BVV Sangha's

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DEPARTMENT OF MATHEMATICS

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Numerical Techniques and Integral Transforms

SUBJECT : MATHEMATICS **SUBJECT CODE:** UMA391C

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Numerical Differentiation

Definition: - Suppose $Y_0, Y_1, Y_2, \ldots\ldots\ldots, Y_n$ are the values of an unknown function y=f(x) corresponding to $x:x_o$, x_1, x_2, ………… , x_n the process of computing f ' (x) , f ' '(x),……. At some particular value of independent variable x is known as numerical differentiation.

In the case of differentiation, we first write the interpolating formula on the interval and the differentiate the polynomial term by term to get an approximated polynomial to the derivative of the function. When the tabular points are equidistant, one uses either the Newton's Forward/ Backward Formula or Sterling's Formula; otherwise, Lagrange's formula is used. Newton's Forward/ Backward formula is used depending upon the location of the point at which the derivative is to be computed. In case the given point is near the mid-point of the interval, Sterling's formula can be used. We illustrate the process by taking

(i) Newton's Forward formula, and
(ii) Sterling's formula.

- Newton's Forward Numerical Difference Method

Formula

1. For $x = x_0$

$$\left[\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h} \cdot \left(\Delta Y_0 - \frac{1}{2} \cdot \Delta^2 Y_0 + \frac{1}{3} \cdot \Delta^3 Y_0 - \frac{1}{4} \cdot \Delta^4 Y_0 + \ldots\right)$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_0} = \frac{1}{h^2} \cdot \left(\Delta^2 Y_0 - \Delta^3 Y_0 + \frac{11}{12} \cdot \Delta^4 Y_0 + \ldots\right)$$

2. For any value of x

$$\left[\frac{dy}{dx}\right] = \frac{1}{h} \cdot \left(\Delta Y_0 + \frac{2t-1}{2!} \cdot \Delta^2 Y_0 + \frac{3t^2-6t+2}{3!} \cdot \Delta^3 Y_0 + \frac{4t^3-18t^2+22t-6}{4!} \cdot \Delta^4 Y_0 + \ldots\right)$$

$$\left[\frac{d^2y}{dx^2}\right] = \frac{1}{h^2} \cdot \left(\Delta^2 Y_0 + (t-1) \cdot \Delta^3 Y_0 + \frac{12t^2-36t+22}{24} \cdot \Delta^4 Y_0 + \ldots\right)$$

Examples
1. Find Solution using Newton's Forward Difference formula

x	f(x)
1891	46

1901	66
1911	81
1921	93
1931	101

x = 189

Solution:
The value of table for x and y

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

Newton's forward difference interpolation method to find solution

Newton's forward difference table is

x	**y**	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	**46**				
		20			
1901	66		**-5**		
		15		**2**	
1911	81		-3		**-3**
		12		-1	
1921	93		-4		
		8			
1931	101				

The value of x at you want to find the $f(x): x=1895$

$h=x_1-x_0=1901-1891=10$

$p=\frac{x-x_0}{h}=\frac{1895-1891}{10}=0.4$

Newton's forward difference interpolation formula is
$y(x)=y_0+p\Delta y_0+\frac{p(p-1)}{2!}\cdot\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\cdot\Delta^3 y_0+\frac{p(p-1)(p-2)(p-3)}{4!}\cdot\Delta^4 y_0$

$y(1895)=46+0.4\times20+\frac{0.4(0.4-1)}{2}\times-5+\frac{0.4(0.4-1)(0.4-2)}{6}\times2+\frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24}\times-3$

$y(1895)=46+8+0.6+0.128+0.1248$

$y(1895)=54.8528$

Solution of newton's forward interpolation method $y(1895)=54.8528$

Newton's Backward Numerical Difference Method

Stirling Formula is obtained by taking the average or mean of the Gauss Forward and Gauss Backward Formula. Both the Gauss Forward and Backward formula are formulas for obtaining the value of the function near the middle of the tabulated set.

1. Using Newton's Backward Difference formula to find solution

Formula

1. For $x = x_n$

$$\left[\frac{dy}{dx}\right]_{x=x_n} = \frac{1}{h} \cdot \left(\nabla Y_n + \frac{1}{2} \cdot \nabla^2 Y_n + \frac{1}{3} \cdot \nabla^3 Y_n + \frac{1}{4} \cdot \nabla^4 Y_n + ...\right)$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_n} = \frac{1}{h^2} \cdot \left(\nabla^2 Y_n + \nabla^3 Y_n + \frac{11}{12} \cdot \nabla^4 Y_n + ...\right)$$

