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Q2. Show that the proportion of the Permutatition of $\{1, 2, ..., n\}$ which contains no consecutive pair $(i, i+1)$ for any i is approximately $\dfrac{n+1}{ne}$ as n becomes large.

PARTITIONS AND COMPOSITIONS

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Ordered divisions of a positive integer n is called **compositions** while un-ordered divisions is called as **partitions**.

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When we write partitions or compositions we will omit the + signs, thus 2+1+1+1 will be written 2111, or 21^3, and for partitions the largest parts will be written first.

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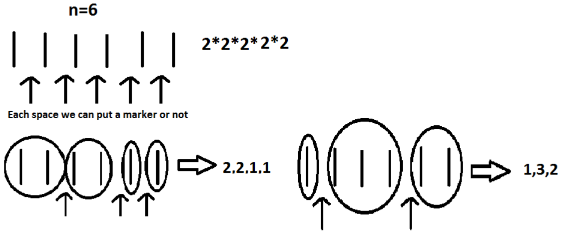

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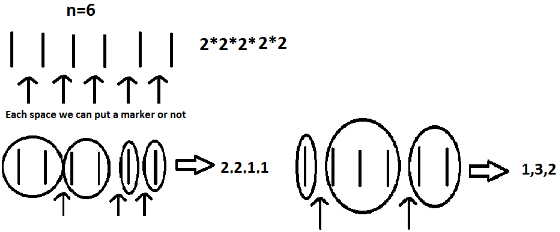

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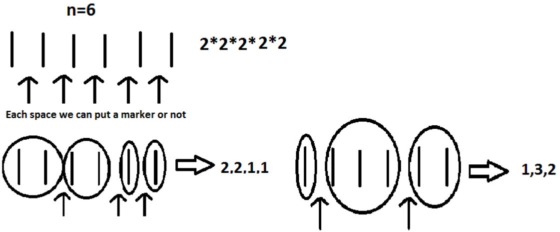

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Such compositions are equal to ${}^{n+m-1}C_{m-1} = {}^{6}C_{1} = 6$.

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$C(x) = x(1 + 2x + (2x)^2 + ...) = \sum_{n=1}^{\infty} 2^{n-1}x^n$.

The enumerating GF for **composition with no restriction on the number of parts** $C(x)$ can be obtained from $C_m(x)$.

$C(x) = \sum_{m=1}^{\infty} C_m(x) = \sum_{m=1}^{\infty} x^m(1-x)^{-m} = \sum_{m=1}^{\infty}(\frac{x}{1-x})^m.$

Put $(\frac{x}{1-x}) = t.$

So, $C(x) = \sum_{m=1}^{\infty} t^m = t + t^2 + t^3 + ... = t(1 + t + t^2 + ...) = (\frac{t}{1-t})$

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Number of compositions with no restriction on the number of parts is the coefficient of x^n in $\sum_{n=1}^{\infty} 2^{n-1}x^n$ which is equal to 2^{n-1}.

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