Homework 8a.

#1.

- $\Delta P(n) = P(n+1) - P(n)$
- $SP(n) = P(n-1)$

$$\Delta S = (P(\cancel{n+1}) - P(n))\cdot P(\cancel{n-1}) = P(\cancel{n-1})\cdot P(n+1) - P(\cancel{n-1})\cdot P(n) =$$

$$= P(\cancel{n-1})\cdot(P(n+1) - P(n)) = \cancel{\Delta S}$$

$$\Delta(SP(n)) = \Delta P(n-1) = P(n) - P(n-1)$$

$$S(\Delta P(n)) = S(P(n+1) - P(n)) = P(n) - P(n-1)$$

$\Rightarrow$ $\boxed{\text{коммутируют.}}$

#2.

$P(n)$ – мн-н степени m с коэфф. α.

Старший коэф. в $\Delta^t P(n)$ при $0 \le t \le m$ – ? $\Delta^m n^m$ – ?

$P(n) = \alpha \cdot n^m + \beta$ (β не будет рассматривать)

$$\Delta^0 = \Delta^0 P(n) = \alpha \cdot n^m$$

$$\Delta P(n) = \Delta(\alpha n^m) = (\alpha n^m)' = \alpha m \cdot n^{m-1}$$

$$\Delta^2 P(n) = \Delta^2(\alpha n^m) = (\alpha \cdot n^m)'' = (\alpha m \cdot n^{m-1})' = \alpha m \cdot (m-1) n^{m-2}$$

...

$$\Delta^m P(n) = \alpha \cdot m \cdot (m-1) \cdot \ldots \cdot (m-m+1) n^{m-m} = \alpha \cdot m! \cdot n^0 = \boxed{\alpha \cdot m!}$$

Тогда в $\Delta^t P(n) = \alpha \cdot \frac{m!}{(m-t)!} \cdot n^{m-t} \Rightarrow$ ст. коэф. $\boxed{\alpha \cdot \frac{m!}{(m-t)!}}$

#3.

$$\sum_{k=0}^{m} (-1)^k C_m^k (m-k)^m = m!$$

Из задачи 2, $\Delta^m(n^m) = m! \cdot n^0 = m!$

$$\Delta^m(n^m) = \sum_{k=0}^{m} (-1)^k C_m^k (n+k)^m$$

При $n=0$: $\underset{\underset{m!}{\|}}{\Delta^m(0^m)} = \sum_{k=0}^{m} (-1)^k C_m^k k^m$

Замена $m-k=j$, тогда $k=m-j$, $k=\overline{0,m} \Rightarrow j=\overline{0,m}$:

$$\Delta^m(n^m) = m! = \sum_{j=0}^{m} (-1)^{m-j} \cdot C_m^{m-j} (m-m+j)^m = \sum_{j=0}^{m} (-1)^j C_m^j \cdot j^m = m! \ \boxed{\text{ч.т.д.}}$$

#4.

$\Delta \frac{a_n}{b_n}$ Замена: $\frac{a_n}{b_n} = q_n$. Знаем: $a_{n+1} = a_n + \Delta a_n$

$$\Delta q_n = q_{n+1} - q_n = \frac{a_{n+1}}{b_{n+1}} - \frac{a_n}{b_n} = \frac{b_n a_{n+1} - a_n b_{n+1}}{b_n \cdot b_{n+1}} =$$

$$= \frac{b_n(a_n + \Delta a_n) - a_n(b_n + \Delta b_n)}{b_n b_{n+1}} = \frac{b_n \Delta a_n - a_n \Delta b_n}{b_n \cdot b_{n+1}}$$

ч.т.д.

#5.

$\Delta \frac{n^2}{(-3)^n}$ Здесь $a_n = n^2$, $b_n = (-3)^n$

А ещё, решив #4, мы поумнели и теперь знаем, что $\Delta \frac{a_n}{b_n} = \frac{b_n \Delta a_n - a_n \Delta b_n}{b_n \cdot b_{n+1}}$. Тогда:

$$\Delta \frac{n^2}{(-3)^n} = \frac{(-3)^n \cdot \Delta n^2 - n^2 \cdot \Delta(-3)^n}{(-3)^n \cdot (-3)^{n+1}} = \frac{(-3)^n \cdot (2n+1) - n^2((-3)^{n+1} - (-3)^n)}{(-3)^{2n+1}} =$$

$$= \frac{(-3)^n(2n+1) + (-3)^n \cdot 4n^2}{(-3)^{2n+1}} = \boxed{\frac{4n^2+2n+1}{(-3)^{n+1}}}$$

#6.

$$\Delta \cos(\alpha n + \beta) = \cos(\alpha(n+1) + \beta) - \cos(\alpha n + \beta) =$$

$$= -2\sin\left(\frac{\alpha(n+1) + \beta + \alpha n + \beta}{2}\right) \cdot \sin\left(\frac{\alpha(n+1) + \beta - \alpha n - \beta}{2}\right) =$$

$$= \boxed{-2\sin\left(\alpha\left(n+\frac{1}{2}\right) + \beta\right) \cdot \sin\frac{\alpha}{2}}$$

#7.

$$\Delta n^4 = (n+1)^4 - n^4 = n^4 + 4n^3 + 6n^2 + 4n + 1 - n^4 = 4n^3 + 6n^2 + 4n + \underline{1}$$

$$\Delta^2 n^4 = \Delta(4n^3 + 6n^2 + 4n + 1) = 4(n^3 + 3n^2 + 3n + 1) + 6(n^2 + 2n + 1) + 4n + 5 -$$

$$- 4n^3 - 6n^2 - 4n - 1 = 12n^2 + 20n + 4n + 14 = 12n^2 + 24n + \underline{14}$$

$$\Delta^3 n^4 = \Delta(12n^2 + 24n + 14) = 12n^2 + 12 \cdot 2n + 12 + 24n + 24 + 14 - 12n^2 - 24n - 14 =$$

$$= 24n + \underline{36}$$

$$\Delta^4 n^4 = \Delta(24n + 36) = 24n + 24 + 36 - 24n - 36 = \underline{24}$$

$\Delta^k n^4 = 0$ при $k > 4$

$$\sum n^4 = \frac{1}{1!}n + \frac{14}{2!}n^2 + \frac{36}{3!}n^3 + \frac{24}{4!}n^4 = \boxed{n + 7n^2 + 6n^3 + n^4}$$

#8.

(a) $a_n = \dfrac{1}{(n+1)(n+2)} = \dfrac{1}{n+1} - \dfrac{1}{n+2}$

$$\sum a_n = \sum\left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = 1$$

(b) $a_n = \dfrac{n^2}{(-3)^n}$ ....