III sem B.Tech(CCE)
2171 Digital systems and computer organisation

- **Topics covered**
- MINTERM, MAXTERM
- Writing Boolean expressions for the given truth table
- Design of a combinational circuit for the given problem

MINTERMS & MAXTERMS

- MIN and MAX are means to represent the inputs through logical AND /OR operations
- Consider a 3-variable Boolean function, F(a,b,c)
- Prepare a table with 6 columns
- In first column, write all possible binary combinations, possible with three input variables, one below the other. Ex:000,001…., .. . 111
- In 2nd column, write decimal equivalent of all the corresponding input combinations.

MSB LSB

Rough

2^2 2^1 2^0 → Least significant bit

a (=4)	b (=2)	c (=1)		MINTERM		MAXTERM	
0	0	0	0	$(\bar{a}\bar{b}\bar{c})$	→ m_0	$(a+b+c)$	M_0
0	0	1	1	$\bar{a}\bar{b}c$ =1	m_1	$(a+b+\bar{c})$ =0	M_1
0	1	0	2	$\bar{a}b\bar{c}$	m_2	$(a+\bar{b}+c)$	M_2
0	1	1	3	$\bar{a}bc$	m_3	$(a+\bar{b}+\bar{c})$	M_3
1	0	0	4	$a\bar{b}\bar{c}$	m_4	$(\bar{a}+b+c)$	M_4
1	0	1	5	$a\bar{b}c$	m_5	$(\bar{a}+b+\bar{c})$	M_5
1	1	0	6	$ab\bar{c}$	m_6	$(\bar{a}+\bar{b}+c)$	M_6
1	1	1	7	abc	m_7	$(\bar{a}+\bar{b}+\bar{c})$	M_7

Minterms (Standard products) and Maxterms (standard sums)

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

Write the Boolean function for f(x, y, z) given below

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z}$$

$$= m_0 + m_1 + m_4 + m_5 + m_6$$

$$= \underline{\underline{\sum m(0,1,4,5,6)}}$$

Boolean function can be represented as (i) sum of minterms and (ii) product of maxterms.

Sum of Minterms and product of maxterms expressions

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

4

6

$\bar{y} + x\bar{z}$

$xy\bar{z}$

$=$ 0, 1, 4, 5

Sum of minterms : Canonical form

$$F(x, y, z) = x'y'z' + x'y'z + xy'z' + xy'z + xyz'$$
$$= m_0 + m_1 + m_4 + m_5 + m_6$$
$$= \Sigma_m(0, 1, 4, 5, 6)$$

$a + a = a$

$a = a + a + a + a$

Sum of product :

$$F(x, y, z) = x'y' + xy' + x'z'$$

$= y' + x\bar{z}$

SOP

Product of maxterms : Canonical form

$$F(x, y, z) = (x+y'+z).(x+y'+z').(x'+y'+z')$$
$$= M_2 . M_3 . M_7$$
$$= \Pi_M(2,3,7)$$

$a + bc = (a+b).(a+c)$

Product of sum :

$$F(x, y, z) = (x+y')(y'+z')$$

Rough

$$(x+\bar{y}+z)(x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+\bar{z})$$

$$(a+z\cdot\bar{z})\quad(b+x\cdot\bar{x})$$

$$a$$

$$(x+\bar{y})(\bar{y}+\bar{z})$$

Relationship between minterms and maxterms

- Sum of minterms : Canonical form
- F(x , y, z)= x’y’z’ + x’y’ z +xy’z’+xy’z+ xyz’

$= m_0 + m_1 + m_4 + m_5 + m_6$

F $= \Sigma(0, 1,4,5,6)$ $= \Pi M(2,3,7)$

$F'(x, y, z) = \Sigma(2, 3, 7) = m2+m3+m7$ $= \Pi_M(0,1,4,5,6)$

Taking complement on both sides and applying DeMorgan’s theorem

$F(x, y, z) = (m2+m3+m7)' = m2'.m3'.m7'$

$= (x'yz')'.(x'yz)'.(xyz)' = (x+y'+z).(x+y'+z').(x'+y'+z')$

F $= M2.M3.M7 = \Pi(2,3,7)$

$$\mathbf{m_j' = M_j}$$

Express the Boolean function F(a,b,c)= ab'+c' using Sum of minterms and Product of maxterms

