1. Permutation and Combinations

The number of r-permutations of n objects in

- 1. With no repetition: ${}^nP_r$
- 2. With unlimited repetition : n^r
- 3.Permutation with restricted repetition: $\frac{n!}{m_1!m_2!\dots m_k!}$ if m_1 objects are of the first kind, m_2 are of the second kind, ..., m_k are of the k^{th} kind.

The number of r-combinations of n objects in

- 1. With no repetition: ${}^nC_r$
- 2. With unlimited repetition : ${}^{n+r-1}C_r$

2. Distribution

Distributing r distinct objects to n distinct cells such that

- 1. Each cell has at most one object: ${}^nP_r$
- 2. Each cell can have any number of objects: n^r

When the r objects to be distributed are not all distinct (identical objects)

- 1. Such that each cell has at most one object: ${}^nC_r$
- 2. If we allow each cell to hold any number of objects: ${}^{n+r-1}C_r$

3. Generating functions

Generating functions for Combinations (Enumerators)

- Without repetition: $(1+x)^n \quad \to (1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$
- With repetition: $(1-x)^{-n} \quad \to (1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r x^r$

Generating functions for Permutations (Exponential gf)

- Without repetition: $\to (1+x)^n \quad \to (1-x)^{-n} = \sum_{r=0}^{n} \frac{{}^nP_r}{r!} x^r$
- With repetition: $e^{nx} \quad \to e^{nx} = \sum_{r=0}^{\infty} \frac{nx^r}{r!}$

4. Formulae:

- $1 + x + x^2 + ... = (1-x)^{-1}$.
- $1 + x + x^2 + \cdots + x^{r-1} = \frac{1-x^r}{1-x}$
- $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$
- $(1-x^m)^n = \sum_{r=0}^{n} (-1)^r \ {}^nC_r (x^m)^r$.
- $(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r x^r$

- If $f(x) = a_0 + a_1 x + \cdots + a_r x^r + \ldots$ and $g(x) = b_0 + b_1 x + \cdots + b_r x^r + \ldots$, then the product is given by
  $f(x)g(x) = \cdots + (a_r b_0 + a_{r-1} b_1 + a_{r-2} b_2 + \cdots + a_0 b_r) x^r + \ldots$
- $e^x = \sum_{r=0}^{\infty} \frac{x^r}{r!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$
- $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$
- $\frac{e^x - e^{-x}}{2} = \frac{x}{1!} + \frac{x^3}{3!} + \ldots$