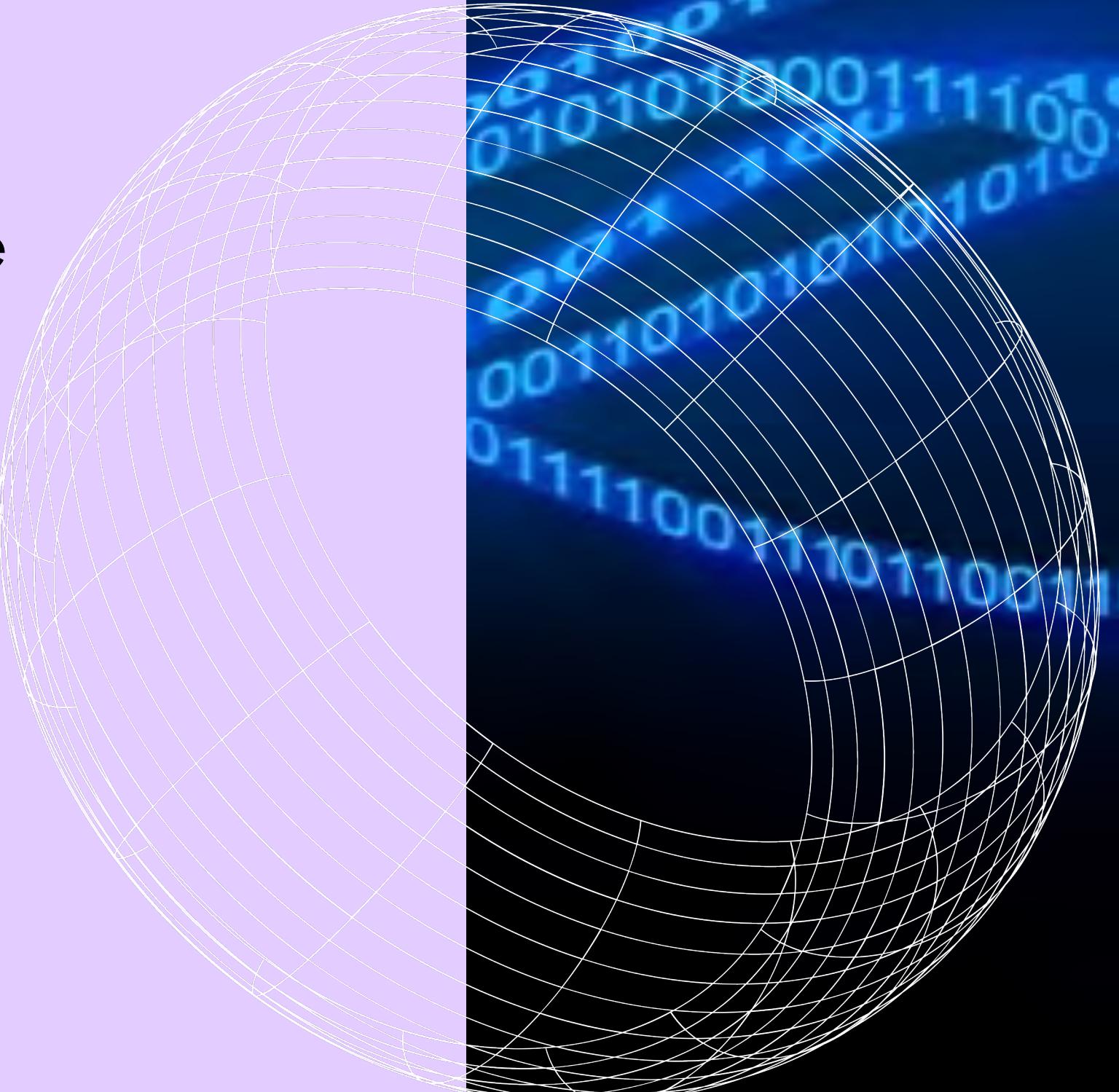


The background features a dense grid of binary digits (0s and 1s) in white, set against a dark blue gradient. Overlaid on this are several thin, light blue wavy lines that curve and flow across the frame, creating a sense of motion and depth.