- Two methods
  1. Identify the missing term and include them in the expression using the postulates : x+0 =x , x.1 =x , x+x'=1, x.x'=0

  2. Write the truth table from the given expression and then write sum of minterms and product of maxterms

$$a\bar{b}\cdot(c+\bar{c})$$

$$a\bar{b}c+a\bar{b}\bar{c}$$

F(a,b,c)= ab'+c' using method 1

$$a\bar{b}(c+\bar{c}) + \bar{c}(a+\bar{a})(b+\bar{b})$$

$$= a\bar{b}c + a\bar{b}\bar{c} + a\bar{c} + \bar{a}\bar{c}$$

$$= \qquad + a\bar{c}(b+\bar{b}) + \bar{a}\bar{c}(b+\bar{b})$$

$$a\bar{b}c + a\bar{b}\bar{c} + ab\bar{c} + a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}\bar{c}$$

100   010

$$= m_5 + m_4 + m_6 + m_2 + m_0$$

$F(a,b,c) = ab' + c'$ using method 2

$c = 0$

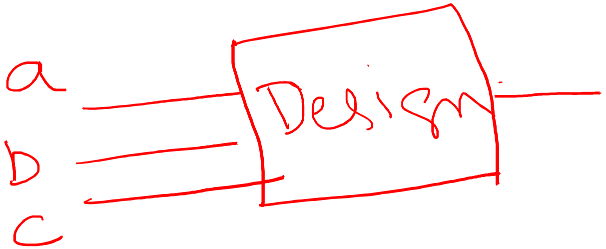
a	b	c	$\overline{F}$	F
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

ab'

$$F(a,b,c) = \Sigma m(0,2,4,5,6) = \Pi M(1,3,7)$$

$$\overline{F}(a,b,c) = \Sigma m(1,3,7) = \Pi M(0,2,4,5,6)$$

Design a combinational circuit that takes 3-bit input and generates an output high whenever the input is a prime number.
Draw the circuit using basic logic gates.



$2^3 = 8$

a	b	c	P	
0	0	0	0	
0	0	1	0	
0	1	0	1	2
0	1	1	1	3
1	0	0	0	
1	0	1	1	5
1	1	0	0	
1	1	1	1	7

