Department of Electrical and Electronics Engineering
National Institute of Technology, Tiruchirappalli
Project Report on
EEPE 18 – Digital Signal Processing
January 2021 Session - Section 'A'

FILTER DESIGN

DSP MINI-PROJECT

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CONTENTS

BANDPASS FILTER DESIGN

1. AIM:

To design a filter centered at 50 Hz, with at least 60 dB attenuation at 25 Hz and 75 Hz. We assume the sampling frequency to be 500 Hz.

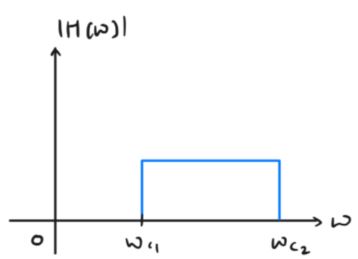
2. THEORY:

Digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. They are implemented by logical components/gates. In this project we try to design a bandpass filter for the given specs.
There are two board categories of filter based on behavior of impulse response – IIR and FIR. In our project we use IIR filter design methods due to following reasons –

1. IIR has lower sidelobes in stopband.
2. Computationally IIRs are easier.
3. Requires less memory.

Major advantage FIR has over IIR is that it has linear phase response. However, due to higher elimination of stopband frequencies IIR was preferred over FIR in cases where linear phase is not required. In our project we choose IIR over FIR due to the control it gives us over stopband and passband attenuations.

2.1 Ideal Filter:



Ideally, we want frequency response of our filter to look like figure given on the left. However, this is not possible due restriction placed by causality. Our system needs to be causal system for us to be able apply all the mathematical tools we learnt. The impulse response of this filter (fig. left) comes out to be non-causal. Thus, due causality restrictions we cannot get ideal frequency response. For any frequency response to be causal it must follow *Paley Wiener Criterion* [1] – if h(n) = 0, for all n<0. Then it should follow [1]. It acts like a checking mechanism for causality.

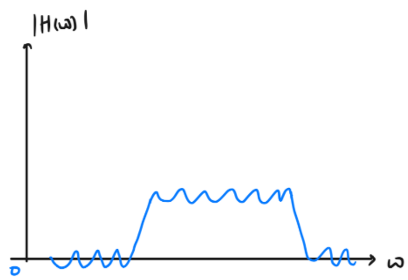
$$\int_{-\pi}^{\pi} |\, \boldsymbol{ln}\, |\boldsymbol{H}(\omega)||\, \partial\omega < \infty \quad [1]$$

2.2. Practical Filter:

From the above discussions we know some restrictions are applied by the causality of the system. One of the most important implication of equation [1] is that magnitude function of H(ω) cannot be zero over a band of frequencies (only some finite points). Hence, we will get frequency response like the figure given below. Moreover, due gibs phenomenon we have ripples in response and as the order of the filter increases so does the ripples. Hence, due to Gibb's phenomenon response of the filter can't be constant over a band of frequencies. However, this effect is very prominent in higher order filters. Due to causality, there has to be a strong relation between $H_R(\omega)$ and $H_I(\omega)$—discrete Hilbert Transform. Finally, we know our system is LTI causal system which can be represented by:

$$y(n) = -\sum_{k=1}^{N} a[k]\, y(n-k) + \sum_{k=}^{M-1} b[k]\, x(n-k) \qquad [2]$$

$$H(\omega) = \frac{\sum_{k=1}^{M-1} b[k]\ e^{-j\omega k}}{1+\sum_{k=1}^{N} a[k] e^{-j\omega k}} \qquad [3]$$



2.3 Filter Design Technique (IIR):

Now, after having reviewed the restrictions of LTI causal system, we move to design of the filter. Design of IIR filter design methodology is to first design analog filters and then maps the continuous frequencies to discreate domain which finally converts the filter to discrete domain. What we get is approximation of analog filter. IIR filter have infinite-duration impulse responses, hence they can be matched to analog filters, all of which generally have infinitely long impulse responses. The basic techniques of IIR filter design transforms well-known analog filters into digital filters using complex-valued mapping. So basically, we first make the desired the filter in analog domain and map it to discrete domain. This is mainly done by three methods – Backward/Forward Difference, Impulse Variance, Bilinear Transform.

Here we use IIR filter design by Bilinear Transform method. As other methods suffer factors like aliasing etc. and mostly appropriate for a class of filters. On the other hand, Bilinear transform can work any specs and is comfortable with any order of the filter.