# Smoothing

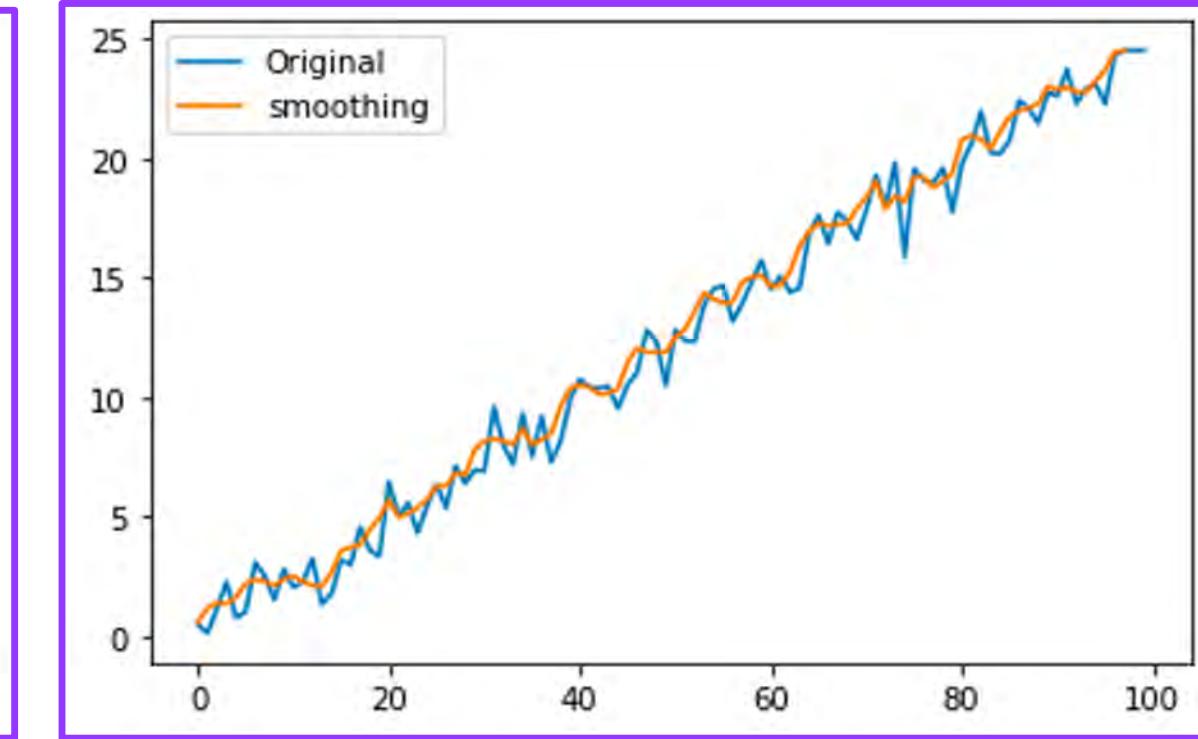
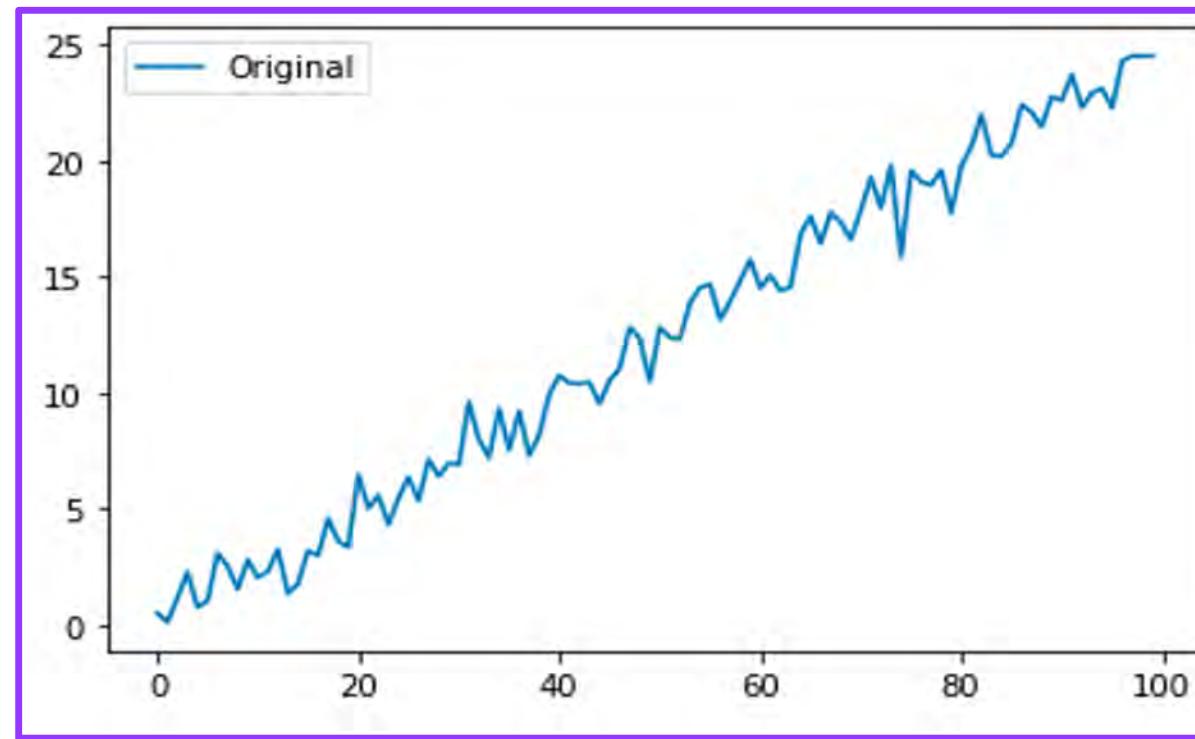
# Intro

- time series transformation
- smoothing
  - moving average
  - exponential moving average
    - single
    - double
    - triple



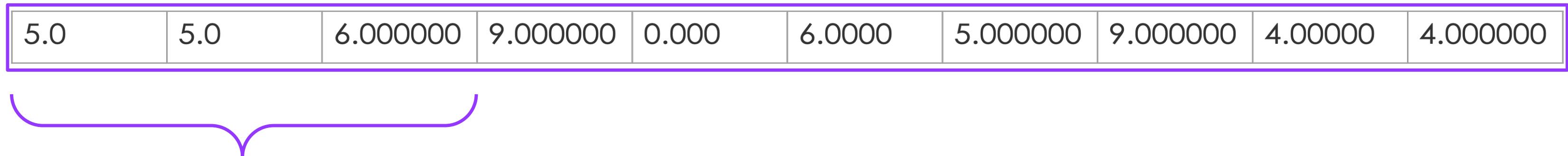
# SMOOTHING

- smoothing is a simple method for reducing noise in the data
- can also be used to provide simple forecasts



# Moving average

$$s_t = \frac{x_t + x_{t-1} + \dots + x_{t-n+1}}{n}$$



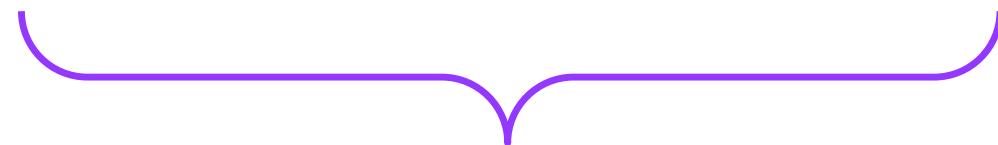
$n = 3$

5.333333	6.666667	5.000	5.0000	3.666667	6.666667	6.000000	5.666667
----------	----------	-------	--------	----------	----------	----------	----------

# Moving average

$$s_t = \frac{x_t + x_{t-1} + \dots + x_{t-n+1}}{n}$$

5.0	5.0	6.000000	9.000000	0.000	6.0000	5.000000	9.000000	4.00000	4.000000
-----	-----	----------	----------	-------	--------	----------	----------	---------	----------



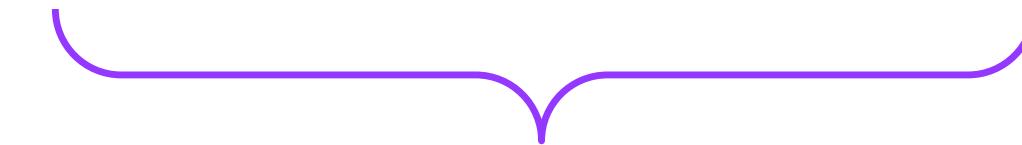
$n = 3$

5.333333	6.666667	5.000	5.0000	3.666667	6.666667	6.00000	5.666667
----------	----------	-------	--------	----------	----------	---------	----------

# Moving average

$$s_t = \frac{x_t + x_{t-1} + \dots + x_{t-n+1}}{n}$$

5.0	5.0	6.000000	9.000000	0.000	6.0000	5.000000	9.000000	4.00000	4.000000
-----	-----	----------	----------	-------	--------	----------	----------	---------	----------

