Семинар 27, 16.04.24 – Бельдиев

$\boxed{1190}$ $f = 2x_1^2 + 9x_2^2 + 3x_3^2 + 8x_1x_2 - 4x_1x_3 - 10x_2x_3 =$

$= 2(x_1^2 + 2x_1(2x_2 - x_3)) + 9x_2^2 + 3x_3^2 - 10x_2x_3 =$

$= 2(x_1^2 + 2(2x_2 - x_3) + (2x_2 - x_3)^2) + x_2^2 + x_3^2 - 2x_2x_3 =$

$= 2(x_1 + 2x_2 - x_3)^2 + (x_2 - x_3)^2 = 2t_1^2 + t_2^2$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left| \begin{array}{l} t_1 = x_1 + 2x_2 - x_3 \\ t_2 = +x_2 - x_3 \\ t_3 = x_2 \end{array} \right.$$

$g = (2y_1^2 + 3y_2^2 + 6y_3^2 - 4y_1y_2 - 4y_1y_3 + 8y_2y_3 =$

$= 2(y_1^2 - 2y_1(y_2 + y_3) + (y_2 + y_3)^2) + y_2^2 + 4y_3^2 + 4y_2y_3 =$

$= 2(y_1 - y_2 - y_3)^2 + (y_2 + 2y_3)^2 = 2t_1'^2 + t_2'^2 = 2t_1^2 + t_2^2$

$$\left| \begin{array}{l} t_1' = t_1 = y_1 - y_2 - y_3 \\ t_2' = t_2 = y_3 + 2y_3 \\ t_3' = t_3 = y_2 \end{array} \right.$$

$x_2 = t_3$

$x_3 = t_3 - t_2$

$x_1 = t_1 - 2x_2 + x_3 = t_1 - 2t_3 - t_3 - t_2 = t_1 - t_2 - t_3$

$x_1 = t_1 - t_2 - t_3 = y_1 - y_2 - y_3 - y_2 - 2y_3 - y_2 = \boxed{y_1 - 3y_2 - 3y_3}$

$x_2 = \boxed{y_2}$

$x_3 = t_3 - t_2 = \boxed{-2y_3}$

V – вект. пространство над F, $F = \mathbb{R}$; $q: V \to F$; $b: V \times V \to F$, $q(v) = b(v,v)$

положит. опред.: $q(v) > 0$, если $v \neq 0$

отрицат. опред.: $q(v) < 0$, если $v \neq 0$.

$\underbrace{x_1^2 + x_2^2 + \ldots + x_p^2}_{p} \underbrace{- x_{p+1}^2 - \ldots - x_{p+q}^2}_{q} \to B = \begin{pmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 1 & & & & & & \\ & & & -1 & & & & & \\ & & & & \ddots & & & & \\ & & & & & -1 & & & \\ & & & & & & 0 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{pmatrix} \begin{matrix} \}p \\ \\ \\ \}q \\ \\ \\ \}n-p-q, \; n = \dim V \end{matrix}$

Сигнатура: (p, q)

$B = \begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & \ddots & \\ & & & & & -1 \end{pmatrix} \begin{matrix} \}p \\ \\ \}q \end{matrix} \qquad B' = \begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & \ddots & \\ & & & & & -1 \end{pmatrix} \begin{matrix} p+m \\ \\ q-m \end{matrix} \quad \begin{matrix} e_1' \\ e_2' \\ \vdots \\ e_n' \end{matrix}$

$U_- = \langle e_{p+1}, \ldots, e_{p+q} \rangle \subset V = (e_1, \ldots, e_{p+q})$

$\begin{pmatrix} -1 & & \\ & \ddots & \\ & & -1 \end{pmatrix}$ – на пр-ве U в отриц. опредл. в ограничении на U_-

$U_+' = \langle e_1', \ldots, e_{p+q}' \rangle \subset V$

$\begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} \}p+m$ в положит. опред. на U_+'

$\dim(U_-) + \dim(U_+') = p + q + m > \dim V$

$\dim(U_- + U_+') + \dim(U_- \cap U_+') > 0$

$v \in U_- \cap U_+'$

$b(v,v) < 0 \quad b(v,v) > 0$

Метод Якоби:

$$B_e = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}$$

$\Delta_1 = b_{11}$

$\Delta_2 = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$

$\Delta_k = \begin{vmatrix} b_{11} & \dots & b_{1k} \\ \vdots & & \vdots \\ b_{k1} & \dots & b_{kk} \end{vmatrix}$

$\Delta_n = \det B_e$

$\Delta_1, \Delta_2, \dots, \Delta_n \neq 0$.

$V_1 = \langle e_1 \rangle$

$V_2 = \langle e_1, e_2 \rangle$

$\vdots$

$V_n = \langle e_1, e_2, \dots, e_n \rangle = V$

$\Delta_1 > 0 \quad b|_{V_1} - (1)$

$\Delta_2 > 0 \quad b|_{V_2}$

$b|_{V_2} \quad \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \cancel{\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}}, \cancel{\begin{pmatrix} -1 & \\ & -1 \end{pmatrix}}$

$\boxed{38.8}$ (a) $2x_1y_1 - x_1y_2 + x_1y_3 - x_2y_1 + x_3y_1 + 3x_3y_3$

$B_e = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 3 \end{pmatrix}$ $\quad \Delta_1 = 2$, $\Delta_2 = -1$, $\Delta_3 = -3$ $\quad \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$

(b) $x_1^2 + x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$

$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ $\quad \Delta_1 = 1 > 0$, $\Delta_2 = -3 < 0$, $\Delta_3 = -7 < 0$

$x_1^2 + x_2^2 - x_3^2$ – нормальный вид

$\boxed{38.11}$ (a) $5x_1^2 + x_2^2 + \lambda x_3^2 + 4x_1x_2 - 2x_1x_3 - 2x_2x_3$

$\begin{pmatrix} 5 & 2 & -1 \\ 2 & 1 & -1 \\ -1 & -1 & \lambda \end{pmatrix}$

$\Delta_1 = 5$

$\Delta_2 = 1$

$\Delta_3 = 5\lambda + 2 + 2 - 9 - 5 - 4\lambda = \lambda - 2 > 0 \Rightarrow \lambda > 2$