Simulation of Rods Undergoing Twist and Analyzing Resulting Plectonemes

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I. Introduction

When elastic rods are subjected to large twist, entangled structures are formed called plectonemes. There are many different forms of plectonemes depending on the amount of twist applied, from a basic helical shape to fully developed loops and knots resulting from sudden buckling. As you can see in Figure 1, as more twist is applied the rod forms increasingly more complex shapes. A common daily example of this can be seen in a rubber band. If cut and held from both sides, a twisting can be applied on both sides (opposite directions). This results in an ordinary helix form at first and then suddenly, the rubber band will "jump" and form some sort of looped structure. If twisting is continued to be applied the plectonemes get increasingly more complex and "jumpy." This phenomenon is what this project will be focused on studying.

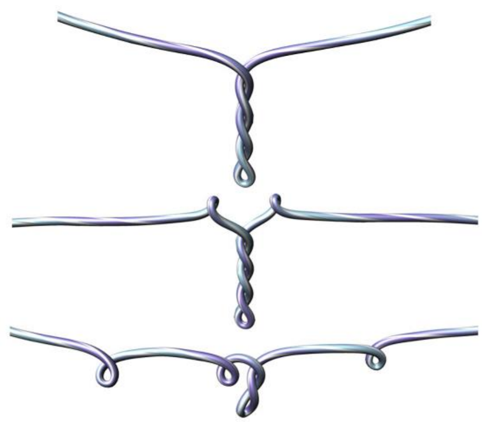

Figure 1. Increasing complexity of plectonemes resulting from ongoing twist [Bergou 2008]

II. Related Literature

A. *Underwater Marine Cable*

The phenomenon of plectonemes can occur in underwater marine cables. Sachin Goyal [2007] studied the effects of these plectonemes, specifically when one end is fixed on a boat, therefore no twisting on that end. This is an area of interest as these tangles can cause significant issues such as damage and signal transmission interruptions. He models the cable as a discrete elastic rod that is fixed from one end. Twist is applied to the other end (free end) and one of his results can be seen in Figure 3. He plots the strain energy density over time. The rod transitions from a helical form (time = 20 seconds) to a buckled intertwined form (time = 32 seconds). This form has a very relatively high strain energy at the top of the loop showing how harmful these plectonemes can be for underwater marine cables.

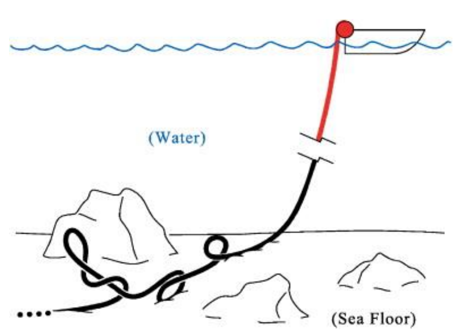

Figure 2. Marine Cable forming intertwining tangles

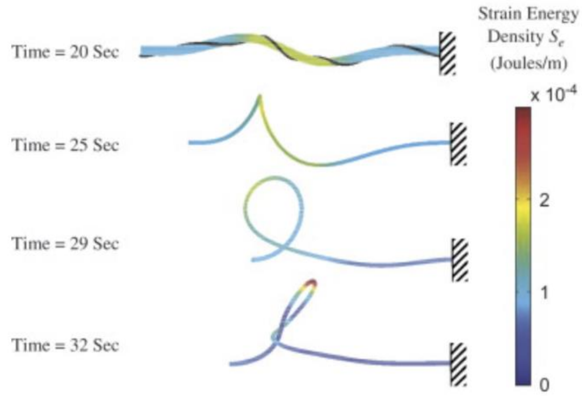

Figure 3. Transition from helical form to an intertwined form and plotting of strain energy density

B. DNA supercoiling

Another area of interest for plectonemes is the supercoiling of DNA when forming chromosomes. Because of the prolific importance of DNA in organic life, this has been heavily studied. Because a strand of DNA is so long (2 meters), the plectonemes resulting from the supercoiling process can be incredibly complex. Yang Yang [1993] modeled a DNA strand as a discrete elastic rod to observe the resulting plectonemic structures. An example of these structures can be seen in Figure 4.

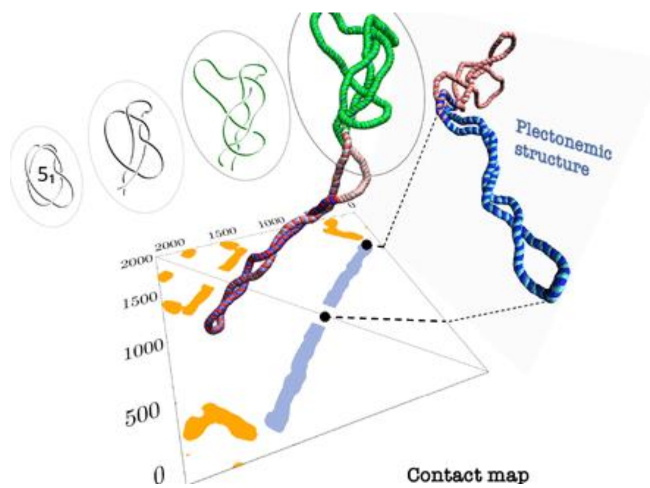

Figure 4. Plectonemic structures of DNA

C. Plectonemes in an Electrically Charged Rod

Lastly, an electrically charged rod can present a new set of behaviors for plectonemes. Because plectonemes are a self-contact phenomenon, the repulsion of the rod contacting itself causes the rod to form a new set of plectonemes. The behavior can vary depending on the charge of the rod, the charge of the solvent, etc. [Lim 2010]. An example in everyday life of an electrically charged rod/ plate is a ribbon that's been rubbed against one's hair.

III. Simulation of Plectonemes

The goal of this project is to simulate the behavior of plectonemes in an elastic rod that is fixed from both ends and experiences twisting on both ends (opposite directions). The twisting energies will be analyzed especially during sudden buckling or "jumps" where loops are suddenly formed. Once a basic simulation has been set up, many variables of interest will be tested to see their effect on the behavior of the plectonemes: radius of the rod, length of the rod, speed of twisting, material, viscosity of surround liquid/ gas, etc.

IV. METHODOLOGY

To simulate this, discrete elastic rod algorithms must be applied using the time parallel reference frame and also an implicit contact model in this framework since plectonemes are self-contact phenomena. Clearly, more research needs to be done to properly simulate this project.

APPENDIX

Bergou, M., Wardetzky, M., Robinson, S., Audoly, B., Grinspun, E. 2008. Discrete Elastic Rods. *ACM Transactions on Graphics (SIGGRAPH)*

Goyal, S., Perkins, N.C., AND Lee, C. L. 2007. Non-linear dynamic intertwining of rods with self-contact. *International Journal of Non-Linear Mechanics In Press, Corrected Proof.*

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