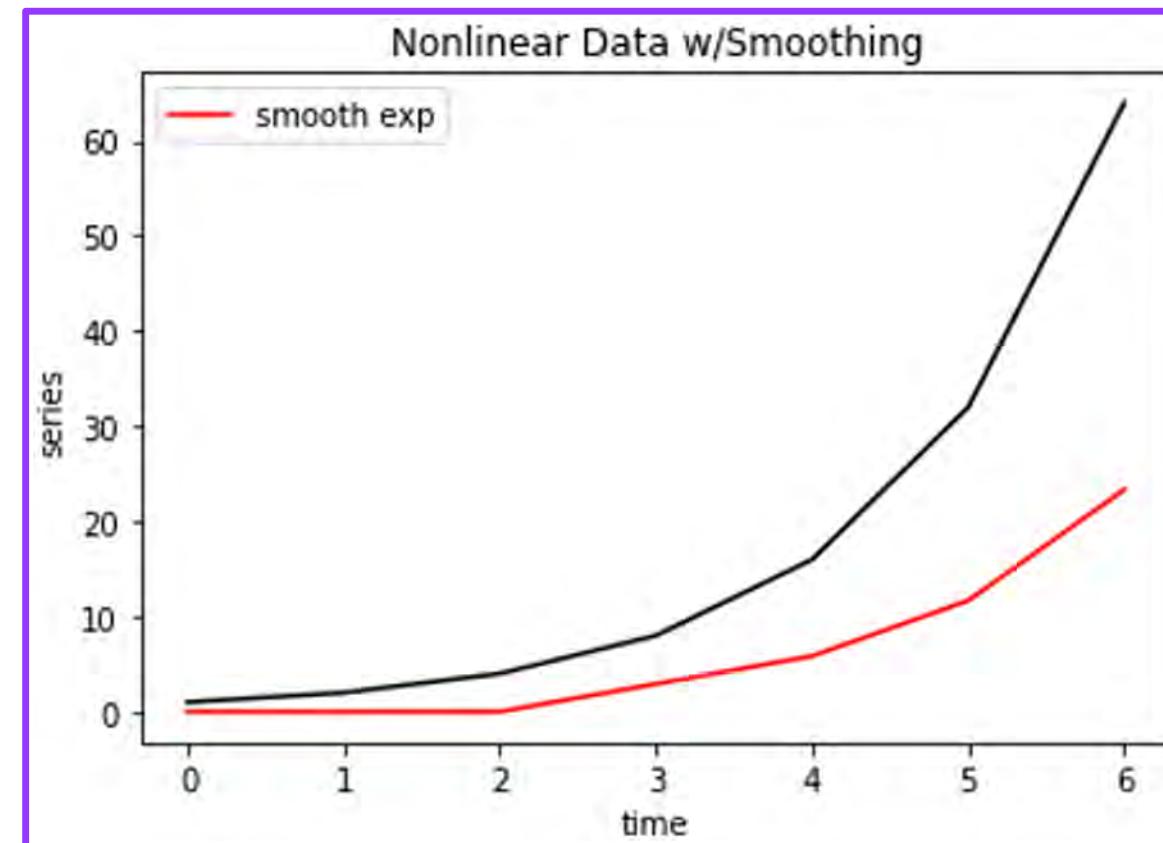
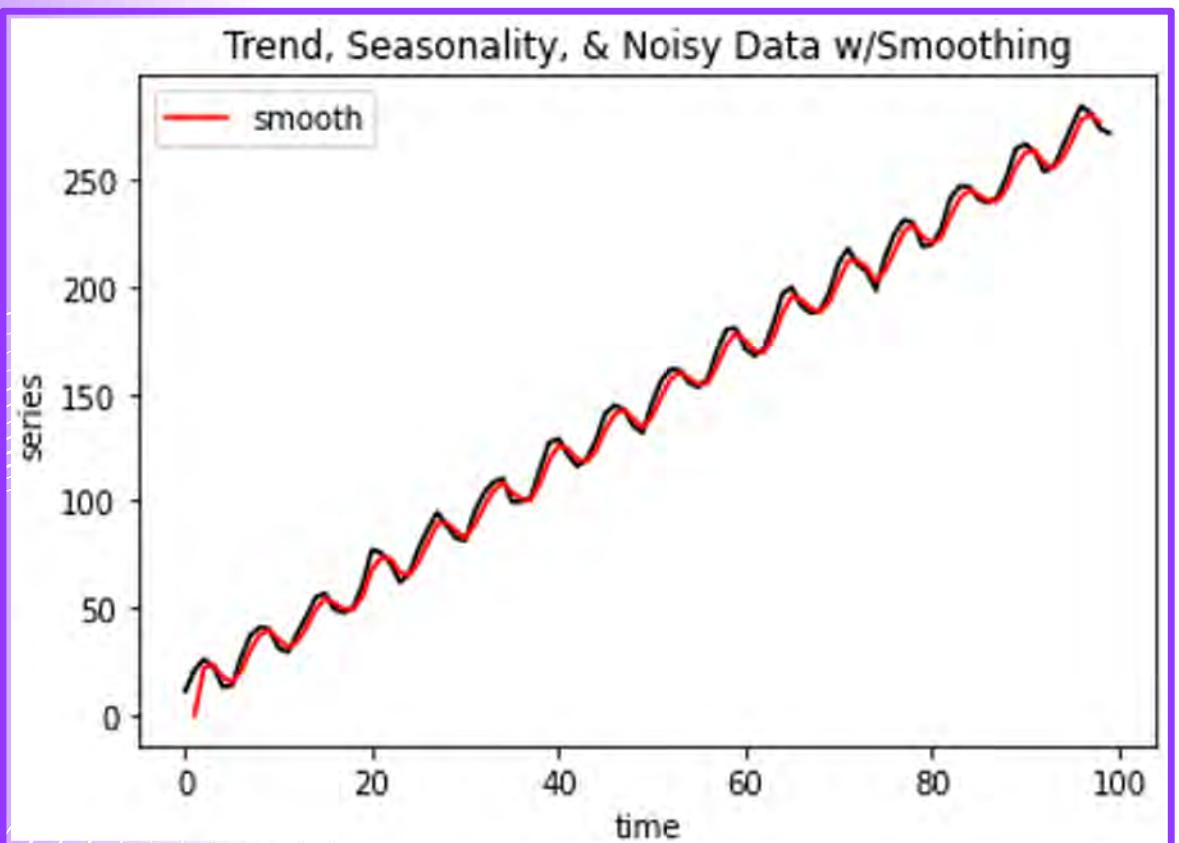


$n = 3$

5.333333	6.666667	5.000	5.0000	3.666667	6.666667	6.00000	5.666667
----------	----------	-------	--------	----------	----------	---------	----------

# Moving average

- weights every data point equally
- cannot track exponential growth well



# Exponential Moving average

$$s_0 = x_0$$

$$s_t = \alpha x_t + (1 - \alpha)s_{t-1}$$

$$\alpha = 0.1$$

x	5.0	6.0	9.0	0.0	6.0	5.0	9.0	4.0	4.0	
s	5.0	5.1	5.490000	4.941	5.0469	5.042210	5.437989	5.29419	5.164771	

A pink arrow points from the first value in the x series (5.0) to the second value in the s series (5.1). A pink box highlights the first two values in each series.

