LECTURE 5 & 6

KARNAUGH MAP (K – MAP)

KARNAUGH– MAP (K-MAP)

- Pictorial form of a truth table / Boolean function.
- Graphical tool to simplify a logical equation by forming groups of cells.
- Each cell corresponds to a input(minterm/maxterm)
- Content of cell is the output for the corresponding input, i.e. output is ‘0’,’1’ or don’t care

K-MAP CONTINUED…

TWO VARIABLE K – MAP

MSB $\rightarrow$ LSB

A	B	SOP	POS
0	0	$\overline{A}\,\overline{B}$	$A + B$
0	1	$\overline{A}B$	$A + \overline{B}$
1	0	$A\overline{B}$	$\overline{A} + B$
1	1	AB	$\overline{A} + \overline{B}$

A. SOP: -

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	$\overline{A}.\overline{B}$	$\overline{A}.B$
A 1	$A.\overline{B}$	$A.B$

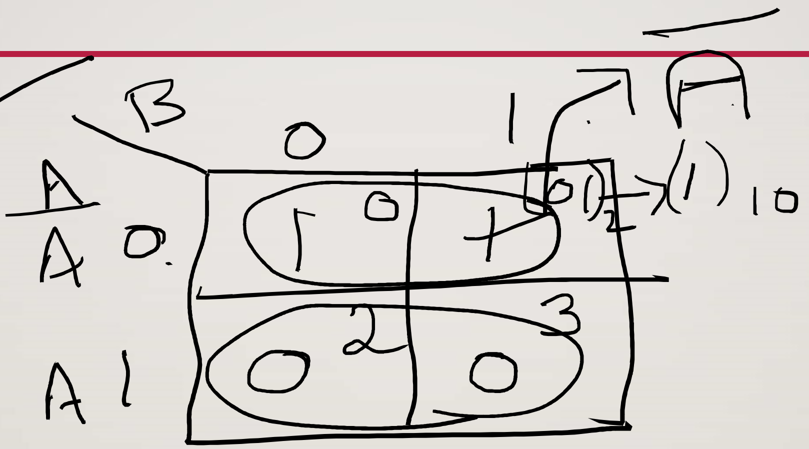
B. POS: -

A \ B	B 0	$\overline{B}$ 1
A 0	$A+B$	$A+\overline{B}$
$\overline{A}$ 1	$\overline{A}+B$	$\overline{A}+\overline{B}$

EXAMPLE 1:

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

$\Sigma m(0,1)$



$2^1 = 2$

$2^2 = 4$

$2^3 = 8, 16, 32$

$\Sigma \bar{A}$

SOP = (1s)

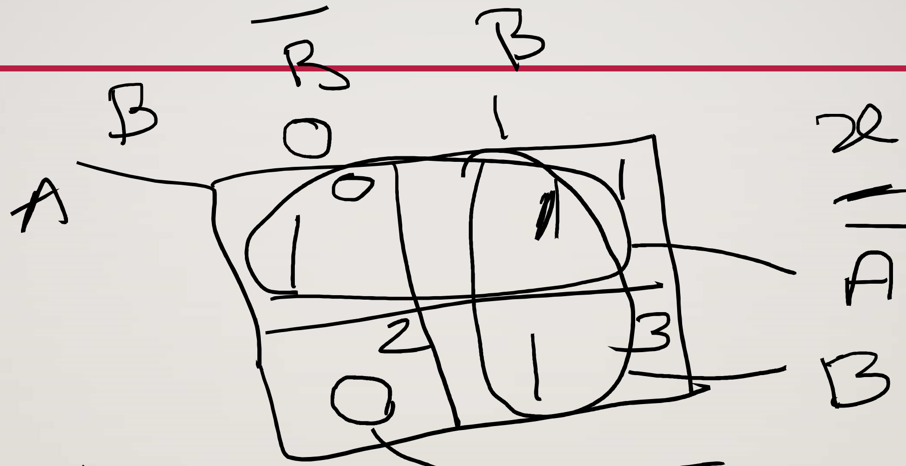
POS = (0)

$\bar{F} = -$

$F = \bar{A}$

EXAMPLE 2:

A	B	F
0	0	I
0	I	I
I	0	0
I	I	I

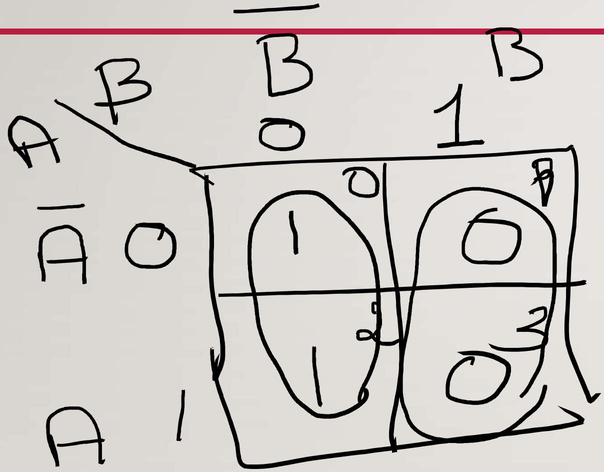


$x + x = x$

$\Rightarrow \bar{A} + B = SO$

$F = \Sigma m(0, 1, 3)$

$POS = (\bar{A} + B)$



$SOP = 1's$

$F = \bar{B}$

$POS = 0's$

$\bar{F} = B \Rightarrow F = \bar{B}$

EXAMPLE 3:

