

D. Pairs of Segments

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Given

Given N Segments

Constraint

$N \leq 2000$

$L_i \leq R_i, R_i \leq 10^9$

To Compute

The minimum number of pairs to be removed from the array s.t. array is beautiful.

The Array is beautiful if and only if array of size k can be divided into $k/2$ pairs s.t. in each pair the segment intersect with each other whereas no two pair intersect with each other.

Algorithm

Let's take any two segment from the array s.t. both segment intersect with each other, We will take Union of the both the segments

There would be atmost $\binom{n}{2}$ new segments formed after performing the above operation.

Now we want this newly created segment to not intersect with each other and our aim is to maximize this number of segment so that the number of segment should be removed from the array can be minimized.

Greedy Approach

Aim - To maximize the number of the non intersecting segments.

Greedy strategy

We will sort the array of segment on the basis of left value then we will take the segment which will end first

Proof

Let our Optimal solution be :-

$$[\{ l_1, r_1 \}, \{ l_2, r_2 \}, \{ l_3, r_3 \} \dots \{ l_t, r_t \}]$$

If we exchange the pair $\{ l_1, r_1 \}$ with r_p such that $r_p \leq r_1$, this will not change our optimal solution.

Now we will do this iteratively thus getting our optimal solution.