$P = \Sigma m(2, 3, 5, 7)$

$= \Pi M(0, 1, 4, 6)$

$\bar{a}b$

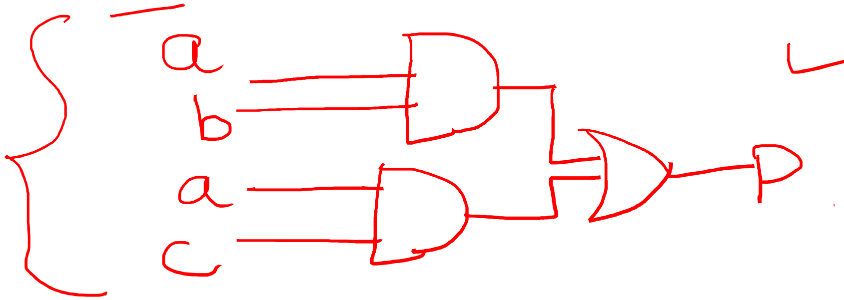
ac

Rough

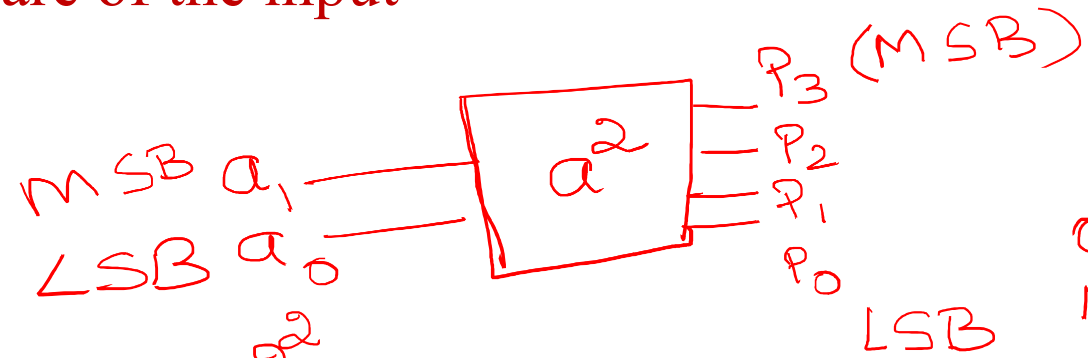
$$P(a,b,c) = \Sigma m(2,3,5,7)$$

$$= \cancel{ac+b}\ \overbrace{\underbrace{\bar{a}b\bar{c}}_{2} + \underbrace{\bar{a}bc}_{3}}^{\bar{a}b} + \underbrace{\underbrace{a\bar{b}c}_{5} + \underbrace{abc}_{7}}_{ac}$$

$$= \bar{a}b + ac$$



Design a combinational circuit that takes 2-bit input and outputs the square of the input



$2^2 = 4$

(0 − 3)