2. For any value of x

$$\left[\frac{dy}{dx}\right] = \frac{1}{h} \cdot \left(\nabla Y_n + \frac{2t+1}{2} \cdot \nabla^2 Y_n + \frac{3t^2+6t+2}{6} \cdot \nabla^3 Y_n + \frac{4t^3+18t^2+22t+6}{24} \cdot \nabla^4 Y_n + ...\right)$$

$$\left[\frac{d^2y}{dx^2}\right] = \frac{1}{h^2} \cdot \left(\nabla^2 Y_n + (t+1) \cdot \nabla^3 Y_n + \frac{12t^2+36t+22}{24} \cdot \nabla^4 Y_n + ...\right)$$

x	f(x)
1.4	4.0552
1.6	4.9530
1.8	6.0496
2.0	7.3891
2.2	9.0250

x = 2.2

Solution:
Numerical differentiation method to find solution.
The value of table for x and y

x	1.4	1.6	1.8	2	2.2
y	4.0552	4.953	6.0496	7.3891	9.025

Newton's backward differentiation table is

x	**y**	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.4	4.0552				
		0.8978			
1.6	4.953		0.1988		
		1.0966		0.0441	
1.8	6.0496		0.2429		**0.0094**
		1.3395		**0.0535**	
2	7.3891		**0.2964**		
		1.6359			
2.2	**9.025**				

The value of x at you want to find $f(x): x_n = 2.2$

$h = x_1 - x_0 = 1.6 - 1.4 = 0.2$

$$\left[\frac{dy}{dx}\right]_{x=x_n} = \frac{1}{h}\cdot\left(\nabla y_n + \frac{1}{2}\cdot\nabla^2 y_n + \frac{1}{3}\cdot\nabla^3 y_n + \frac{1}{4}\cdot\nabla^4 y_n\right)$$

$$\therefore \left[\frac{dy}{dx}\right]_{x=2.2} = \frac{1}{0.2}\times\left(1.6359 + \frac{1}{2}\times 0.2964 + \frac{1}{3}\times 0.0535 + \frac{1}{4}\times 0.0094\right)$$

$$\therefore \left[\frac{dy}{dx}\right]_{x=2.2} = 9.02142$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_n} = \frac{1}{h^2}\cdot\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\cdot\nabla^4 y_n\right)$$

$$\therefore \left[\frac{d^2y}{dx^2}\right]_{x=2.2} = \frac{1}{0.04}\cdot\left(0.2964 + 0.0535 + \frac{11}{12}\times 0.0094\right)$$

$$\therefore \left[\frac{d^2y}{dx^2}\right]_{x=2.2} = 8.96292$$

$\therefore P_n'(2.2) = 9.02142$ and $P_n''(2.2) = 8.96292$

APPLICATIONS

There are many. Numerical differentiation is used any time an analytical solution is not possible. One application is edge detection in image processing. If you differentiate an image, the edges of objects tend to stand out. Here is a random image I grabbed from the internet of some sunglasses. I calculated a gradient (2D derivative) which you see on the right. The edges are quite apparent!

Numerical Analysis is a technique of mathematical analysis that uses numerical approximation in particular to obtain accurate results for some of the problems that are hard to resolve otherwise. You must have had Numerical Analysis Questions and Answers in your graduate years. It is a part of engineering, architecture and scientific studies that involve applied mathematics. Streams like differentiation and integration, differential equations, and linear programming are also included in numerical analysis methods.

We will talk about some major applications of Numerical Analysis in daily-day life that are both intriguing and easy to understand. Trying to figure out these applications will give you a sound understanding of the concept in general.

1. Making Weather Predictions
Advanced computer simulations have made it possible to make weather predictions by computing numerical data from weather forecasting equipment such as weather satellites. This is done by making a mathematical model of a particular location and using computer based Numerical Analysis to obtain precise numerical values that are used for determining weather changes.

2. Car Safety Enhancement
Car manufacturers also use Numerical Analysis to make numerical models of car crash safety simulations. These models serve as the platform to unravel optimal results that signify various aspects of a car crash test. These are then fed to an advanced computer to gain insights into improving car safety. Mostly, these are partial differential equations.

3. Machine Learning
Just like other domains, Machine Learning tends to use Numerical Analysis Questions and Answers for optimization of numerical functions that it is presented with. A few examples of the same are Newton's Method and Nestorov Method. Machine learning needs numerical analysis for problems that don't have an analytically optimize-able solution. Artificial Intelligence is yet another field where machine learning is applied by the use of Numerical Analysis in stochastic environments.

4. Spacecraft Dynamics
Space travel also uses numerical methods to make approximations on the spacecraft trajectories. Space has no gravity in it and therefore the dynamics of motion operate very differently than on Earth. Numerical Analysis comes handy in making these estimations with the help of linear as well as non-linear equations that are computed by onboard computers. Solving simple differential equations is the key in this application of Numerical Analysis.

5. Price Estimation by Airline Companies

It might not seem obvious but Airline companies also use Numerical Methods to optimize their ticket prices, keep a check on their fuel needs, payroll and many other activities. Sophisticated Numerical Analysis technologies have made it possible to generate accurate approximations of quantities that can't be computed otherwise. A field of study stemming from this application; operations research also deals in finding optimised results for some daily day issues. This is done making intelligent algorithms that work on **Numerical Analysis Questions and Answers** to solve problems.

Advantages

Differentiation is one of the most important concepts in calculus, which has been used almost everywhere in many fields of mathematics and applied mathematics. It is natural that numerical differentiation should be an important technique for the engineers.