Семинар 31, 21.05.24 – Бельдиев

$V, (\cdot,\cdot)$

КБШ: $|(x,y)| \le |x|\cdot|y| = \sqrt{(x,x)}\cdot\sqrt{(y,y)}$

$=$: $x \parallel y$; $x = ky$

$\mathbb{R}^n$, $(x,y) = x_1y_1 + x_2y_2 + \ldots + x_ny_n$

#1383. $|x_1y_1 + \ldots + x_ny_n| \le \sqrt{x_1^2 + \ldots + x_n^2}\cdot\sqrt{y_1^2 + \ldots + y_n^2}$

$$(x_1y_1 + \ldots + x_ny_n)^2 \le (x_1^2 + \ldots + x_n^2)(y_1^2 + \ldots + y_n^2)$$

#1384. $C[a;b]$

$f,g \in C[a;b]$; $(f,g) = \int_a^b f(x)\cdot g(x)\,dx$

(1) $(\alpha f_1 + \beta f_2, g) \overset{?}{=} \alpha(f_1,g) + \beta(f_2,g)$

$$\int_a^b (\alpha f_1 + \beta f_2)(x)g(x)\,dx = \alpha\int_a^b f_1(x)g(x)\,dx + \beta\int_a^b f_2(x)g(x)\,dx = \alpha(f_1,g) + \beta(f_2,g)$$

(2) $(f,g) = (g,f)$ – очев.

(3) $(f,f) = \int_a^b f^2(x)\,dx \overset{?}{>} 0$, если $f \ne 0$

$$\left|\int_a^b f(x)g(x)\,dx\right| \le \sqrt{\int_a^b f^2(x)\,dx}\cdot\sqrt{\int_a^b g^2(x)\,dx}$$

$$\int_a^b f(x)g(x)\,dx \le \int_a^b f(x)\,dx\cdot\int_a^b g(x)\,dx$$

#1385. $x, y \in V$

$$\cos\angle(x,y) = \frac{(x,y)}{|x|\cdot|y|}$$

$A(2,4,2,4,2)$ $\quad B(6,4,4,4,6)$ $\quad C(5,7,5,7,2)$

$$|AB| = \sqrt{16+4+16} = \sqrt{36} = 6$$
$$|BC| = \sqrt{1+9+1+9+16} = \sqrt{36} = 6$$
$$|AC| = \sqrt{9+9+9+9} = \sqrt{36} = 6$$

углы равны 60°.

$\dim V < \infty$; $U \subset V$

$U^\perp = \{v \in V \mid (v, u) = 0 \quad \forall u \in U\}$

$\{0\}^\perp = V$; $\quad V^\perp = \{0\}$

$v \in V^\perp \Rightarrow (v, v) = 0$, $v = 0$.

$v_1, v_2 \in U^\perp \Rightarrow \alpha v_1 + \beta v_2 \in U^\perp$

$u \in U$; $(\alpha v_1 + \beta v_2, u) = \alpha(v_1, u) + \beta(v_2, u) = 0$.

$U \subset V$; $\quad \langle u_1, \dots, u_k \rangle = U$

$\langle u_1, \dots, u_k, u_{k+1}, \dots, u_n \rangle = V$ – базисы

$\langle u_1, \dots, u_n \rangle \to \langle w_1, \dots, w_n \rangle$ – о/н базис в V

$\langle u_1, \dots, u_k \rangle$ – о/н базис в U

$U^\perp = \langle w_{k+1}, \dots, w_n \rangle$

$a_1 w_1 + \dots + a_n w_n \in U^\perp$

$(a_1 w_1 + \dots + a_n w_n, w_i) = a_i = 0 \quad \forall i = \overline{1, k}$

$U \cap U^\perp = \{0\}$

#1366. $L \subset \mathbb{R}^4$; $L = (a_1, a_2, a_3)$

$X = (x_1, x_2, x_3, x_4) \in L^\perp$

$$\begin{aligned}(a_1, x) &= \\ (a_2, x) &= \\ (a_3, x) &= \end{aligned}\begin{cases} x_1 + 2x_3 + x_4 = 0 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 0 \\ x_2 - 2x_3 + x_4 = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 2 & 3 \\ 0 & 1 & -2 & 1 \end{pmatrix} \xrightarrow{\text{II}-2\text{I}} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \end{pmatrix}$$

x_1	x_2	x_3	x_4
-2	2	1	0
-1	-1	0	1

$$\Rightarrow \begin{cases} x_1 + 2x_3 + x_4 = 0 \\ x_2 - 2x_3 + x_4 = 0 \end{cases} \qquad x_3 \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

#1367.

