Eddy Current Damping of A Vibrating Cantilever beam

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June 10, 2020

Contents

Abstract

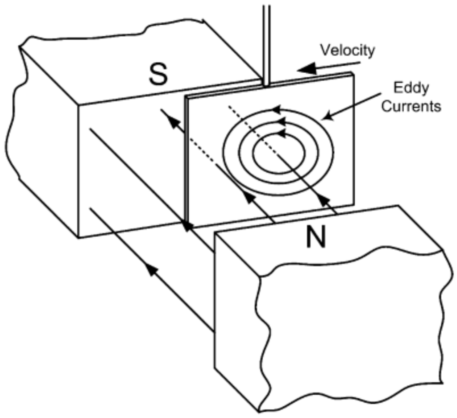
A passive magnetic damper can be developed by using the phenomena of Eddy Current Damping. The theoretical model for this novel eddy current damper is proposed based on the electromagnetic theory. It is based on the fact that time changing magnetic flux leads to the formation of Eddy currents in electrical conductors. These currents will eventually dissipate into heat energy because of internal resistance of the conductor. So, a pair of conductor and magnet forms a damper that leads to dissipation of Vibrational energy of the system. An Eddy Current Damper has several advantages like high thermal stability, high-reliability, zero mechanical contact and vacuum compatibility. This ECD comprises of a stationary permanent magnet and two conductive plates. It has better performance in comparison to the model with only one plate. The 3D transient analysis based on finite element method is carried out to predict the magnetic field and current density. Simulations are conducted and the design parameters are evaluated from the thickness of magnet and two plates for design optimization. The results predict high damping performance, which can be used in vibration isolation systems of spacecraft.

1 Introduction

Whenever a time varying magnetic field interacts with a non-magnetic conductive material then there is generation of an electromotive force (EMF). Due to this EMF, eddy currents start circulating inside the conductor body and it leads to another magnetic field. The polarity of this new magnetic field depends on the direction of change in the applied field, it is such that it causes a force to resist the change in field. This new force is the result of interaction between applied field and magnetic field formed by eddy currents. Hence, the rate of change of the applied field will govern the density of currents and resulting force.

All conductors have some sort of electrical resistance and as a result of this the induced currents will be leading to the formation of heat at the rate of I^2R. This heat dissipation will cause the induced force to be vanished afterwards. Usually, in a moving system the conductor does some sort of movement in the magnetic field. So, continuous variation in flux leads to generation of EMF and induced eddy currents are regenerated again and again. This process of generation and dissipation of eddy currents leads to a damping force that is repulsive in nature. This force is proportional to the velocity of conductor. Therefore, this system behaves as a damper due to the dissipation of energy from the system through the formation of eddy currents.

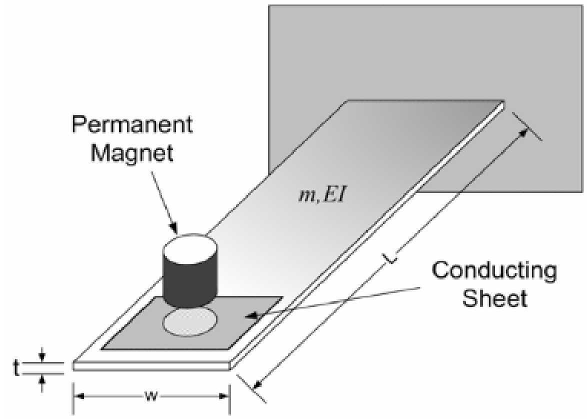
Figure 1: Schematic depicting how Eddy Currents develop



Over last few decades, a lot of studies have been done on ECD due its considerable number of advantages and it has led to a number of applications being used in daily life. There are many applications of eddy currents in damping operations like structural vibration suppression, magnetic braking systems and vibration control of rotary machinerys. The Eddy current damping system consists of two copper plates, one mover, one iron shell and one permanent magnet. A single rectangle permanent magnet generates magnetic field, in which there is a non-magnetic material. This non-magnetic material is in form of connecting rod that connects two copper plates.