A	B	F
0	0	1
0	1	1
1	0	1
1	1	1

THREE VARIABLE K – MAP

$2^3 = 8$ cells

$\left.\begin{array}{l} 4 \text{ row} \times 2 \text{ col's} \\ 2 \text{ row} \times 4 \text{ col's} \end{array}\right\}$

THREE VARIABLE K – MAP

00 → 01 → 11 → 10

00 01 11 10

MSB LSB Min Max

A	B	C	SOP	POS
0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	$A+B+C$
0	0	1	$\overline{A}\,\overline{B}C$	$A+B+\overline{C}$
0	1	0	$\overline{A}B\overline{C}$	$A+\overline{B}+C$
0	1	1	$\overline{A}BC$	$A+\overline{B}+\overline{C}$
1	0	0	$A\overline{B}\,\overline{C}$	$\overline{A}+B+C$
1	0	1	$A\overline{B}C$	$\overline{A}+B+\overline{C}$
1	1	0	$AB\overline{C}$	$\overline{A}+\overline{B}+C$
1	1	1	ABC	$\overline{A}+\overline{B}+\overline{C}$

01 → 10

A. SOP: -

A \ BC	$\overline{B}\,\overline{C}$ 0 0	$\overline{B}C$ 0 1	BC 1 1	$B\overline{C}$ 1 0
$\overline{A}$ 0	$\overline{A}\,\overline{B}\,\overline{C}$ 0	$\overline{A}\,\overline{B}C$ 1	$\overline{A}BC$ 3	$\overline{A}B\overline{C}$ 2
A 1	$A\overline{B}\,\overline{C}$ 4	$A\overline{B}C$ 5	ABC 7	$AB\overline{C}$ 6

111

7

B. POS: -

A \ B+C	$B+C$ 0 0	$B+\overline{C}$ 0 1	$\overline{B}+\overline{C}$ 1 1	$\overline{B}+C$ 1 0
A 0	$A+B+C$ 0	$A+B+\overline{C}$ 1	$A+\overline{B}+\overline{C}$ 3	$A+\overline{B}+C$ 2
$\overline{A}$ 1	$\overline{A}+B+C$ 4	$\overline{A}+B+\overline{C}$ 5	$\overline{A}+\overline{B}+\overline{C}$ 7	$\overline{A}+\overline{B}+C$ 6

3-VARIABLE K-MAP

MSB

ABC LSB

AB \ C	C 0: $\bar{C}$	1: C
00 $\bar{A}\bar{B}$	m_0	m_1
01 $\bar{A}B$	m_2	m_3
11 AB	m_6	m_7
10 $A\bar{B}$	m_4	m_5

BC \ A	$\bar{A}$	A
$\bar{B}\bar{C}$	m_0	m_4
$\bar{B}C$	m_1	m_5
BC	m_3	m_7
$B\bar{C}$	m_2	m_6

EXAMPLES

ABC

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1			1
A	1	1	1	1

$\bar{C}$

$= A + \bar{C}$

SOP

A

pair $= 2^{1} = 2$ cells

Quad $= 2^{2} = 4$ cells

EXAMPLE 1:

Given the Boolean function:

$$F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$$

- Express it in Sum of minterms form.
- Find the minimal sum of products expression using k-map.

$= \bar{A}C(B+\bar{B}) + \bar{A}B(C+\bar{C}) + A\bar{B}C + BC(A+\bar{A})$

$= \Sigma m(\quad)$

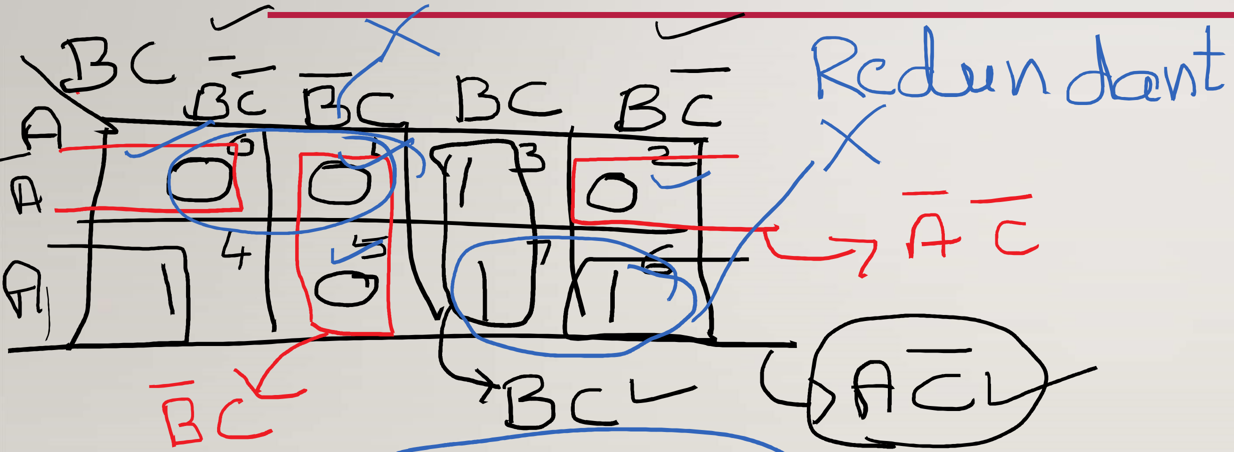
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1^{1}	1^{3}	1^{2}
A	0	1^{5}	1^{7}	0

