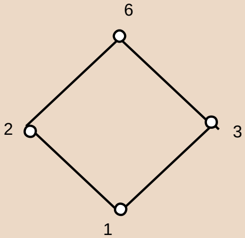
- **Comparable elements:** Let $(A, \leq)$ is a poset. Two elements $a, b \in A$ are said to be comparable if either $a \leq b$ or $b \leq a$.
- **Chain:** Let $(A, \leq)$ is a poset. A subset of A is called a chain if every two elements in the subset are comparable. The number of elements in a chain is known as the length of the chain.
- **Antihain:** Let $(A, \leq)$ is a poset. A subset of A is called an antichain if no two distinct elements in the subset are comparable.
- **Totally ordered set:** A poset $(A, \leq)$ is called a totally ordered set if A is a chain. In this case, the binary relation $\leq$ is called a total ordering relation.
- **Cover of an element:** Let $(A, \leq)$ be a poset. An element $b \in A$ is said to cover an element $a \in A$ if $a \leq b$ and there is no element $c \in A$ such that $a \leq c \leq b$.

Hasse diagram

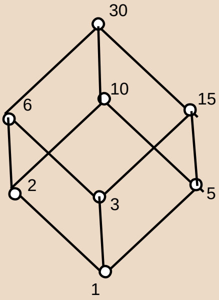
A poset $(A, \leq)$ is graphically represented by Hasse diagram. The following steps are to be followed to draw Hasse diagram corresonding to a given poset $(A, \leq)$.

- Each element of A is represented by a small circle or a dot.
- The circle for $x \in A$ is drawn below the circle for $y \in A$ if $x \leq y$. A line is drawn if y covers x.
- If $x \leq y$ but y doesn't cover x, then x and y are not connected directly by a single line.

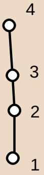
Example 0.1.



The poset (A, |) where A={1,2,3,6}



The poset (D, |) where D={1,2,3,5,6,10,15,30}



The poset (C, $\leq$) where C={1,2,3,4}

Here $|$ is the relation "divides" and $\leq$ is the relation "less than or equal to".

We note the following terminologies for a given poset $(A, \leq)$.

- **Maximal element:** An element $a \in A$ is said to be a maximal element of A if there is no $b \in A$ such that $a \neq b$ and $a \leq b$. We note that 6,30 and 4 are the maximal elements of $(A, |), (D, |)$ and $(C, \leq)$ respectively.
- **Minimal element:** An element $a \in A$ is said to be a minimal element of A if there is no $b \in A$ such that $a \neq b$ and $b \leq a$.1 is the minimal element of $(A, |), (D, |)$ and $(C, \leq)$.

Theorem 0.2.

Let $(P, \leq)$ be a partially ordered set. Suppose the length of the longest chains in P is n. Then the elements in P can be partitioned into n disjoint antichains.

Proof.

We shall prove the theorem by induction on n. For $n = 1$, no two elements in P are related. Clearly, they constitue an antichain.
We assume that the theorem holds when the length of the longest chains in partially ordered set is $n-1$. Let P be a partially ordered set with the length of its longest chains being n. Let M denote the set of maximal elements in P. Clearly, M is a nonempty antichain. Consider now the partially ordered set $(P - M, \leq)$. Since there is no chain of length n in $P - M$, the length of the longest chains is at most $n - 1$. on the other hand, if the length of the longest chains in $P - M$ is less than $n - 1$, M must contain two or more elements that are members of the same chain, which is certainly an impossibility. Consequently, we conclude that the length of the longest chain in $P - M$ is $n - 1$. According to the induction hypothesis, $P - M$ can be partitioned into $n - 1$ disjoint antichains. Thus P can be partitioned into n disjoint antichains. $\square$

- **Upper bound:** Let $a, b \in A$. An element $c \in A$ is said to be an upper bound of a and b if $a \leq c$ and $b \leq c$.
- **Lower bound:** An element $c \in A$ is said to be a lower bound of a and b if $c \leq a$ and $c \leq b$.
- **Least upper bound (lub):** An element $c \in A$ is said to be a least upper bound of a and b if c is an upper bound for a and b, and there is no upper bound d of a and b such that $d \leq c$.
  In $(D, |)$ of example 1.1, the element 30 is an upper bound of 2 and 3, but it is not the least upper bound. The lub for 2 and 3 is 6.
- **Greatest lower bound (glb):** An element $c \in A$ is said to be an greatest lower bound of a and b if c is a lower bound for a and b, and there is no lower bound d of a and b such that $c \leq d$.