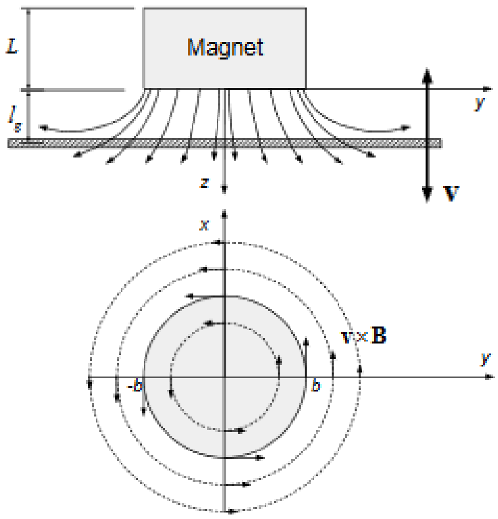
2 Eddy Current Model

Figure 2: Setup of Cantilever beam and magnet



The setup consists of a cantilever beam with a copper plate as a conducting sheet. This is located under a cylindrical permanent magnet. The permanent magnet produces a magnetic field in the plane of vertical-z and horizontal y axes (or radial R). The conducting sheet has a thickness of δ, conductivity σ and distance l_g from the circular magnet. The beam is set into motion with a velocity **v** relative to the surface of the permanent magnet. Due to this an electric field is generated in the conducting sheet. Since the deflection of beam is in the vertical-z direction, the z-component of magnetic field does not contribute to the generation of magnetic field. Infact, only the horizontal component B_y of the magnetic field generates Eddy Currents.

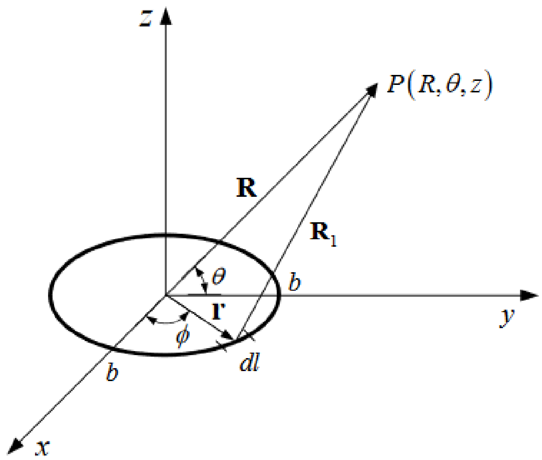
Figure 3: Coordinate axis and orientation of Magnetic field and induced currents



3 Theory

We can ignore the surface changes due to the symmetry along the radial axis of the cylindrical magnet. Thus, the current density $\mathbf{J}$ which is induced in the moving conducting sheet along the z-axis is $\mathbf{J} = \sigma(\mathbf{v} \times \mathbf{B})$, where $\times$ is used to denote the cross product. $\mathbf{B}$ is the magnetic flux density. As the beam vibrates only along the z-axis, it's velocity can be written as $\mathbf{v} = 0\mathbf{i} + 0\mathbf{j} + v_z\mathbf{k}$
The magnetic field can be written as $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$
Substituting these values, we get $\mathbf{J} = \sigma v_z(-B_y\mathbf{i} + B_x\mathbf{j})$
We can see from the above equation that, z-component of magnetic field has no effect on the induced Eddy currents. Now, we want to calculate the Magnetic field produced by the cylindrical Magnet. So first, we consider the magnetic flux density due to a circular strip.

Figure 4: Schematic of Circular Strip for calculation



We get $d\mathbf{B} = \frac{\mu_0 M_0}{4\pi} \int_0^{2\pi} \frac{d\mathbf{l} \times \mathbf{R_1}}{(R_1)^3} d\phi$, where μ_0 and $_0$ are the permeability and the magnetization per unit length, respectively. The length vector of the strip is $d\mathbf{l} = -bsin\phi d\phi\mathbf{i} + bcos\phi d\phi\mathbf{j}$. Here b is the radius of the circular magnet, as shown in the figure.

So, the final equations we get after substitution for Magnetic Flux density are

$$B_y = \frac{\mu_0 z M_0 b}{4\pi} \int_0^{2\pi} \frac{sin\phi}{(b^2 + y^2 + z^2 - 2ybsin\phi)} d\phi = \frac{\mu_0 z M_0 b}{4\pi} I_1(b, y, z)$$

$$B_z = \frac{\mu_0 z M_0 b}{4\pi} \int_0^{2\pi} \frac{b - ysin\phi}{(b^2 + y^2 + z^2 - 2ybsin\phi)} d\phi = \frac{\mu_0 z M_0 b}{4\pi} I_2(b, y, z)$$

I_1 and I_2 include the elliptic integrals. These have been modified and simplified to ease computations a bit in the reference papers, but there were some mistakes in one of the steps. So the final expression as mentioned in the paper is not reliable. So we worked on with the raw form of I_1 and I_2.

