Семинар 16, 30.01.24.

Интегралы

1. $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1)$
2. $\int \frac{1}{x} dx = \ln|x| + C$
3. $\int \sin x \, dx = -\cos x + C$
4. $\int \cos x \, dx = \sin x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C$
6. $\int \frac{dx}{\cos^2 x} = tg x + C$
7. $\int \frac{dx}{\sin^2 x} = -ctg + C$
8. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$
9. $\int \frac{dx}{1+x^2} = arctg x + C$

① **Вычислить:**

a) $\int\frac{(\sqrt{x}-2\sqrt[3]{x})^2}{x}dx = \int\frac{x-4x^{\frac{5}{6}}+4x^{\frac{2}{3}}}{x}dx = \int(1-4x^{-\frac{1}{6}}+4x^{-\frac{1}{3}})dx =$

$= x-4\cdot\frac{x^{1-\frac{1}{6}}}{1-\frac{1}{6}}+4\cdot\frac{x^{1-\frac{1}{3}}}{1-\frac{1}{3}}+C = \boxed{x-\frac{24}{5}x^{\frac{5}{6}}+6x^{\frac{2}{3}}+C}$

b) $\int 3^x\cdot 5^{2x}dx = \int 75^x dx = \boxed{\frac{75^x}{\ln 75}+C}$

c) $\int\cos^2\frac{x}{2}dx = \int\frac{1+\cos x}{2}dx = \frac{1}{2}(x+\sin x)+C = \boxed{\frac{x}{2}+\frac{1}{2}\sin x+C}$

② **Вычислить:**

$\int f(x(t))\cdot x'(t)\,dt = \int f(x)dx \qquad df = f'(x)dx$

a) $\int(3x-5)^{10}dx = \frac{1}{3}\int(3x-5)^{10}d(3x-5) \underset{y=3x-5}{=} \frac{1}{3}\int y^{10}dy = \frac{1}{3}\cdot\frac{y^{11}}{11}+C =$

$= \boxed{\frac{1}{33}(3x-5)^{11}+C}$

b) $\int x^2\sqrt[5]{5x^3+1}\,dx = \frac{1}{15}\int\sqrt[5]{5x^3+1}\,d(5x^3+1) \underset{y=5x^3+1}{=} \frac{1}{15}\int\sqrt[5]{y}\,dy = \frac{1}{15}\cdot\frac{y^{1+\frac{1}{5}}}{1+\frac{1}{5}}+C =$

$= \frac{1}{18}y^{\frac{6}{5}}+C = \boxed{\frac{1}{18}(5x^3+1)^{\frac{6}{5}}+C}$

c) $\int tg\,x\,dx = \int\frac{\sin x}{\cos x}dx = \int\frac{d(-\cos x)}{\cos x} \underset{y=-\cos x}{=} \int\frac{dy}{-y} = -\ln|y|+C = -\ln|\cos x|+C$

d) $\int\frac{x^7}{\sqrt{1-x^{16}}}dx = \frac{1}{8}\int\frac{d(x^8)}{\sqrt{1-x^{16}}} \underset{y=x^8}{=} \frac{1}{8}\int\frac{dy}{\sqrt{1-y^2}} = \frac{1}{8}\arcsin y+C = \boxed{\frac{1}{8}\arcsin(x^8)+C}$

③ **Вычислить:**

a) $\int\frac{dx}{2+\sqrt{x}} \underset{y=\sqrt{x}}{=} \int\frac{d(y^2)}{2+y} = 2\int\frac{dy}{2+y} = 2\int\frac{y+2-2}{y+2}dy = 2\int\left(1+\frac{-2}{y+2}\right)dy =$

$= 2y-4\ln|y+2|+C = \boxed{2\sqrt{x}-4\ln(\sqrt{x}+2)+C}$

b) $\int \frac{dx}{\sqrt{e^x+1}} = \begin{bmatrix} y=\sqrt{e^x+1} \\ x=\ln(y^2-1) \end{bmatrix} = \int \frac{d(\ln(y^2-1))}{y} = \int \frac{\frac{2y}{y^2-1}dy}{y} =$

$= 2\int \frac{dy}{y^2-1} = 2\int \frac{dy}{(y-1)(y+1)} = 2\int \left(\frac{1/2}{y-1} + \frac{-1/2}{y+1}\right) dy =$

$= \int \left(\frac{1}{y-1} - \frac{1}{y+1}\right) dy = \ln|y-1| - \ln|y+1| + C =$

$= \ln\left(\frac{y-1}{y+1}\right) + C = \boxed{\ln\left(\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}\right) + C}$

③ Вычислить:

$\int f dg = fg - \int g df$

a) $\int x^2 \cdot e^x dx = \int x^2 d(e^x) = x^2 \cdot e^x - \int e^x d(x^2) =$

$= x^2 e^x - 2\int e^x dx = x^2 e^x - 2\int x d(e^x) =$

$= x^2 e^x - 2(e^x \cdot x - \int e^x dx) = \boxed{x^2 e^x - 2xe^x + 2e^x + C}$

b) $\int \arccos^2 x \, dx = x\arccos^2 x - \int x d(\arccos^2 x) =$

$= x\arccos^2 x + \int \frac{x \cdot 2\arccos x}{\sqrt{1-x^2}} dx =$

$= x \cdot \arccos^2 x - 2\int \arccos x \, d(\sqrt{1-x^2}) =$

$= x\arccos^2 x - 2(\sqrt{1-x^2} \cdot \arccos x - \int \sqrt{1-x^2} \, d(\arccos x)) =$

$= \boxed{x \cdot \arccos^2 x - 2\sqrt{1-x^2} \cdot \arccos x - 2x + C}$