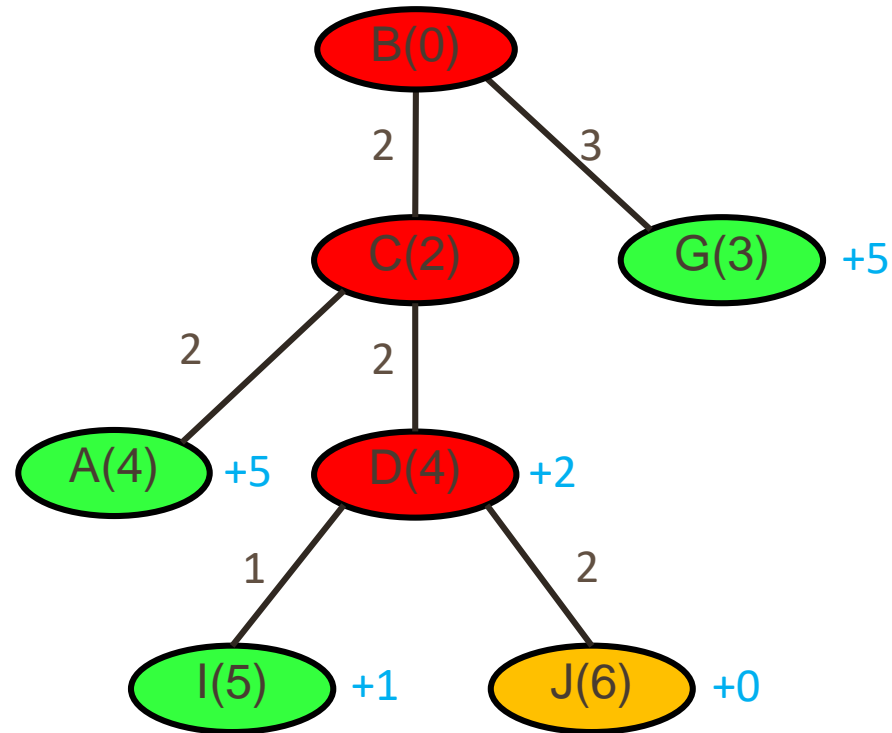
Open (Q):

{ J (6+0),
I (5+1),
G (3+5),
A (4+5) }

Closed:

{ B (0),
C (2),
D (4) }

A* Algorithm Example



Open (Q):

{ I (5+1),
G (3+5),
A (4+5) }

Closed:

{ B (0),
C (2),
D (4),
J (6) }

Final path solution: $B \rightarrow C \rightarrow D \rightarrow J$
with path cost 6

A* Heuristic

- The heuristic must be **admissible**

- It never overestimates the cost

$$h(x) \leq d(x, goal)$$



True cost to goal

- The heuristic must be **consistent**

- For any pair of adjacent nodes x and y, where $d(x,y)$ is the cost of edge between them

$$h(x) \leq d(x, y) + h(y)$$

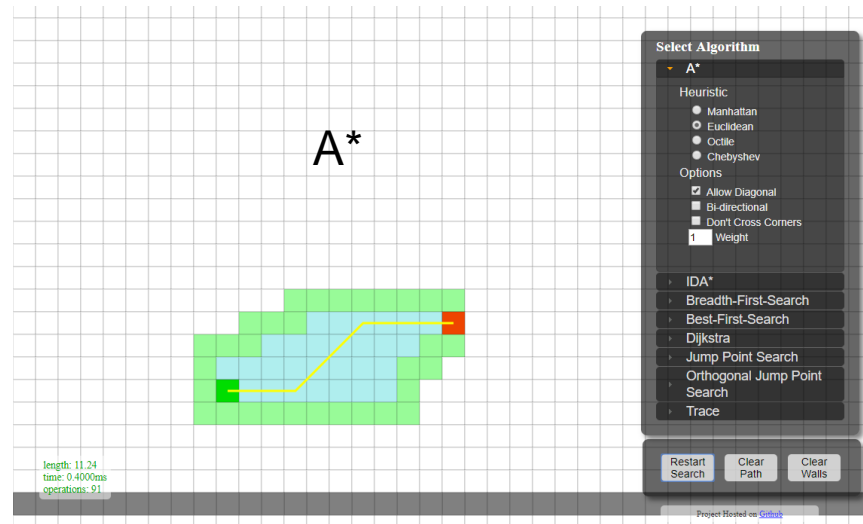
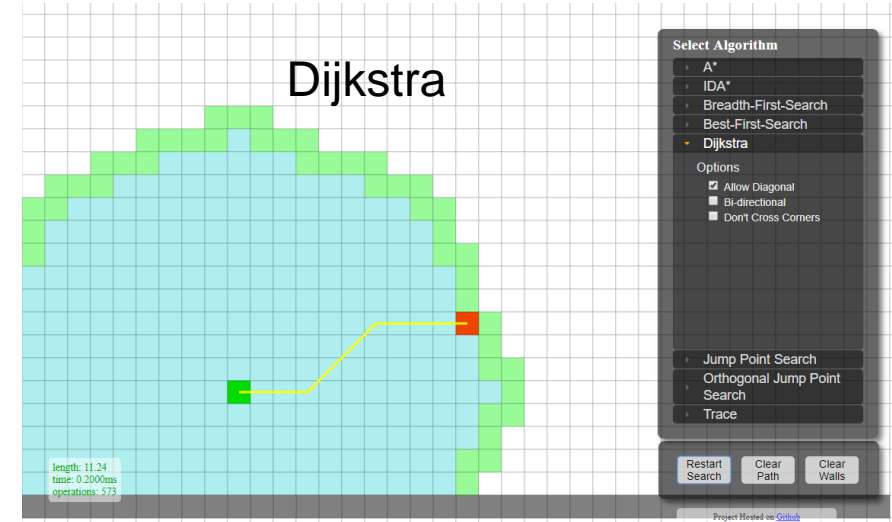
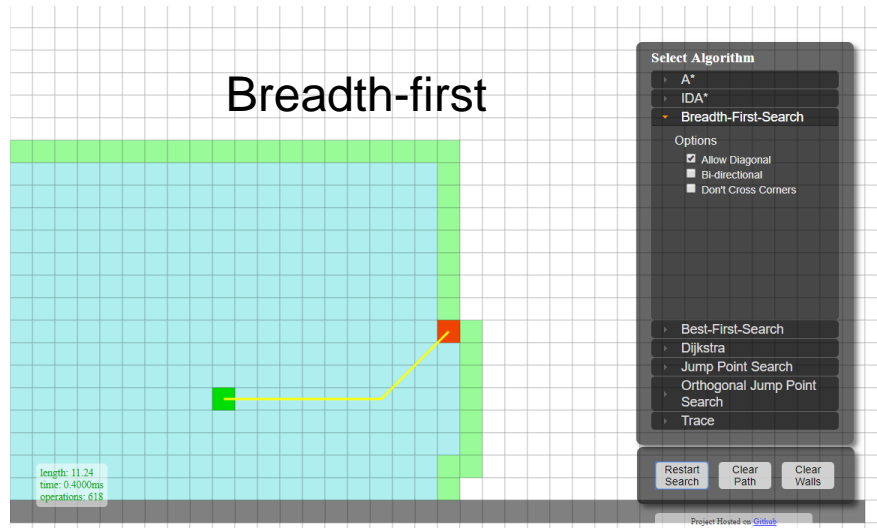
- Typical valid heuristics:

- Euclidean distance
- Manhattan distance
- Zero (Dijkstra's algorithm)

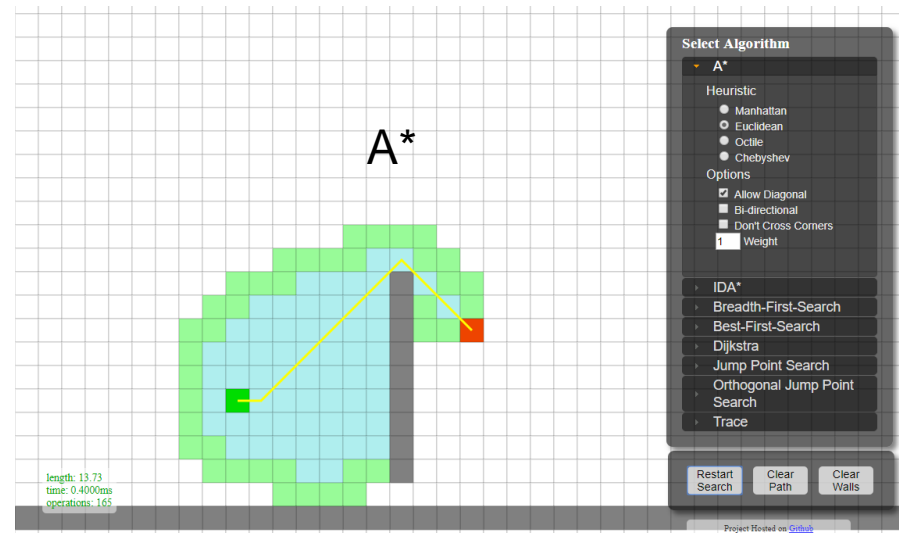
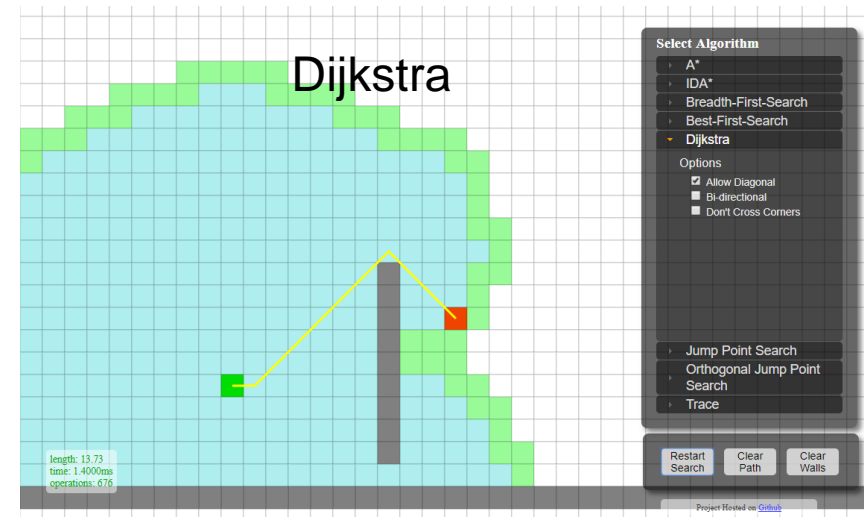
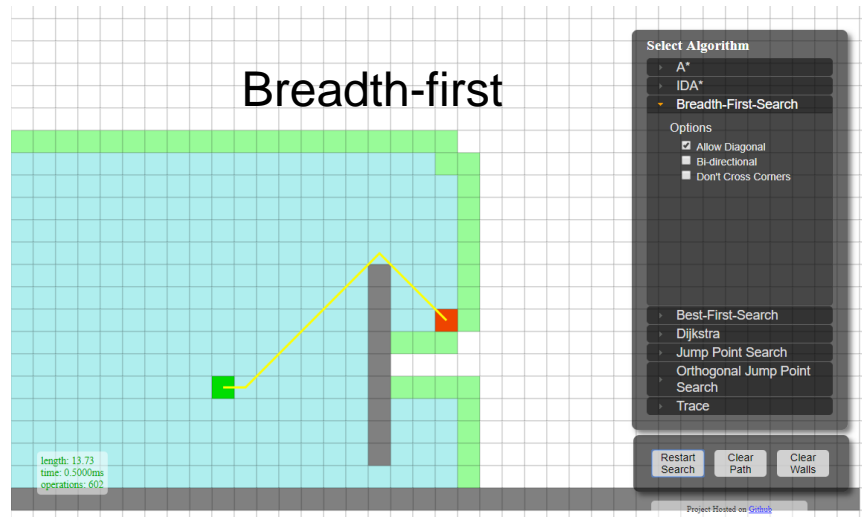
A* Search Algorithm