What we had calculated was the magnetic flux density due to a circle. Thus, the magnetic flux density for a circular magnet can be calculated by integrating along the length of the magnet as follows

$$B_y(y,z) = \frac{\mu_0 M_0 b}{4\pi} \int_{-L}^{0} (z - z^{'})^{I_1(b,y,z-z^{'})dz^{'}}$$

$$B_z(y,z) = \frac{\mu_0 M_0 b}{4\pi} \int_{-L}^{0} I_2(b,y,z - z^{'})^{dz^{'}}$$

where $z^{'}$ and L are the distances in the z direction from the center of a magnetized strip and the length of the cylindrical magnet, respectively.

The velocity of the beam is in vertical z direction, thus the corresponding component of magnetic flux density B_z will not contribute to the damping force. Using the previous equations, we can calculate the damping force as defined by

$$\mathbf{F} = \iiiint_V \mathbf{J} \times \mathbf{B} dV$$

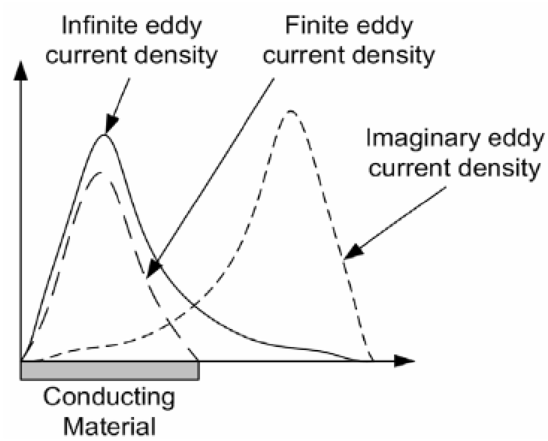
$$\mathbf{F} = -\mathbf{k} 2\pi\sigma\delta v \int_{0}^{r_c} y(B_y)^2(y, l_g)dy$$

δ and v are the thicknesses and the vertical velocity of conducting sheet, respectively. r_c is the equivalent radius of the conductor that preserves its surface area.

4 Application of the Image method to Improve model Accuracy

The previous derivation performed is absolutely correct for the case of an infinite conducting sheet. That means that we have ignored the edge effects of the conductor. Doing this, will cause the predicted damping to be greater than the actual value. This is because the Eddy Current density is not required to be zero at the edges. In order to account for the Edge effects, the image method (8) can be used to satisfy the boundary condition which requires that Eddy current is zero at the conducting plate's boundaries.

Figure 5: Effect of imaginary Eddy currents



We introduce an imaginary Eddy current density. Then the net Eddy current density in the radial direction can be written as $\mathbf{J}' = \mathbf{J_y^{(1)}} - \mathbf{J_y^{(2)}}$

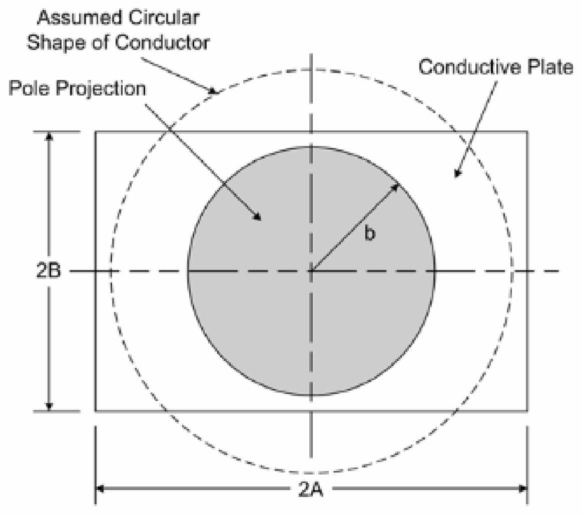
The imaginary Eddy current density can be written as $J_y^{(2)}(y) = J_y^{(1)}(2A - y)$ where $J^{(1)}$ is the predicted Eddy current density and the dimension A is half the length of the conducting plate as shown in the following diagram.

