COMBINATIONS AND PERMUTATIONS

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Reference books;

1. C L Liu, Elements of Discrete Mathematics.
2. E S Page and L B Wilson, An introduction to Computational Combinatorics.
3. Alan Takkar, Applied Combinatorics.

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- **5.Combinations with repetition:** The number of r-combinations of n objects with repetition is $C(n+r-1,r) = {}^{n+r-1}C_{r}$

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Problems

Q1. Find the no of ways in which 3 exams can be scheduled in a 5 day period s.t
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Q3. In how many ways 3 integers can be selected from $3n$ consecutive integers such that the sum is a multiple of 3?

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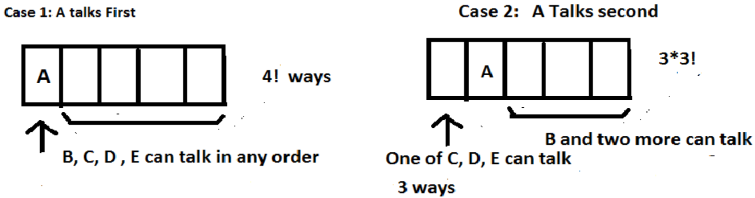

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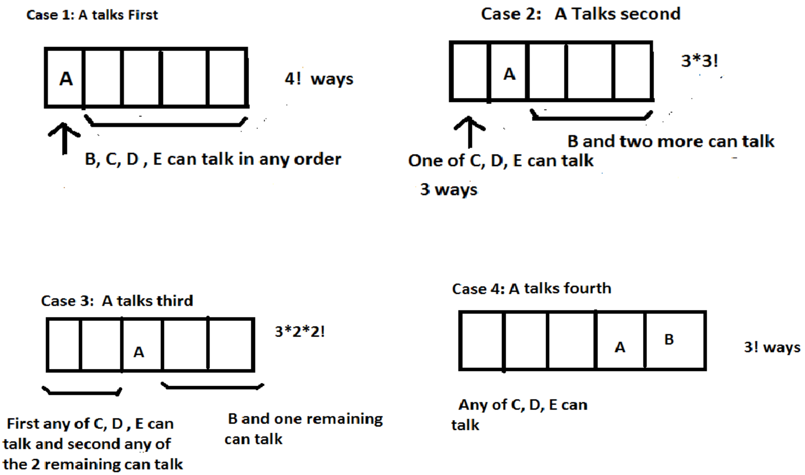

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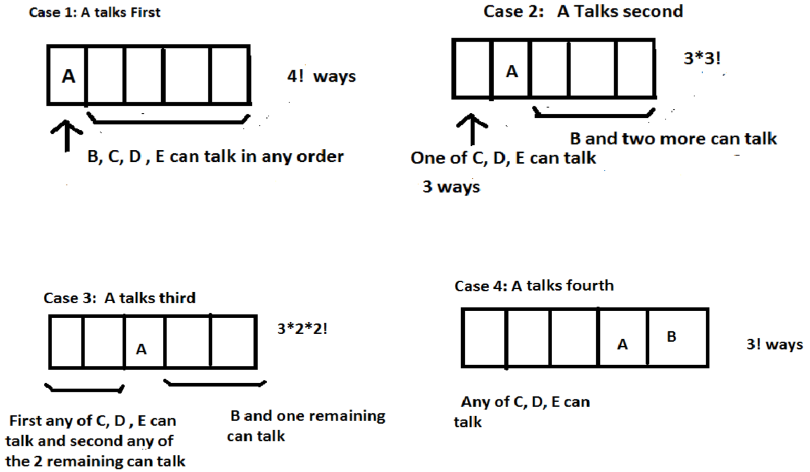

Answer is $4! + 3 \times 3! + 3 \times 2 \times 2! + 3!$

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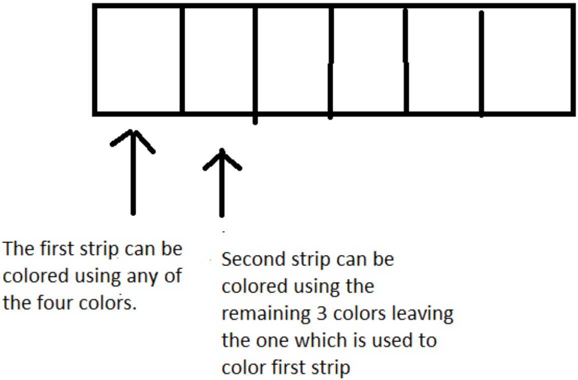

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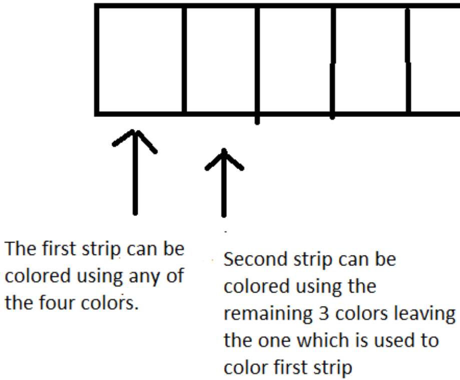

Answer is 4×3^5.