- A* is an extension of Dijkstra's algorithm, and achieves faster performance by using heuristics
- **Best-first search:** A* traverses a graph following a path of lowest expected total cost or distance
- The cost function is a sum of two functions:
 - Past path-cost function, which is a **known** cost from the starting node to the current node
 - Future path-cost function, which is a “heuristic estimate” of the distance from the current node to the goal

<https://qiao.github.io/PathFinding.js/visual/>



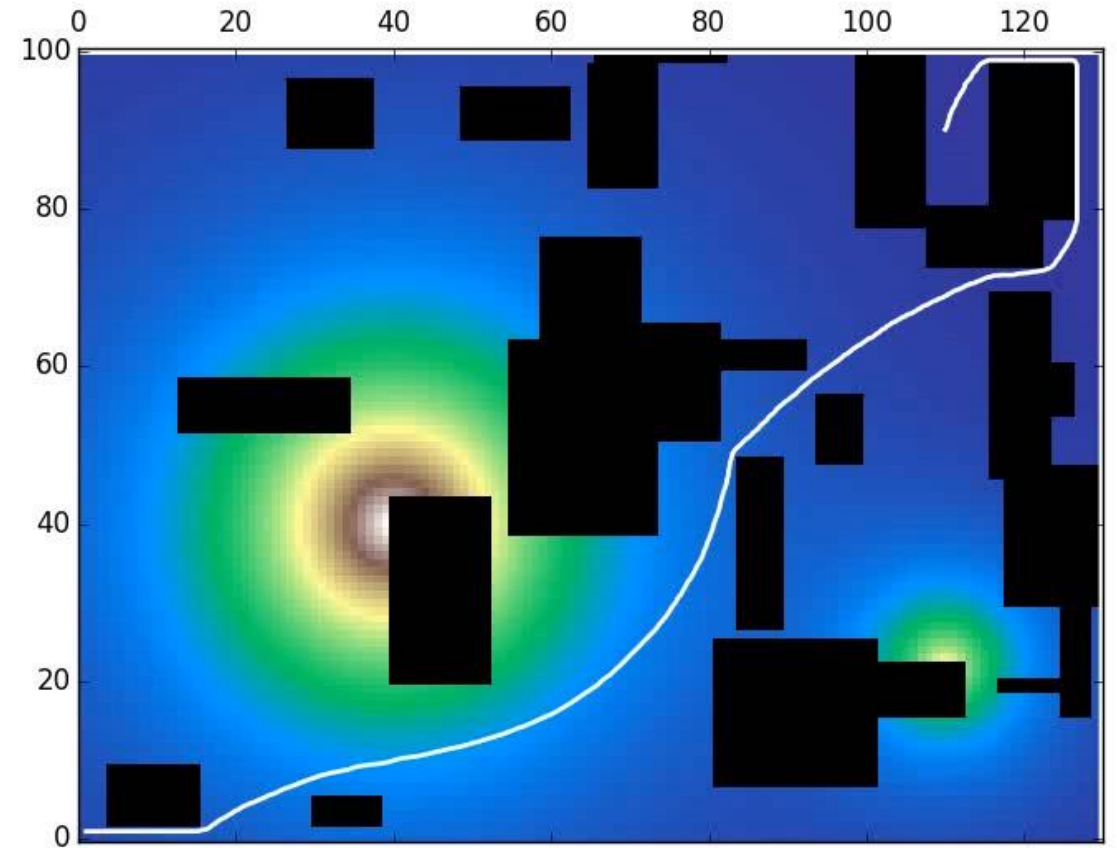
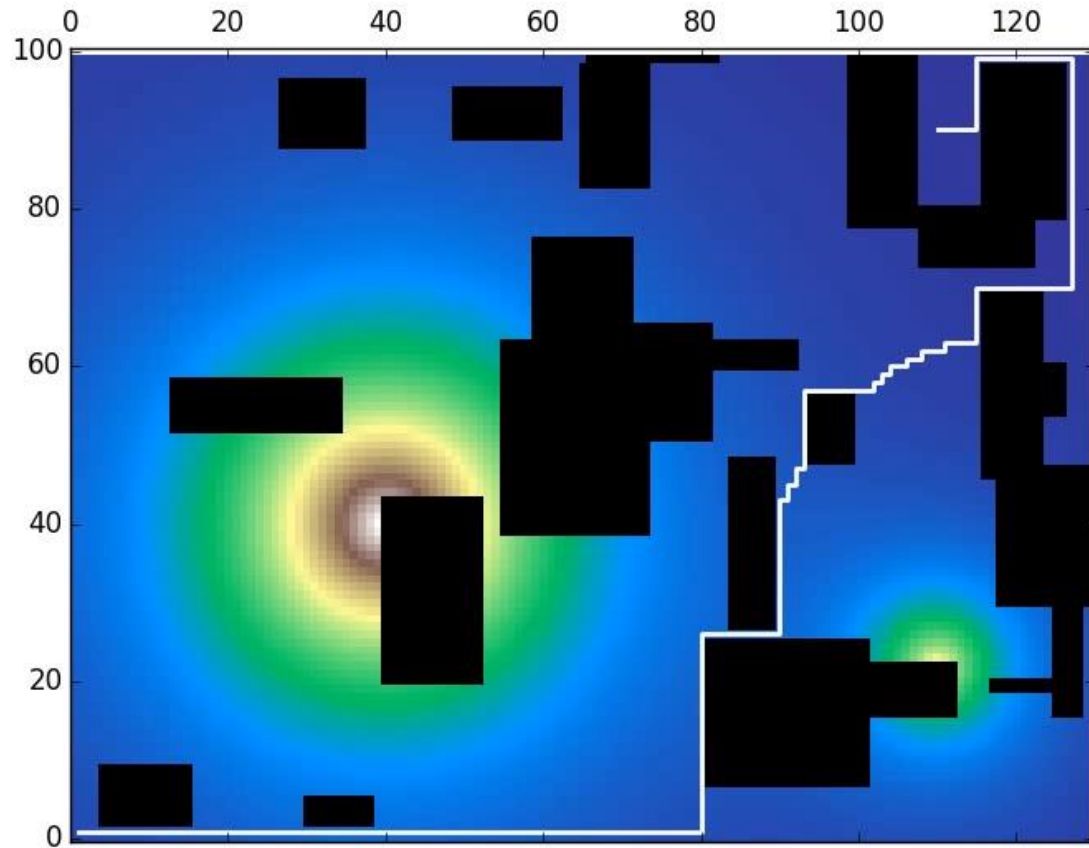
<https://qiao.github.io/PathFinding.js/visual/>



