4 Non-variational (?) data driven computational mechanics

Another idea to introduce a length scale into DD mechanics comes from the 'implicit strain gradient' models. In those models, an effective strain is first defined, which is then used to compute the damage variable. For instance,

$$\sigma = g(d)E_0\epsilon, \tag{31}$$

where $\epsilon = u_{,x}$. The damage variable is now a function of an effective strain variable, $\tilde{\epsilon}$ (defined later on below). An equivalent energetic representation of this formulation is not possible (*I will give it a try later on*). The intent is to draw some inspiration from such techniques into the regime of DD mechanics.

For instance, the DDM can be described as rendering the following functional stationary:

$$\Pi = \frac{1}{2}(\sigma - \sigma^*)^T C^{-1} M(\sigma - \sigma^*) + \frac{1}{2}(Bu - \epsilon^*)^T CM(Bu - \epsilon^*) + B^T M\sigma. \tag{32}$$

Taking its variations for the mechanical update, assuming that the material states are given,

$$\delta^u\Pi = B^T MC(Bu - \epsilon^*) = 0, \tag{33}$$

$$\delta^\sigma\Pi = 0 \implies \sigma = \sigma^* - CB\eta, \tag{34}$$

$$\delta^\eta\Pi = B^T M\ \sigma = 0. \tag{35}$$

The last two of the above equations result in

$$B^T MCB\eta = B^T M\sigma^*. \tag{36}$$

The material update is now defined as

$$(\epsilon_e^*, \sigma_e^*, \tilde{\epsilon}_e^*) \in \arg\min_{(\epsilon^*,\sigma^*,\tilde{\epsilon}^*)} \frac{1}{2}(\sigma - \sigma^*)^T C^{-1} M(\sigma - \sigma^*) + \frac{1}{2}(\epsilon - \epsilon^*)^T CM(\epsilon - \epsilon^*) + \frac{1}{2}(\tilde{\epsilon} - \tilde{\epsilon}^*)^T DM(\tilde{\epsilon} - \tilde{\epsilon}^*). \tag{37}$$

It shall be noted that the distance used here is different from what has been used during the mechanical update. This is what destroys the variational structure of the data driven mechanics in this setting, but introducing the length scale into the problem. This minimization can be performed analytically (*or may be not*) at each integration point.

The equivalent strain, $\tilde{\epsilon}$ is defined as the solution of the following PDE:

$$\tilde{\epsilon} - \ell^2 \tilde{\epsilon}_{,xx} = \epsilon, \tag{38}$$

where $\epsilon = u_{,x}$. The above equation can be written in a *weak form* as

$$\int (\delta\tilde{\epsilon})\tilde{\epsilon}\,\mathrm{d}x + \ell^2 \int (\delta\tilde{\epsilon}_{,x})\tilde{\epsilon}_{,x}\,\mathrm{d}x = \int (\delta\tilde{\epsilon})\epsilon\,\mathrm{d}x. \tag{39}$$

In matrix form, $(\tilde{M} + \ell^2\tilde{K})\tilde{\epsilon} = \tilde{M}\epsilon = \tilde{M}Bu$. The equivalent strain is then used to compute the material state together with σ and ϵ.

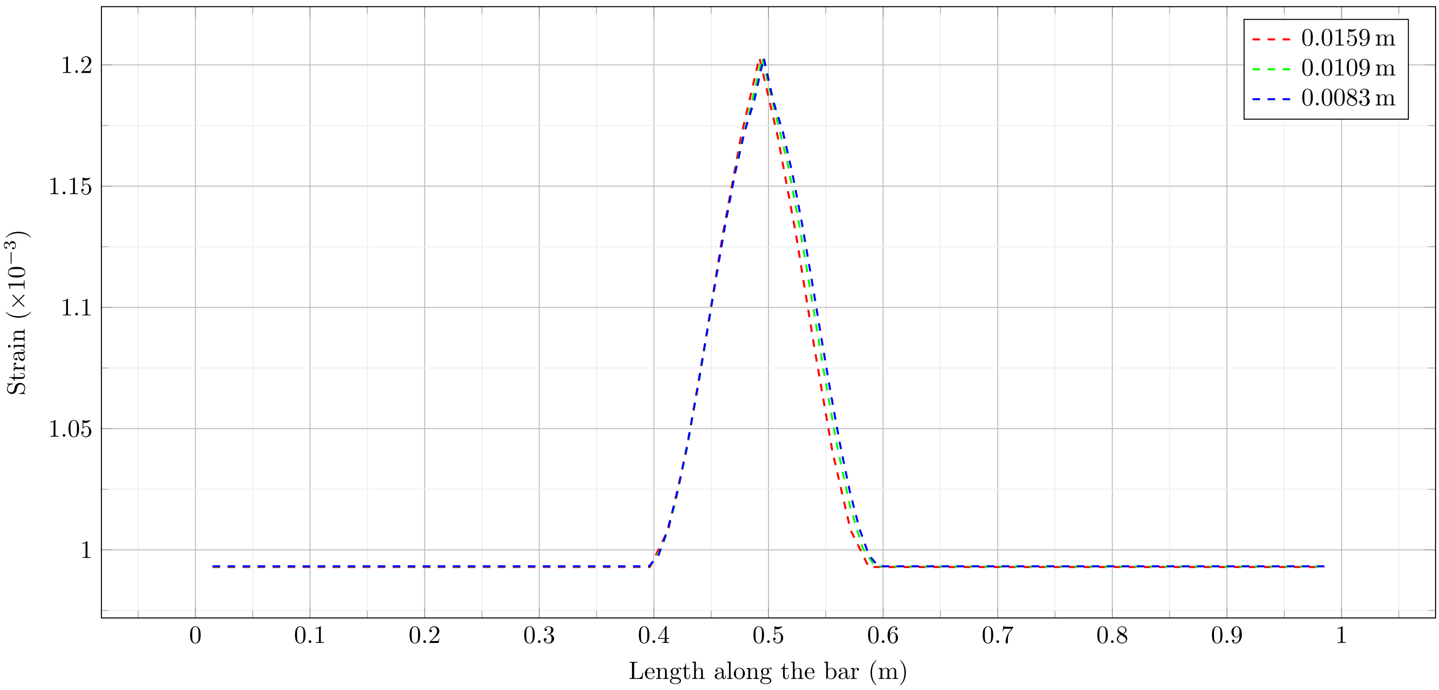

Figure 2: Strain distribution for different mesh sizes.

4.1 Some results

The problem of a bar with a defect at the center has been analyzed. The strain distribution for different meshes when the damage initiated can be seen in figure 2.