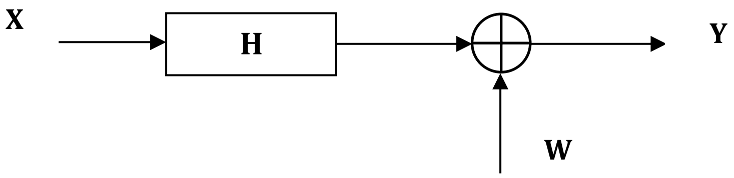
ELEC 462/562 Statistical Signal Processing
Project
Due: 05.01.2021

Signal Restoration

1. Assume that we observe the signal $\mathbf{Y} = \mathbf{HX} + \mathbf{W}$ over an LTI noncausal channel given in the following figure. Here $\mathbf{H}$ is a convolution matrix constituted by the system impulse response $h[n] = [8 \;\; 9 \;\; \underset{\uparrow}{10} \;\; 9 \;\; 8]/44$. The noise $\mathbf{W} \in \mathbb{R}^{256}$ is a white Gaussian vector with zero mean and variance 25×10^{-4}. $\mathbf{X}$ and $\mathbf{W}$ are assumed to be uncorrelated.



We assume that the signal $\mathbf{X}$ is a Gaussian random vector with zero mean and covariance matrix $(\lambda \mathbf{L}^{T}\mathbf{L})^{-1}$ where $\mathbf{L}$ is a Tikhonov regularization matrix constituted by the Laplacian operator (or impulse response) $l[n] = [-1 \;\; \underset{\uparrow}{2} \;\; -1]$. The parameter λ is the regularization parameter that controls the smoothing level. The observation vector $\mathbf{Y} = \mathbf{y} \in \mathbb{R}^{256}$, the ground-truth signal $\mathbf{x} \in \mathbb{R}^{256}$, the system matrix $\mathbf{H} \in \mathbb{R}^{256 \times 256}$ and the matrix $\mathbf{L} \in \mathbb{R}^{256 \times 256}$ are given in the attached mat files. In this project, your task is to estimate the signal $\mathbf{X}$ using three different estimators and to compare their performances.

a) Derive the maximum likelihood (ML) estimator for $\mathbf{X}$.

b) For given observation vector $\mathbf{Y} = \mathbf{y}$, obtain the ML estimate $\hat{\mathbf{x}}_{ML}$. Plot the observation $\boldsymbol{y}$. Plot the estimate $\hat{\mathbf{x}}_{ML}$ along with the ground-truth $\boldsymbol{x}$ in the same figure. Interpret the result.

c) Derive the maximum-a-posteriori (MAP) estimator for $\mathbf{X}$.

d) For given observation vector $\mathbf{Y} = \mathbf{y}$, obtain the MAP estimate $\hat{\mathbf{x}}_{MAP}(\lambda)$ for $\lambda = \{1, 10, \; 100, \; 1000\}$. Plot each $\hat{\mathbf{x}}_{MAP}(\lambda)$ for different values of λ along with the ground-truth $\mathbf{x}$. Interpret the effect of λ on the MAP estimate.

e) Derive the minimum mean square error (MMSE) linear estimator for $\mathbf{X}$.

f) For given observation vector $\mathbf{Y} = \mathbf{y}$, obtain the linear estimate $\hat{\mathbf{x}}_{MMSE}(\lambda)$ for $\lambda = \{1, 10, \; 100, \; 1000\}$. Plot each $\hat{\mathbf{x}}_{MMSE}(\lambda)$ for different values of λ along with the ground-truth $\mathbf{x}$. Interpret the effect of λ on the Linear estimate.

g) Calculate peak signal-to-noise ratio (PSNR) of each estimate such that

$$PSNR = 20 \log_{10} \frac{\max(\mathbf{x})\sqrt{256}}{||\mathbf{x} - \hat{\mathbf{x}}||}$$

List the best results in a table

	ML	MAP ($\lambda =$)	MMSE ($\lambda =$)
PSNR (dB)			