Limitations

- A* is very commonly used in robot planning, especially for low-dimensional state spaces
- Limitations:
 - You need to construct a graph
 - Sometimes an admissible heuristic function is difficult to find (as hard as the problem)
 - A grid may not be a good representation of your problem

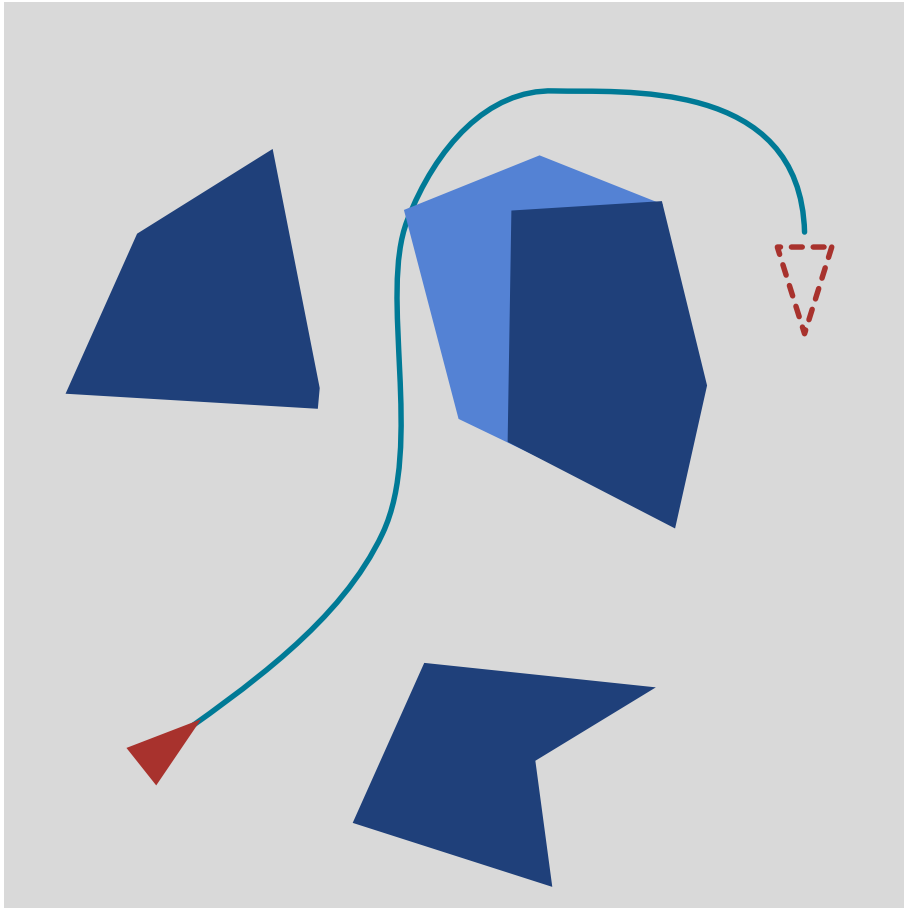
Fast Marching



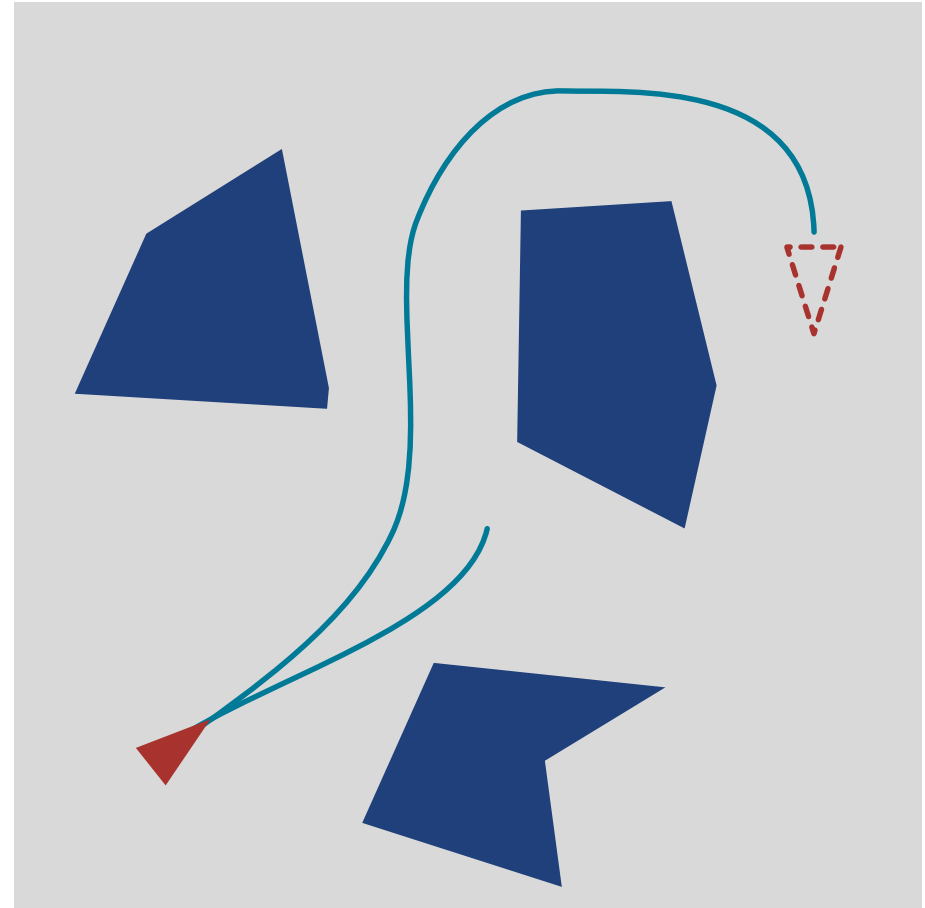
What happens when it doesn't all go to plan?

- So far, we've basically ignored the plan **execution**
- Our plans are a series of states that we assume the robot is capable of visiting sequentially and reliably
- However, there are many reasons that, in practice, this may not always be the case

- Environment uncertainty



- Motion uncertainty



Collision Avoidance

- A simple solution is to try to move back onto the planned trajectory (global plan), while avoiding collisions (local planning)
