Homework 9.

#1.

(a) Δa_n

$$A(s)=\sum_{n=0}^{\infty} a_n s^n$$

$$B(s)=(a_1-a_0)+(a_2-a_1)s+(a_3-a_2)s^2+\ldots=$$

$$=(a_1+a_2s+a_3s^2+\ldots)-(a_0+a_1s+a_2s^2+\ldots)=$$

$$=(a_0+a_1s+a_2s+a_3s^3+\ldots)\cdot\frac{1}{s}-\frac{a_0}{s}-(a_0+a_1s+a_2s^2+\ldots)=$$

$$=\frac{A(s)}{s}-\frac{a_0}{s}-A(s)=\boxed{\frac{A(s)-A(0)}{s}-A(s)}$$

(б) Σa_n

$$B(s)=0+a_0s+(a_0+a_1)s^2+(a_0+a_1+a_2)s^3+\ldots=$$

$$=s(a_0+(a_0+a_1)s+(a_0+a_1+a_2)s^2+\ldots)=$$

$$=s((a_0+a_1s+a_2s^2+\ldots)+(a_0+a_1s+a_2s^2+\ldots)s+(a_0+a_1s+a_2s^2+\ldots)s^2+\ldots)=$$

$$=s(a_0+a_1s+a_2s^2+\ldots)(1+s+s^2+\ldots)=s\cdot A(s)\cdot\frac{1}{1-s}=\boxed{\frac{s\cdot A(s)}{1-s}}$$

(в) $a_0, 0, 0, a_1, 0, 0, a_2, 0, 0, a_3, \ldots$

$$B(s)=a_0+a_1s^3+a_2s^6+a_3a^9+\ldots=\boxed{A(s^3)}$$

(г) $a_1, a_3, a_5, \ldots, a_{2n+1}, \ldots$

$$B(s)=a_1+a_3s+a_5s^2+\ldots=\frac{\sqrt{s}\,a_1+a_3(\sqrt{s})^3+a_5(\sqrt{s})^5+\ldots}{\sqrt{s}}=$$

$$=\frac{2\sqrt{s}\,a_1+2a_3(\sqrt{s})^3+2a_5(\sqrt{s})^5+\ldots}{2\sqrt{s}}=\frac{a_0-a_0+a_1\sqrt{s}+a_1\sqrt{s}+a_2(\sqrt{s})^2-a_2(\sqrt{s})^2+a_3(\sqrt{s})^3+a_3(\sqrt{s})^3+\ldots}{2\sqrt{s}}=$$

$$=\frac{(a_0+a_1\sqrt{s}+a_2(\sqrt{s})^2+a_3(\sqrt{s})^3+\ldots)-(a_0-a_1\sqrt{s}+a_2(\sqrt{s})^2-a_3(\sqrt{s})^3+\ldots)}{2\sqrt{s}}=\boxed{\frac{A(\sqrt{s})-A(-\sqrt{s})}{2\sqrt{s}}}$$

#2.

$1, -1, 2, -2, 3, -3, 4, -4, \dots$

$$B(x) = 1 - x + 2x^2 - 2x^3 + 3x^4 - 3x^5 + \dots = (1-x) + 2x^2(1-x) + 3x^4(1-x) + \dots =$$

$$= (1-x)(1 + 2x^2 + 3x^4 + \dots) = (1-x)(1 + x^2 + x^4 + \dots)^2 = \boxed{\frac{1-x}{(1-x^2)^2}}$$

#3.

$a_0 = 1 \qquad a_1 = 2 \qquad a_{n+2} = 4a_{n+1} - 5a_n \Rightarrow a_n = 4a_{n-1} - 5a_{n-2}$

$$A(x) = \sum_{n=0}^{+\infty} a_n s^n$$

$$4x \cdot A(x) = \sum_{n=0}^{+\infty} 4a_n x^{n+1} = 4a_0 x + \sum_{n=1}^{+\infty} 4a_n x^{n+1} = 4a_0 x + \sum_{n=2}^{+\infty} 4a_{n-1} x^n$$

$$-5x^2 \cdot A(x) = \sum_{n=0}^{+\infty} (-5x^{n+2} \cdot a_n) = -\sum_{n=2}^{+\infty} 5a_{n-2} x^n$$

$$4xA(x) - 5x^2 A(x) = 4a_0 x + \sum_{n=2}^{+\infty} 4a_{n-1} x^n - \sum_{n=2}^{+\infty} 5a_{n-2} x^n =$$

$$= 4a_0 x + \sum_{n=2}^{+\infty} (4a_{n-1} - 5a_{n-2}) x^n =$$

$$= 4a_0 x + \sum_{n=2}^{+\infty} a_n \cdot x^n = 4a_0 x - a_0 - a_1 x + \sum_{n=0}^{+\infty} a_n x^n = 4a_0 x - a_0 - a_1 x + A(x)$$

$$A(x)(4x - 5x^2 - 1) = 4a_0 x - a_1 x - a_0$$

$$A(x) = \frac{4a_0 x - a_0 - a_1 x}{4x - 5x^2 - 1} = \frac{4x - 2x - 1}{4x - 5x^2 - 1} = \boxed{\frac{2x - 1}{4x - 5x^2 - 1}}$$

#4.

$a_0 = -3 \qquad a_{n+1} = 7a_n + 4$

$$7xA(x) = \sum_{n=0}^{+\infty} 7a_n x^{n+1} = \sum_{n=1}^{+\infty} 7a_{n-1} x^n$$

$$7xA(x) + 4\sum_{n=0}^{+\infty} x^n = \sum_{n=1}^{+\infty} 7a_{n-1} x^n + \sum_{n=0}^{+\infty} 4x^n = 4 + \sum_{n=1}^{+\infty} (7a_{n-1} + 4) x^n =$$

$$= 4 + \sum_{n=0}^{+\infty} (7a_n + 4) x^{n+1} = 4 + \sum_{n=0}^{+\infty} a_{n+1} x^{n+1} = 4 - a_0 + A(x)$$

$$(7x-1)A(x) = 4 - a_0 - \sum_{n=0}^{+\infty} 4x^n = 4 - a_0 - \frac{4}{1-x}$$

$$A(x) = \frac{4 - a_0 - \frac{4}{1-x}}{7x-1} = \frac{(4-a_0)(1-x)-4}{(7x-1)(1-x)} = \frac{7(1-x)-4}{(7x-1)(1-x)} = \boxed{\frac{3-7x}{(7x-1)(1-x)}}$$

#5.

$$A_1(x) = \sum_{k=1}^{9} x^k$$

$$A_2(x) = x^2 + x^3 + x^5 + x^7$$

$$A_3(x) = A_4(x) = A_6(x) = A_7(x) = A_8(x) = A_{10}(x) = \sum_{k=0}^{9} x^k$$

$$A_5(x) = x^0 + x^3 + x^6 + x^9$$

$$A_9(x) = x^7$$

Всего : $\boxed{\prod_{k=1}^{10} A_k(x)}$