Bilinear Transform is a conformal mapping of jΩ axis (of S-plane) into the unit circle in the Z-plane. The bilinear transform leads to highly nonlinear mapping but it maps all continuous domain frequencies to discrete domain (other methods only do not map the whole continuous domain). Also, note the IIR filter cannot have a linear phase response, from equation [4] it clear that for IIR filter to have linear phase it would have pole outside unit circle which is not possible for IIR filter as all methods maps analog frequencies inside the unit circle.

$$H(Z) = \pm Z^{-D} H(Z^{-1}), \text{ where D: delay} \quad [4]$$

2.3.1. Solving by Bilinear Transform:

First, we convert distracted filter spec to analog by using equation [6] or [7]. Here we are going Butterworth filter. Then, we would make prototype lowpass filter and convert it to bandpass.

$$\omega = 2 \arctan \frac{\Omega T}{2} \quad [6]$$

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2} \quad [7]$$

The order of the filter is determined by [8]. After we have the order is determined and we have H(s) from standard set of formulas set for Butterworth filter. Then we would convert the lowpass filter to bandpass by frequency transformation techniques [9]. This gives the bandpass analog filter transfer function $H_a(s)$. we convert it to digital domain by bilinear transform [10]. Thus giving, H(Z) the digital bandpass filter transfer function.

$$N = \frac{1}{2} \frac{\log \frac{(\frac{1}{A_2^2} - 1)}{(\frac{1}{A_1^2} - 1)}}{\log \frac{\Omega_s}{\Omega_p}} \quad [8]$$

$$s \rightarrow \Omega_p \frac{s(\Omega_l - \Omega_u)}{s^2 + \Omega_u \Omega_l} \quad [9]$$

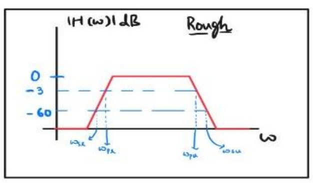
$$Bilinear\ Tranform: s \rightarrow \frac{2}{T}\,\frac{1-Z^{-1}}{1+Z^{-1}}\ [10]$$

3.NUMERICAL COMPUTATION:

$A_p = -3dB \quad f_{sl} = 25Hz \quad f_{pl} = 40Hz \quad F_s = 500Hz$

$A_s = -60\ dB \quad f_{su} = 75Hz \quad f_{pu} = 60Hz$

$\omega = \frac{2\pi f}{F_s} \Rightarrow \omega_{sl} = 0.1\pi \quad \Rightarrow \omega_{su} = 0.3\pi \quad \Rightarrow \omega_{pl} = 0.16\pi \quad \Rightarrow \omega_{pu} = 0.24\pi$



$\Omega = \tan\frac{\omega}{2} \Rightarrow \Omega_{sl} = 0.158384 \quad \Rightarrow \Omega_{su} = 0.5095254$

$\Rightarrow \Omega_{pl} = 0.256756 \quad \Rightarrow \Omega_{pu} = 0.395928$

Protopype to get low pass:

$\Omega_p = 1 \qquad \Omega_s = \frac{\Omega_{su}^2 - \Omega_{pl}\cdot\Omega_{pu}}{(\Omega_{pu}-\Omega_{pl})\,\Omega_{su}} = 2.27549372$

$$N = \frac{\log\left([10^{|A_s|/10} - 1] / [10^{|A_p|/10} - 1]\right)}{2 \times \log\left(\frac{\Omega_s}{\Omega_p}\right)} = 6.81$$

$\Rightarrow N \simeq 7$

Transfer function of low pass prototype: $\Omega_c = 1$

$$\Rightarrow H_a(s)_{lp} = \left[\frac{1}{S_n+1}\right] \prod_{k=1}^{6} \frac{1}{(S_n^2 + b_k S_n + 1)} \qquad \text{where } S_n = \frac{s}{\Omega_c}$$

$$b_k = 2\sin\left[\frac{2(k-1)\pi}{2N}\right]$$

$$\Rightarrow H_a(s)_{lp} = \left(\frac{1}{S+1}\right)\left\{\left(\frac{1}{S^2+1}\right)\left(\frac{1}{S^2+0.87S+1}\right)\left(\frac{1}{S^2+1.56s+1}\right)\left(\frac{1}{S^2+1.95s+1}\right)\left(\frac{1}{S^2+1.956s+1}\right)\left(\frac{1}{S^2+1.56s+1}\right)\right\}$$