$SOP = C + \bar{A}B$

$POS = (\quad)(\quad)$

Simplify the Boolean expression to SOP and POS, draw the circuit using (i) NAND only (ii)NOR only:

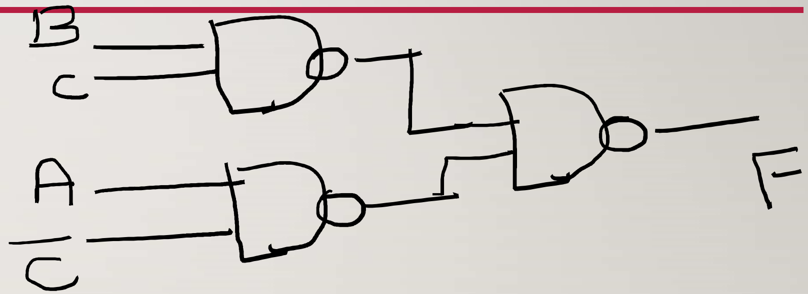
Example 2: $F(A, B, C) = \Sigma m(3,4,6,7) = \Pi M(0,1,2,5)$

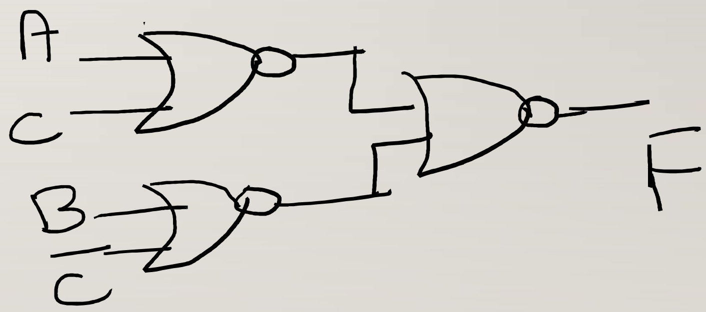


$SOP = BC + A\bar{C}$

$POS \Rightarrow \bar{F} = \bar{A}\bar{C} + \bar{B}C$

$F = (A+C)(B+\bar{C})$





EXAMPL2 3: $F(x,y,z) = \sum(0,2,4,5,6)$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{x}$				
x				

✓ SOP $\Rightarrow \bar{C} + A\bar{B}$

✓ POS $\Rightarrow (A+\bar{C}) \cdot (\bar{B}+\bar{C})$

EXAMPLE 4: $F(A, B, C) = \Pi M(0,2,5,7) = \Sigma m(1,3,4,6)$

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0^{0}	1^{1}	1^{3}	0^{2}
A	1^{4}	0^{5}	0^{7}	1^{6}