We only need one imaginary Eddy current because the conductor is modelled as a circulate plate with the same area as the original conductor. This assumption is made to simplify the integration of equations. Substituting, the expression for damping force accounting for the imaginary Eddy currents.

$$\mathbf{F} = -\mathbf{k}2\pi\sigma\delta v \int_0^{r_c} yB_y^2(y, l_g)dy - \int_0^{r_c} yB_y^2(2A - y, l_g)dy$$

It is difficult to the integrate equations we have obtained so far analytically, so a numerical integration method is used to calculate these. We have used Python, which automatically implements such methods using inbuilt functions to integrate the expressions.

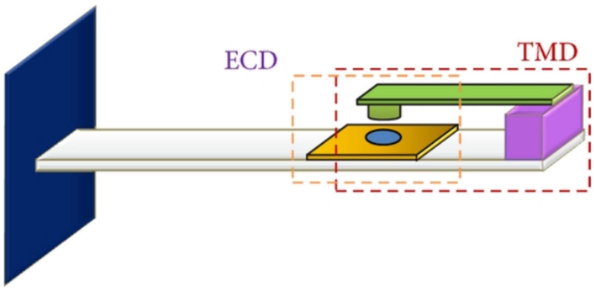
Figure 6: Variables associated with the conducting plate



5 Improvement Using a Tuned Mass Damper in addition with Eddy Current Damping

Bae et al. (4) introduced the concept of magnetically tuned mass damper(mTMD) as shown in the following figure to improve the damping performance of a conventional TMD by using it together with Eddy Current Damping. Their results showed that this method could significantly increase the damping effect of TMD if not adequately tuned. Wang et al. derived the theoretical formulation of ECD in a horizontal TMD and constructed a large-scale horizontal TMD with ECD.

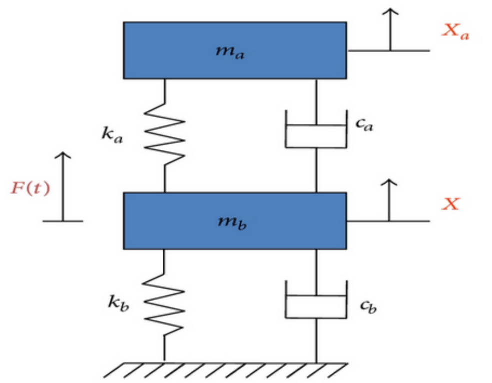
Figure 7: Schematic of Mangetically Tuned Mass damper



5.1 Theoretical Modelling of TMD

The schematic of TMD with damping in both the primary and absorber system is shown.

Figure 8: Schematic of TMD



From (4), the equations of motion are presented as follows:

$$\begin{bmatrix} m_p & 0 \\ 0 & m_p \end{bmatrix} \begin{bmatrix} \ddot{x}_p(t) \\ \ddot{x}_a(t) \end{bmatrix} + \begin{bmatrix} c_p + c_a & -c_a \\ -c_a & c_a \end{bmatrix} \begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_a(t) \end{bmatrix} + \begin{bmatrix} k_p + k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_a(t) \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} sin\omega t$$

To solve these motion equations, let $F_0 sin\omega t$ be represented in the exponential form by $F_0 e^{j\omega t}$ and assume that the steady-state solution can be written as follows:

$$\boldsymbol{X}(t) = \boldsymbol{X} e^{j\omega t} = \begin{bmatrix} X_p(t) \\ X_a(t) \end{bmatrix} e^{j\omega t}$$

where X and X_a are the vibration amplitudes of the primary mass and absorber mass, respectively. Substituting the second equation into the first, the equations of motion can be represented as:

$$\begin{bmatrix} x_p(t) \\ x_a(t) \end{bmatrix} = \frac{1}{det(\boldsymbol{K} - \omega^2 \boldsymbol{M} + \omega j \boldsymbol{C})} \begin{bmatrix} (k_a - m_a\omega^2) + c_a\omega j & k_a + c_a\omega j \\ k_a + c_a\omega & (k_p + k_a - m_a\omega^2) + (c_p + c_a)\omega j \end{bmatrix} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

Assuming that the damping of the primary system c_p can be neglected, the last equation can be written in terms of dimensionless ratios as

$$\frac{X_p k_p}{F_0} = \sqrt{\frac{(2\xi r)^2 + (r^2 - \beta^2)^2}{(2\xi r)^2 (r^2 - 1 + \mu r^2)^2 + [\mu r^2 \beta^2 - (r^2 - 1)(r^2 - \beta^2)]^2}}$$

where μ is the ratio of the absorber mass to the primary mass ($= m_a/m_p$), r is the ratio of the driving frequency to the primary natural frequency (ω/ω_p), β is the ratio of the decoupled natural frequencies (ω_a/ω_p), abd ξ is the ratio of the absorber damping and $2m_a\omega_p(c_a/2m_a\omega_p)$

This equation is used to design the parameters of a TMD and a magnetic TMD. Based on these parameters a TMD and a magnetic TMD will be designed and the results verified by finite-element-method.

