Homework 17.

#3.

a) $\int \frac{x^2-x+1}{\sqrt{x}}dx = \int (x^{2-\frac{1}{2}} - x^{1-\frac{1}{2}} + x^{-\frac{1}{2}})dx = \int (x^{\frac{3}{2}} - x^{\frac{1}{2}} + x^{-\frac{1}{2}})dx =$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + C = \boxed{2\sqrt{x}\left(\frac{x^2}{5} - \frac{x}{3} + 1\right) + C}$

b) $\int \sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int \sqrt{x\sqrt{\sqrt{x^3}}}\,dx = \int \sqrt{\sqrt{\sqrt{x^7}}}\,dx = \int x^{\frac{7}{8}}dx = \frac{x^{\frac{15}{8}}}{\frac{15}{8}} + C = \boxed{\frac{8}{15}x^{\frac{15}{8}} + C}$

c) $\int \sin^2\frac{x}{2}dx = \int \left(1 - \cos^2\frac{x}{2}\right)dx = \int \left(1 - \frac{1}{2}(1+\cos x)\right)dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos x\right)dx =$

$= \frac{1}{2}\int (1-\cos x)dx = \frac{1}{2}(x - \sin x) + C = \boxed{\frac{x}{2} - \frac{\sin x}{2} + C}$

#4.

a) $\int \frac{6x-7}{3x^2-7x+1}dx = \int \frac{d(3x^2-7x+1)}{3x^2-7x+1} = [t = 3x^2-7x+1] = \int \frac{dt}{t} = \ln|t| + C =$

$= \boxed{\ln|3x^2-7x+1| + C}$

b) $\int x^3\sqrt{x^2-1}\,dx = \int \frac{1}{2}x^2\sqrt{x^2-1}\,d(x^2-1) = [t = x^2-1] = \frac{1}{2}\int (t+1)\sqrt{t}\,dt =$

$= \frac{1}{2}\int (t^{\frac{3}{2}} + t^{\frac{1}{2}})dt = \frac{1}{2}\left(\frac{2t^{\frac{5}{2}}}{5} + \frac{2t^{\frac{3}{2}}}{3}\right) + C = \sqrt{t}\left(\frac{t^2}{5} + \frac{t}{3}\right) + C =$

$= \sqrt{x^2-1}\left(\frac{x^4-2x^2+1}{5} + \frac{x^2-1}{3}\right) + C = \boxed{\sqrt{x^2-1}\cdot\frac{1}{15}\cdot(3x^4-x^2-2) + C}$

c) $\int e^{2x^2+2x-1}(2x+1)dx = \int \frac{1}{2}\,d(e^{2x^2+2x-1}) = \boxed{\frac{1}{2}e^{2x^2+2x-1} + C}$

d) $\int \frac{2^x}{\sqrt{1-4^x}}dx = \int \frac{1}{\ln 2}\cdot\frac{1}{\sqrt{1-4^x}}\,d(2^x) = [2^x = t] = \int \frac{1}{\ln 2}\cdot\frac{1}{\sqrt{1-t^2}}dt =$

$= \frac{1}{\ln 2}\arcsin t + C = \boxed{\frac{1}{\ln 2}\cdot\arcsin(2^x) + C}$

e) $\int \frac{dx}{x(\ln x)\cdot\ln(\ln x)} = \int \frac{d(\ln x)}{\ln x\cdot\ln(\ln x)} = [t = \ln x] = \int \frac{dt}{t\cdot\ln(t)} = \int \frac{d(\ln t)}{\ln t} =$

$= [y = \ln t = \ln(\ln x)] = \int \frac{dy}{y} = \ln|y| + C = \boxed{\ln|\ln(\ln x)| + C}$

f) $\int \sin^6 x\cdot\cos x\,dx = \int \sin^6 x\,d(\sin x) = [t = \sin x] = \int t^6 dt = \frac{t^7}{7} + C = \boxed{\frac{\sin^7 x}{7} + C}$

g) $\int \frac{1}{x^2}\cos\frac{1}{x}\,dx = \int -1\cdot d\left(\sin\frac{1}{x}\right) = \boxed{-\sin\frac{1}{x} + C}$

h) $\int \frac{\sin x\,dx}{\sqrt{1+2\cos x}} = \int -1\cdot d\left(\sqrt{1+2\cos x}\right) = -\sqrt{1+2\cos x} + C$

i) $\int \frac{dx}{\sqrt{1-x^2}\arcsin x} = \int \frac{d(\arcsin x)}{\arcsin x} = [\arcsin x = t] = \int \frac{dt}{t} = \ln|t| + C =$

