Семинар 16, 23.01.24.

$\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$

$|S_n| = n!$

A_n – группа всех чётных подстановок $\{1, 2, \dots, n\}$

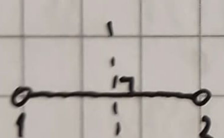
$|A_n| = \frac{n!}{2}$ $(n > 1)$

$\left.\begin{array}{l}\text{поворот: } id,\ \varphi_{\frac{\pi}{2}},\ \varphi_{\pi},\ \varphi_{\frac{3\pi}{2}} \\ \text{симметрия: } s_1,\ s_2,\ s_3,\ s_4\end{array}\right\}$ движения

правильный n-угольник; n поворотов
n осевых симметрий

D_n – группа диэдра

$|D_n| = 2n$



– отрезок

$\mathbb{Z}_2 = \{0, 1\}$

$G = \{id, s\}$

$id \circ id = id \mapsto 0 + 0 = 0$

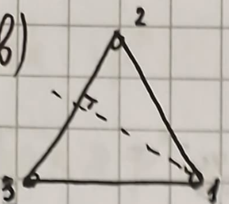
$id \circ s = s \mapsto 0 + 1 = 1$

$s \circ s = id \mapsto 0$

$\mapsto 1 + 1 = 0$

#55.25 (Кострикин)

в)



$D_3 \to S_3$

$id \mapsto id$

$r_{\frac{4\pi}{3}} \mapsto (1\,3\,2)$

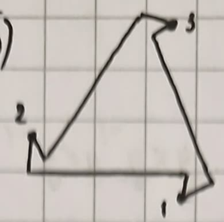
$r_{\frac{\pi}{3}} \mapsto (1\,2\,3)$

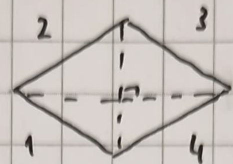
$s_1 \mapsto (2\,3)$

$s_2 \mapsto (1\,3)$

$s_3 \mapsto (1\,2)$

б)





$id, (12)(34), (23)(14), (13)(24)$

г) $V_4 = \{(12)(34), (13)(24), (14)(23), id\}$

$(12)(34)(12)(34) = (14)(23)$

$\mathbb{Z}_4$ - не изоморфна

Есть G, H, тогда $G \times H = \{(g,h) \mid g \in G,\ h \in H\}$

$(g_1, h_1)(g_2, h_2) = (g_1 g_2, h_1 h_2)$

$\mathbb{Z}_2 \times \mathbb{Z}_2 = \{(0,0), (0,1), (1,0), (1,1)\}$

$V_4 \to \mathbb{Z}_2 \times \mathbb{Z}_2$

$id \mapsto (0,0)$

$(12)(34) \mapsto (0,1)$

$(13)(24) \mapsto (1,0)$

$(14)(23) \mapsto (1,1)$

#56.7.

a) x, yxy^{-1}

$(yxy^{-1})^n = yxy^{-1}yxy^{-1}yxy^{-1}\ldots yxy^{-1}yxy^{-1} = yx^ny^{-1} = yy^{-1} = e$

$(yxy^{-1})^k = yx^ky^{-1} = e$

$y^{-1}yx^ky^{-1}y = y^{-1}y$

$x^k = e$

b) ab, ba

$b(ab)b^{-1} = babb^{-1} = ba$

#56.13.

$\mathbb{Z}_{p^n}$; $m \le n$

$\mathbb{Z}_{p^n} = \{0, 1, \ldots, p^n-1\}$

$k \in \{0, 1, 2, \ldots, p^n-2, p^n-1\}$

$ord(k) = d = p^n$

$\underbrace{k+k+k+\ldots+k}_{d} = dk = p^n k \underset{p^n}{\equiv} 0 \quad (p^n k \vdots p)$

$ak \underset{p^n}{\equiv} 0$; $a < p^n$

Если $k \vdots p$, то $p^{n-1} \vdots p^{n-1} \cdot k \vdots p^n$

Если $k \not\vdots p$, то $ak \vdots p^n \Leftrightarrow a \vdots p^n \Rightarrow a \geq p^n$

$m < n$: $ord(k) = p^m$

$k \vdots p^{n-m}$, $k \not\vdots p^{n-m+1}$

$k \cdot p^m \vdots p^n$; $ka \vdots p^n \Rightarrow a \vdots p^m \Rightarrow a \geq p^m$

Кол-во:

$\frac{p^n}{p^{n-m}} - \frac{p^n}{p^{n-m+1}} = p^m - p^{m-1}$

HSE

$\operatorname{ord}(k) = a = p^s \cdot b, \quad b \not\vdots p$

$ak \vdots p^n$

$\cancel{b} p^s k \vdots p^n$

$p^s k \vdots p^n \Rightarrow a = \operatorname{ord}(k) \leq p^s$

#56.15.

$\mathbb{Z}_{24} = \{0, 1, 2, \dots, 23\}$

$\underbrace{g + g + g + \dots + g}_{k \text{ раз}} = kg = 6g \vdots 24$

$g \vdots 4$

$g = 4, \cancel{8}, \cancel{12}, \cancel{16}, 20, \cancel{0}$

#56.16.

a) $\mathbb{Z}_{24} = \{0, 1, 2, \dots, 23\}$

$24 \vdots d$

$d = 2: \{0, 12\}$

$d = 3: \{0, 8, 16\}$

$d = 4: \{0, 6, 12, 18\}$

$d = 6: \{0, 4, 8, 12, 16, 20\}$

$d = 8: \{0, 3, \dots\}$

$d = 12: \{0, 2, 4, \dots\}$

$d = 24: \{0, 1, 2, \dots\}$

$\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$

$n \vdots d \to \{0, \frac{n}{d}, \frac{2n}{d}, \dots \frac{(d-1)n}{d}\}$

#56.19.

$U_\infty = \{z \in \mathbb{C} : \exists n \in \mathbb{N}, z^n = 1\}$

HSE