Contents

- A binary relation R from a set A to B is a subset of $A \times B$. That is, $R = \{(a, b) | a \in A, b \in B\} \subseteq A \times B$. If $(a, b) \in R$, then we say that the element 'a is related to b' and write aRb.
- A binary relation R on a set A is said to be a binary relation on A.

Types of relations:

1. **Reflexive relation:** A binary relation R on a A is said to be a reflexive relation if $(a, a) \in R$ for all $a \in A$.
   Ex: Let A be the set of positive integers and R be the binary relation on A defined by $(a, b) \in R$ if and only if a divides b. Then R is reflexive as every integer divides itself.
2. **Symmetric relation:** A binary relation R on a A is said to be symmetric if $(a, b) \in R \implies (b, a) \in R$ for all $a, b \in A$.
   Ex: The relations "is parallel to "and "is perpendicular to "are symmetric relations on the set of all straight lines.

3. **Antisymmetric relation:** A binary relation R on a set A is said to be antisymmetric if $(a, b) \in R \implies (a, b) \notin R$ unless $a = b$.
   Ex: The binary relation R defined by $(a, b) \in R$ if and only if $a \geq b$ is antisymmetric on the set of positive integers.
4. **Transitive relation:** A binary relation R on A is said to be transitive if $(a, c) \in R$ whenever both $(a, b) \in R$ and $(b, c) \in R$.
   Ex: The relation "is parallel to" is transitive, but the relation "is perpendicular to" is not transitive on the set of straight lines.
5. **Equivalence relation:** A binary relation on a set is said to be an equivalence relation if it is reflexive, symmetric and transitive.

3. **Partial ordering relation:** A binary relation on a set is said to be a partial ordering relation if it is reflexive, antisymmetric and transitive. A nonempty set A with a partial ordering relation R is a partially ordered set (abbreviated as poset). **For each ordered pair** $(a, b) \in R$**, we write** $a \leq b$ **instead of** aRb **where** $\leq$ **is a generic symbol and commonly read as "less than or equal to "**. It is often denoted as (A, R) or $\langle A, R \rangle$ or $(A, \leq)$.
Ex: Let A be the set of positive integers and R be the binary relation on A defined by $a \leq b$ if and only if a divides b. Then $(A, \leq)$ is a poset.