

Homework 31.

#1.

$$u_1 = (1, 2, 2, -1)$$

$$u_2 = (1, 1, -5, 3)$$

$$u_3 = (3, 2, 8, -7)$$

$$\Rightarrow v_1 = (1, 2, 2, -1)$$

$$v_2 = u_2 - \frac{(u_2, v_1)}{(v_1, v_1)} v_1 = (1, 1, -5, 3) - \frac{-10}{10} (1, 2, 2, -1) = (2, 3, -3, 2)$$

$$v_3 = u_3 - \frac{(u_3, v_1)}{(v_1, v_1)} v_1 - \frac{(u_3, v_2)}{(v_2, v_2)} v_2 = (3, 2, 8, -7) - \frac{30}{10} (1, 2, 2, -1) - \frac{-26}{26} (2, 3, -3, 2) = (2, -1, -1, -2)$$

$$\langle u_1, u_2, u_3 \rangle : \begin{cases} v_1 = (1, 2, 2, -1) \\ v_2 = (2, 3, -3, 2) \\ v_3 = (2, -1, -1, -2) \end{cases}$$

#2.

$$u_1 = (1, -2, 2, -3), \quad u_2 = (2, -3, 2, 4)$$

Базис $\langle u_1, u_2 \rangle^\perp$:

$$\begin{pmatrix} 1 & -2 \\ -2 & -3 \\ 2 & 2 \\ -3 & 4 \end{pmatrix} \xrightarrow{II+I} \begin{pmatrix} 1 & -2 \\ 0 & -1 \\ 0 & 1 \\ 0 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$u_1 = (0, 0, 0, 1) \quad u_2 = (0, 0, 1, 0)$$

Ортогонализируем u_1, u_2 .

$$v_3 = u_1 - \frac{(u_1, u_1)}{(u_1, u_1)} u_1 - \frac{(u_1, u_2)}{(u_2, u_2)} u_2 = (0, 0, 0, 1) - \frac{13}{18} (1, -2, 2, -3) - \frac{4}{33} (2, -3, 2, 4) = (-5, 2, 6, 1)$$

$$v_4 = (0, 0, 1, 0) - \frac{2}{18} (1, -2, 2, -3) - \frac{2}{33} (2, -3, 2, 4) = (-2, -2, 1, 0)$$

$$-\frac{6}{66} (-5, 2, 6, 1)$$

#3.

$$u_1 = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right)$$

$$u_2 = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right)$$

Bazuc $\langle u_1, u_2 \rangle^\perp$:

$$\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \end{pmatrix} \xrightarrow{\substack{II \leftrightarrow I \\ I - 2II}} \begin{pmatrix} 1 & 2 & -2 \\ 0 & -3 & 6 \end{pmatrix} \xrightarrow{\substack{II \cdot (-\frac{1}{3}) \\ I - 2II}} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{pmatrix}$$

$$u_1 = (-2, 2, 1) \Rightarrow u_3 = \left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right)$$

#5.

$$u_1 = (1, 0, 2, 1), \quad u_2 = (2, 1, 2, 3), \quad u_3 = (0, 1, -2, 1)$$

$$\begin{pmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 2 & 3 \\ 0 & 1 & -2 & 1 \end{pmatrix} \xrightarrow{\substack{II - 2I \\ II - III}} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \end{pmatrix}$$

$$\text{Bazuc } U^\perp: \begin{pmatrix} -1 & -1 & 0 & 1 \\ -2 & 2 & 1 & 0 \end{pmatrix}$$

#4.

$$\{1, x, x^2, x^3\} \quad (f, g) = \int_0^1 f(x)g(x)dx$$

$$1) (1, x) = \int_0^1 x dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$$

$$2) (1, 1) = \int_0^1 1 dx = x \Big|_0^1 = 1$$

$$3) (1, x^2) = \int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}$$

$$4) (1, x^3) = \int_0^1 x^3 dx = \frac{x^4}{4} \Big|_0^1 = \frac{1}{4}$$

$$5) (x, x) = \int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}$$

$$6) (x, x^2) = \int_0^1 x^3 dx = \frac{x^4}{4} \Big|_0^1 = \frac{1}{4}$$

$$7) (x, x^3) = \int_0^1 x^4 dx = \frac{x^5}{5} \Big|_0^1 = \frac{1}{5}$$

$$8) (x^2, x^2) = \int_0^1 x^4 dx = \frac{x^5}{5} \Big|_0^1 = \frac{1}{5}$$

$$9) (x^2, x^3) = \int_0^1 x^5 dx = \frac{x^6}{6} \Big|_0^1 = \frac{1}{6}$$

$$10) (x^3, x^3) = \int_0^1 x^6 dx = \frac{x^7}{7} \Big|_0^1 = \frac{1}{7}$$

$$G = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

- Answer