Домашняя работа #1.

#7

$$\begin{cases} 2x_1 + 3x_2 + x_3 + 2x_4 = 4 \\ \lambda x_1 + 3x_2 + x_3 + 4x_4 = 5 \\ 2x_1 + 11x_2 + 3x_3 + 5x_4 = 2 \\ 2x_1 + 9x_2 + x_3 + 3x_4 = 2 \end{cases}$$

$$\left(\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 2 & 3 & 1 & 2 & 4 \\ \lambda & 3 & 1 & 4 & 5 \\ 2 & 11 & 3 & 5 & 2 \\ 2 & 9 & 1 & 3 & 2 \end{array}\right) \xrightarrow{\text{столбцы } I \leftrightarrow IV \leftrightarrow III} \left(\begin{array}{cccc|c} x_3 & x_2 & x_4 & x_1 & \\ 1 & 3 & 2 & 2 & 4 \\ 1 & 3 & 4 & \lambda & 5 \\ 3 & 11 & 5 & 2 & 2 \\ 1 & 9 & 3 & 2 & 2 \end{array}\right) \xrightarrow{\substack{II-I \\ III-3I \\ IV-I}} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 4 \\ 0 & 0 & 2 & \lambda-2 & 1 \\ 0 & 2 & -1 & -4 & -10 \\ 0 & 6 & 1 & 0 & -2 \end{array}\right) \xrightarrow{\substack{IV-3III \\ III \leftrightarrow II}}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 4 \\ 0 & 2 & -1 & -4 & -10 \\ 0 & 0 & 2 & \lambda-2 & 1 \\ 0 & 0 & 4 & 12 & 28 \end{array}\right) \xrightarrow{\substack{IV \cdot \frac{1}{4} \\ II + IV \\ III - 2 \cdot IV}} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 4 \\ 0 & 2 & 0 & -1 & -3 \\ 0 & 0 & 0 & \lambda-8 & -13 \\ 0 & 0 & 1 & 3 & 7 \end{array}\right) \xrightarrow{\substack{I - II \\ I - 2IV}} \left(\begin{array}{cccc|c} 1 & 1 & 0 & -3 & -7 \\ 0 & 2 & 0 & -1 & -3 \\ 0 & 0 & 0 & \lambda-8 & -13 \\ 0 & 0 & 1 & 3 & 7 \end{array}\right)$$

$$\begin{cases} x_3 + x_2 - 3x_1 = -7 \\ 2x_2 - x_1 = -3 \\ (\lambda-8)x_1 = -13 \\ x_4 + 3x_1 = 7 \end{cases} \Rightarrow \begin{cases} x_3 = -7 - \frac{39}{\lambda-8} + \frac{3}{2} + \frac{13}{2\lambda-16} \\ x_2 = -\frac{3}{2} + \frac{-13}{2\lambda-16} \\ x_1 = \frac{-13}{\lambda-8} \\ x_4 = 7 + \frac{39}{\lambda-8} \end{cases} \Rightarrow \begin{cases} x_3 = \frac{23-11\lambda}{2\lambda-16}, \\ x_2 = \frac{11-3\lambda}{2\lambda-16}, \\ x_1 = \frac{-13}{\lambda-8}, \\ x_4 = \frac{7\lambda-17}{\lambda-8}, \end{cases} \quad \begin{array}{l} \lambda \in \mathbb{R} \\ \lambda \neq 8 \text{ (при } \lambda = 8 \text{ нет реш.)} \end{array}$$

Ответ

#8.

$(1; 1; 1)$ $A + B + C + D = 0$

$(2; 3; -1)$ $2A + 3B - C + D = 0$

$(3; -1; -1)$ $3A - B - C + D = 0$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 2 & 3 & -1 & 1 & 0 \\ 3 & -1 & -1 & 1 & 0 \end{array}\right) \xrightarrow{\substack{II-2I \\ III-3I}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 \\ 0 & 2 & -4 & -2 & 0 \end{array}\right) \xrightarrow{III - II \cdot 2} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{array}\right) \xrightarrow{\substack{I+III \\ I+II}} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{array}\right)$$

$$\begin{cases} A+2B=0 \\ B-C-D=0 \\ 2C=0 \end{cases} \Rightarrow \begin{cases} A=-2B=-2D \\ B=D \\ C=0 \end{cases}$$

$$Ax+By+Cz+D=0$$

$$-2Dx+Dy+0z+D=0 \quad |:D$$

$$-2x+y+0\cdot z+1=0$$

Ответ: $-2x+y+0\cdot z+1=0$.

#9.

$$\frac{A}{x+2}+\frac{B}{x+3}+\frac{C}{x-1}=\frac{x^2-19x+6}{(x-1)(x+2)(x+3)}$$

$$A(x+3)(x-1)+B(x+2)(x-1)+C(x+2)(x+3)=x^2-19x+6$$

$$Ax^2+2Ax-3A+Bx^2+Bx-2B+Cx^2+5Cx+6C=x^2-19x+6$$

$$(A+B+C)x^2+(2A+B+5C)x+(-3A-2B+6C)=x^2-19x+6$$

$$\begin{cases} A+B+C=1 \\ 2A+B+5C=-19 \\ -3A-2B+6C=6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 5 & -19 \\ -3 & -2 & 6 & 6 \end{array}\right) \xrightarrow[\text{III}+3\text{I}]{\text{II}-2\text{I}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 3 & -21 \\ 0 & 1 & 9 & 9 \end{array}\right) \xrightarrow[\text{III}\cdot(-1)]{\text{III}+\text{II}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 21 \\ 0 & 0 & 12 & -12 \end{array}\right)$$

$$\begin{cases} A+B+C=1 \\ B-3C=21 \\ 12C=-12 \end{cases} \qquad \begin{cases} A=-16 \\ B=18 \\ C=-1 \end{cases}$$

Ответ: $A=-16$; $B=18$; $C=-1$.