Семинар 25, 02.04.24 – Бельдиев

$U \subset V = \mathbb{R}^n$

$\langle v_1, v_2, \dots, v_n \rangle = \{a_1 v_1 + \dots + a_n v_n \mid a_n \in F\}$

$$\begin{cases} a_{11} x_1 + \dots + a_{1n} x_n = 0 \\ a_{21} x_1 + \dots + a_{2n} x_n = 0 \\ \dots \\ a_{n1} x_1 + \dots + a_{nn} x_n = 0 \end{cases}$$

#1312. $\langle (1,-1,1,0), (1,1,0,1), (2,0,1,1) \rangle = U$

$a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = 0$

$$\begin{cases} a_1 - a_2 + a_3 = 0 \\ a_1 + a_2 + a_4 = 0 \\ 2a_1 + a_3 + a_4 = 0 \end{cases} \quad \begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 2 & -1 & 1 \end{pmatrix} \rightsquigarrow$$

$$\rightsquigarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{pmatrix}$$

$2a_2 - a_3 + a_4 = 0$

$a_2 = \dfrac{a_3 - a_4}{2}$

$a_1 = a_2 - a_3 = -\dfrac{a_3 + a_4}{2}$

$a_3 = 2,\ a_4 = 0: \quad a_1 = -1,\ a_2 = 1$

$a_3 = 0,\ a_4 = 2: \quad a_1 = -1,\ a_2 = -1$

$$\begin{cases} -x_1 + x_2 + 2x_3 = 0 \\ -x_1 - x_2 + 2x_4 = 0 \end{cases} = \tilde{U}$$

$U \subset \tilde{U} \Rightarrow \dim U \le \dim \tilde{U}$ $\quad \boxed{\dim U = \dim \tilde{U} \Leftrightarrow U = \tilde{U}}$

$\dim U = 2 \qquad \dim \tilde{U} = 2$

$\dim$ пр-ва реш. $= n - \mathrm{rk} A$

$U_1, U_2 < V$

$U_1 \cap U_2 = \{ u \mid u \in U_1,\ u \in U_2 \}$

$U_1 + U_2 = \{ u_1 + u_2 \mid u_1 \in U_1,\ u_2 \in U_2 \}$

#1318. $((1,1,1,1),\ (1,-1,1,-1),\ (1,3,1,3)) = U_1$

$((1,2,0,2),\ (1,2,1,2),\ (3,1,3,1)) = U_2$

$U_1 + U_2 = (a_1, a_2, a_3, b_1, b_2, b_3)$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & 1 & 3 \\ 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ \cancel{0} & \cancel{-2} & \cancel{0} & \cancel{-2} \\ \cancel{0} & \cancel{2} & \cancel{0} & \cancel{2} \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ \cancel{0} & \cancel{-2} & \cancel{0} & \cancel{-2} \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$\dim(U_1 + U_2) = 3$

$$\boxed{\dim U_1 + \dim U_2 = \dim(U_1 + U_2) + \dim(U_1 \cap U_2)}$$

$\dim U_1 = 2 \qquad \dim U_2 = 3$

$\dim(U_1 \cap U_2) = -\dim(U_1 + U_2) + \dim U_1 + \dim U_2 = -3 + 2 + 3 = 2$

#1321.

$$\begin{pmatrix} 1 & 2 & 1 & -2 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & 2 & -3 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 3 & 0 & -4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & -2 \\ 0 & -1 & -1 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 3 \\ 0 & -2 & 0 & 1 \\ 0 & 1 & -1 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -7 \\ 0 & 0 & -2 & 2 \end{pmatrix} \longrightarrow \begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \\ \end{array} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\dim(U_1 + U_2) = 4 \qquad (U_1 + U_2 = \mathbb{R}^4)$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 9 \end{pmatrix} \begin{array}{c} \beta_1 \\ \beta_2 \\ \beta_3 \end{array}$$

$\dim(U_1) = 3 \qquad \dim U_2 = 3$

$\dim(U_1 \cap U_2) = 2$

$$\begin{cases} \alpha_1 = \beta_1 \\ 2\alpha_1 + \alpha_2 = \beta_1 + \beta_2 \\ \alpha_1 + \alpha_2 + \alpha_3 = \beta_1 + \beta_3 \\ 2\alpha_1 - 4\alpha_2 - \alpha_3 = \beta_1 + 2\beta_2 + 9\beta_3 \end{cases}$$

$$\begin{cases} \alpha_1 + \alpha_2 = \beta_2 \\ \alpha_3 + \beta_2 = \beta_1 + \beta_3 \\ \alpha_1 - 4\alpha_2 - \alpha_3 = 2\beta_1 + 2\beta_3 - 2\alpha_3 + 9\beta_3 \end{cases}$$

$$\alpha_1 - 4\alpha_2 - \alpha_3 = 2\alpha_1 - 2\alpha_3 + 9(\alpha_3 - \alpha_1 + \beta_2) + 2(\alpha_3 + \beta_2 - \alpha_1)$$

$$5\alpha_1 - \alpha_3 + 4\alpha_2 + 9\alpha_3 + 9\alpha_2 + 2\alpha_3 + 2\alpha_2 = 0$$

$$5\alpha_1 + 15\alpha_2 + 10\alpha_3 = 0$$

$$\alpha_1 + 3\alpha_2 + 2\alpha_3 = 0$$

$$\boxed{\begin{aligned} &\alpha_2,\ \alpha_3 - \text{своб.} \\ &\alpha_1 = -3\alpha_2 - 2\alpha_3 \\ &\beta_1 = -3\alpha_2 - 2\alpha_3 \\ &\beta_3 = \alpha_2 + \alpha_3 \\ &\beta_2 = -2\alpha_2 - 2\alpha_3 \end{aligned}}$$

$$(-3\alpha_2 - 2\alpha_3)\begin{pmatrix} 1 \\ 2 \\ 1 \\ -2 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 1 \\ -4 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3\alpha_2 - 2\alpha_3 \\ -5\alpha_2 - 4\alpha_3 \\ -2\alpha_2 - \alpha_3 \\ 2\alpha_2 + 3\alpha_3 \end{pmatrix} = \alpha_2 \begin{pmatrix} -3 \\ -5 \\ -2 \\ 2 \end{pmatrix} + \alpha_3 \begin{pmatrix} -2 \\ 4 \\ -1 \\ 3 \end{pmatrix}$$

$\alpha_2 = 1,\ \alpha_3 = 0 : (-3, -5, -2, 2)$

$\alpha_2 = 0,\ \alpha_3 = 1 : (-2, 4, -1, 3)$