$\rightarrow \bar{A}C \Rightarrow SOP = \bar{A}C + A\bar{C}$

$\hookrightarrow A\bar{C}$ $\quad POS \Rightarrow (\bar{A}+\bar{C}) \cdot (A+C)$

FOUR VARIABLE K – MAP

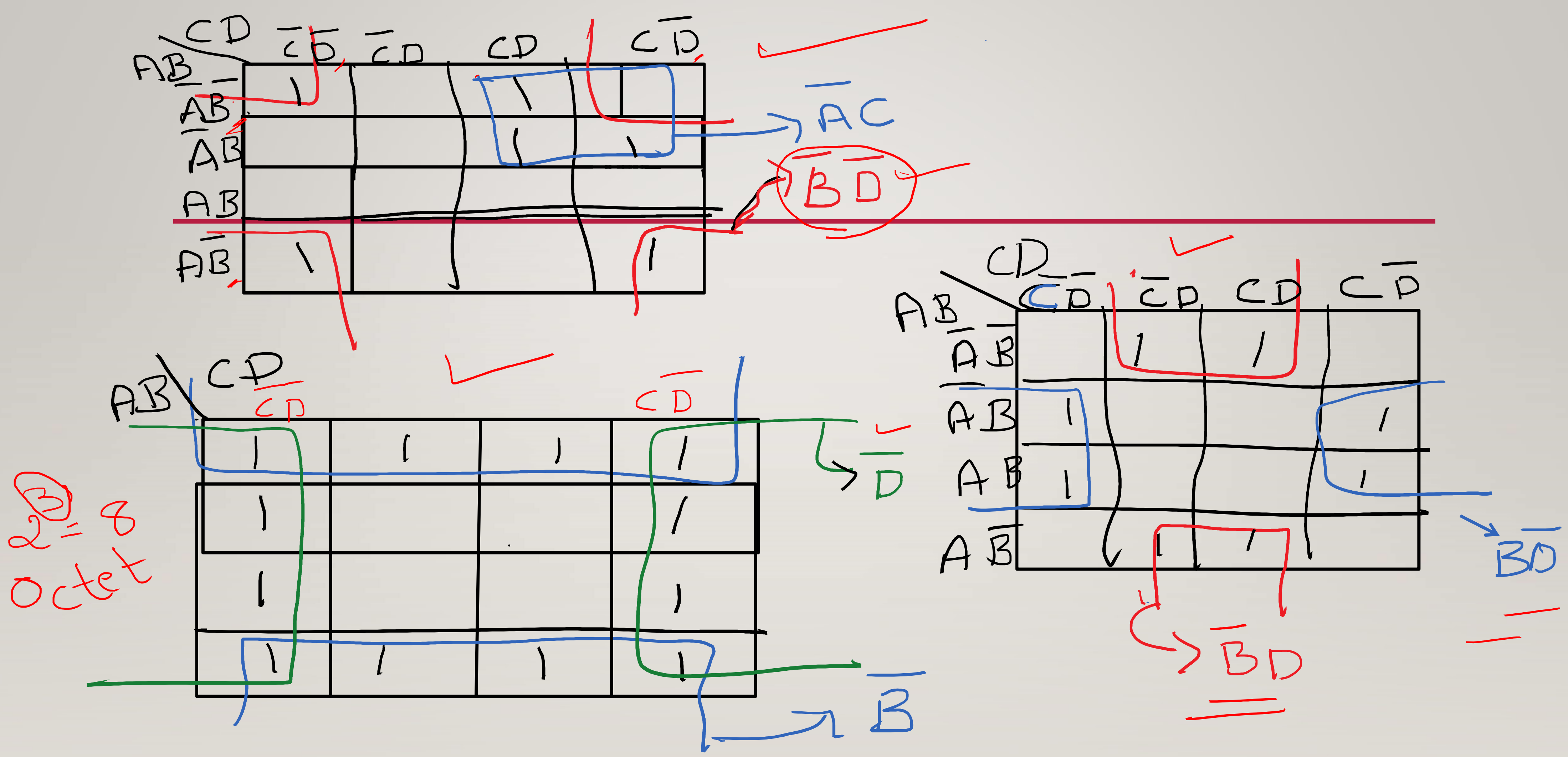
A. SOP: -

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$ 0	$\overline{A}\,\overline{B}\,\overline{C}D$ 1	$\overline{A}\,\overline{B}CD$ 3	$\overline{A}\,\overline{B}C\overline{D}$ 2
$\overline{A}B$ 01	$\overline{A}B\overline{C}\,\overline{D}$ 4	$\overline{A}B\overline{C}D$ 5	$\overline{A}BCD$ 7	$\overline{A}BC\overline{D}$ 6
AB 11	$AB\overline{C}\,\overline{D}$ 12	$AB\overline{C}D$ 13	$ABCD$ 15	$ABC\overline{D}$ 14
$A\overline{B}$ 10	$A\overline{B}\,\overline{C}\,\overline{D}$ 8	$A\overline{B}\,\overline{C}D$ 9	$A\overline{B}CD$ 11	$A\overline{B}C\overline{D}$ 10

FOUR VARIABLE K – MAP

POS: -

$A+B$ \ $C+D$	$C+D$ 0 0	$C+\overline{D}$ 0 1	$\overline{C}+\overline{D}$ 1 1	$\overline{C}+D$ 1 0
$A+B$ 0 0	$A+B+C+D$ 0	$A+B+C+\overline{D}$ 1	$A+B+\overline{C}+\overline{D}$ 3	$A+B+\overline{C}+D$ 2
$A+\overline{B}$ 0 1	$A+\overline{B}+C+D$ 4	$A+\overline{B}+C+\overline{D}$ 5	$A+\overline{B}+\overline{C}+\overline{D}$ 7	$A+\overline{B}+\overline{C}+D$ 6
$\overline{A}+\overline{B}$ 1 1	$\overline{A}+\overline{B}+C+D$ 12	$\overline{A}+\overline{B}+C+\overline{D}$ 13	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$ 15	$\overline{A}+\overline{B}+\overline{C}+D$ 14
$\overline{A}+B$ 1 0	$\overline{A}+B+C+D$ 8	$\overline{A}+B+C+\overline{D}$ 9	$\overline{A}+B+\overline{C}+\overline{D}$ 11	$\overline{A}+B+\overline{C}+D$ 10



AB CD
C̄D̄ C̄D CD CD̄
ĀB̄ ĀB AB AB̄
ĀC
B̄D̄
D̄
B̄
2³ = 8
Octet
BD

EXAMPLE 1:

- Simplify the following expression into
  - SOP ⇒ 1s
  - POS ⇒ 0s

$$F(A,B,C,D) = \sum(1,5,6,7,8,9,10,14)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 [0]	1 [1]	0 [3]	0 [2]
$\bar{A}B$	0 [4]	1 [5]	1 [7]	1 [6]
AB	0 [12]	0 [13]	0 [15]	1 [14]
$A\bar{B}$	1 [8]	1 [9]	0 [11]	1 [10]

$$SOP = A\bar{B}\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + AC\bar{D}$$

$$POS \Rightarrow \bar{F} = \bar{A}\bar{C}\bar{D} + AB\bar{C} + \bar{A}BC + ACD$$

$$F = (A+C+D)(\bar{A}+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{C}+\bar{D})$$