# Exponential Moving average

$$s_0 = x_0$$

$$s_t = \alpha x_t + (1 - \alpha)s_{t-1}$$

$$\alpha = 0.1$$

x	5.0	6.0	9.0	0.0	6.0	5.0	9.0	4.0	4.0	
s	5.0	5.1	→	5.490000	4.941	5.0469	5.042210	5.437989	5.29419	5.164771

# Exponential Moving average

$$s_0 = x_0$$

$$s_t = \alpha x_t + (1 - \alpha)s_{t-1}$$

$$\alpha = 0.1$$

x	5.0	6.0	9.0	0.0	6.0	5.0	9.0	4.0	4.0	
s	5.0	5.1	5.490000	4.941	5.0469	5.042210	5.437989	5.29419	5.164771	

The diagram illustrates the calculation of an Exponential Moving Average (EMA) over time steps t. The x-series represents the raw data, and the s-series represents the EMA values. The formula for the EMA is  $s_t = \alpha x_t + (1 - \alpha)s_{t-1}$ , where  $\alpha = 0.1$ . The initial value  $s_0 = x_0 = 5.0$ . Subsequent values are calculated as follows:

- Step 1:  $s_1 = 0.1 \cdot 6.0 + 0.9 \cdot 5.0 = 5.1$
- Step 2:  $s_2 = 0.1 \cdot 9.0 + 0.9 \cdot 5.1 = 5.490000$  (highlighted with a pink arrow)
- Step 3:  $s_3 = 0.1 \cdot 0.0 + 0.9 \cdot 5.490000 = 4.941$
- Step 4:  $s_4 = 0.1 \cdot 6.0 + 0.9 \cdot 4.941 = 5.0469$
- Step 5:  $s_5 = 0.1 \cdot 5.0 + 0.9 \cdot 5.0469 = 5.042210$
- Step 6:  $s_6 = 0.1 \cdot 9.0 + 0.9 \cdot 5.042210 = 5.437989$
- Step 7:  $s_7 = 0.1 \cdot 4.0 + 0.9 \cdot 5.437989 = 5.29419$
- Step 8:  $s_8 = 0.1 \cdot 4.0 + 0.9 \cdot 5.29419 = 5.164771$

# Single exponential moving average

$$s_0 = x_0$$

$$s_t = \alpha x_t + (1 - \alpha) s_{t-1}$$

- to provide varying weights to recent values
- Expanded equation can be used for recursively forecasting future values

$$s_t = \alpha x_t + \alpha(1 - \alpha)x_{t-1} + \alpha(1 - \alpha)^2 x_{t-2} + \dots + (1 - \alpha)^t x_0$$