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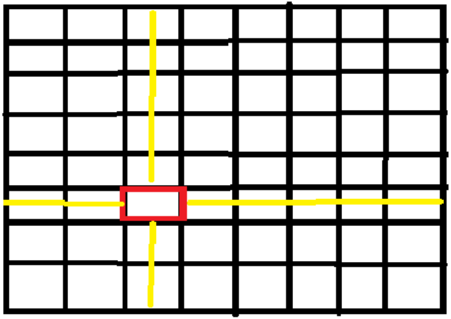

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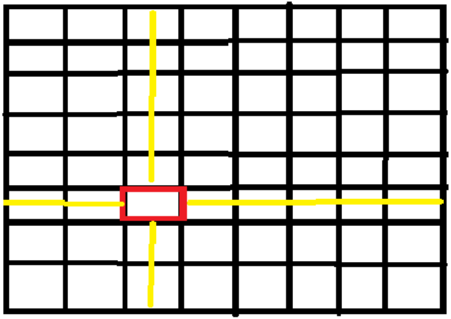

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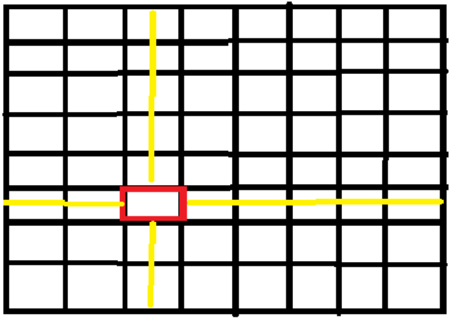

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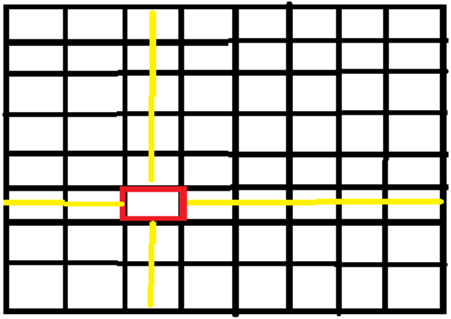

For the first sqaure we have 64 choices.
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Therefore Ans is 64×49.

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1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
1 3 2 4	2 3 1 4	3 2 1 4	4 2 1 3
1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
1 4 2 3	2 4 1 3	3 4 1 2	4 3 1 2
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1243	2143	3142	4132
1324	2314	3214	4213
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1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
1 4 2 3	2 4 1 3	3 4 1 2	4 3 1 2
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The answer is

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$= 66660.$

What is the sum if repetition of the 4 digits allowed?

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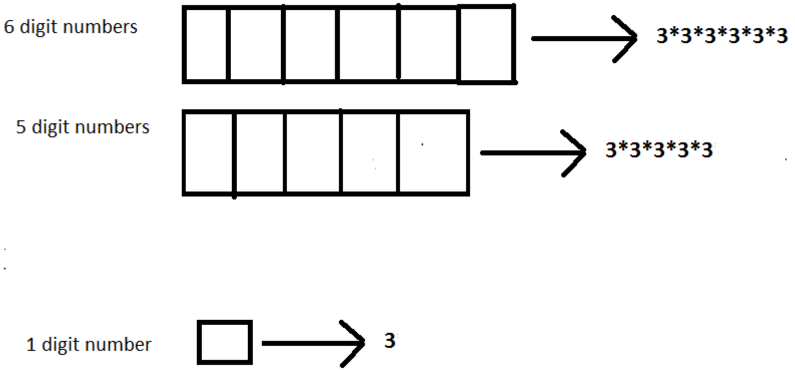