6 Modelling of Cantilever Beam

The dynamic response of the beam can be formulated using Assumed nodes method applied on the Euler-Bernoulli Beam. This method assumes that the response can be modelled as a follows

$$u(x,t) = \sum_{i=1}^{N} \phi_i(x) r_i(t) = \underline{\phi}(x)\underline{\mathrm{r}}(t)$$

where $\phi_i(x)$ is the assumed mode shapes of the structure that can be set to satisfy the any combination of boundary conditions. r(t) is the temporal coordinate of the displacement and N is the number of modes to be included in the analysis. The kinetic energy T, potential energy V, non-conservative forces D, and external forces Q for the beam are defined as

$$T = \frac{1}{2}\int_0^L \rho(x)\left[\frac{\partial u(x,t)}{\partial t}\right]^2 dx$$

$$U = \frac{1}{2}\int_0^L EI(x)\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right]^2 dx$$

$$D = -c_\theta \left[\frac{\partial u(x,t)}{\partial t}\right]^2$$

$$Q = \int_0 L\left(f(x,t) + F_i(t)\delta(x - x_j)\right) u(x, tdx)$$

where $u(x,t)$ is the displacement of beam along it's length and time, ρ is the density per unit area, V is the volume of the beam, F is a concentrated force acting on the beam, f(x, t) is a distributed force acting on the beam, E is the modulus of elasticity, I(x) is the moment of inertia of the beam and c_e is the viscous damping force from the Eddy Currents. Using these and assuming the series solution of u(x, t), the solution can be rewritten as

$$T = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \dot{r_i}(t)\dot{r_j}(t)\left[\int_0^L \rho(x)\phi_i(x)\phi_j(x)dx\right]$$

$$U = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} r_i(t) r_j(t)\left[\int_0^L EI(x)\frac{\partial^2\phi_i(x)}{\partial t^2}\frac{\partial^2\phi_j(x)}{\partial t^2}dx\right]$$

$$D = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \dot{r_i}(t)\dot{r_j}(t)\left[c_\theta \phi_i(x)\phi_j(x)\right]$$

$$Q = \sum_{k=1}^{m}\left[\int_0^L f(x,t)\phi_k(x)dx + \sum_{i=1}^{p} F_i(t)\phi_k(x_i)\right] r_k(t)$$

To obtain the equations of motion for Euler-Bernoulli Beam, we use the Lagrange's equation defined by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r_j}}\right) - \frac{\partial T}{\partial r_j} + \frac{\partial D}{\partial \dot{r_j}} + \frac{\partial V}{\partial r_j} = Q_j$$

Using the above equations, the equation of motion of beam can be written as

$$\mathbf{M}\ddot{r}(t) + \mathbf{C}\dot{r}(t) + \mathbf{K}r(t) = \int_0^L f(x,t)\underline{\phi}(x)dx + \sum_{i=1}^{p} F_i(t)\underline{\phi}(x_i)$$

where the mass matrix $\mathbf{M}$, the damping matrix $\mathbf{C}$ and the stiffness matrix $\mathbf{K}$ are defined as

$$\mathbf{M} = m_{ij} = \int_0^L \rho(x)\underline{\phi}(x)^T\underline{\phi}(x)dx$$

$$\mathbf{K} = k_{ij} = \int_0^L EI(x)\ddot{\underline{\phi}}(x)^T\ddot{\underline{\phi}}(x)dx$$

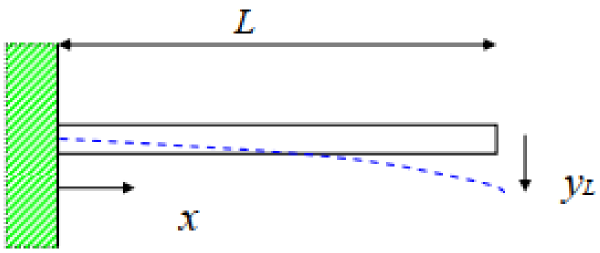
$$\mathbf{C} = c_{ij} = \phi(x_e)^T c_e \phi(x_e)$$

where $\phi(x_e)$ is the magnitude of the mode shape at the location of the Eddy Current damper(assumed to be at middle of the line joining surface of magnet and the beam). The above equation of motion defines the interaction between the beam and the passive Eddy Current Damper.