$s_t$  = Smooth the value at time  $t$

$x_t$  = Actual value at time  $t$

$\alpha$  = Parameter optimized to fit past data

# Double exponential moving average

$$s_t = \alpha x_t + (1 - \alpha)(s_{t-1} + b_{t-1})$$

Smooth the **value** of the series

$$b_t = \beta(s_t - s_{t-1}) + (1 - \beta)b_{t-1}$$

Smooth the **trend** of the series

$$\hat{x}_{t+1} = s_t + b_t$$

**Future prediction** of the series = sum of value and trend

# Triple exponential moving average

$$s_t = \alpha(x_t - c_{t-L}) + (1 - \alpha)(s_{t-1} + b_{t-1})$$

Smooth the **value** of the series

$$b_t = \beta(s_t - s_{t-1}) + (1 - \beta)b_{t-1}$$

Smooth the **Trend** of the series

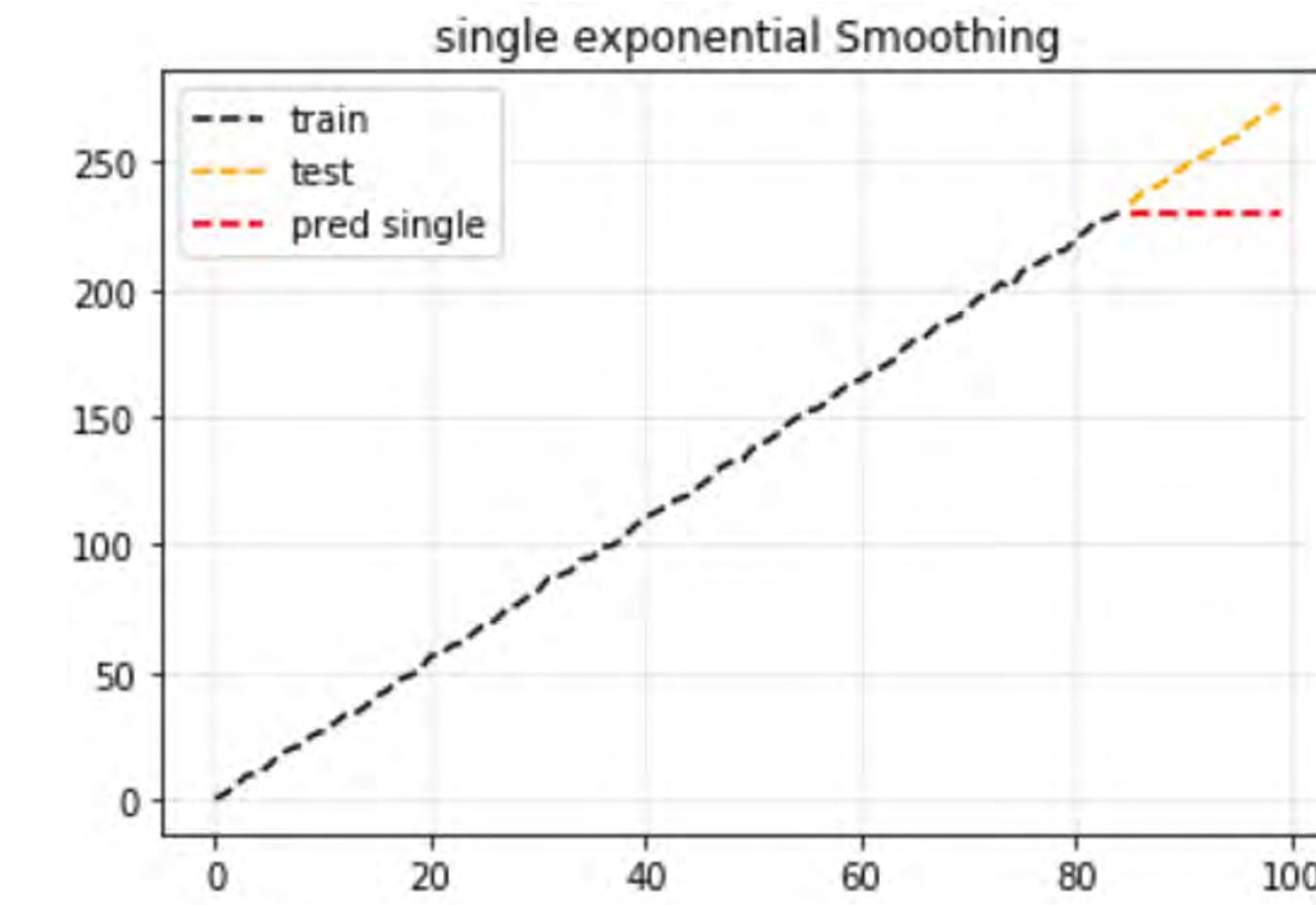
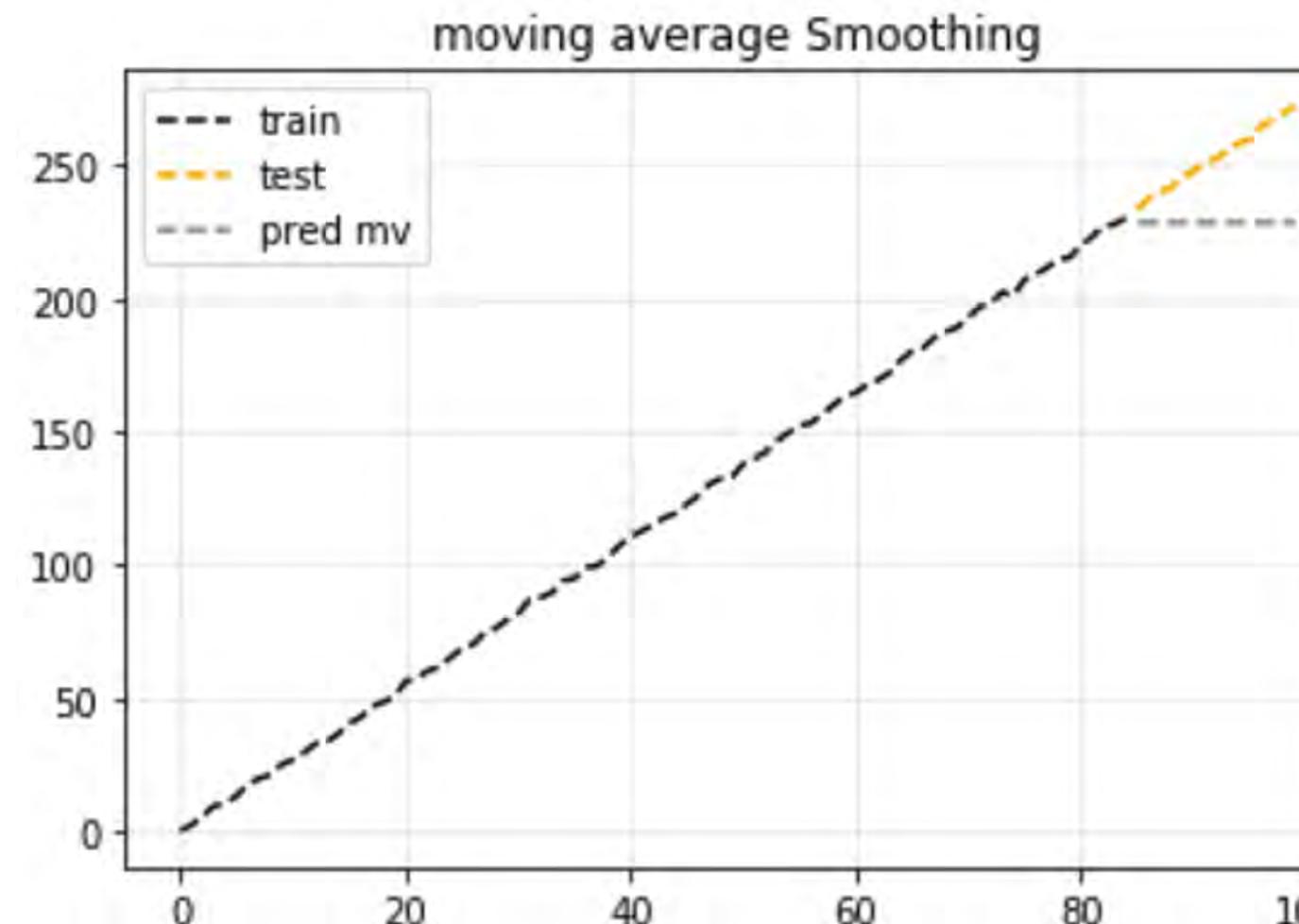
$$c_t = \gamma(x_t - s_{t-1} - b_{t-1}) + (1 - \gamma)c_{t-L}$$