EXAMPLE 2:

$$F(A,B,C,D)=\sum(0,1,2,4,5,6,8,9,10,12,13)$$

	C'D'	C'D	CD	CD'
A'B'	1	1	0	1
A'B	1	1	0	1
AB	1	1	0	0
AB'	1	1	0	

$SOP = \bar{C} + \bar{A}\bar{D} + \bar{B}\bar{D}$

$\bar{A}\bar{D}$

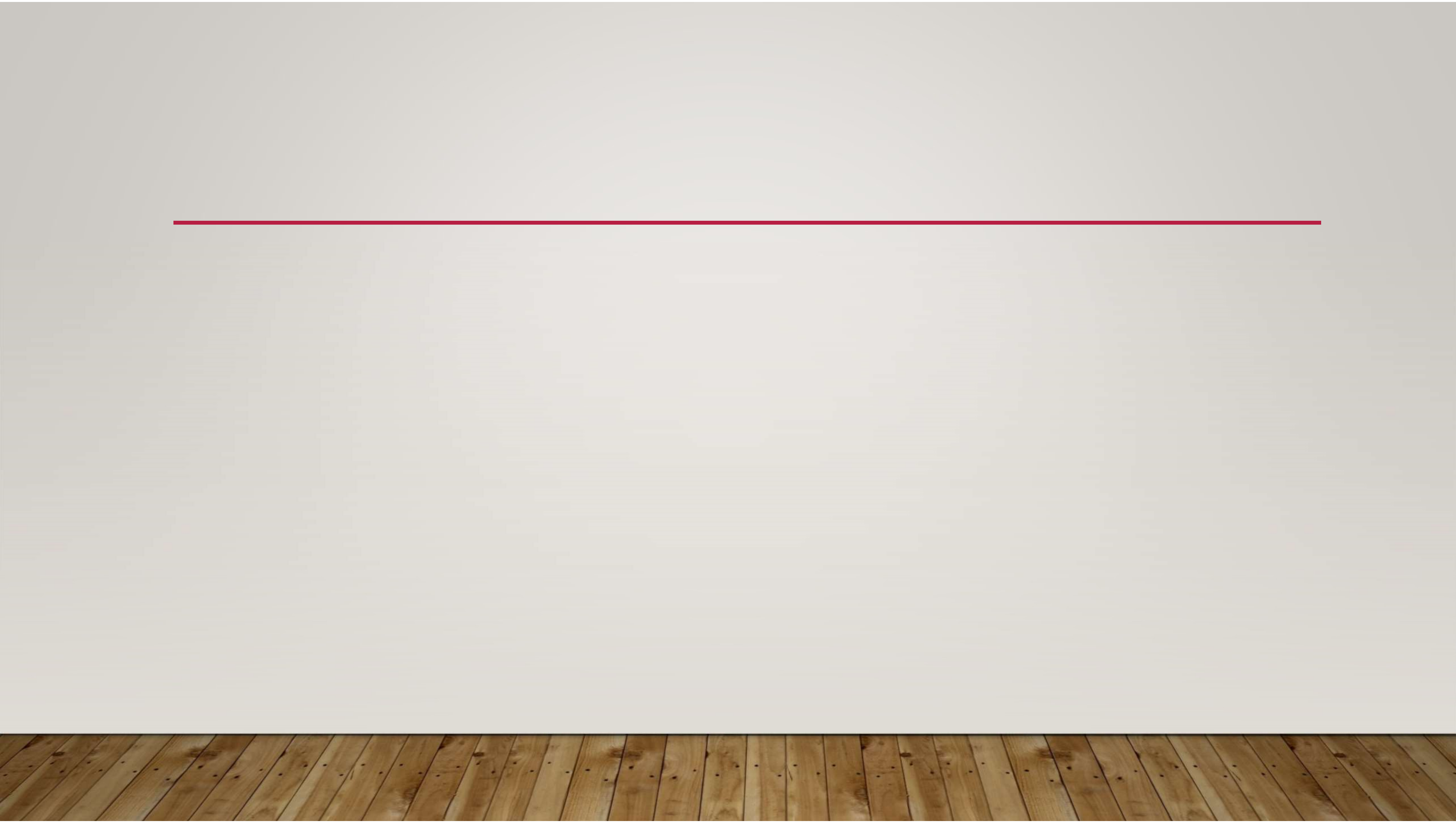
$\bar{F} = \bar{C}D + ABC$

$POS \Rightarrow (\bar{C}+\bar{D})\cdot(\bar{A}+\bar{B}+\bar{C})$

$\bar{A}\bar{B}\bar{C}$

$(A+B+C)$

$2^3 = 8$



EXAMPLE 3:

F(A,B,C,D)= Π M(0,2,3,4,6,8,9,10,14)

2i/P

	C'D'	C'D	CD	CD'
A'B'	0	1	0	0
A'B	0	1	1	0
AB	1	1	1	0
AB'	0	0	1	0

SOP = $BD + \bar{A}\bar{C}D + AB\bar{C} + ACD$

BCD 3i/P

POS = $(\bar{C}+D)(A+D)(\bar{A}+B+C)(A+B+\bar{C})$

$(B+D)$ ← 4 corner 0's

EXAMPLE 4:

$$F(A,B,C,D)=\overline{C}(\overline{A}\,\overline{B}\,\overline{D}+\overline{D})+A\overline{B}C+\overline{D}$$

$$=\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}+\overline{C}D+A\overline{B}C+\overline{D}$$

0 0 0 0

Quad

	C'D'	C'D	CD	CD'
A'B'	1	1	0	1
A'B	1	1	0	1
AB	1	1	0	1
AB'	1	1	1	1

SOP $=\overline{C}+\overline{D}+A\overline{B}$

POS $=(A+\overline{C}+\overline{D})\cdot(\overline{B}+\overline{C}+\overline{D})$

CONTINUED..

