Q1

(a) $G(s) = \dfrac{50}{s(s^2+10s+50)(s+5)}$; $L_d = \dfrac{100}{s}$

$$K = LG^{-1} = \frac{100}{s} \times \frac{s(s^2+10s+50)(s+5)}{50}$$

$$= 2(s^2+10s+50)(s+5)$$

Make a proper controller

$$\Rightarrow \boxed{K = \frac{2(s^2+10s+50)(s+5)}{s^2 \times \left(\frac{s}{10000}+1\right)}}$$

[Adding Integral term & high freq pole]

We know that $G_p = G(1+\alpha\Delta)$ for $\|\Delta\|_\infty < 1$

$\Rightarrow$ $M = KSGW$ $[W = \alpha]$

$$= KSG\alpha$$

$$S = \frac{1}{1+GK} = \frac{1}{1 + \frac{50}{s(s^2+10s+50)(s+5)} \times \frac{2\times(s^2+10s+50)(s+5)}{s^2\left(\frac{s}{10000}+1\right)}}$$

$$= \frac{1}{1 + \frac{100\times 10^4}{s^3(s+10^4)}} = \frac{s^3(s+10^4)}{s^3(s+10^4)+10^6}$$

$$M = \frac{2(s^2+10s+50)(s+5)\times 10^4}{s^2 \times (s+10^4)} \times \frac{s^3(s+10^4)}{s^3(s+10^4)+10^6} \times \frac{50}{s(s^2+10s+50)(s+5)}\alpha$$

$$\Rightarrow M = \frac{10^6 \alpha}{s^3(s+10^4)+10^6}$$

$$\therefore \|M\|_\infty < 1 \Rightarrow \alpha < \frac{1}{\left\| \frac{10^6}{s^3(s+10^4)+10^6} \right\|_\infty} = \frac{1}{\text{Inf}}$$

$$\Rightarrow \boxed{\alpha < 0}$$

Maximum value of α is not positive

$\therefore$ The system cannot be robustly stable for $\forall$ ~~all~~ $\alpha \in R^+$