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Theorem 1.1

Consider the N objects, of which $N(a_1)$ having the property a_1, $N(a_2)$ the property a_2,..., $N(a_r)$ the property a_r, $N(a_1a_2)$ having both the properties a_1 and a_2, ..., $N(a_1a_2..a_r)$ having all the properties $a_1, a_2, ..., a_r$, then the number $N(a_1'a_2'...a_r')$ with none of these properties is given by

$N(a_1'a_2'...a_r') =$

$N - N(a_1) - N(a_2) - ... - N(a_r) + N(a_1a_2) + N(a_1a_3) + ... + (-1)^r N(a_1...a_r)$

$= N - \sum N(a_i) + \sum_{i<j} N(a_ia_j) - \sum N(a_ia_ja_k) + ... + (-1)^r N(a_1a_2...a_r).$

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$$N(a_1' a_2' a_3') = 70 - (35 + 14 + 10) + (7 + 5 + 2) - 1 = 24.$$

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Derangements

It is a permutation of objects that leaves no object in its position.
Example: (2,3,1) and (3,1,2) are the derangements of 1,2,3.
For n $= 4$ the total number of derangements are—— 9.
i.e., total number of possibilities such that i^{th} digit not in i^{th} position.
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So, $N(a_1' a_2' a_3' a_4') = 4! - 4 \times 6 + \ {}^4C_2 \times 2 - \ {}^4C_3 \times 1 + 1 = 9.$

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$$N(a_1' a_2' ... a_n') = N - \sum_{i=1}^{n} N(a_i) + \sum_{i<j} N(a_i a_j) - \sum N(a_i a_j a_k) ... + (-1)^n N(a_1 ... a_n).$$

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For $n = 6$ the total number of derangements are
$6!(\frac{1}{2} - \frac{1}{3!} + \ldots + \frac{1}{6!}) = 265$.

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$$= (n-1)! \{ (1 - \tfrac{2}{2!} + \tfrac{3}{3!} - \tfrac{4}{4!} + \ldots) + n (1 - \tfrac{1}{1!} + \tfrac{1}{2!} - \tfrac{1}{3!} + \tfrac{1}{4!} - \ldots) \}$$

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Proportion of permutation is $\dfrac{(n-1)!(\frac{n+1}{e})}{n!} = \dfrac{n+1}{ne}$.

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$N(a_1) = 6$ which are $1234,\ 1243,\ 3124,\ 3412,\ 4123,\ 4312$.

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1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

Here, a_1 is the property that 12 is together.
$N(a_1) = 6$ which are $1234,\ 1243,\ 3124,\ 3412,\ 4123,\ 4312$.
Then, $a_1 a_2$ is the property that 12 is together and 23 is together which means 123 is together.

Consider $n = 4$. Here, a_i is the property that $(i, i+1)$ occurs consecutively, $1 \leq i \leq 3$.

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
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$N(a_1 a_2) = 2$, which are 1234, 4123.

Consider $n = 4$. Here, a_i is the property that $(i, i+1)$ occurs consecutively, $1 \le i \le 3$.

1234	2134	3124	4123
1243	2143	3142	4132
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$a_1 a_3$ is the property that 12 is together and 34 is together.

Consider $n = 4$. Here, a_i is the property that $(i, i+1)$ occurs consecutively, $1 \le i \le 3$.

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
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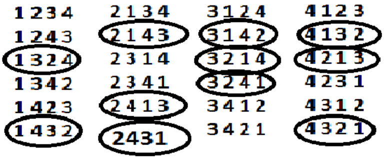
$a_1 a_3$ is the property that 12 is together and 34 is together.
$N(a_1 a_3) = 2$ which are 1234, 3412.

Consider $n = 4$. Here, a_i is the property that $(i, i+1)$ occurs consecutively, $1 \leq i \leq 3$.



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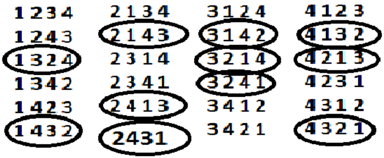
$N(a_1 a_3) = 2$ which are 1234, 3412.
$N(a_1 a_2 a_3) = 1$.

Consider $n = 4$. Here, a_i is the property that $(i, i+1)$ occurs consecutively, $1 \leq i \leq 3$.



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$N(a_1 a_2 a_3) = 1$.

$$N(a_1' a_2' a_3' a_4') = 4! - {}^3C_1(3!) + {}^3C_2(2!) - 1 = 11.$$