Homework 4

#8

$$\begin{vmatrix} 7 & 3 & 2 & 6 \\ 8 & -9 & 4 & 9 \\ 7 & -2 & 7 & 3 \\ 5 & -3 & 3 & 4 \end{vmatrix} \overset{\cdot(-1)}{=} \begin{vmatrix} 7 & 3 & 2 & 6 \\ 1 & -12 & 2 & 3 \\ 0 & -5 & 5 & 3 \\ 5 & 3 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 87 & -12 & -15 \\ 1 & -12 & 2 & 3 \\ 0 & -5 & 5 & -3 \\ 0 & 57 & -7 & -11 \end{vmatrix} \overset{\text{разл. по I стр.}}{=} -\begin{vmatrix} 87 & -12 & -15 \\ -5 & 5 & -3 \\ 57 & -7 & -11 \end{vmatrix} =$$

$$= -\begin{vmatrix} 30 & -5 & -4 \\ -5 & 5 & -3 \\ 57 & -7 & -11 \end{vmatrix} = -\begin{vmatrix} 25 & 0 & -7 \\ -5 & 5 & -3 \\ 57 & -7 & -11 \end{vmatrix} = -\begin{vmatrix} 25 & 0 & -7 \\ -5 & 5 & -3 \\ 7 & -7 & 3 \end{vmatrix} = -(25 \cdot 15 - 5 \cdot 49 + 5 \cdot 49 - 21 \cdot 25) =$$

$$= -25 \cdot (-6) = \boxed{150} \text{ - Answer}$$

#9

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 & \dots & n-1 & n \\ 2 & 2 & 3 & 4 & 5 & \dots & n-1 & n \\ 3 & 3 & 3 & 4 & 5 & \dots & n-1 & n \\ 4 & 4 & 4 & 4 & 5 & \dots & n-1 & n \\ 5 & 5 & 5 & 5 & 5 & \dots & n-1 & n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & n-1 & n-1 & n-1 & n-1 & \dots & n-1 & n \\ n & n & n & n & n & \dots & n & n \end{vmatrix} \overset{\cdot(-1)}{=} \begin{vmatrix} 1 & 1 & 2 & 3 & 4 & \dots & n-2 & n-1 \\ 2 & 0 & 1 & 2 & 3 & \dots & n-3 & n-2 \\ 3 & 0 & 0 & 1 & 2 & \dots & n-4 & n-3 \\ 4 & 0 & 0 & 0 & 1 & \dots & n-5 & n-4 \\ 5 & 0 & 0 & 0 & 0 & \dots & n-6 & n-5 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ n & 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{vmatrix} \overset{\cdot(-1)}{=} \begin{vmatrix} n & 1 & 2 & 3 & 4 & \dots & n-2 & n-1 \\ n & 0 & 1 & 2 & 3 & \dots & n-3 & n-2 \\ n & 0 & 0 & 1 & 2 & \dots & n-4 & n-3 \\ n & 0 & 0 & 0 & 1 & \dots & n-5 & n-4 \\ n & 0 & 0 & 0 & 0 & \dots & n-6 & n-5 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ n & 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & 1 & 2 & 3 & 4 & \dots & n-2 & n-1 \\ 0 & 0 & 1 & 2 & 3 & \dots & n-3 & n-2 \\ 0 & 0 & 0 & 1 & 2 & \dots & n-4 & n-3 \\ 0 & 0 & 0 & 0 & 1 & \dots & n-5 & n-4 \\ 0 & 0 & 0 & 0 & 0 & \dots & n-6 & n-5 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ n & 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{vmatrix} \overset{\text{разл. по I стр.}}{=} n \cdot (-1)^{n+1} \cdot \begin{vmatrix} 1 & 2 & 3 & \dots & n-1 \\ 0 & 1 & 3 & \dots & n-2 \\ 0 & 0 & 1 & \dots & n-3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{vmatrix} = n \cdot (-1)^{n+1} \cdot 1 = \boxed{n \cdot (-1)^{n+1}}$$

Answer:

#10

$$\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ a_0 & x & a_2 & \dots & a_n \\ a_0 & a_1 & x & \dots & a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_0 & a_1 & a_2 & \dots & x \end{vmatrix} = \begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ 0 & x-a_1 & 0 & \dots & 0 \\ 0 & 0 & x-a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & x-a_n \end{vmatrix} = a_0 \cdot (x-a_1) \cdot (x-a_2) \cdot \dots \cdot (x-a_n) =$$

$$= \boxed{a_0 \cdot \prod_{i=1}^{n} (x-a_i)} \text{ - Answer}$$