- This errs towards a control problem, but the distinction blurs since you often have to consider both problems simultaneously
- A conceptually simple and relatively common approach for this type of problem are **potential field** methods
- Basically create a function that pushes the robot away from obstacles, and towards the goal

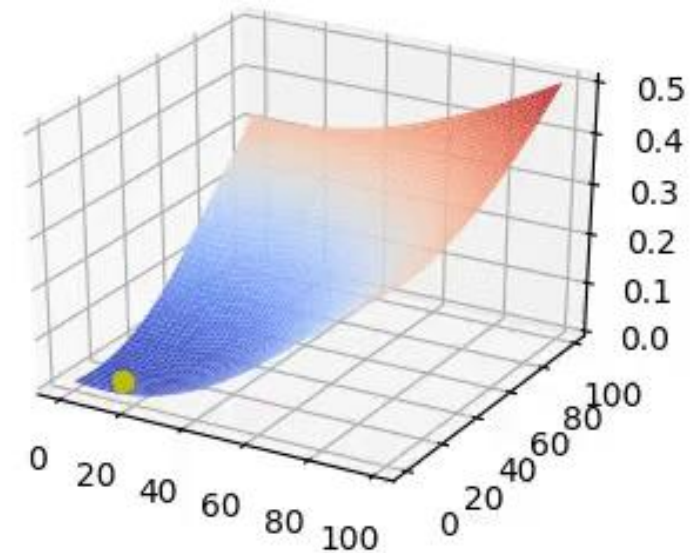
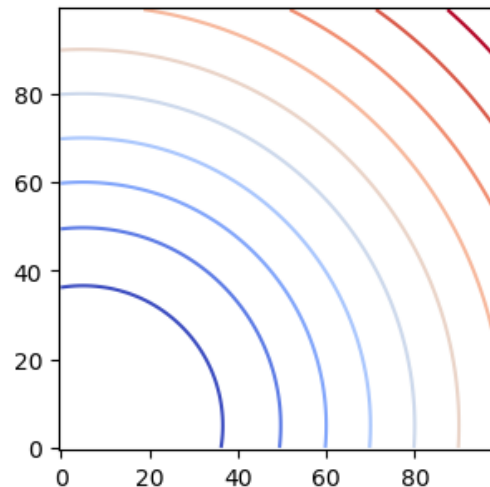
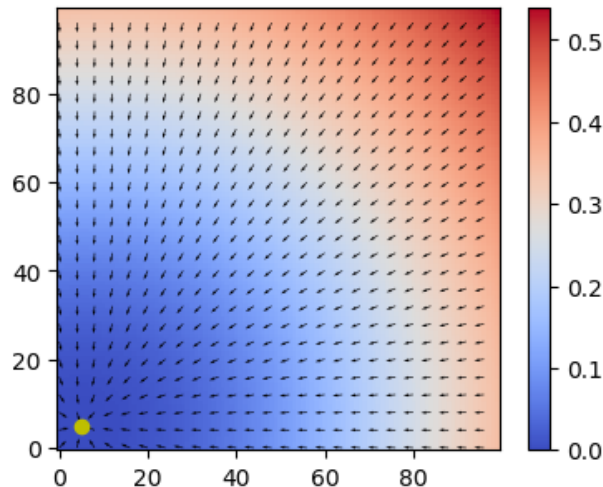
Potential Field Methods – Global potential

- Imagine we had a function that related an **elevation** with some kind of distance to the goal (a ‘potential’ function, as in potential energy)
- By taking control actions that direct the robot in the direction of **maximum gradient (down)**, the robot should ‘fall’ towards the goal (minimum energy state)
- The global potential function should ‘attract’ the robot towards the goal, from any valid state

