1616D - Keep the Average High

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Given

Array A of size N

Integer x

Constraint

$1 \leq N \leq 50000$

$-100000 \leq a_i \leq 100000$

$-100000 \leq x \leq 100000$

Problem

You need to select the maximum number of elements in the array, such that for every subsegment $a_l, a_{l+1}, \ldots, a_r$ containing strictly more than one element ($l < r$), either:

- At least one element in this subsegment is not selected
- $a_l + a_{l+1} + \cdots + a_r \geq x \cdot (r - l + 1)$

Algorithm

Note that

$$a_l + a_{l+1} + \cdots + a_r \geq x \cdot (r - l + 1) \implies (a_l - x) + (a_{l+1} - x) + \cdots + (a_r - x) \geq 0$$

After subtracting x our problem reduces to choose maximum number of contiguous subbarray such that each subsegment is positive in it.

Greedy Algorithm

Argument-1 0 index element will always be there

Using Exchange Argument

Let us assume we have a optimal solution and it do not have 0 index element

$$a_0 \ \underbrace{a_1 \ a_2 \ a_3} \ a_4 \ a_5 \ \underbrace{a_6 \ a_7} \cdots a_n$$

Now even If we exhange the a_0 with a_1 the optimal answer would not be worsen

Argument-2 In the optimal answer index of two selected consecutive element should not be greater than 1

Let there be optimal solution with difernece between two selected element greater than 1

Using Same Exchange Argument

$$a_0 \ \underbrace{a_1 \ a_2 \ a_3} \ a_4 \ a_5 \ \underbrace{a_6 \ a_7} \cdots a_n$$

Let it be a_4 If we exhange the element with it unselected neighbour element then there will be no change in the optimal solution.

Thus our problem reduces to select maximum consecutive element such that the sum of each subsegment inside it is positive

If there is a subsegment inside the negative then there will surely exist an element of size 2 or 3 such that there sum is negative so we have to check two condition for the element to be included

If $a[i] + a[i-1] \geq 0 \wedge selected[i-1] == True \wedge selcted[i-2] == False$ then $selected[i] = true$

If $a[i]+a[i-1]+a[i-2] \geq 0 \wedge selected[i-1] == True \wedge selected[i-2] == True$ then $selected[i] = true$

Answer the count of the selected elements.