	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

EXAMPLE 5:

$$F(A,B,C,D) = D(\overline{A} + B) + \overline{B}(C + AD)$$

DON'T CARE CONDITION

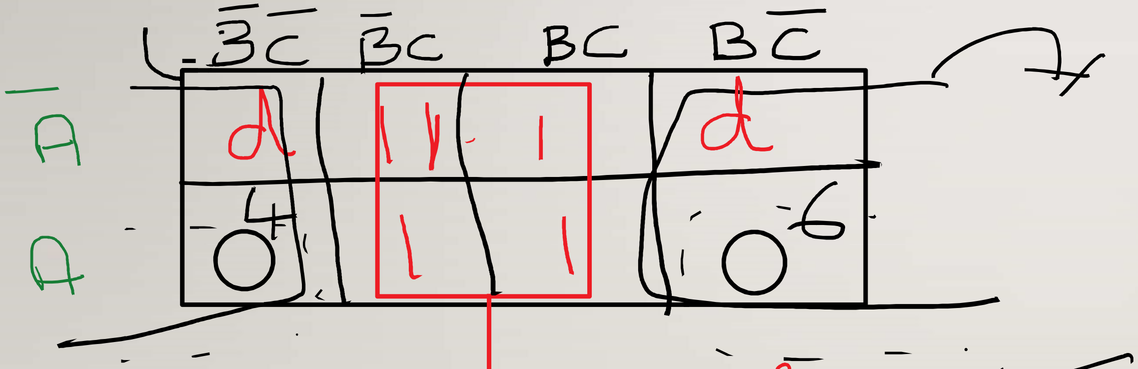
- The "Don't Care" conditions indicate the input combinations which are invalid for a particular circuit.
- While forming groups of cells, we can consider a "Don't Care" cell as either 1 or 0 or we can simply ignore that cell.
- Therefore, "Don't Care" condition are used to form a larger group of cells.

ϕ, D, d, φ, X

EXAMPLE 1:

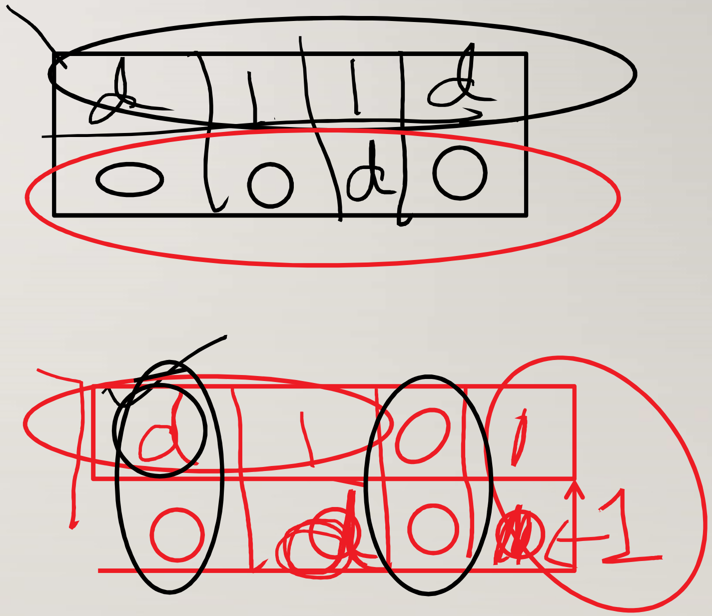
$$F(A, B, C) = \sum_{m} (1,3,5,7) + \sum_{d} (0,2)$$

Ex:



SOP = C

POS = C



EXAMPLE 2:

$$F(W,X,Y,Z)=\sum_{m}(1,3,7,11,15)+\sum_{d}(0,2,5)$$

	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

EXAMPLE 3:

$$F(W,X,Y,Z)=\prod_{M}(0,1,3,5,8,9,14)\prod_{D}(2,6,10)$$

	C'D'	C'D	CD	CD'
A'B'	O	O	O	d
A'B	1	O	1	d
AB	1	1	1	O
AB'	O	O	1	d

$SOP = B\bar{C}\bar{D} + \bar{A}BC + ABD$

$+ A\bar{B}C$

$POS = (A+B)(B+C)(\bar{C}+D)$

$\cancel{(A+B)}\ (A+C+\bar{D})$

EXAMPLE 4:

$$F(W, X, Y, Z) = \sum_{m} (1,4,5,7,9,10,11) + D(14,15)$$

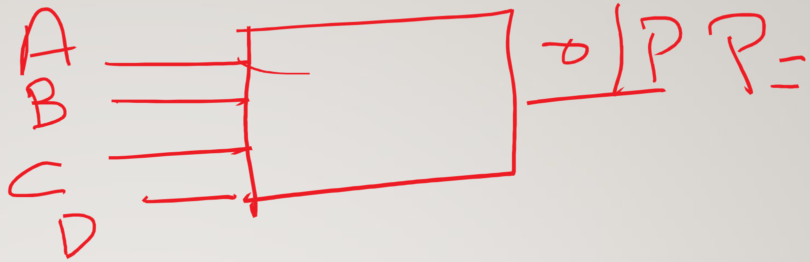
	$\bar{Y}\bar{Z}$ C'D'	$\bar{Y}Z$ C'D	YZ CD	$Y\bar{Z}$ CD'
$\bar{W}\bar{X}$ A'B'	0^{0}	1^{1}	0^{3}	0^{2}
$\bar{W}X$ A'B	1^{4}	1^{5}	1^{7}	0^{6}
WX AB	0^{12}	0^{13}	d^{15}	d^{14}
$W\bar{X}$ AB'	0^{8}	1^{9}	1^{11}	1^{10}

$$SOP = WY + \bar{X}\bar{Y}Z + \boxed{\begin{matrix} \bar{W}XZ \\ \text{or} \\ XYZ \end{matrix}} + \bar{W}X\bar{Y}$$

$$POS = (\bar{W} + \bar{X})(X + Y + Z)(W + X + \bar{Y}) (W + \bar{Y} + Z)$$

EXAMPLE 5:

Design a 4-bit odd parity bit generator circuit.



