Семинар 28, 14.05.24

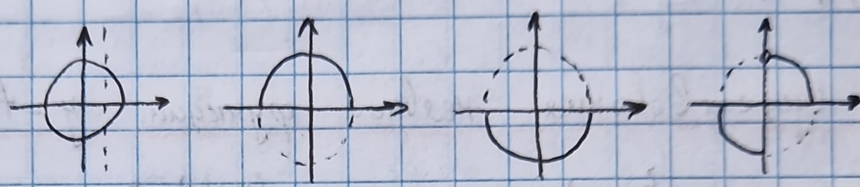
$F: \Omega \to \mathbb{R}$, $\Omega \subset \mathbb{R}\times\mathbb{R}$

$y = f(x)$ – „явная" функция

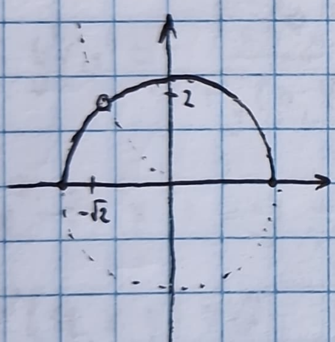
$F(x, y) = 0$

$ax + by + c = 0 \quad (b \neq 0)$

$x^2 + y^2 - 1 = 0$



① $\dfrac{x^2 + y^2 - 6}{\sqrt{y} + x} = 0$



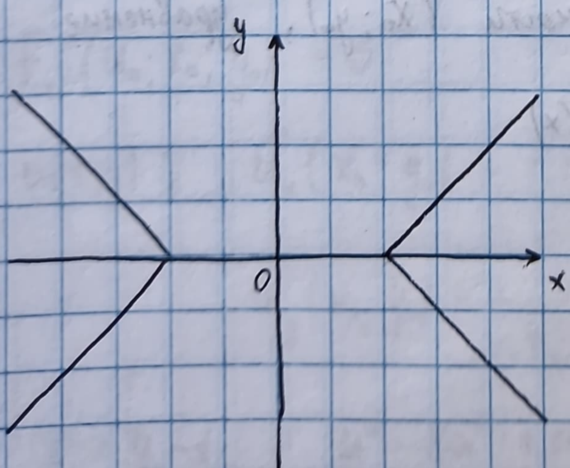
$y \geq 0$

$x \neq -\sqrt{y}$

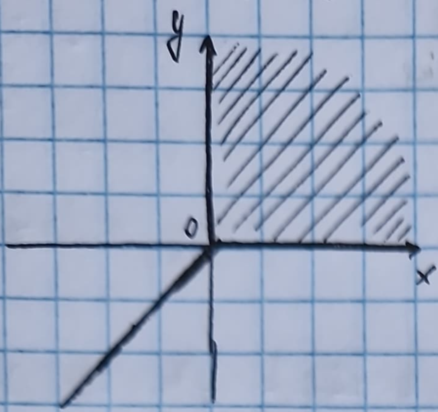
$y^2 + y - 6 = 0$

$y = -3 \;;\; y = 2$

② $|x| - |y| = 2$

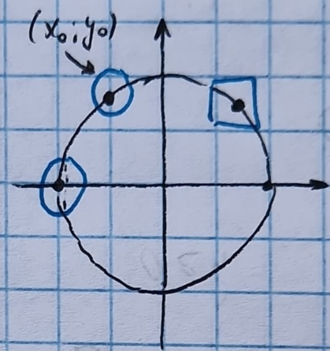


③ $|x| - x = |y| - y$



Условие существования неявной функции $y = f(x)$:

$F(x, y) = 0 \qquad x^2 + y^2 - 1 = 0$



(1) $(x_0; y_0): \ F(x_0; y_0) = 0$

(2) $F'_y(x_0; y_0) \neq 0$

(3) $F'_y(x_0; y_0)$ - непрер., $F(x, y)$ - непрер.

$F(x, y) = x^2 + y^2 - 1$

$F'_y(x, y) = 2y$

если $2y_0 \neq 0$, то локально, в окрестности $(x_0; y_0)$, уравнение $F(x, y) = 0$ задаёт функцию $y = y(x)$

$$\boxed{y'(x) = -\frac{F'_x(x_0, y_0)}{F'_y(x_0, y_0)}}$$

$F'_x(x, y) = 2x$

$F'_y(x, y) = 2y$

$y'(x) = \frac{-2x}{2y} = -\frac{x}{y}$

$y'(x_0) = -\frac{x_0}{y_0}$

$$e^{y} + xy = e \qquad (y>0)$$

$$y'(0) = ?$$

$$F(x,y) = e^{y} + xy - e$$

$$F'_x(x,y) = y$$

$$F'_y(x,y) = e^{y} + x$$

$$x_0 = 0, \quad y_0 = ? \; ; \quad e^{y_0} + x_0 y_0 - e = 0 \Rightarrow y_0 = 1$$

$$(x_0; y_0) = (0; 1):$$

$$F'_y(0,1) = e \neq 0$$

$$y'(0) = -\frac{F'_x(0;1)}{F'_y(0;1)} = -\frac{1}{e} = -e^{-1}$$

$$u^3 - 2u^2x + uxy - 2 = 0 \qquad (x_0, y_0) = (1, 1)$$

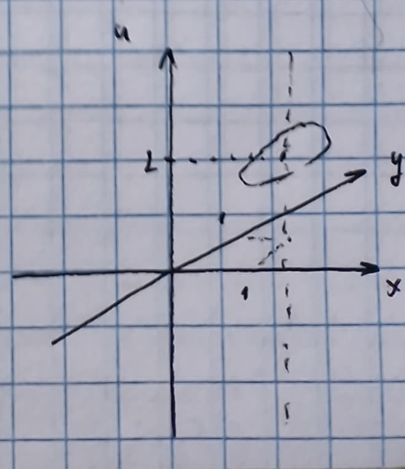
$$F(u, x, y) = 0$$

$$(u_0; x_0; y_0) : F(u_0, x_0; y_0) = 0$$

$$F'_u(u_0; x_0; y_0) \neq 0$$

$$u(x,y); \quad u'_x(x_0; y_0) = -\frac{F'_x(u_0, x_0, y_0)}{F'_u(u_0, x_0, y_0)}$$

$$u'_y(x_0, y_0) = -\frac{F'_y(u_0, x_0, y_0)}{F'_u(u_0, x_0, y_0)}$$



$$x_0 = y_0 - 1 : \quad u^3 - 2u^2 + u - 2 = 0$$

$$(u^2+1)(u-2) = 0 \Rightarrow u = 2 \; ; \quad u_0 = 2$$

$$(u_0; x_0; y_0) = (2, 1, 1)$$

$$F'_u = 3u^2 - 4ux + xy$$

$$F'_u(2,1,1) = 12 - 8 + 1 = 5 \neq 0$$

$$F'_x = -2u^2 + uy \qquad F'_x(2,1,1) = -6$$

$$F'_y = ux \qquad F'_y(2,1,1) = 2$$

$$u'_x(1,1) = -\frac{-6}{5} = \frac{6}{5} \qquad u'_y(1,1) = \frac{-2}{5}$$