When is Poisson a good approximation for binomial (HT [www.oxfordmathcenter.com/drupal7/node/297](http://www.oxfordmathcenter.com/drupal7/node/297) for getting me started):

Binomial PMF:

Substitute , and expand the binomial coefficient:

Rearrange terms, simplify factorials, and split the final term (N.B., if k = 0, all but last two terms are 1 by above binomial PMF):

As a reminder, Poisson PMF looks like this:

There are at least two ways for the binomial to approximate the Poisson,

1. (in a binomial context, this means with arbitrary n), arbitrary k
2. ,

In case 1, a few things happen:

1. Final two terms approach 1 in binomial PMF
2. in Poisson PMF
3. (really ) dominates remaining terms in both PMFs, with minor effects from n and k
4. For small k, the two distributions are nearly equal, as k increases, the approximations diverge slightly, but probabilities are so low that the relative effect is minimal (i.e. P(X = x) is generally below any reasonable p-value cutoff)
5. We’re generally incapable of ever nearing the mean for smaller values of n

In case 2, we need to be a bit more careful:

1. Since , second to last term approaches
2. Since , first term in binomial PMF approaches 1
3. Can decide if binomial is approximated by Poisson with two criteria:
   1. (the first term in the binomial PMF)
4. Approximations are much less exact than in case 1 if k is near the mean