	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

	ABCD	P
	0000	1
	0001	0
	0010	0
	0011	1
	0100	0
	0101	1
	0110	1
7	0111	0
8	1000	0

	ABCD	P
	~~1000~~	
9	1001	1
10	1010	1
	1011	0
	1100	1
	1101	0
	1110	0
	1111	1

PARITY BIT

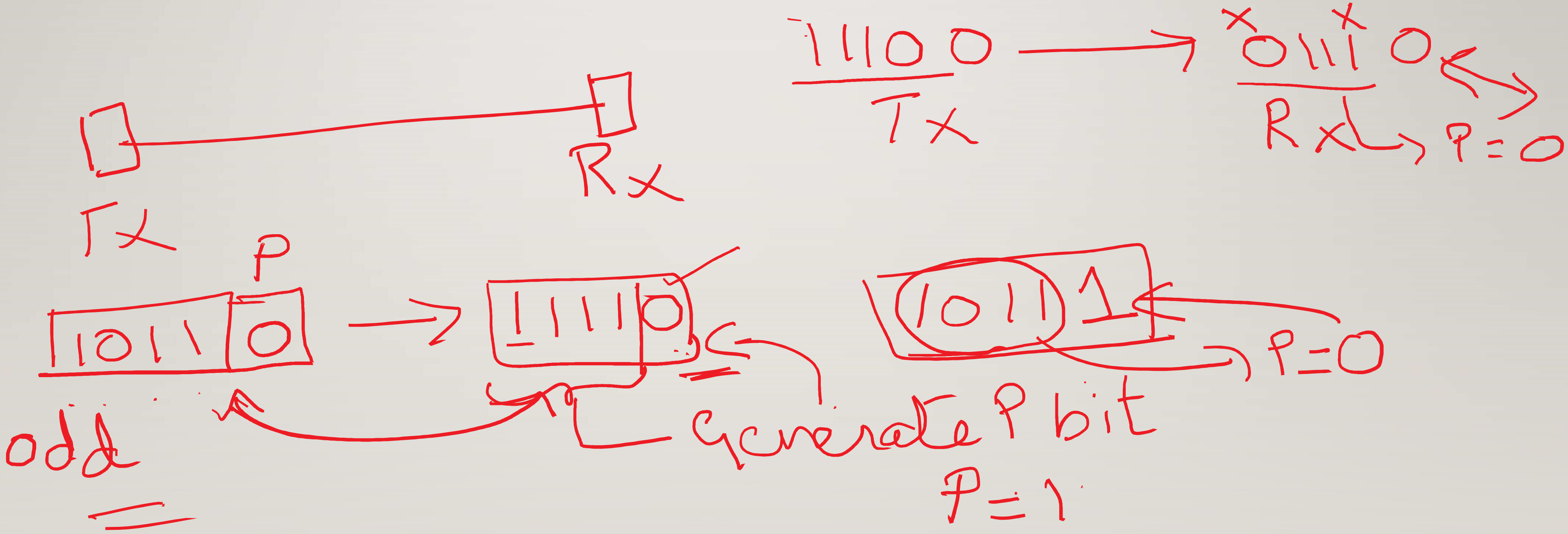
- Used for error detection in data communication
- Two types: Odd parity and even parity system.
- In odd parity: Total number of logic '1' s including parity bit should be odd

  Ex: input:1001 p=1, $\boxed{10011}$

  input 1101 p=0; $\boxed{11010}$
- In even parity: Total number of logic '1' s including parity bit should be even
- Ex: input:1001 p=0,

  input 1101 p=1;

USE OF PARITY BIT



XOR GATE

XNOR

A	B	$Y = A \oplus B$	$A \odot B = \overline{A \oplus B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$+\bar{A}\bar{B}$

AB

$$Y = A \oplus B = \bar{A}B + A\bar{B}$$

$$A \odot B = \bar{A}\bar{B} + AB$$

$$= \overline{A \oplus B}$$

$$P = \Sigma m(0, 3, 5, 6, 9, 10, 12, 15)$$

$$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD$$

$$= \bar{A}\bar{B}(\overline{C \oplus D}) + \bar{A}B(C \oplus D) + A\bar{B}(C \oplus D) + AB(\overline{C \oplus D})$$

$$= \bar{X}(\overline{A \oplus B}) + X(A \oplus B)$$

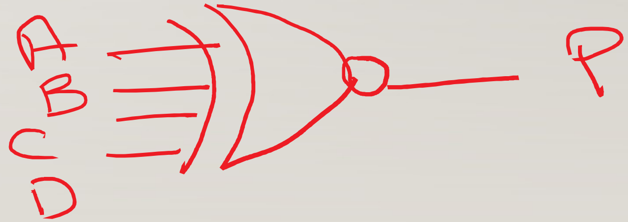
$$X = C \oplus$$

$$X = C \oplus D \qquad Y = A \oplus B$$

$$P = \bar{X}\,\overline{(A \oplus B)} + X \cdot (A \oplus B)$$

$$= \bar{X} \cdot \bar{Y} + X \cdot Y = \overline{X \oplus Y} \checkmark$$

$$= \overline{A \oplus B \oplus C \oplus D}$$



EXAMPLE 6:

Design a combinational circuit with 4- input lines that represents a decimal digit in BCD and 4- output lines that generates 2's complement of input digit.