$B = \Omega_{pu} - \Omega_{pl} = 0.139$

$\Omega_o^2 = \Omega_{pu} \times \Omega_{pl} = 0.10165$

$\Omega_c = \frac{\Omega_s}{(10^6 - 1)} = 2.27 \times 10^{-6}$

$\Omega_u = 0.0505$

$\Omega_l = 0.0504$

to convert $H_a(s)_{lp} \longrightarrow H_a(s)_{bp} \Rightarrow s \longrightarrow \Omega_p \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)} \Rightarrow s \longrightarrow \frac{s^2 + 0.255}{(0.001)s}$

then: $s \rightarrow \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$

This will give us the transfer function of the bandpass filter.

4. SIMMULATION:

Here, we have used MATLAB and signal processing toolbox in MATLAB to give us bit more control over filters design.

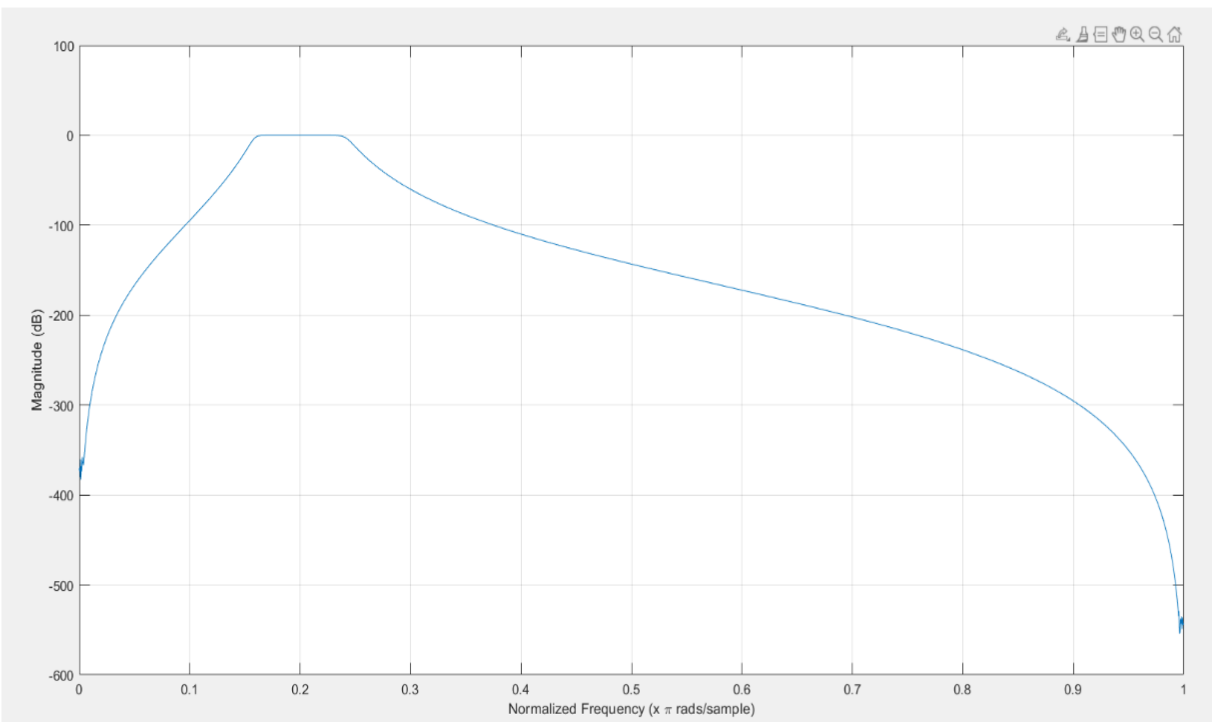
4.1. MATLAB CODE:

```
clear all;
close all;

%% Filter Design

A_p = 3;     % passband attenuation in dB
A_s = 60;    % stopband attenuation in dB

ws = [ 0.1*pi , 0.3*pi ];    % stopband frequency
wp = [0.16*pi , 0.24*pi ];   % passband frequency

[ n , wc ] = buttord( wp/pi ,ws/pi , A_p , A_s ); % calculation of
order of the filter(n) and cutt off(wn)

[ b , a ] = butter( n , wc , 'bandpass'  );        % Transfer function

w = 0:0.001:pi;

%% Digital Domain

%z domain conversion

    [ h , ph ] = freqz(b , a , w);       % freq response in digital
domain
    magn = 20*log10(abs(h));
    phase = angle(h);                    % calculates phase angle


    subplot(1,3,1);
    title('MAGNITUDE RESPONSE');
    plot(ph/pi, magn);
    xlabel('Normalized Frequency (x \pi rads/sample)');
    ylabel('Magnitude (dB)');
    grid on;

    subplot(1,3,3);
```

```
title('POLE-ZERO PLOT');
zplane(b,a);
xlabel('Real part');
ylabel('Imaginary part');
grid on;

subplot(1,3,2);
title('PHASE RESPONSE');
plot(ph/pi,phase);
xlabel('Normalized Frequency (x \pi rads/sample)');
ylabel('Phase');
grid on;
hold on;
```