Smooth the **seasonality** of the series

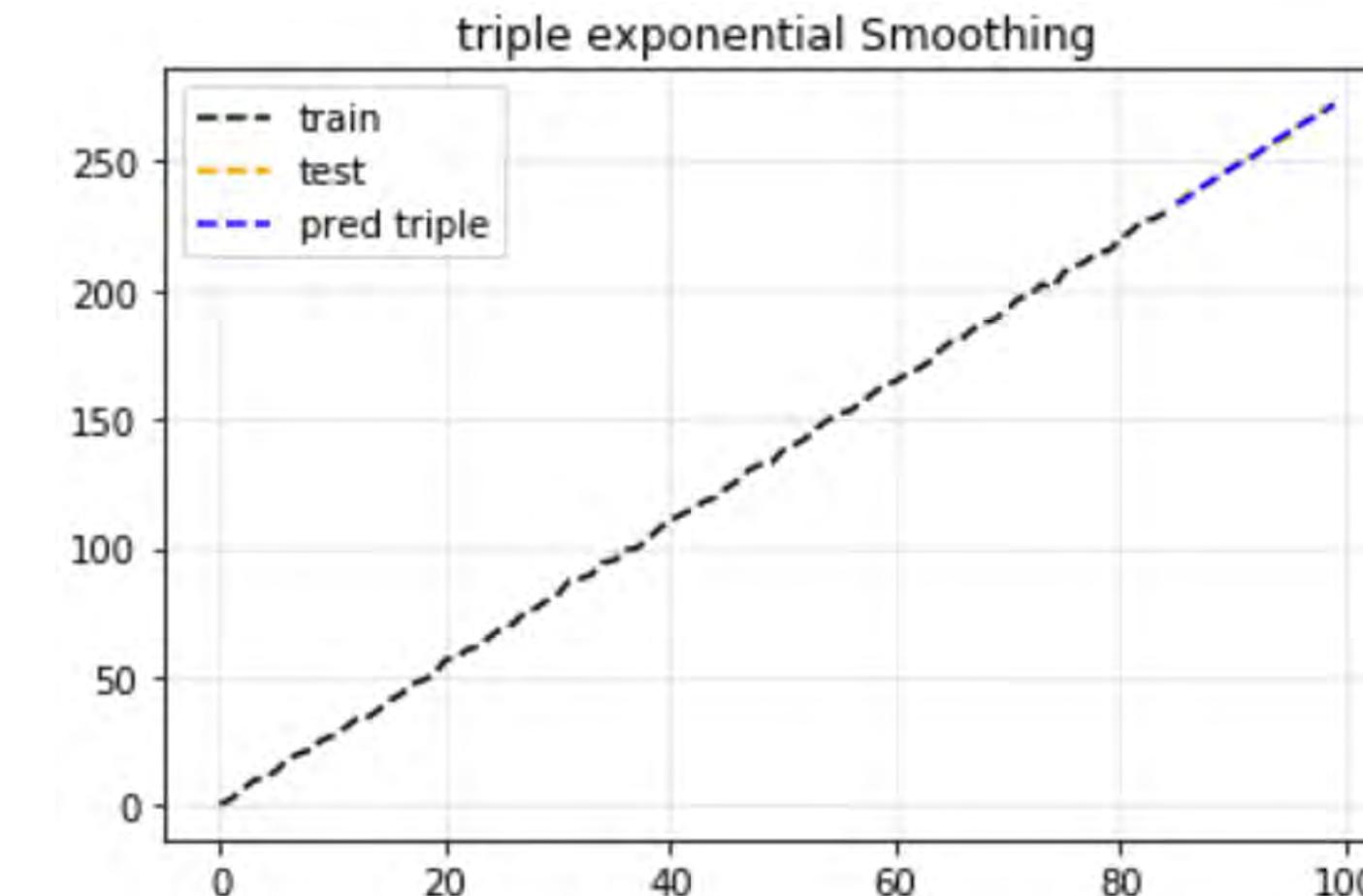
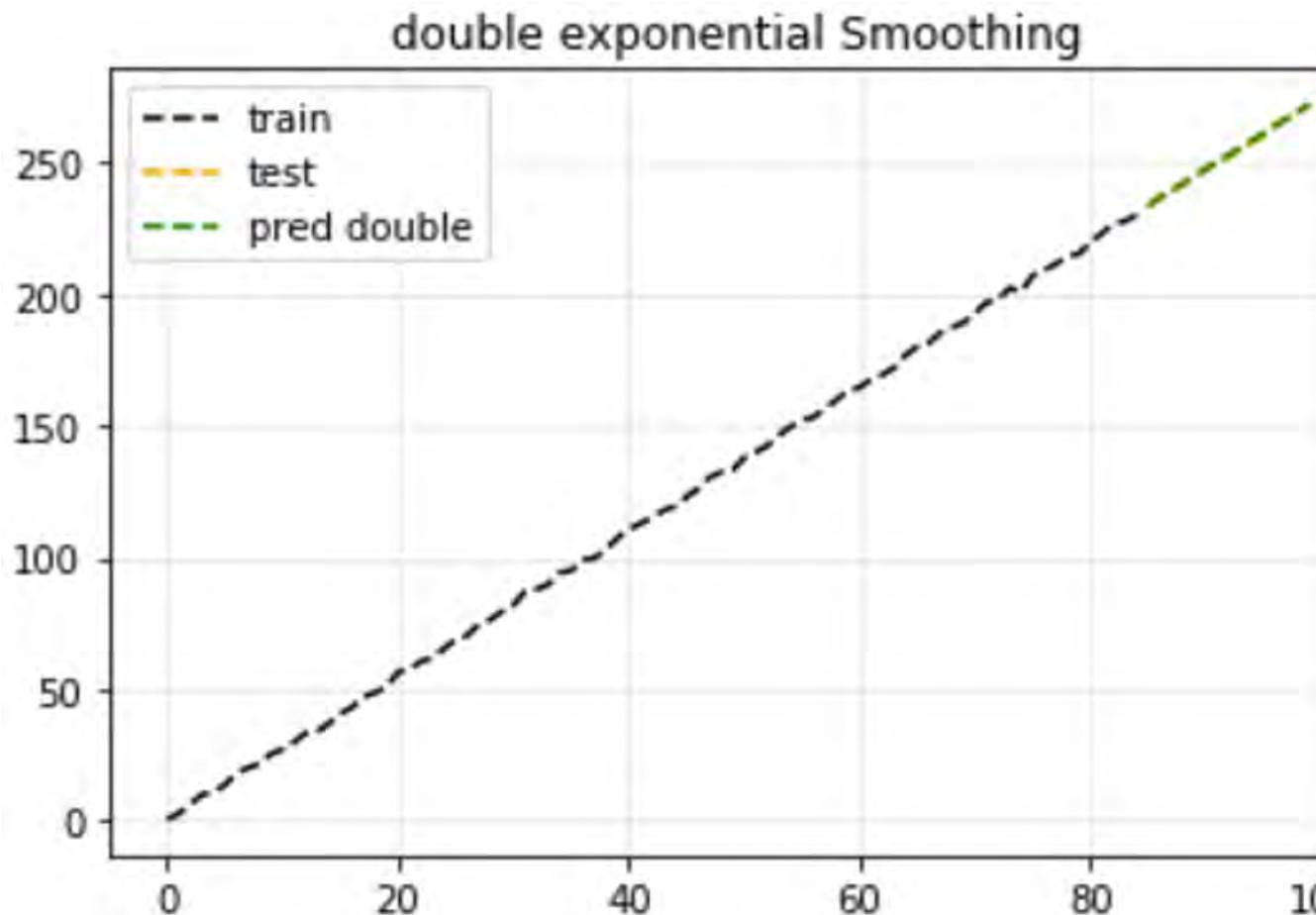
$$\hat{x}_{t+m} = (s_t + mb_t)c_{t-L+(m-1) \bmod L}$$

