

# Homework-3a.

#1.

$$a) A = \{1\}, B = \{\{1\}\}, C = \{\{1\}, \{\{1\}\}\}$$

$$A \in B, \text{ т.к. } \{1\} = \{1\} - \text{true}$$

$$B \in C, \text{ т.к. } \{\{1\}\} = \{1\} \vee \{\{1\}\} = \{\{1\}\} - \text{true}$$

$$A \in C, \text{ т.к. } \{1\} = \{1\} \vee \{1\} = \{\{1\}\} - \text{true}$$

$$b) A = \{1\}, B = \{\{1\}\}, C = \{\{\{1\}\}\}$$

$$A \in B, \text{ т.к. } \{1\} = \{1\} - \text{true}$$

$$B \in C, \text{ т.к. } \{\{1\}\} = \{\{1\}\} - \text{true}$$

$$A \notin C, \text{ т.к. } \{1\} \neq \{\{1\}\}$$

#2.

$$\{x \in \mathbb{N} \mid (2 \mid x) \vee (\forall y \ y \mid x \rightarrow \sin(y) < \frac{9}{10})\}$$

#3.

$$\text{Дано: } \forall x (x \in S) \Leftrightarrow \exists y (x = \{y\})$$

$$\text{Тогда } \exists S \ x = \{S\} \Rightarrow x \in \{S\} \Rightarrow \text{т.к. } \forall x, \text{ то } S \in \{S\}$$



$$\{S\} \in S$$

$$\textcircled{1} \text{ и } \textcircled{2} \Rightarrow \dots S \in \{S\} \in S \in \{S\} \in \dots$$

⊥ по аксиоме основания.



#4.

$$c) A \subseteq B \cup C$$

$$\begin{aligned} (x \in A \Rightarrow x \in B \vee x \in C) &\Leftrightarrow (x \notin A \vee x \in B \vee x \in C) \Leftrightarrow \\ &\Leftrightarrow (\neg(x \in A \wedge x \notin B) \vee x \in C) \Leftrightarrow ((x \in A \wedge x \notin B) \Rightarrow x \in C) \\ &\Leftrightarrow A \cap \bar{B} \subseteq C \end{aligned}$$

ч.т.г.

$$b) (A \subseteq B \cap C) \Leftrightarrow (x \in A \Rightarrow x \in B \wedge x \in C) \Leftrightarrow$$

$$\begin{aligned} &\Leftrightarrow (x \notin A \vee x \in B \wedge x \in C) \Leftrightarrow ((x \notin A \vee x \in B) \wedge (x \notin A \vee x \in C)) \\ &\Leftrightarrow (x \in A \Rightarrow x \in B) \wedge (x \in A \Rightarrow x \in C) \Leftrightarrow A \subseteq B \text{ и } A \subseteq C \end{aligned}$$

$$a) (A \setminus B) \cup B = A$$

$$(x \in A \wedge x \notin B \vee x \in B) \Leftrightarrow (x \in A \wedge x \notin B \vee x \in B \vee \underbrace{x \in B \wedge x \in A}_{\subseteq A}) \Leftrightarrow$$

$$\Leftrightarrow (x \in A \wedge (x \notin B \vee x \in B) \vee x \in B) \Leftrightarrow (x \in A \vee x \in B) \Leftrightarrow$$

$$\Leftrightarrow \underline{A \cup B = A} \text{ (по условию)}$$

↓ по лемме 5 (о равносильности  $A \subseteq B$ ,  $A \cap B = A$ ,  $A \cup B = B$ )  
 $B \subseteq A$  ч.т.г.

#5.

$$A \subseteq C \text{ и } B \subseteq D$$

$(x, y) \in A \times B$  - определение

$$A \subseteq C \Leftrightarrow (x \in A \Rightarrow x \in C) \Leftrightarrow (x \notin A \vee x \in C)$$

$$B \subseteq D \Leftrightarrow (y \in B \Rightarrow y \in D) \Leftrightarrow (y \notin B \vee y \in D)$$

$$x \in A \times B \Leftrightarrow ((x, y) \in A \times B) \Leftrightarrow (x \in A \wedge y \in B) \Leftrightarrow$$

$$\Leftrightarrow (x \in A \wedge x \in C) \wedge (y \in B \wedge y \in D) \Leftrightarrow$$

$$\Leftrightarrow (x \in A \wedge y \in D) \wedge (y \in B \wedge x \in C) \Leftrightarrow$$

$$\Leftrightarrow (x, y \in (A \times D)) \wedge (x, y \in C \times B) \Leftrightarrow$$

$$\Leftrightarrow \alpha \in (A \times D) \cap (C \times B) \quad \text{u. i. g.}$$