o/p − 0 − 9 → 4 lines

1001

	a_1	a_0	P_3	P_2	P_1	P_0
0	0	0	0	0	0	0
1	0	1	0	0	0	1
2	1	0	0	1	0	0
3	1	1	1	0	0	1

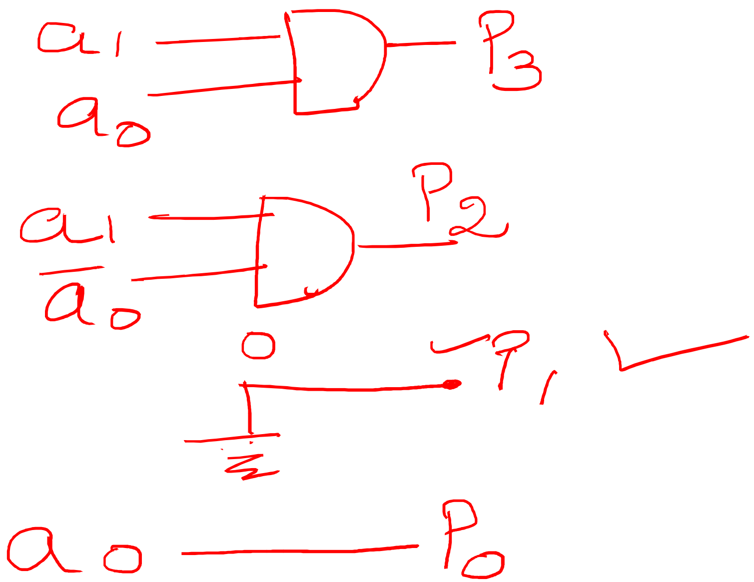
$P_3 = a_1 a_0$

$P_2 = a_1 \bar{a}_0$

$P_1 = 0$

$P_0 = \bar{a}_1 a_0 + a_1 a_0 = a_0$

Rough



Drawing the circuit using only universal gates

- 1. F(A,B,C,D) = AB + CD using only NAND Gates

Drawing the circuit using only universal gates

$$\overline{X + Y} = \overline{X} \cdot \overline{Y}$$

- 1. F(A,B,C,D) = AB + CD using only NAND Gates

$$F = \overline{\overline{F}} = \overline{\overline{AB + CD}} \checkmark$$

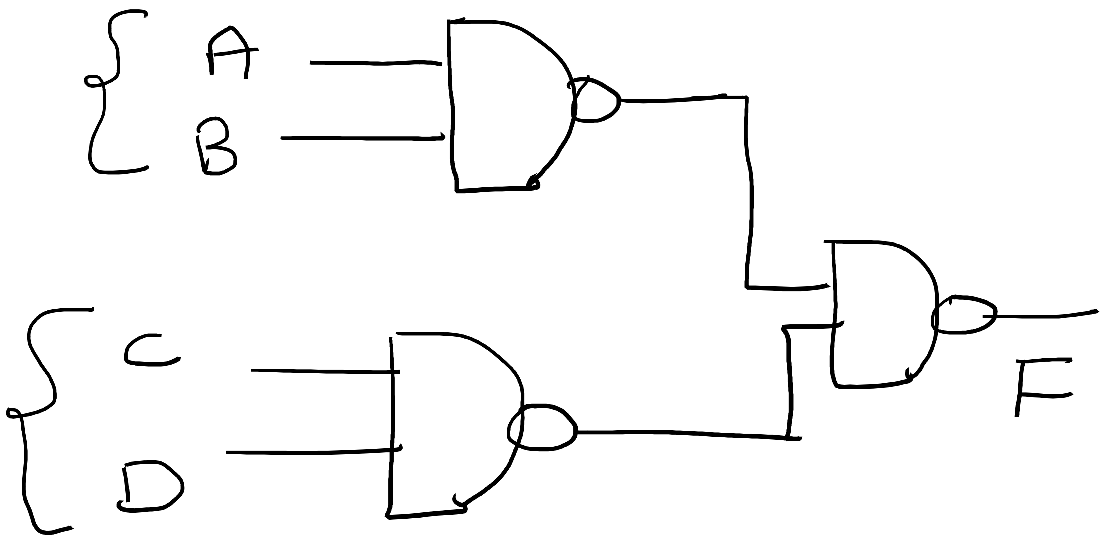
$$= \overline{\overline{AB} \cdot \overline{CD}} = \overline{X \cdot Y} \checkmark$$

