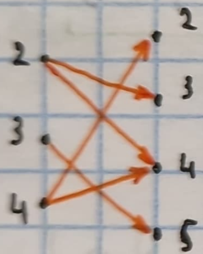
# Homework - 3b.

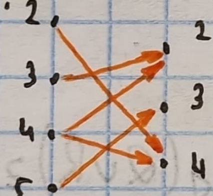
## #1.

$R = \{(2,3),\ (3,5),\ (2,4),\ (4,4),\ (4,2)\}$

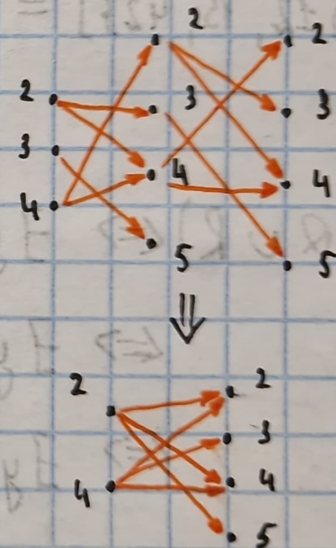
$R$ :



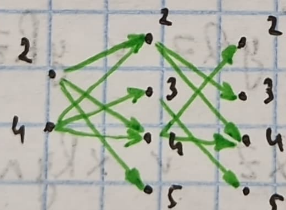
$R^{-1}$ :



$R \circ R$ :



$R \circ R \circ R = (R \circ R) \circ R$ :



$\operatorname{dom}(R \circ R \circ R) = \{2, 4\}$

$\operatorname{rng}(R \circ R \circ R) = \{2, 3, 4, 5\}$

## #3.

$x R y \iff x | y$

$x R^{-1} y \iff y | x \implies R^{-1}[x]$ – множество чисел, на которые делится $x$

$R^{-1}[\{12, 15, 42\}] = \boxed{\{1, 2, 3, 4, 5, 6, 7, 12, 14, 15, 21, 42\}}$

## #4.

$R \circ (Q \cup P) \iff \exists y\, (xRy \wedge y(Q \cup P)z)$

$\iff \exists y\, (xRy \wedge (yQz \vee yPz))$

$\iff \exists y\, (xRy \wedge yQz \vee xRy \wedge yPz)$

$\iff \exists y\, (xRy \wedge yQz) \vee \exists y\, (xRy \wedge yPz)$

$$\Leftrightarrow (R\circ Q)\cup(R\circ P)$$ ч.т.д.

## #5.

$$(R\circ P)\cap(R\circ Q) \Leftrightarrow \exists y(xRy \wedge yPz) \wedge \exists w(xRw \wedge wQz)$$

$$\begin{tikzcd} & y \arrow[dr, "P"] & \\ x \arrow[ur, "R"] \arrow[dr, "R"'] & & z \\ & w \arrow[ur, "Q"'] & \end{tikzcd}$$

$$R\circ(P\cap Q) \Leftrightarrow \exists k\,(xRk \wedge k(P\cap Q)z)$$

$$\Leftrightarrow \exists k\,(xRk \wedge kPz \wedge kQz)$$

$$\begin{tikzcd} x \arrow[r, "R"] & k \arrow[r, bend left, "P"] \arrow[r, bend right, "Q"'] & z \end{tikzcd}$$

т.е. $y$ и $w$ должны совпадать

Антипример:

$R=\{(1,2),(1,3)\}$ $P=\{(2,0)\}$ $Q=\{(3,0)\}$

$(R\circ P)\cap(R\circ Q)=\{(1,0)\}\cap\{(1,0)\}=\{(1,0)\}$

$R\circ(P\circ Q)=R\circ\varnothing=\varnothing$

Но $\{(1,0)\}\not\subseteq\varnothing$

$\boxed{\text{Ответ: не всегда.}}$

## #6.

$$R[X]\cap R[Y] \overset{?}{\subseteq} R[X\cap Y]$$

$R[X]=\{b\in \operatorname{rng}R \mid \exists a_1(a_1\in X \wedge a_1Rb)\}$

$R[Y]=\{b\in \operatorname{rng}R \mid \exists a_2(a_2\in Y \wedge a_2Rb)\}$

$R[X\cap Y]=\{b\in \operatorname{rng}R \mid \exists a\,(a\in(X\cap Y) \wedge aRb)\}=$