$$\begin{pmatrix} 2 & 1 & 3 & -1 \\ 3 & 2 & 0 & -2 \\ 3 & 1 & 9 & -1 \end{pmatrix} \xrightarrow[\text{II}-\text{I}]{\text{III}-\text{II}} \begin{pmatrix} 2 & 1 & 3 & -1 \\ 1 & 1 & -3 & -1 \\ 0 & -1 & 9 & 1 \end{pmatrix} \xrightarrow{\text{I}-2\text{II}} \begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & -1 & 9 & 1 \\ 0 & -1 & 9 & 1 \end{pmatrix} \longrightarrow$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 6 & 0 \\ 0 & -1 & 9 & 1 \end{pmatrix} \qquad \begin{aligned} x_1 &= -6x_3 \\ x_2 &= 9x_3 + x_4 \end{aligned} \qquad \begin{pmatrix} -6x_3 \\ 9x_3 + x_4 \\ x_3 \\ x_4 \end{pmatrix} = x_3 \begin{pmatrix} -6 \\ 9 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$L = \langle (-6, 9, 1, 0), (0, 1, 0, 1) \rangle$

$L^\perp : (x_1, x_2, x_3, x_4) \in L^\perp$

$$\begin{cases} -6x_1 + 9x_2 + x_3 = 0 \\ x_2 + x_4 = 0 \end{cases}$$

$U \subset V, \ U^\perp \subset V$

$$\begin{cases} \dim U + \dim U^\perp = \dim V \\ U \cap U^\perp = \{0\} \end{cases}$$

$V = U \oplus U^\perp$

$v \in V: \ v = u_1 + u_2$

#1370. $x = (4, -1, -3, 4)$; $L = \langle a_1, a_2, a_3 \rangle$

$x = u_\perp + u = \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 + u_\perp$

$u_\perp = x - \lambda_1 a_1 - \lambda_2 a_2 - \lambda_3 a_3 \perp a_1, a_2, a_3$

$(x - \lambda_1 a_1 - \lambda_2 a_2 - \lambda_3 a_3, a_1) = 0 = (x, a_1) - \lambda_1(a_1, a_1) - \lambda_2(a_2, a_1) - \lambda_3(a_3, a_1) =$

$= 4 - 4\lambda_1 - 4\lambda_2 - 4\lambda_3 = 0$

$(u_\perp, a_2) = 0 = -8 - 4\lambda_1 - 10\lambda_2 + 2\lambda_3 = 0$

$(u_\perp, a_3) = 0 = 16 - 4\lambda_1 + 2\lambda_2 - 10\lambda_3 = 0$

$$\begin{cases} \lambda_1 + \lambda_2 + \lambda_3 = 1 \\ 2\lambda_1 + 5\lambda_2 - \lambda_3 = -4 \\ 2\lambda_1 - \lambda_2 + 5\lambda_3 = 8 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 5 & -1 & -4 \\ 2 & -1 & 5 & 8 \end{array}\right) \xrightarrow[\text{III}-2\text{I}]{\text{II}-2\text{I}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 3 & -3 & -6 \\ 0 & -3 & 3 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -2 \end{array}\right)$$

Пусть $\lambda_3 = 0$, тогда $\lambda_1 = 3$, $\lambda_2 = -2$

$u = 3a_1 - 2a_2 = (1, -1, -1, 5) \in U$

$u_\perp = x - u = (3, 0, -2, -1)$

$\boxed{\lambda_i = \frac{(v, u_i)}{(u_i, u_i)}}$ $\boxed{u = \sum_{i=1}^{k} \frac{(v, u_i)}{(u_i, u_i)} \cdot u_i}$

О/Г базис: (a_1, a_2, a_3)

$u_1 = a_1$

$u_2 = a_2 - \frac{(a_2, u_1)}{(u_1, u_1)} u_1 = (1, 2, 2, -1) - \frac{4}{4}(1, 1, 1, 1) = (0, 1, 1, -2)$

$u_3 = a_3 - \frac{(a_3, u_1)}{(u_1, u_1)} u_1 - \frac{(a_3, u_2)}{(u_2, u_2)} u_2 = (1, 0, 0, 3) - \frac{4}{4}(1, 1, 1, 1) - \frac{-6}{6}(0, 1, 1, -2) = (0, 0, 0, 0)$

$$x = u + u_{\perp}$$

$$u = \frac{(x, u_1)}{(u_1, u_1)} u_1 + \frac{(x, u_2)}{(u_2, u_2)} u_2 = \frac{4}{4}(1, 1, 1, 1) + \frac{-12}{6}(0, 1, 1, -2) = (1, -1, -1, 5)$$