L = Length of time of the seasonality

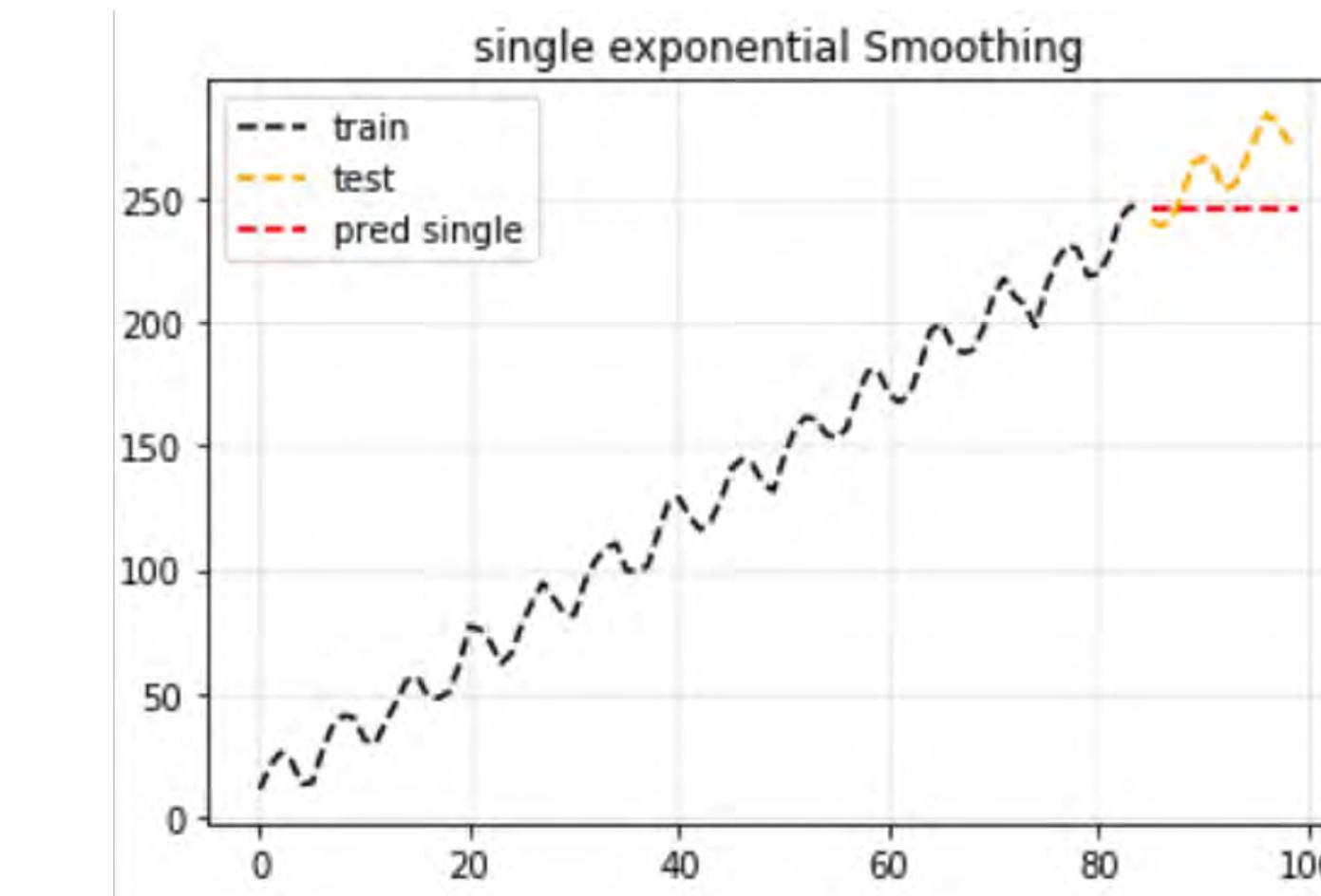
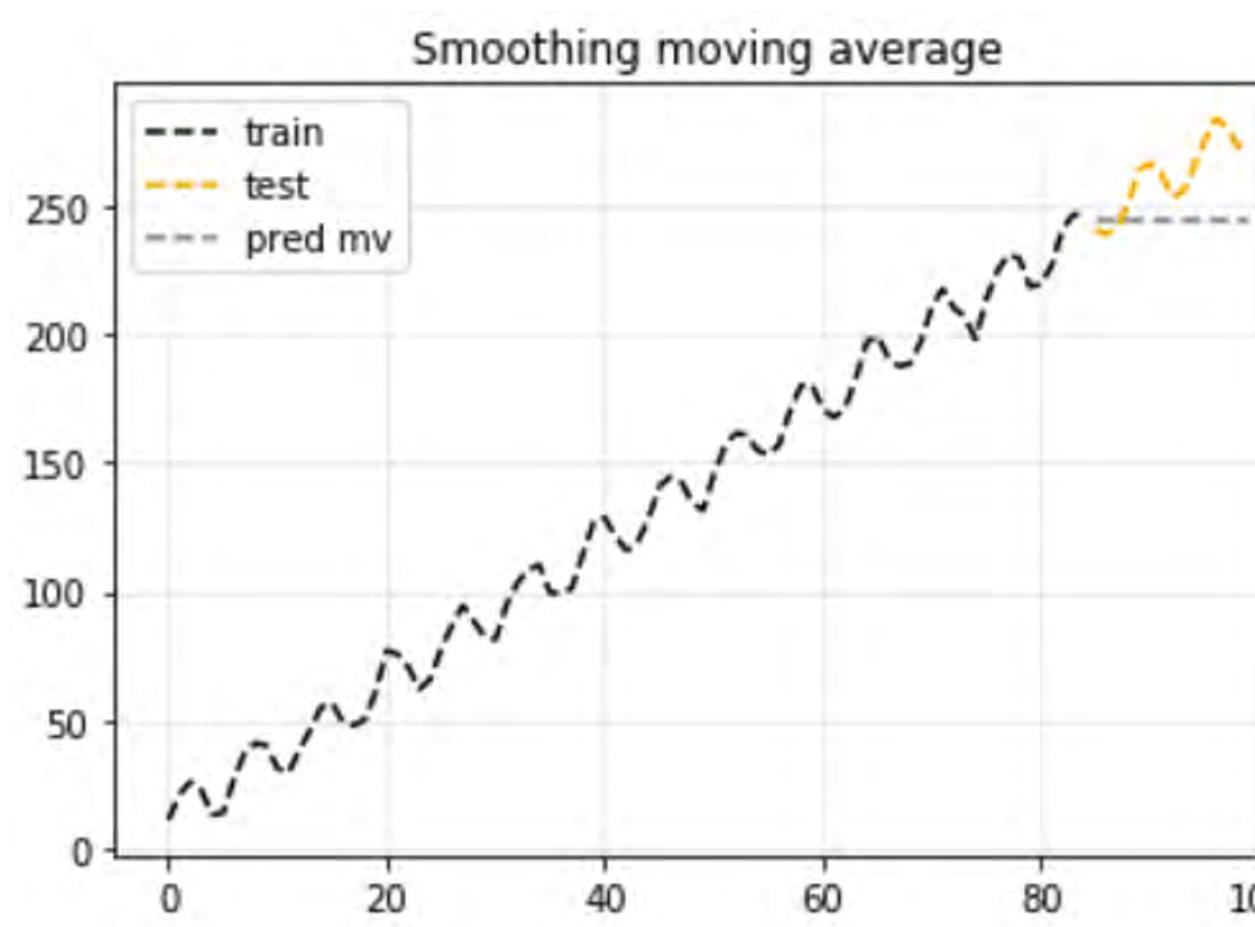
# Prediction with smoothing (trend)