	4-bit BCD B3 B2 B1 B0	2'S COMPLEMENT Y3 Y2 Y1 Y0
0	0 0 0 0	0000
	0 0 0 1	1111
	0 0 1 0	1110
	0 0 1 1	1101
	0 1 0 0	1100
	0 1 0 1	1011
	0 1 1 0	1010
	0 1 1 1	1001
8	1 0 0 0	1000
9	1001	0111

0 — 9   10

0000   1001   1010 ← binary

0000   1001   10 ← BCD

0001   0000

i/p $= (0110)_2 \Rightarrow (1001)$ ← 1's

+ve   -ve   1

1010 ← 2's

$(0110)_2 \xrightarrow{2's} (1010)_2$

$$\left(\underbrace{011011010}_{\text{Invert}}\,\underbrace{1000}_{\text{Same upto first '1'}}\right)_2$$

$$2\text{'s comp} \Rightarrow (1001001011\underline{000})_2$$

$$Y_3 = \Sigma m(1,2,3,4,5,6,7,8) + d(10,11,12,13,14,15)$$

$$Y_2 = \Sigma m(1,2,3,4,9) + d(\quad '' \quad)$$

$$Y_1 = \Sigma m(1,2,5,6,9) + d(\quad '' \quad)$$

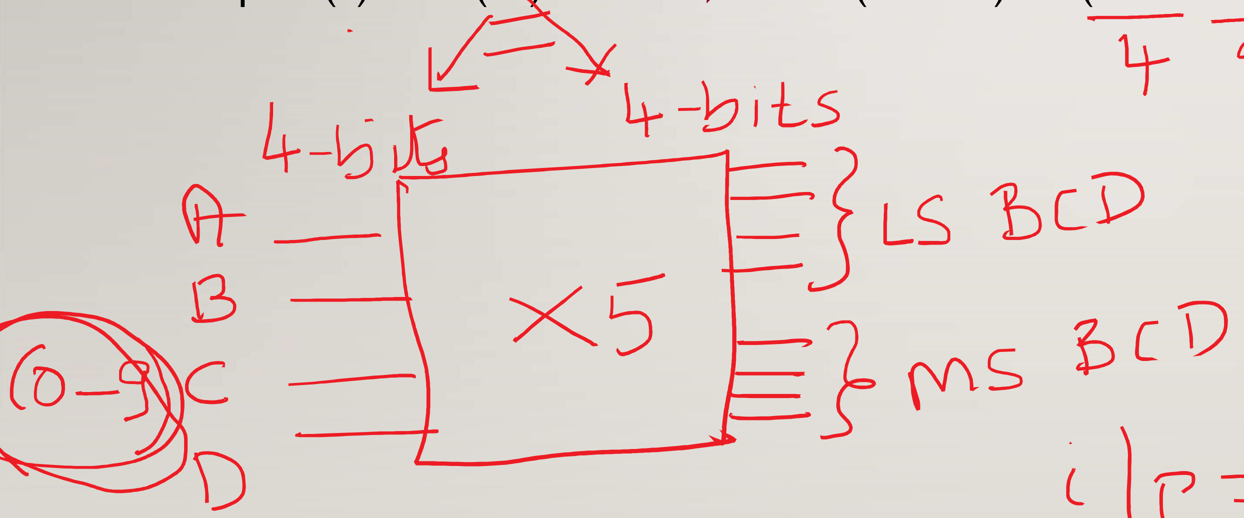
$$Y_0 = \Sigma m(1,3,5,7,9) + d(\quad '' \quad)$$

EXAMPLE 7:

Design a combinational circuit that multiplies by ‘5’ an input decimal digit represented in BCD. The output is also in BCD.

Input : $(0)_{10}$ to $(9)_{10}$ ⟶ (0000) to (1001) in BCD

output : $(0)_{10}$ to $(45)_{10}$ ⟶ (000000) to $(\underbrace{0100}_{4}\ \underbrace{0101}_{5})$ in BCD



don't cares : 10 to 15

$i/p = (5)_{10} \Rightarrow (0101)$

$o/p = (25)_{10} = (0010\ 0101)$

SIMPLIFICATION PROBLEMS

- 1. $F(A,B,C,D) = \Sigma m(1,3,4,5,10,11,12,13,14,15)$
- 2. $F(A,B,C,D) = \Sigma m (0,2,5,7,8,10,13,15) + D(1,4,11,14)$
- 3. $F(A,B,C,D) = \Pi M(0,1,4,5,8,9,11) . D(2,10)$
- 4. . $F(A,B,C,D) = \Pi M(0, 2,6,11,13,15) . D(1,9,10,14)$
- 5. $F(w,x,y,z) = w'x'z+xyz+wx'z+xy'z'$
- 6. $F(w,x,y,z)= (w+y+z')(x'+y+z')(w'+x'+y)(w+x+y+z).(w+x'+y'+z')(w'+x'+y'+z)$

- Any Qns?