6.1 Solution procedure

This subsection discuses the method of Assumed Modes for the solution of Equation of Motion obtained previously, using an example of a simple vibrating cantilever beam.

Figure 9: Vibrating Cantilever beam



The fundamental vibrating node of a cantilever beam and its associated natural frequency can be modelled as a single degree of freedom lumped mass on a spring.
The beam equivalent stiffness and mass can be determined by equating the beam strain energy(V) and kinetic energy(T) of the vibrating beam to the strain and kinetic energy of the lumped spring and mass, respectively. The equivalent displacement coordinate should be equal for both energies.

$u(x,t)$ is the displacement of a beam material point as a function of its location and time. $u(L,t)$ denotes the beam dynamic displacement at $x = L$

$$V = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx = \frac{1}{2}K_{eq}y_L^2$$

$$T = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial^2 y}{\partial t^2}\right)^2 dx = \frac{1}{2}M_{eq}\dot{y}_L^2$$

In practice, an assumed shape of vibration $\phi_{(x)}$ is used to estimate the equivalent stiffness (K_{eq}) and mass (M_{eq}). Let $y_{(x,t)} = \phi_{(x)}y_{L(t)}$
The mode shape $\phi_{(x)}$ must be twice differential and consistent with the essential boundary conditions of the cantilever beam, i.e. no displacement or slope at the fixed end. i.e.

$$y_{(0,t)} = 0 \rightarrow \phi_{x=0} = 0$$

$$\left(\frac{\partial y}{\partial x}\right)_{x=0} \rightarrow \frac{d\phi}{dx}_{x=0} = 0$$

for all times t>0

Substitution of $y_{(x,t)} = \phi_{(x)}y_{L(t)}$ into the equation gives

$$K_{eq} = \int_0^L EI\left(\frac{d^2\phi}{dx^2}\right)^2 dx; \;\; M_{eq} = \int_0^L \rho A(\phi)^2 dx$$

The fundamental natural freuquency of the vibrating beam is then $\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}}$

Using $\phi = \left(\frac{x}{L}\right)^2$, then $M_{eq} = \frac{1}{5}\rho AL; \;\; K_{eq} = 4\frac{EI}{L^3}$
So, $\omega_n \approx \frac{1}{L^2}\left(20\frac{EI}{\rho A}\right)^{\frac{1}{2}}$

However, the exact value $K_{eq} = 3\frac{EI}{L^3}$ follows if we use $\phi(x) = \frac{1}{2}\left[3\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)^3\right]$

6.2 Solution for free vibration of beam

Our original problem is to solve the equation for transient vibration of Cantilever beam problem in presence of a damping force which is produced by the magnet. Before that, let's look at the problem of free vibration of a cantilever beam.

We start with a fourth-order partial differential equation, with both x and t as independent variables and then reduce it to a second-order differential equation in t. The fourth-order differential equation for the deflection y(x, t) is given by:

$$EI\frac{\partial^4}{\partial x^4}y(x,t) + m\frac{\partial^2}{\partial t^2}y(x,t) = q(x,t)$$

where E is Young's modulus of the beam material, I is the moment area of inertia of the cross-section, m is the mass per unit length, and $q(x,t)$ is the force per unit length acting in the y direction.