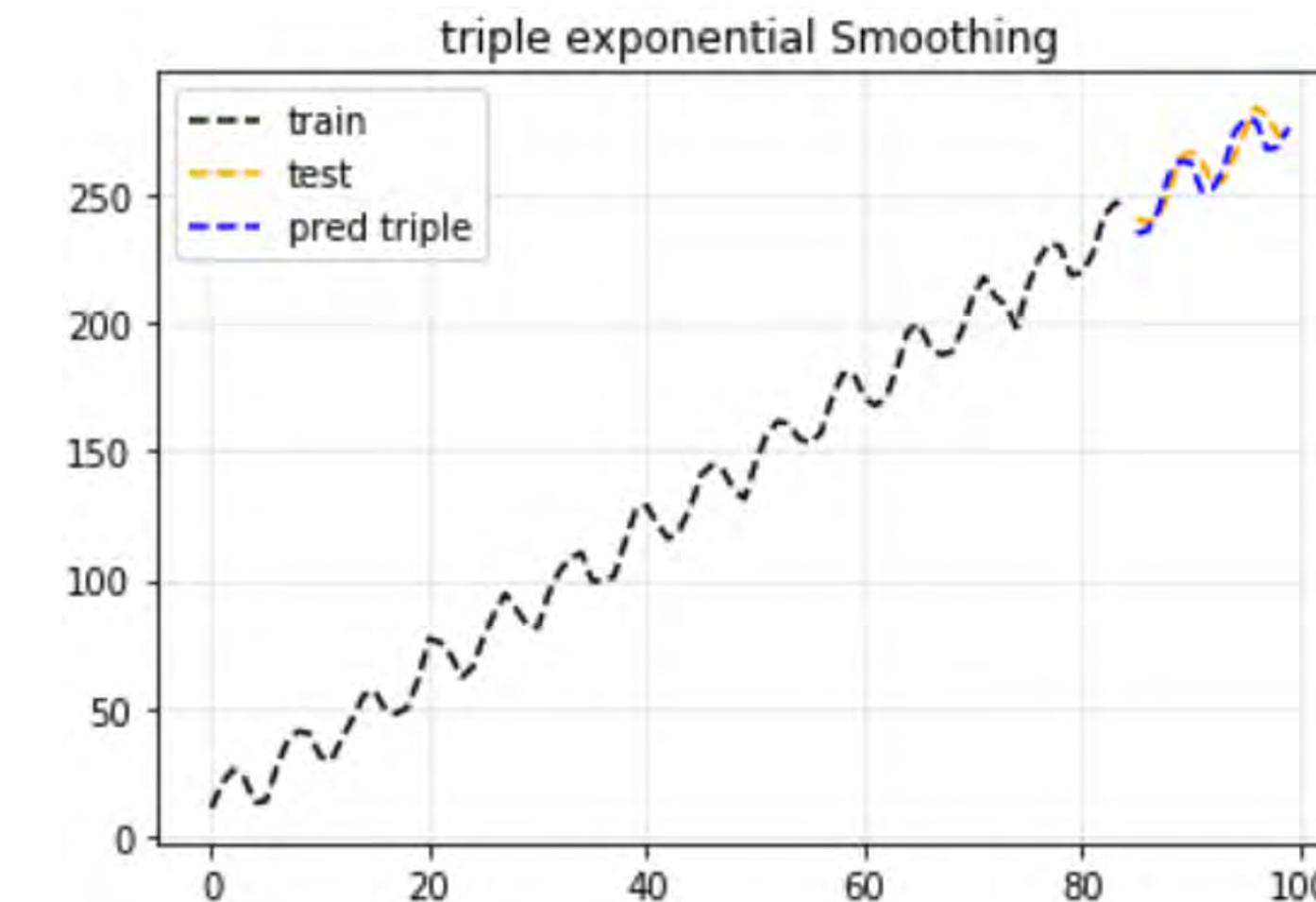
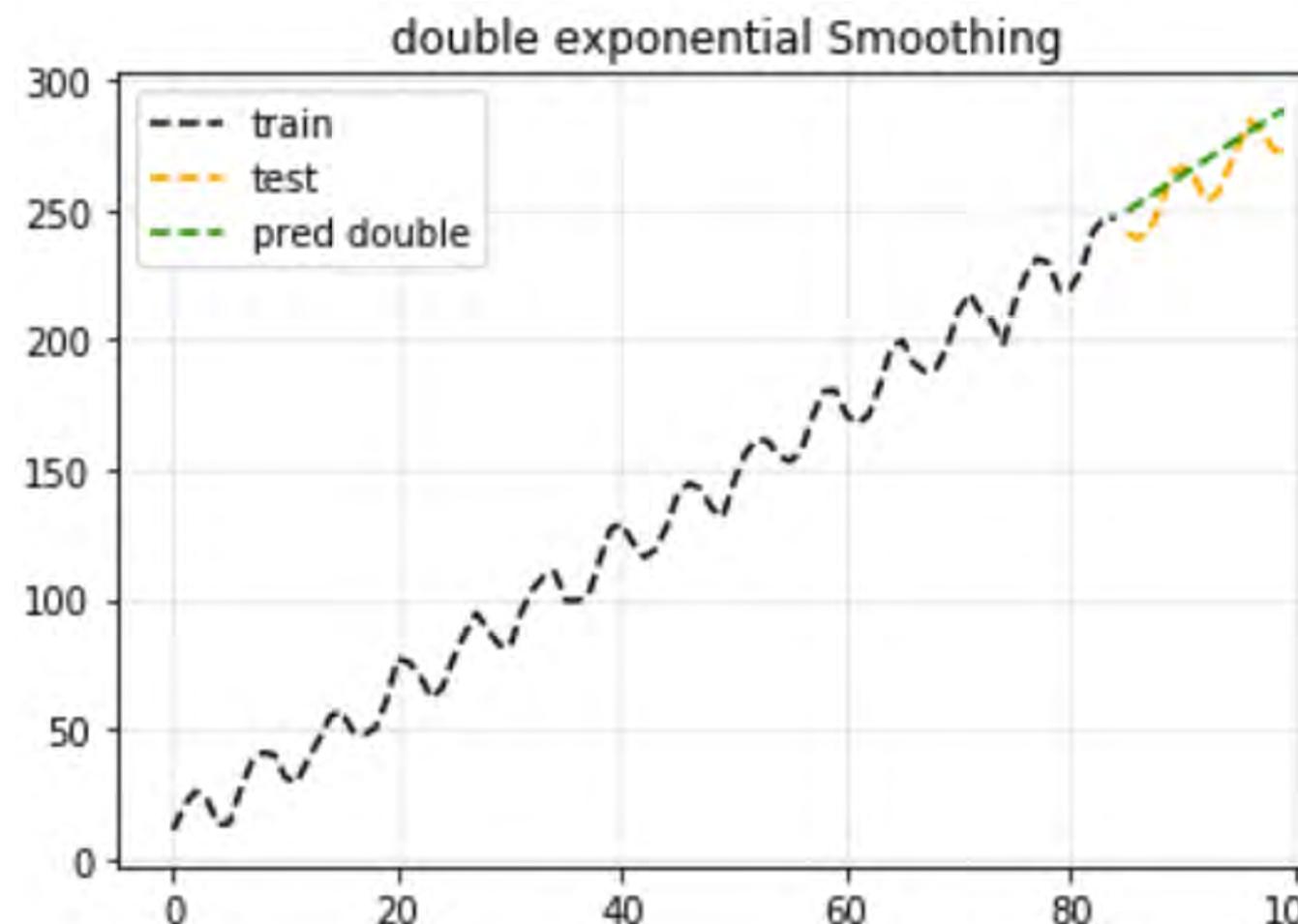
# Prediction with smoothing (trend)



# Prediction with smoothing (trend + seasonal)



# Prediction with smoothing (trend + seasonal)



# Conclusion

