An Information–Theoretic Diagnostic of Dark Matter Halo Anisotropy

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Abstract

We introduce a novel, information–theoretic methodology for characterizing the three–dimensional structure and dynamical state of dark matter halos in numerical simulations. The method quantifies the angular organization of mass on concentric spherical shells using normalized Shannon entropy, providing a bounded and physically interpretable measure of anisotropy and substructure. Unlike standard halo diagnostics based on density estimation or ellipsoidal fitting, this approach makes no assumptions about equilibrium, symmetry, or smoothness. It is therefore well suited to studying highly non–relaxed systems such as major galaxy mergers. In this work, we present the theoretical motivation, formal definition, and numerical implementation of the Angular Information Entropy diagnostic, and outline its intended application to the Milky Way–Andromeda merger.

1 Introduction

The shape and internal structure of dark matter halos encode essential information about their formation history and dynamical state. In the standard cosmological framework, halos grow through hierarchical accretion and mergers, processes that generically drive departures from spherical symmetry and equilibrium. Quantifying these departures is therefore a central task in numerical studies of galaxy formation.

Commonly used halo diagnostics include spherically averaged density profiles, inertia–tensor–based axis ratios, and tessellation or kernel–based density estimators. While these techniques are powerful, they primarily characterize the spatial distribution of mass and often impose implicit assumptions of smoothness or ellipsoidal geometry. During major mergers, when halos are strongly disturbed and exhibit prominent tidal features, these assumptions can obscure physically meaningful structure.

In this work, we propose an alternative diagnostic that focuses on the angular organization of mass rather than its spatial density. By measuring the information content of the angular mass distribution on spherical shells, we obtain a direct and assumption–minimal probe of anisotropy, substructure, and dynamical mixing.

2 Physical Motivation

In a relaxed, spherically symmetric halo, the angular distribution of particles at fixed radius is approximately isotropic. Conversely, departures from equilibrium—such as triaxiality, tidal stripping, shell crossing, and the presence of streams or bridges—introduce coherent angular structure. These effects are particularly pronounced during major mergers, where the halo cannot be well described by a single smooth ellipsoid.

Traditional density–based methods quantify how much mass occupies a region of space, but are often insensitive to how that mass is directionally organized. An information–theoretic approach provides a natural alternative: isotropic distributions maximize entropy, while structured or anisotropic distributions reduce it. This principle motivates the use of Shannon entropy as a diagnostic of halo angular structure.

3 Angular Information Entropy Diagnostic

3.1 Conceptual Definition

We treat the dark matter halo as a three–dimensional point cloud sampled by collisionless particles. At a given radius, the angular positions of these particles define a probability distribution on the unit sphere. The Angular Information Entropy measures the degree to which this distribution deviates from isotropy.

Importantly, the diagnostic is insensitive to the total mass or radial density profile of the halo. It depends only on the relative angular distribution of mass within each radial shell.

3.2 Particle Selection and Reference Frame

For each simulation snapshot, dark matter particles associated with the Milky Way (MW) and Andromeda (M31) systems are combined into a single particle set. The system is recentered using the MW center of mass, removing bulk translational motion while preserving intrinsic merger asymmetries. All subsequent calculations are performed in this centered frame.

3.3 Radial Shell Decomposition

The particle distribution is divided into concentric, logarithmically spaced radial shells. This choice provides adequate resolution of both the inner halo and the extended outskirts while maintaining sufficient particle counts in each shell. Each shell is analyzed independently, allowing the diagnostic to capture radial variation in angular structure.

3.4 Angular Discretization

Within each radial shell, particle positions are converted from Cartesian coordinates to spherical angles (θ, ϕ). The unit sphere is discretized into a fixed number of polar and azimuthal bins, defining a set of equal–area angular cells to first order. Particle masses are accumulated into these cells, producing a mass–weighted angular histogram.

The choice of angular resolution represents a trade–off between sensitivity to small–scale structure and statistical noise due to finite particle sampling. This resolution is held fixed across all snapshots to enable consistent temporal comparisons.

3.5 Entropy Calculation

Let p_i denote the fraction of mass contained in the ith angular bin of a given radial shell. The Shannon entropy of the angular mass distribution is defined as

$$S = -\sum_i p_i \ln p_i. \tag{1}$$

To facilitate comparison across shells and snapshots, the entropy is normalized by its maximum possible value,

$$S_{\max} = \ln N_{\text{bins}}, \tag{2}$$

where N_{bins} is the total number of angular bins. The resulting normalized angular entropy,

$$S_{\text{ang}} = \frac{S}{S_{\max}}, \tag{3}$$

lies in the interval $[0, 1]$.

3.6 Physical Interpretation

Values of S_{ang} close to unity indicate a nearly isotropic angular mass distribution, consistent with a relaxed or well–mixed halo. Lower values reflect increasing angular anisotropy due to triaxiality, tidal debris, streams, or merger–driven distortions.

By tracking $S_{\text{ang}}(r, t)$ as a function of radius and time, the diagnostic provides a direct measure of angular mixing and relaxation throughout the merger process.

4 Numerical Implementation

The diagnostic is implemented using particle data from simulation snapshots. A minimum particle threshold is imposed within each radial shell to suppress shot noise and ensure robust entropy estimates. The method involves no kernel smoothing, tessellation, or ellipsoidal fitting, and is therefore computationally straightforward and geometrically unbiased.

The primary output is a two–dimensional time–radius map of normalized angular entropy, which can be directly compared to other halo diagnostics such as inertia–tensor axis ratios or convex–hull measures.

5 Scope and Intended Applications

The Angular Information Entropy diagnostic is intended for studies of dark matter halo evolution in non–equilibrium environments, particularly major mergers. While developed here for the Milky Way–Andromeda system, the method is general and applicable to any collisionless particle system where angular structure carries physical significance.