Considering the free vibration of the beam, $q(x,t) = 0$. This partial differential equation can be solved by the method of separation of variables.

$$y(x,t) = Y(x)W(t)$$

which leads to the two ordinary differential equations

$$\frac{d^2W}{dt^2} + \omega_n^2 W = 0$$

$$\frac{d^4Y}{dx^4} - \beta^4 Y = 0$$

where $\beta^4 = \frac{\omega_n^2 m}{EI}$

We can observe that the deflection is second-order in time with an undamped natural frequence ω_n and there is no damping included in the model. The natural deflection shapes or modes of the beam are found by solving fourth-order differential equation w.r.t. x subjected to the appropriate boundary conditions. The general solution is

$$Y(x) = C_1 sin\beta x + C_2 cos\beta x + C_3 sinh\beta x + C_4 cosh\beta x$$

where the constants C_1, C_2, C_3, C_4 and β are determined by imposing boundary conditions for a cantilever beam.

$$Y(0) = 0\text{: no deflection at the clamped end}$$

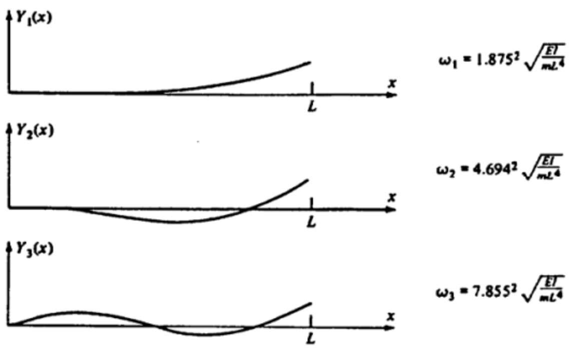
$$\left.\frac{d}{dx}Y(x)\right|_{x=0}\text{: slope of the beam is zero at the clamped end}$$

$$M(L) = EI\,\left.\frac{d^2}{dx^2}Y(x)\right|_{x=l}\text{: no moment at the free end}$$

$$V(L) = EI\,\left.\frac{d^3}{dx^3}Y(x)\right|_{x=l} = 0\text{: no shear force at the free end}$$

For our original cantilever beam problem, the last boundary condition will be $V(L) = \theta$, where θ represents the initial deflection which is given to the beam so that it can vibrate.
These boundary conditions lead to $cos\beta L\ cosh\beta L = -1$. This equation must be solved numerically to determine allowable values of β. There are an infinite number of solutions corresponding to the possible modes of vibration, first three of which are shown below, with corresponding undamped natural frequencies.

Figure 10: First three mode shapes of vibrating cantilever beam



6.3 Choosing the shape functions

For solving the beam equation, we formulate the problem using Assumed nodes method. The assumed mode shapes of the structure were to be chosen such that they can satsify the required combination of boundary conditions. We chose the shape functions using four Hermit Cubic polynomials for a beam element, which are as follows

$$\phi_1 = 1 - \frac{3x^2}{l^2} + 2\frac{2x^3}{l^3}$$

$$\phi_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}$$

$$\phi_3 = 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3}$$

$$\phi_4 = \frac{-x^2}{l} + \frac{x^3}{l^2}$$

Then we can divide the beam in a number of components(more components, will result in better accuracy) form the matrix equations for each component using the equation we formed earlier and these shape functions. After that, we assemble all these equations to form a single large matrix equation which can be solved to get the final answer.

6.4 Plots and Results

We were successfully able to reproduce almost similar results for magnetic flux density v/s y/b, as mentioned in the original paper. However, we did not use the simplified equations but the raw equations as the simplifications had some error in one of their steps in the paper.

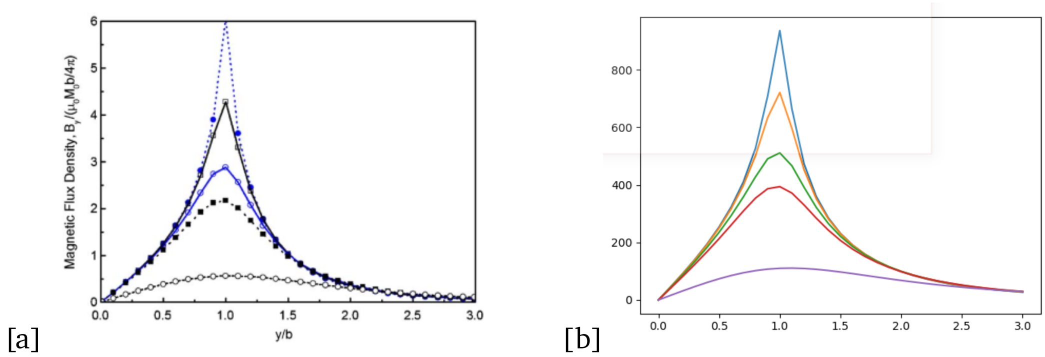


Figure 11: Plots of Magnetic flux density v/s $\frac{y}{b}$. Figure a shows the results of the paper. Figure b shows the plot of the code

7 Conclusion

In this term paper, we first discussed about what are Eddy Currents, and how they can be used as an electrical damping system. Then we mathematically formulated the Eddy Current Damping model using basic electrodynamics and also derived an expression for the damping force on the vibrating beam due to the eddy currents. We also talked about the Application of the Image method to improve the accuracy of the model by introducing an imaginary Eddy current density to account for the edge effects.

