Семинар 30, 14.05.24 – Бельдиев

$\mathbb{R}^n : (x_1, x_2, x_3, \dots, x_n)$

$v = (x_1, x_2, \dots, x_n)$

$u = (y_1, y_2, \dots, y_n)$

$(v, u) = x_1 y_1 + \dots + x_n y_n$ – билинейная форма + симметрич. + положит. опр.

$((u,v) > 0$ при $v \neq 0)$

$(v, u) = (u, v)$

Опр. (Евклидово) <u>Скалярное произведение</u> на векторном пространстве V (над $\mathbb{R}$) – это билин. симм. положит. опред. форма $(\cdot, \cdot) : V \times V \to \mathbb{R}$

$\mathbb{R}^n$; $(v, u) = x_1 y_1 + \dots + x_n y_n$

$e_1, e_2, \dots, e_n$; $(e_i, e_j) = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} = \delta_{ij}$

ортонормированный базис

Ортогональний базис: $(v_1, v_2, \dots, v_n)$: $(v_i, v_j) = 0$ при $i \neq j$

#1357. $\mathbb{R}^4$; $v_1 = (1, -2, 2, -3)$
$v_2 = (2, -3, 2, 4)$

$(v_1, v_2) = 1 \cdot 2 - 2 \cdot (-3) + 2 \cdot 2 - 3 \cdot 4 = 0$

$(x_1, x_2, x_3, x_4) \perp v_1, v_2$

$$\begin{cases} x_1 - 2x_2 + 2x_3 - 3x_4 = 0 \\ 2x_1 - 3x_2 + 2x_3 + 4x_4 = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -2 & 2 & -3 \\ 2 & -3 & 2 & 4 \end{pmatrix} \xrightarrow[-2(I)]{} \begin{pmatrix} 1 & -2 & 2 & -3 \\ 0 & 1 & -2 & 10 \end{pmatrix}$$

$x_3 = a$

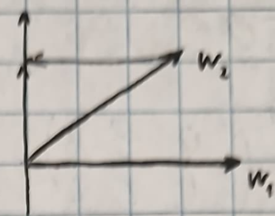
$x_4 = b$

$x_2 - 2a + 10b = 0$; $x_2 = 2a - 10b$

$x_1 - 2x_2 + 2x_3 - 3x_4 = 0$

$$x_1 = 2(2a-10b) - 2a + 3b = 2a - 17b$$

$$\begin{pmatrix} 2a-17b \\ 2a-10b \\ a \\ b \end{pmatrix} = a\begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} -17 \\ -10 \\ 0 \\ 1 \end{pmatrix}$$



$$w_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \end{pmatrix} \qquad w_2 = \begin{pmatrix} -17 \\ -10 \\ 0 \\ 1 \end{pmatrix}$$

$$(w_1, w_2) \leftrightarrow (w_1, w_2 + cw_1)$$

$$(w_1, w_2 + cw_1) = (w_1, w_2) + c(w_1, w_1) = 0$$

$$c = -\frac{(w_1, w_2)}{(w_1, w_1)}$$

$$(w_1, w_2) = -34 - 20 = -54$$

$$\Rightarrow c = 6$$

$$(w_1, w_1) = 4 + 4 + 1 = 9$$

$$w_2 + cw_1 = (-17, -10, 0, 1) + 6(2, 2, 1, 0) = (-5, 2, 6, 1)$$

$$\boxed{\begin{aligned} v_3 &= (2, 2, 1, 0) \\ v_4 &= (-5, 2, 6, 1) \end{aligned}}$$

#1359. $v_1 = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right) = \frac{1}{3}(2, 1, 2)$

$v_2 = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right) = \frac{1}{3}(1, 2, -2)$

$$(v_1, v_2) = \frac{1}{9}(2 + 2 - 4) = 0$$

$$(v_1, v_1) = \frac{1}{9}(4 + 1 + 4) = 1$$

$$(v_2, v_2) = \frac{1}{9}(1 + 4 + 4) = 1$$

$$(x_1, x_2, x_3) \perp v_1, v_2$$

$$\begin{cases} 2x_1 + x_2 + 2x_3 = 0 \\ x_1 + 2x_2 - 2x_3 = 0 \end{cases}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \end{pmatrix} \xrightarrow{-2(\text{II})} \begin{pmatrix} 0 & -3 & 6 \\ 1 & 2 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$x_3 = a$

$x_2 = 2a$

$x_1 + 4a - 2a = 0$

$x_1 = -2a$

$$\begin{pmatrix} -2a \\ 2a \\ a \end{pmatrix} = a \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \qquad a = \frac{1}{3}$$

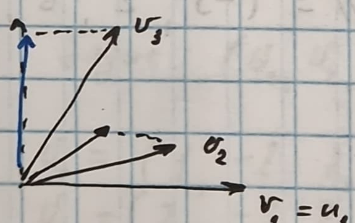
$v_3 = \left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

Процесс Грама-Шмидта

$(v_1, v_2, \dots, v_n)$

(v_1, v_2)

(v_1, v_2, v_3)



#1363. $v_1 = (2, 1, 3, -1)$

$v_2 = (7, 4, 3, -3)$

$v_3 = (1, 1, -6, 0)$

$v_4 = (5, 7, 7, 8)$

$(v_1, v_2, v_3, v_4) \longrightarrow (u_1, u_2, u_3, u_4)$

$v_1 = u_1$

$$u_2 = v_2 + cu_1 = v_2 + \frac{-(v_2, u_1)}{(u_1, u_1)} u_1$$

$(v_2 + cu_1, u_1) = 0$

$$(v_2, u_1) + c(u_1, u_1) = 0$$

$$c = -\frac{(v_2, u_1)}{(u_1, u_1)}$$

$$(v_2, u_1) = 30$$

$$(u_1, u_1) = 15$$

$$u_2 = v_2 - 2u_1 = (7, 4, 3, -3) - 2(2, 1, 3, -1) = (3, 2, -3, -1)$$

$$u_3 = v_3 + c_1 u_1 + c_1 u_2$$

$$0 = (u_3, u_2) = (v_3 + c_1 u_1 + c_1 u_2, u_2) = (v_3, u_2) + c_2(u_2, u_2)$$

$$0 = (u_3, u_1) = (v_3 + c_1 u_1 + c_1 u_2, u_1) = (v_3, u_1) + c_1(u_1, u_1)$$

$$\boxed{u_3 = v_3 - \frac{(v_3, u_1)}{(u_1, u_1)} u_1 - \frac{(v_3, u_2)}{(u_2, u_2)} u_2}$$

$(v_3, u_1) = -15$ $\quad$ $(u_1, u_1) = 15$

$(v_3, u_2) = 23$ $\quad$ $(u_2, u_2) = 23$

$$u_3 = (1, 1, -6, 0) + (2, 1, 3, -1) - (3, 2, -3, -1) = (0, 0, 0, 0) = 0$$

$$u_4 = v_4 - \frac{(v_4, u_1)}{(u_1, u_1)} u_1 - \frac{(v_4, u_2)}{(u_2, u_2)} u_2 - \cancel{\frac{(v_4, u_3)}{(u_3, u_3)}} u_3 \quad = 0$$