Potential Field Methods – Global potential

- We want a smooth, differentiable function so that it is easy to calculate the target vector

$$U_{goal}(x) = \begin{cases} \frac{1}{2} \zeta \|x - x_{goal}\|^2, & \|x - x_{goal}\| < d^* \\ d^* \zeta \left(\|x - x_{goal}\|^2 - \frac{1}{2} d^{*2} \right), & \text{otherwise} \end{cases}$$

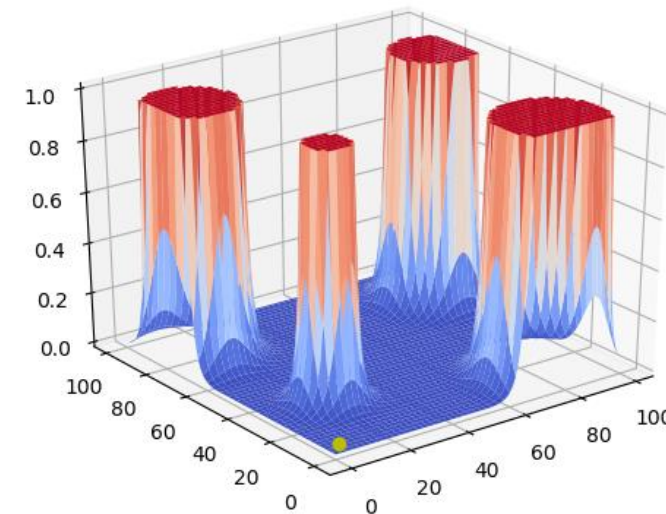
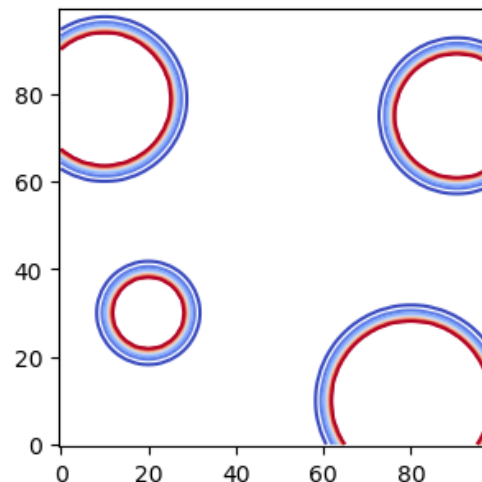
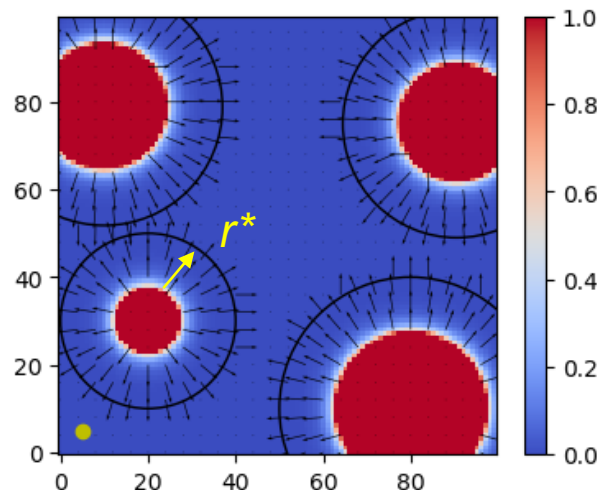


Potential Field Methods – Obstacles

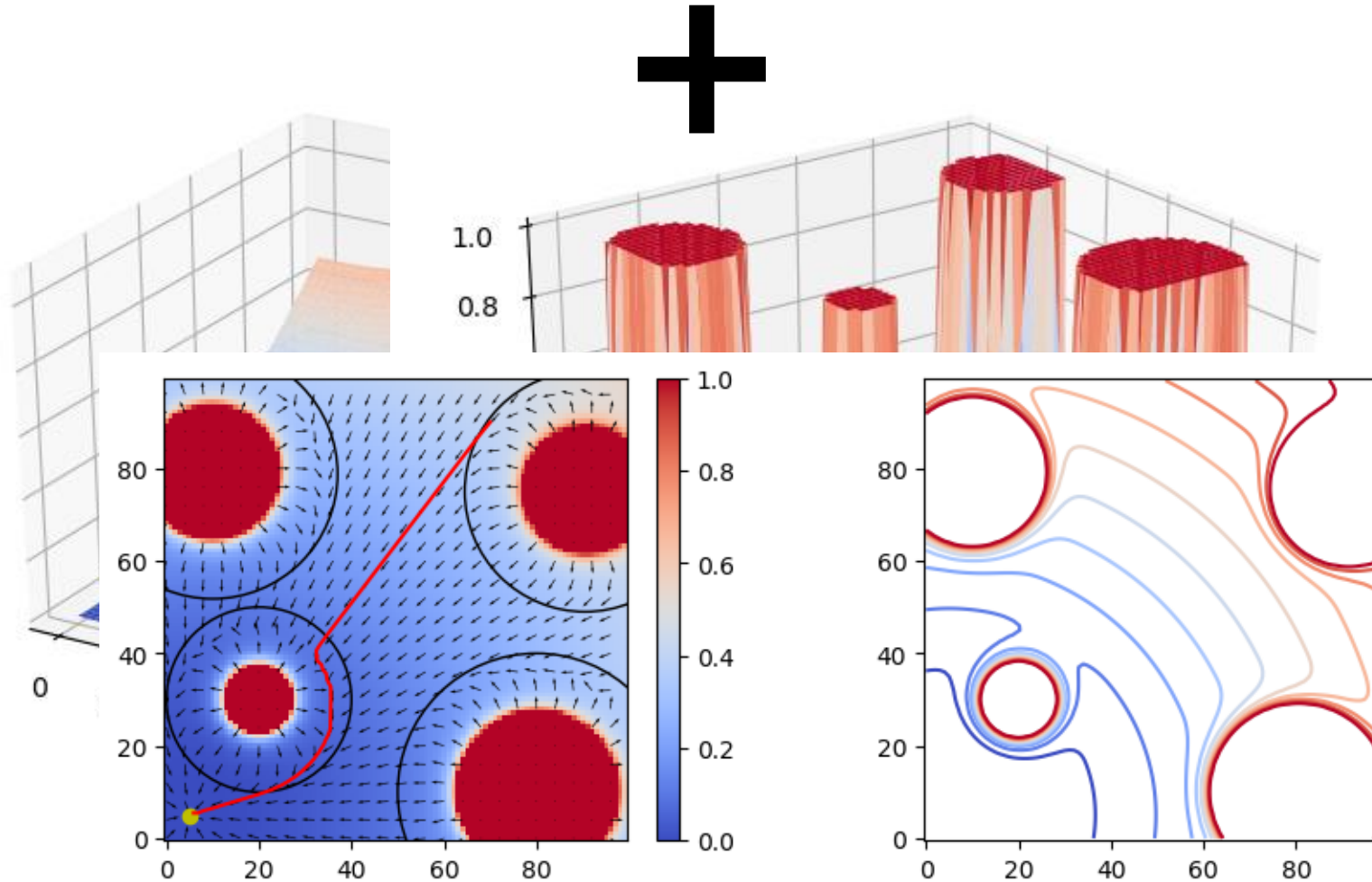
- We also want to avoid obstacles, so we add an additional component that 'repels' from obstacles