Nand3

Nand1 Nand2

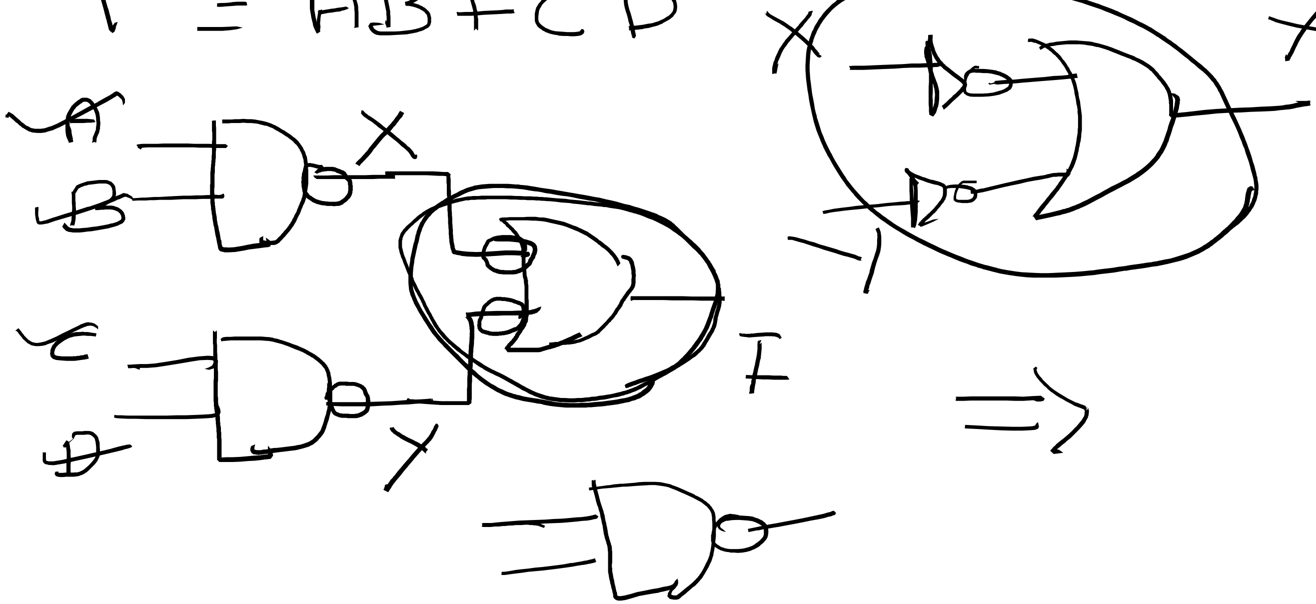
X Y

$F(A,B,C,D) = AB + CD$



Simple way:

$\overline{F} = AB + CD$



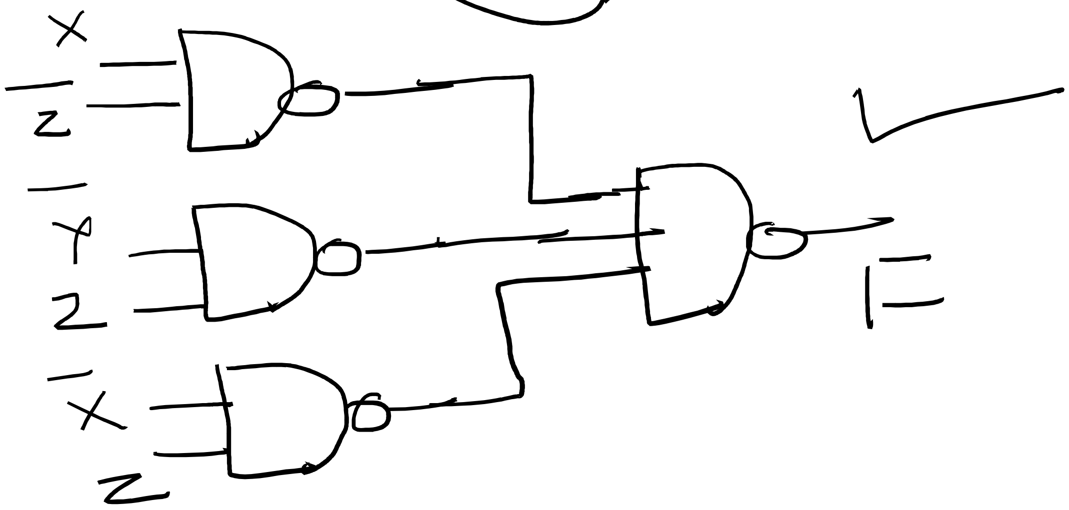
$\overline{X} + \overline{Y} = \ ?$

$\overline{X \cdot Y}$

2. F(x,y,z) = xz’ + y’z+x’z using only NAND gates

$$F(x,y,z) = \overline{\overline{x\bar{z} + \bar{y}z + \bar{x}z}}$$

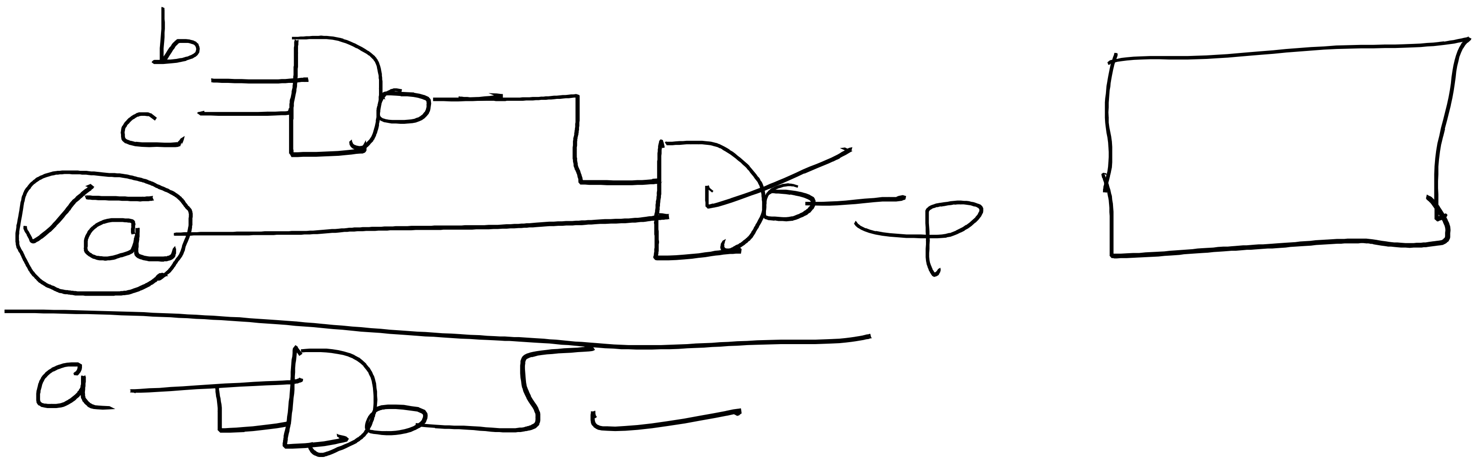
$$= \overline{\overline{x\bar{z}} \cdot \overline{\bar{y}z} \cdot \overline{\bar{x}z}}$$



3. $F(x,y,z) = \Sigma$ (1,2,3,4,5,7) using only NAND gates

4. F(a,b,c)= a+bc using only NAND gates

$$\overline{\overline{F}} = \overline{\overline{a+bc}} = \overline{\bar{a} \cdot \overline{bc}}$$

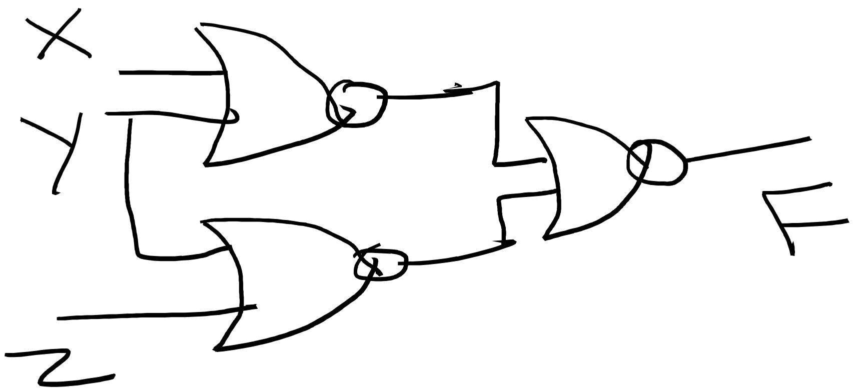


Drawing the circuits using only NOR gates

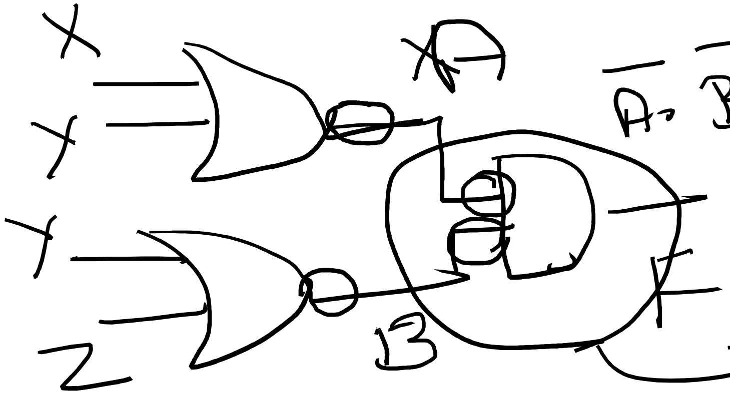
- 1. F(x,y,z) = (x+y) (y+z)

$$\overline{\overline{F}} = \overline{\overline{(x+y)\cdot(y+z)}} = \overline{\underbrace{\overline{(x+y)}}_{A} + \underbrace{\overline{(y+z)}}_{B}}$$