$= \boxed{\ln|\arcsin x| + C}$

#5.

a) $\int x\ln x\,dx = \int \frac{1}{2}\ln x\,d(x^2) = \frac{1}{2}\left(\ln x\cdot x^2 - \int x^2\,d(\ln x)\right) =$

$= \frac{1}{2}\left(x^2\ln x - \int x^2\cdot\frac{1}{x}\,dx\right) = \frac{1}{2}\left(x^2\ln x - \int x\,dx\right) = \boxed{\frac{x^2\ln x}{2} - \frac{x^2}{4} + C}$

b) $\int \operatorname{arctg} x\,dx = x\cdot\operatorname{arctg} x - \int x\cdot d(\operatorname{arctg} x) = x\cdot\operatorname{arctg} x - \int \frac{x}{1+x^2}\,dx =$

$= x\cdot\operatorname{arctg} x - \frac{1}{2}\int \frac{d(x^2+1)}{1+x^2} = \boxed{x\cdot\operatorname{arctg} x - \frac{1}{2}\ln|x^2+1| + C}$

c) $\int \frac{\arcsin x}{x^2}\,dx = \int -\arcsin x\,d\left(\frac{1}{x}\right) = -\frac{\arcsin x}{x} + \int \frac{1}{x}\,d(\arcsin x) =$

$= -\frac{\arcsin x}{x} + \int \frac{1}{x\sqrt{1-x^2}}\,dx = \begin{bmatrix}\sqrt{1-x^2} = t\\ x = \sqrt{1-t^2}\end{bmatrix} = -\frac{\arcsin x}{x} + \int \frac{1}{t\sqrt{1-t^2}}\,d\left(\sqrt{1-t^2}\right) =$

$= -\frac{\arcsin x}{x} + \int \frac{1}{t\sqrt{1-t^2}}\cdot\frac{-2t}{2\sqrt{1-t^2}}\,d(t) = -\frac{\arcsin x}{x} + \int \frac{dt}{t^2-1} =$

$= -\frac{\arcsin x}{x} + \int\left(\frac{1/2}{t-1} + \frac{-1/2}{t+1}\right)dt = -\frac{\arcsin x}{x} + \frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1| + C =$

$= \boxed{-\frac{\arcsin x}{x} + \frac{1}{2}\ln\left|\sqrt{1-x^2}-1\right| - \frac{1}{2}\ln\left|\sqrt{1-x^2}+1\right| + C}$

#1.

$f(x) = |1+x| - |1-x|, \quad x \in \mathbb{R}$

$\int |1+x| - |1-x|\,dx = \int |1+x|\,dx - \int |1-x|\,dx$

1) $x < -1: \quad \int -(1+x)\,dx - \int (1-x)\,dx = -\int dx - \int x\,dx - \int dx + \int x\,dx =$

$= -2\int dx = -2x + C$

2) $x \in [-1;1]$: $\int(1+x)\,dx - \int(1-x)\,dx = \int dx + \int x\,dx - \int dx + \int x\,dx =$

$= 2\int x\,dx = 2\cdot\frac{x^2}{2} + C = x^2 + C$

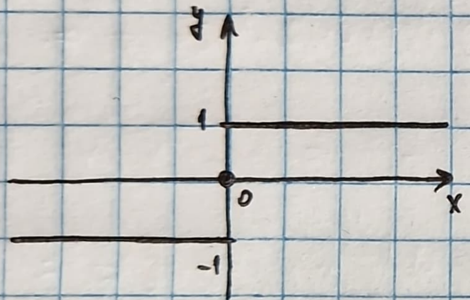
3) $x > 1$: $\int(1+x)\,dx - \int -(1-x)\,dx = \int dx + \int x\,dx + \int dx - \int x\,dx =$

$= 2\int dx = 2x + C$

$$\boxed{\text{Ответ: } \begin{cases} -2x + C, & x \in (-\infty; -1) \\ x^2 + C, & x \in [-1; 1] \\ 2x + C, & x \in (1; +\infty) \end{cases}}$$

#2.

а) $f(x) = \operatorname{sign}(x)$



$$\operatorname{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

В $x = 0$ $f(x)$ прерывается, т.к. $f(0) = \operatorname{sign}(0) = 0$, но

$\lim\limits_{x \to 0^-} \operatorname{sign}(x) = -1$

$\lim\limits_{x \to 0^+} \operatorname{sign}(x) = 1$

$\Rightarrow$ на $\mathbb{R}$ не будет первообразной, т.к. в т. $x = 0$ $(\operatorname{sign} x)'$ не сущ.

б) $f(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}$ Тогда первообразные: $F(x) = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$.