$= \{ b \in rng R \mid \exists a ( a \in X \wedge a \in Y \wedge aRb ) \}$

$R[X] \cap R[Y] = \{ b \in rng R \mid \exists a_1 ( a_1 \in X \wedge a_1 R b ) \wedge$
$\exists a_2 ( a_2 \in Y \wedge a_2 R b ) \}$

Чтобы выполнялось включение, должно выполняться $a_1 = a_2$

Антипример:

$R = \{ (0, 1), (1, 1) \}$

$R[\{0\}] = \{1\}$

$R[\{1\}] = \{1\}$ $\Rightarrow R[\{0\}] \cap R[\{1\}] = \{1\}$

$R[\{0\} \cap \{1\}] = R[\emptyset] = \emptyset$

Но $\{1\} \not\subseteq \emptyset$

$\boxed{\text{Ответ: не всегда.}}$

#7.

$(R \cup Q)[X] \overset{?}{=} R[X] \cup Q[X]$

$(R \cup Q)[X] \Leftrightarrow \{ b \in rng (R \cup Q) \mid \exists a ( a \in X \wedge a (R \cup Q) b ) \}$

$\Leftrightarrow \{ b \in rng R \vee b \in rng Q \mid \exists a ( a \in X \wedge ( aRb \vee aQb ) ) \}$

$\Leftrightarrow \{ b \in rng R \vee b \in rng Q \mid \exists a ( a \in X \wedge aRb \vee a \in X \wedge aQb ) \}$

$$\left[ \begin{array}{l} b \in rng R \Leftrightarrow \exists a \in X \wedge aRb \\ b \in rng Q \Leftrightarrow \exists a \in X \wedge aQb \end{array} \right]$$

$\Rightarrow \{ b \in rng R \mid \exists a ( a \in X \wedge aRb ) \vee$
$b \in rng Q \mid \exists a ( a \in X \wedge aQb ) \}$

$$\Leftrightarrow R[X] \cup Q[X]$$ ч.т.д.

$\boxed{\text{Ответ: всегда}}$

#2.

$$\subseteq \circ \subseteq = \{ (x,y) \in P(A) \times P(A) \mid \exists z \, (x \subseteq z \wedge z \subseteq y) \}$$

$x \subseteq z \wedge z \subseteq y \Rightarrow x \subseteq y$, тогда $\subseteq \circ \subseteq = \{(x,y) \in P(A) \times P(A) \mid x \subseteq y\} = \, \subseteq$

$$A = \{1, 2, 3\}$$

$$P(A) = \{ \varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

Answer:

$$\subseteq = \{ (\varnothing, \{1\}), (\varnothing, \{2\}), (\varnothing, \{3\}), (\varnothing, \{1,2\}), (\varnothing, \{1,3\}), (\varnothing, \{2,3\}),$$
$$(\varnothing, \{1,2,3\}), (\{1\}, \{1,2\}), (\{1\}, \{1,3\}), \cancel{(\{1\}, \{2,3\})}, (\{1\}, \{1,2,3\}),$$
$$(\{2\}, \{1,2\}), (\{2\}, \{2,3\}), (\{2\}, \{1,2,3\}), (\{3\}, \{2,3\}), (\{3\}, \{1,3\}),$$
$$(\{3\}, \{1,2,3\}), (\{1,2\}, \{1,2,3\}), (\{1,3\}, \{1,2,3\}), (\{2,3\}, \{1,2,3\}) \}$$