$$\overline{A+B}$$



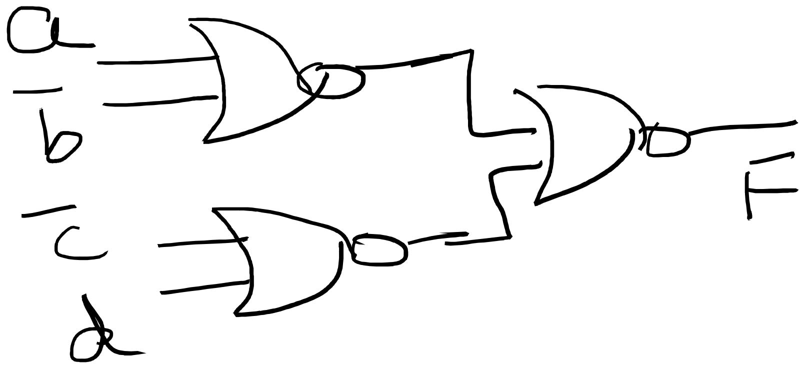
$F = (X+Y)(Y+Z)$



$\bar{A} \cdot \bar{B} = \overline{(A+B)}$

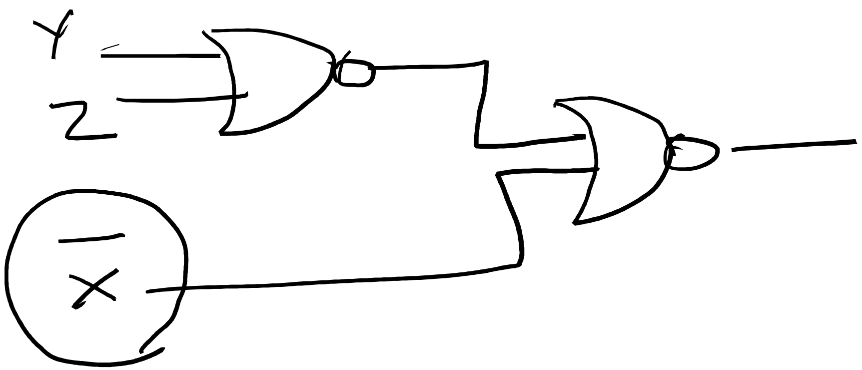
$\rightarrow$ NOR

2. F(a, b, c, d)= (a+b')(c'+d)

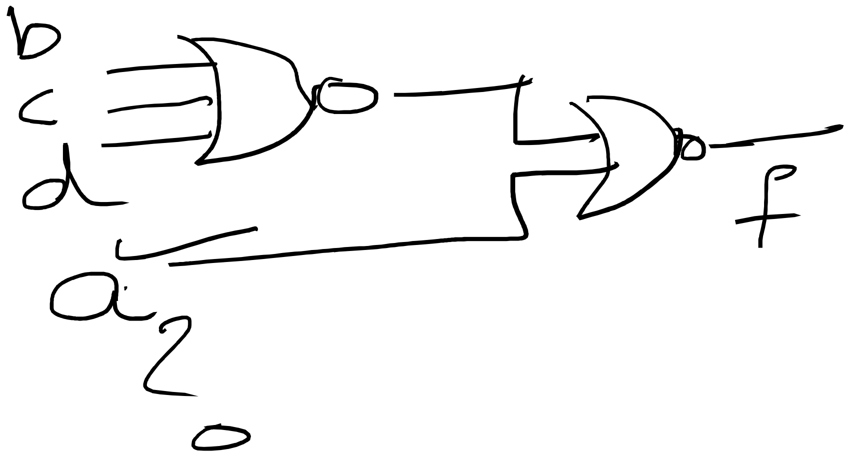


3. f(x,y,z) = x. (y+z)

$\bar{\bar{f}} = \overline{\bar{x} + \overline{(y+z)}}$



$f(x,y,z) = \bar{a} \cdot (b+c+d$



4. f(a, b, c, d)= a.b.(c+d)