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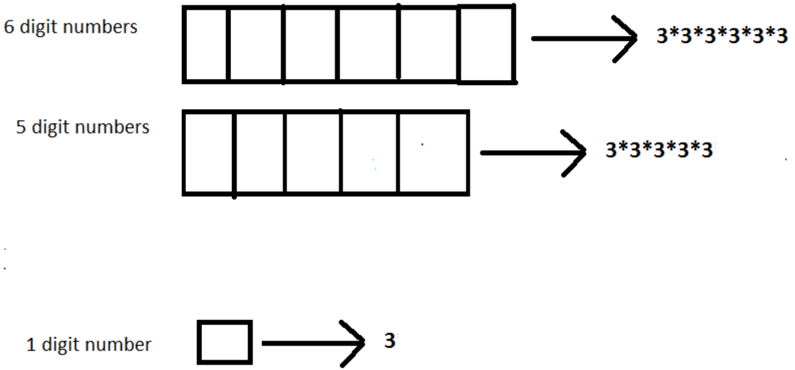

Answer is $3^6 + 3^5 + 3^4 + 3^3 + 3^2 + 3 = 1092.$

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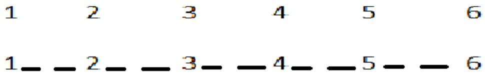

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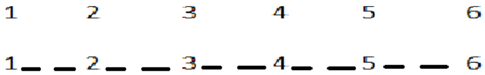

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Distributing 2 identical blanks (objects) to 5 distinct gaps (cells), such that each cell can hold any number of objects (with allowing repetition of gaps). Therefore Answer is ${}^{5+2-1}C_2 = 15$.

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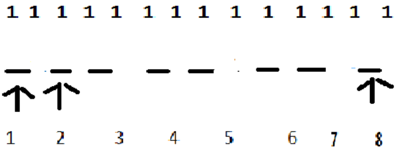

Q11. In how many ways can an examiner assign 30 Marks to 8 questions such that no question recieves less than 2 marks?

Solution: Initially we give 2 marks each to all the 8 questions.

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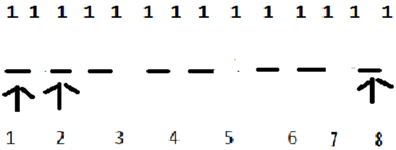

Distribution of 14 identical marks(objects) to 8 distinct questions (cells) such that each cell can hold any number of objects (with allowing repetition of questions).

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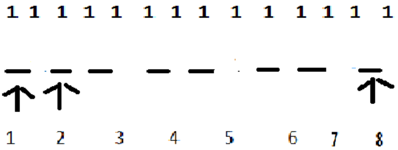

Distribution of 14 identical marks(objects) to 8 distinct questions (cells) such that each cell can hold any number of objects (with allowing repetition of questions).

Answer is ${}^{8+14-1}c_{14} = {}^{21}c_{14}$

Q12. In how many ways can two adjacent squares can be selected from an 8×8 chess board?

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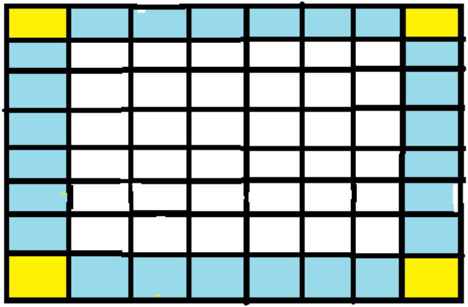

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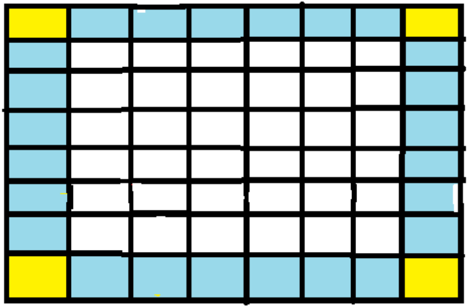

Answer is $4 \times 2 + 24 \times 3 + 36 \times 4$

Q13.Among all 7 digits numbers, how many of them contain exactly three 9s?