$$U_{obs}(x) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{D(x)} - \frac{1}{r^*} \right)^2, & D(x) \leq r^* \\ 0, & \text{otherwise} \end{cases}$$

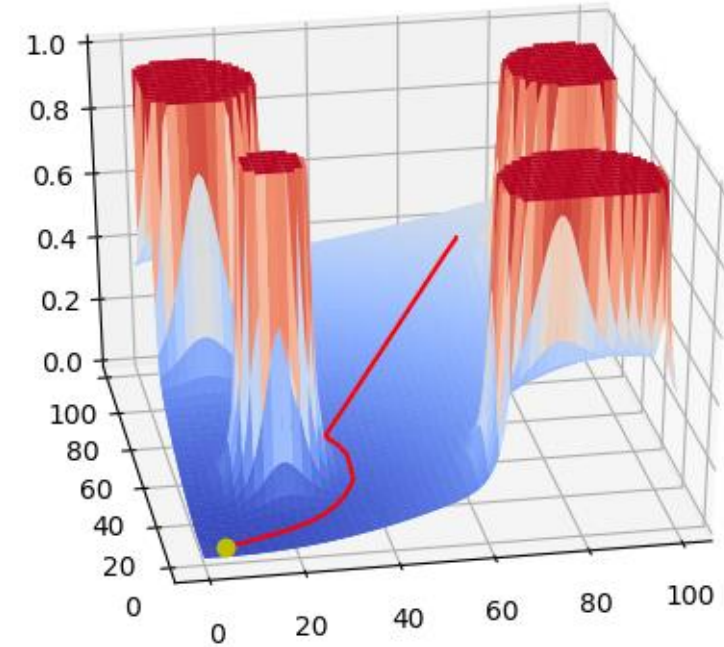
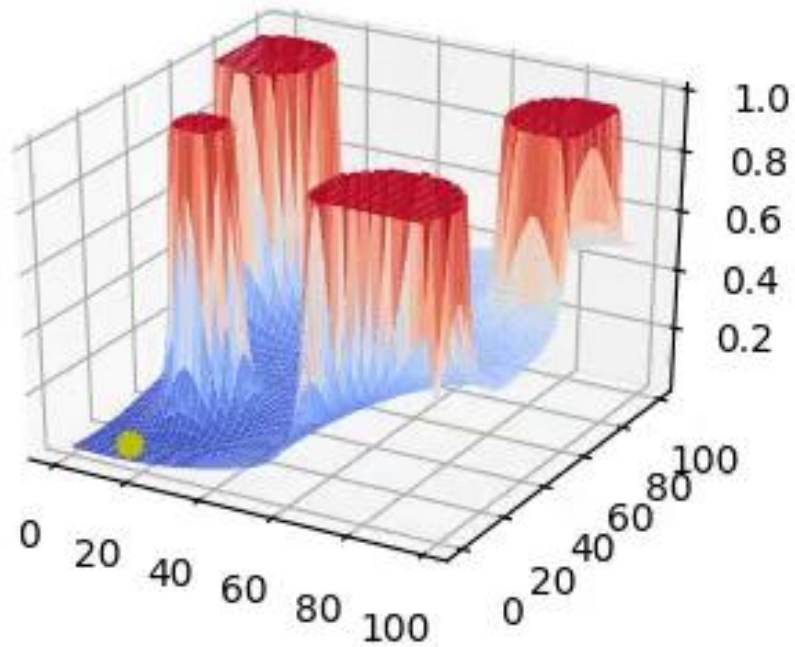
$*D(x)$ is the distance to the nearest obstacle boundary



