#1.

1) $1+i\sqrt{3} = \sqrt{1+3}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \underline{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

2) $-\sqrt{3}+i = \sqrt{1+3}\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \underline{2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}$

#2.

$$(\sqrt{3}+i)^{30} = \left(2\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right)\right)^{30} = \left(2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right)^{30} = 2^{30}\cdot\left(\cos\frac{30\pi}{6} + i\sin\frac{30\pi}{6}\right) =$$

$$= 2^{30}\cdot(\cos 5\pi + i\sin 5\pi) = 2^{30}\cdot(1 + i\cdot 0) = \underline{2^{30}}.$$

#3.

$$\sqrt[3]{i} = \left\{\sqrt[3]{1}\left(\cos\frac{\frac{\pi}{2}+2\pi k}{3} + i\sin\frac{\frac{\pi}{2}+2\pi k}{3}\right)\right\}_{k=0}^{2} = \underline{\left\{\cos\left(\frac{\pi}{6}+\frac{2\pi k}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{2\pi k}{3}\right)\right\}_{k=0}^{2}}$$

#4.

$$\begin{cases} 2z_1 - (2+i)z_2 = -i \\ (4-2i)z_1 - 5z_2 = -1-2i \end{cases}$$

$$\left(\begin{array}{cc|c} 2 & -(2+i) & -i \\ 4-2i & -5 & -1-2i \end{array}\right) \xrightarrow{II - I\cdot(2-i)} \left(\begin{array}{cc|c} 2 & -(2+i) & -i \\ 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 2 & -(2+i) & -i \end{array}\right)$$

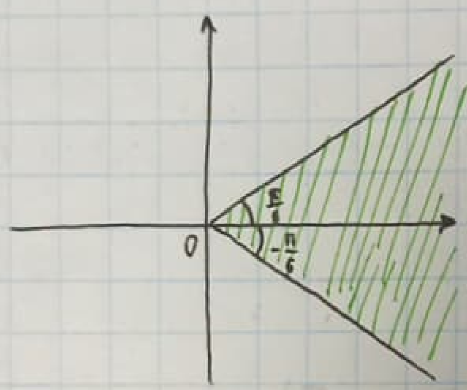
$$z_1 = \frac{(2+i)z_2 - i}{2}$$

$$\boxed{\text{Ответ: } \left(\frac{(2+i)z_2 - i}{2};\ z_2\right),\ \text{где } z_2 \in \mathbb{C}}$$

#6.

$$|\arg z| < \frac{\pi}{6}$$

$$-\frac{\pi}{6} < \arg z < \frac{\pi}{6}$$



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#5.

$z = x+iy$, $\bar{z} = x-iy$, $z^2 = (x+iy)^2 = x^2+2ixy-y^2$

$$z^2 = \bar{z}$$

$$x^2+2ixy-y^2 = x-iy$$

$$(x^2-y^2-x) + i(2xy+y) = 0$$

$$\begin{cases} x^2-y^2-x=0 \\ 2xy+y=0 \end{cases} \quad \begin{cases} x^2-y^2-x=0 \\ \left[\begin{array}{l} y=0 \\ 2x=-1,\ y\neq 0 \end{array}\right. \end{cases} \quad \begin{cases} y^2 = x^2-x \\ \left[\begin{array}{ll} y=0 & (1) \\ x=-\frac{1}{2},\ y\neq 0 & (2) \end{array}\right. \end{cases}$$

(1) $y=0$; $x^2-x=0 \Rightarrow \left[\begin{array}{l} x=1 \\ x=0 \end{array}\right. \Rightarrow z_1=0$; $z_2=1$

(2) $x=-\frac{1}{2}$; $y^2 = \frac{1}{4}+\frac{1}{2} = \frac{3}{4}$; $y = \pm\frac{\sqrt{3}}{2} \Rightarrow z_3 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$; $z_4 = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$

$$\boxed{\text{Ответ: } 0;\ 1;\ -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}}$$

#7.

Док-ть: $\sin x + \sin 2x + \ldots + \sin nx = \dfrac{\sin\frac{nx}{2}\cdot\sin\frac{(n+1)x}{2}}{\sin\frac{x}{2}}$

$z = \cos x + i\sin x$; $\quad \operatorname{Im} z = \sin x$

$\sin(kx) = \operatorname{Im}(z^k)$

$$\sum_{k=1}^{n}\sin(kx) = \sum_{k=1}^{n}\operatorname{Im}(z^k) = \operatorname{Im}\left(\sum_{k=1}^{n} z^k\right) = \operatorname{Im}\left(\frac{z\cdot(z^n-1)}{z-1}\right) =$$

$$= \operatorname{Im}\left(\frac{z(z^n-1)(\bar z-1)}{(z-1)(\bar z-1)}\right) = \operatorname{Im}\left(\frac{(z^n-1)(1-z)}{1-\bar z-z+1}\right) = \operatorname{Im}\left(\frac{-z^{n+1}+z^n+z-1}{2(1-\cos x)}\right) =$$

$$= \frac{-\sin(n+1)x+\sin(nx)+\sin x}{2(1-\cos x)} = \frac{-2\left(\sin\left(\frac{(n+1)x}{2}\right)\cdot\cos\left(\frac{(n+1)x}{2}\right)-\sin\frac{(n+1)x}{2}\cos\frac{(n-1)x}{2}\right)}{4\sin^2\frac{x}{2}} =$$

$$= \frac{\sin\left(\frac{(n+1)x}{2}\right)\left(\cos\left(\frac{(n-1)x}{2}\right)-\cos\left(\frac{(n+1)x}{2}\right)\right)}{2\sin^2\frac{x}{2}} = \frac{\sin\left(\frac{(n+1)x}{2}\right)\cdot 2\cdot\sin\frac{nx}{2}\cdot\sin\frac{x}{2}}{2\sin^2\frac{x}{2}} =$$

$$= \frac{\sin\frac{(n+1)x}{2}\cdot\sin\frac{nx}{2}}{\sin\frac{x}{2}}$$

ч.т.д.