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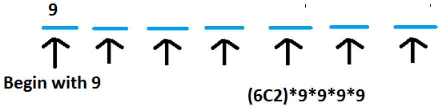

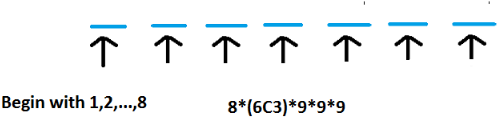

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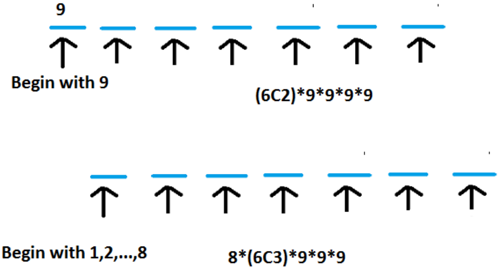

Answer is ${}^{6}C_2 \times 9^4 + 8 \times {}^{6}C_3 \times 9^3$

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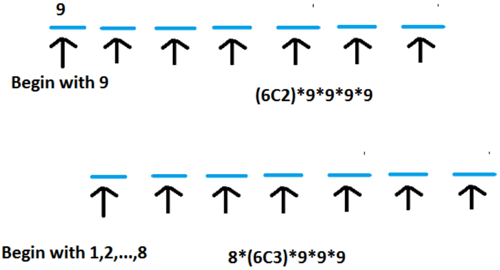

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Q14. The number of ways to choose 3 days out of 7 days (with repetition) is ———

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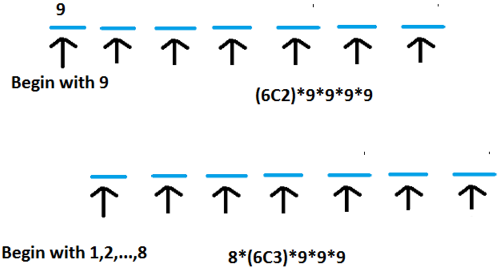

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Q14. The number of ways to choose 3 days out of 7 days (with repetition) is ———

$${}^{7+3-1}C_3 = {}^{9}C_3.$$

Q15. How many points of intersection are formed by n lines drawn in a plane if no two are parallel and no three concurrent? Into how many regions is the plane divided?

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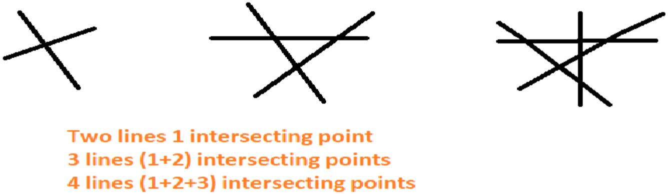

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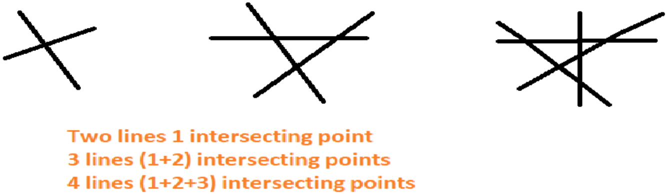
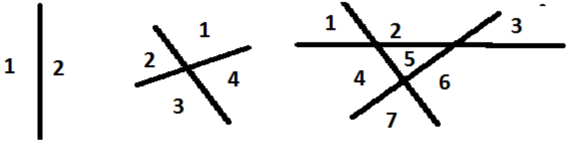
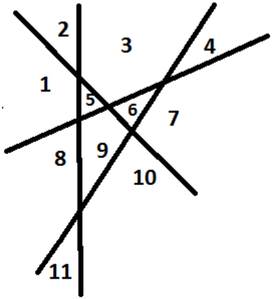


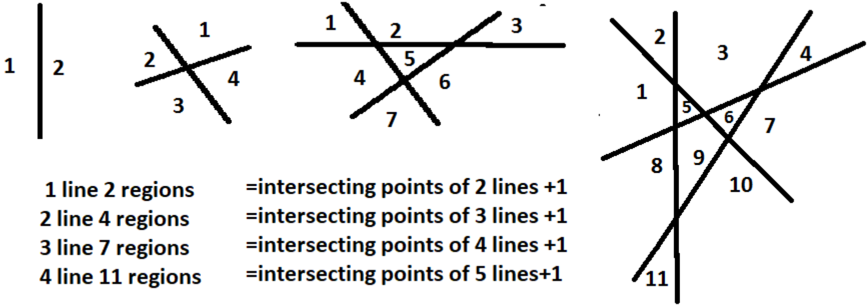
So n lines 1+2+...+(n-1) intersecting points.

Number of intersecting points is ${}^{n}C_{2}$.



1 line 2 regions =intersecting points of 2 lines +1
2 line 4 regions =intersecting points of 3 lines +1
3 line 7 regions =intersecting points of 4 lines +1
4 line 11 regions =intersecting points of 5 lines+1





So, number of regions created by n lines is number of intersecting points created by n+1 lines+1= ${}^{n+1}C_2 + 1 = \dfrac{(n+1)n}{2} + 1$ regions.