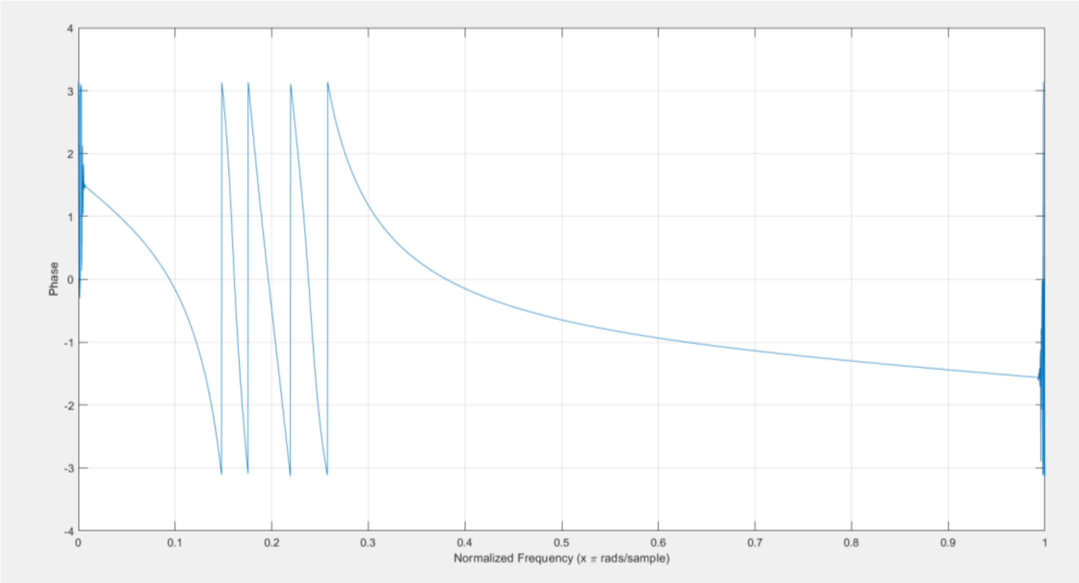
4.2. RESULTS:

- *Magnitude response:*

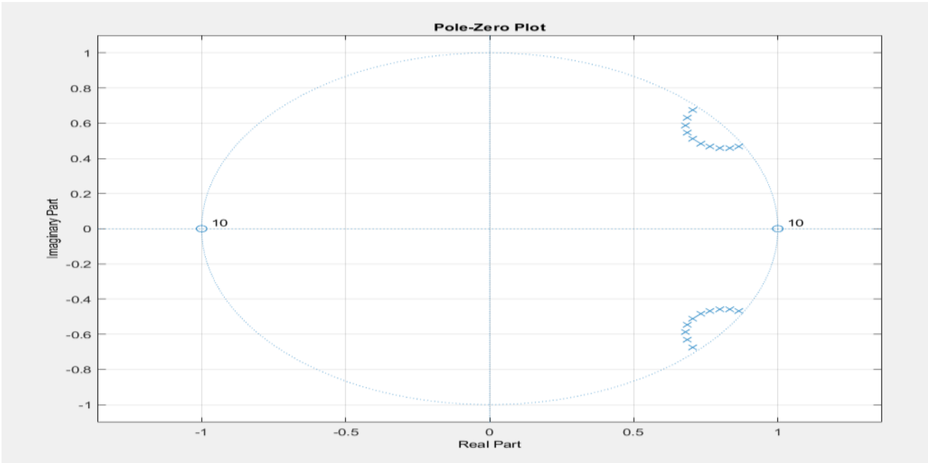


We can see that the attenuation at stopband at 25 Hz (0.1) and 75 Hz (0.3) is less than -60 dB, Hence, we achieved the required specs.