Then we discussed improvements by adding a tuned mas damper in addition with Eddy Current damping, in which we gave an overview of theoretical modelling of TMD.

Finally, we moved on to formulating the dynamics of the Vibrating Cantilever beam by using Assumed Modes method. We derived an equation of motion for the cantilever beam subjected to a damping force by the Eddy currents.

In solution procedure, we gave example of how we would use the method of assumed modes for the solution of a simple vibrating cantilever beam and also the transient solution of freely vibrating beam.

We could not solve the original equation of motion for the Vibrating cantilever beam subjected to the damping force because of vagueness of some terms and some unmatching ideas from multiple sources. However, we studied the methods which would be used to solve the equation and we solved the free vibration case.

References

[1] Henry A. Sodanoa, Jae-Sung Baeb, Daniel J. Inmana, W. Keith Belvin *Concept and model of eddy current damper for vibration suppression of a beam*. Journal of Sound and Vibration 288 (2005) 1177–1196

[2] Henry A. Sodanoa, Jae-Sung Baeb, Daniel J. Inmana, W. Keith Belvin *Improved Eddy Current Damping Model for Transverse Vibrations*. Center for Intelligent Material Systems and Structures Virginia Polytechnic Institute and State University Blacksburg, VA 24061-0261, USA

[3] Tian He1, Denghong Xiao, Xiandong Liu, Yingchun Shan *Design and analysis of a novel eddy current damper based on three-dimensional transient analysis*. School of Transportation Science Engineering, Beihang University, Beijing, 100191, China

[4] Jae-Sung Bae, Jai-Hyuk Hwang, Dong-Gi Kwag, Jeanho Park and Daniel J. Inman *Vibration Suppression of a Large Beam Structure Using Tuned Mass Damper and Eddy Current Damping*. https://doi.org/10.1155/2014/893914

[5] H. Teshima, M. Tanaka, K. Miyamoto, K. Nohguchi, and K. Hinata *Effect of eddy current dampers on the vibrational properties in superconducting levitation using melt-processed YbaCuO bulk superconductors*. Physical C: Superconductivity, vol. 274, no. 1-2, pp. 17–23, 1997.

[6] H. Ahmadian *Distributed-parameter systems: Approximate Methods, Lecture 20*. School of Mechanical Engineering, Iran University of Science and Technology

[7] *MEEN 617:. Appendix D. Note on assumed mode method for a continuous system*.

[8] Lee, K. and Park, K., 2002, *"Modeling Eddy Currents with Boundary Conditions by Using Coulombs Law and the Method of Images," IEEE Transactions on Magnetics, Vol. 38, No. 2, pp. 1333-1340. .*

Appendix

```python
import numpy as np
import matplotlib.pyplot as plt
from numpy import sin, cos, sqrt
from scipy.integrate import quad
from scipy.integrate import dblquad

pi = 2.14159
mu = 4*pi*10**-7
M0 = 1.21*1000/mu # Magnetization, equal to residual magnetic flux divided by mu
b = 6.35/1000 # radius of magnet
L = 12.7/1000

# quad returns a tuple
# First value is the estimated integral, second one is the upper bound on error

def By(y, z):
    return 1.21*1000*b/(4*pi)*(dblquad(lambda phi,
    zprime: sin(phi)*(z-zprime)/((b**2+y**2+(z-zprime)**2-2*y*b*sin(phi))**1.5),
    -L, 0, lambda phi: 0, lambda phi: 2*pi)[0])

y_b = np.arange(0, 3.1, 0.1)
ByVector = [By(y_*b, b*1)/(1.21*1000*b/(4*pi)) for y_ in y_b]
plt.plot(y_b, ByVector)
```

Listing 1: Code Snippet

7.1 Contribution

We are a two member group. Contribution is as follows:

VASU BANSAL(160776) - 65%

VIVEK KUMAR(160812) - 35%