Potential Field

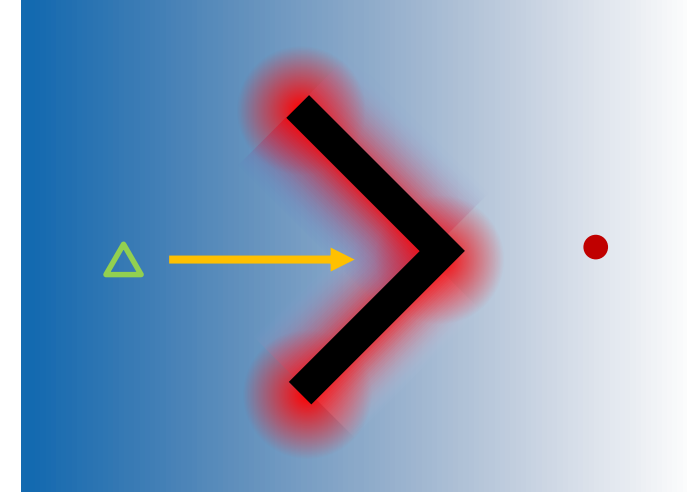


Potential Field



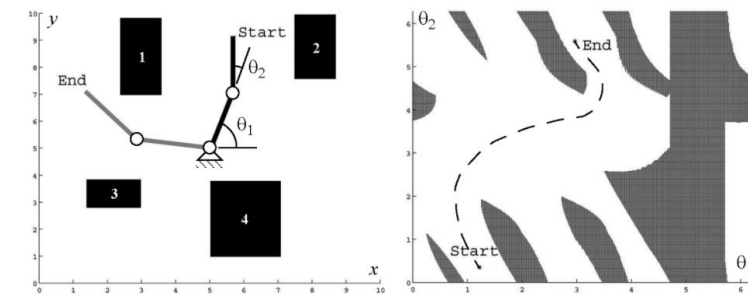
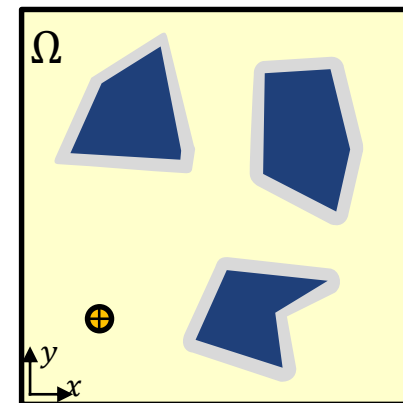
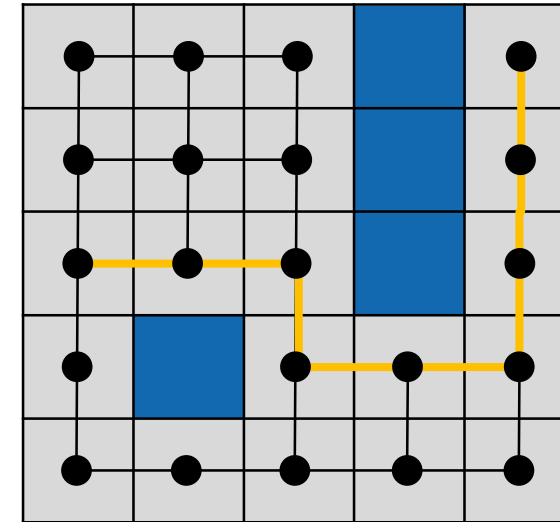
Potential Field Methods

- Relatively simple to implement
- Simplest versions can have issues with stationary points or local minima
- Modifying the potential functions can allow these conditions to be avoided (harmonic potentials, homework!)
- Dealing with higher-order state spaces (arms etc.) can be difficult to grasp/visualise, but potential functions are applicable



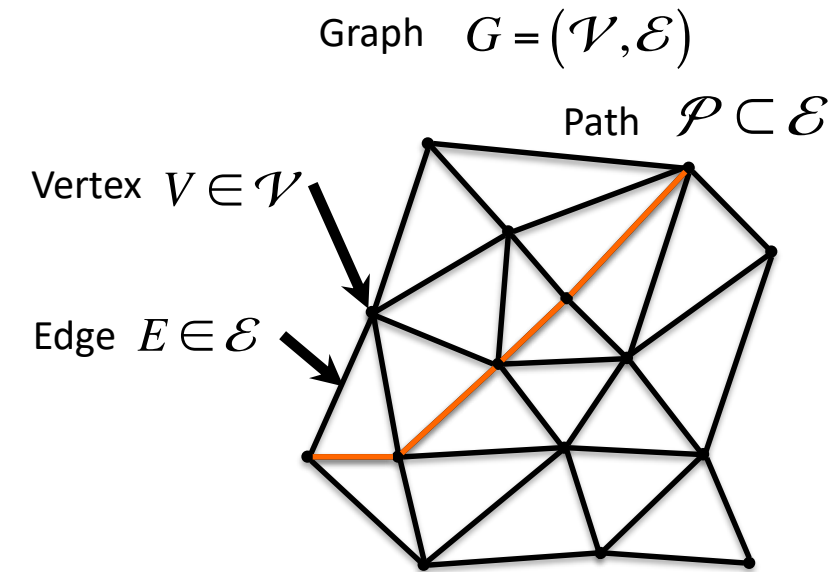
Summary

- Motion planning
 - Representation – how to define the robot's understanding of the world, and ensure that it is sufficient to complete the task
- Work space – the world without the robot
- Configuration space – the robot's configuration (joint angles etc.) in the world



Summary

- Graph search methods
 - Graphs are constructions of vertices and connecting edges
 - Graph search techniques are used to find low-cost paths through graphs
 - Breadth-first and depth-first search – complete searches from start (unweighted graphs)
 - Dijkstra – search outwards in order of cost from start (weighted graphs)
 - A* – focused search that prioritises searching towards the goal using an admissible heuristic



Summary

- Potential fields
 - Design a function such that descending the gradient leads to a collision-free path to the goal
- Additional references
 - Course text book and online lectures
 - Howie Choset (CMU) motion planning lecture notes:
 - https://www.cs.cmu.edu/~motionplanning/lecture/Chap4-Potential-Field_howie.pdf
 - Steven LaValle's Planning Algorithm Textbook
 - <http://planning.cs.uiuc.edu/>