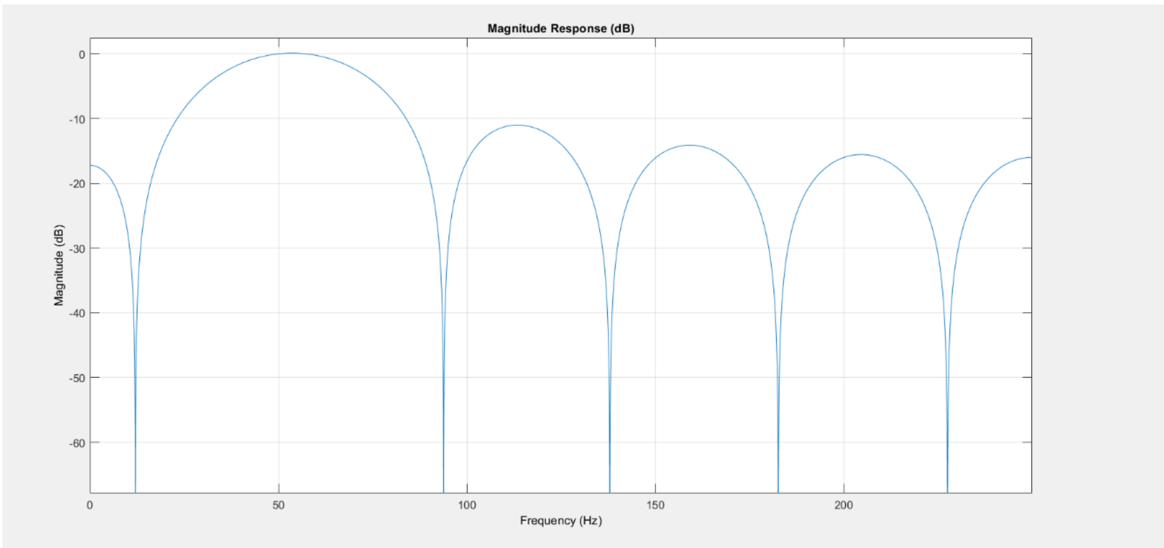
- *Phase Response:*



- Pole-Zero Plot:



- Comparison with FIR (window- rectangular):



We have also tried to design filter with specs but FIR, shown in the figure above. As we can clearly see the differences, FIR filter of somewhat relative specs generates sidelobes with greater heights basically lesser attenuations at the stopband frequencies, but FIR has linear phase response. This FIR filter was made in Filter Designer (MATLAB's signal processing toolbox). But FIR has linear phase response.

5. RESULTS AND CONCLUSIONS:

We have successfully developed bandpass filter centered at 50 Hz and it attenuates frequencies 25 Hz and 75 Hz by ~ 60 dB.

We also did brief comparison on FIR and IIR filters. In our analysis both FIR and IIR filters give BIBO stable response ***(from Paley Wiener Criterion [1]).*** If the h(n) follows ***Paley Wiener Criterion***, then it is also square integrable therefore **BIBO** stability is confirmed.
However, choosing either type of filter over the other require deeper understanding of the filter is supposed to do. For example, if we need a filter that should have linear phase response (***discussed in section 2.3***) and we do not need much attenuation of stopband frequencies than the choice of filter should be FIR. Hence while choosing the filter, one must keep **table (below)** in mind. In our project we did not have the requirement of linear phase response, we made attenuation of stopband frequencies our priority hence we chose IIR approach. Also, from above we see that IIR filter gives more control over the

stopband and passband- frequencies and attenuations. Due these reasons we choose IIR filter to design filter.

FIR	IIR
ADVATAGES:-	ADVATAGES:-
* Linear phase response. * Fixed point performance. * Numerial stability as it doestn't depend on feedback.	* Low memory requirment. Hence, low cost. * Low latency. * Analog equivalent.
DISADVANTAGES:	DISADVANTAGES:
* Higher computation. * Higher Memory requirment * Higher latency. * No analog equivalent.	* Non-linear phase response. * Require more analysis. * Less numerical stability than FIR.

6. INFERENCES:

- Class Lectures of EEPE-18.
- Digital Signal Processing: Principles, Algorithms and Application by John Proakis.
- NPTEL Lecture by Prof. S.C. Dutta Roy.
- Discrete-time Signal Processing by Alan V. Oppenheim.