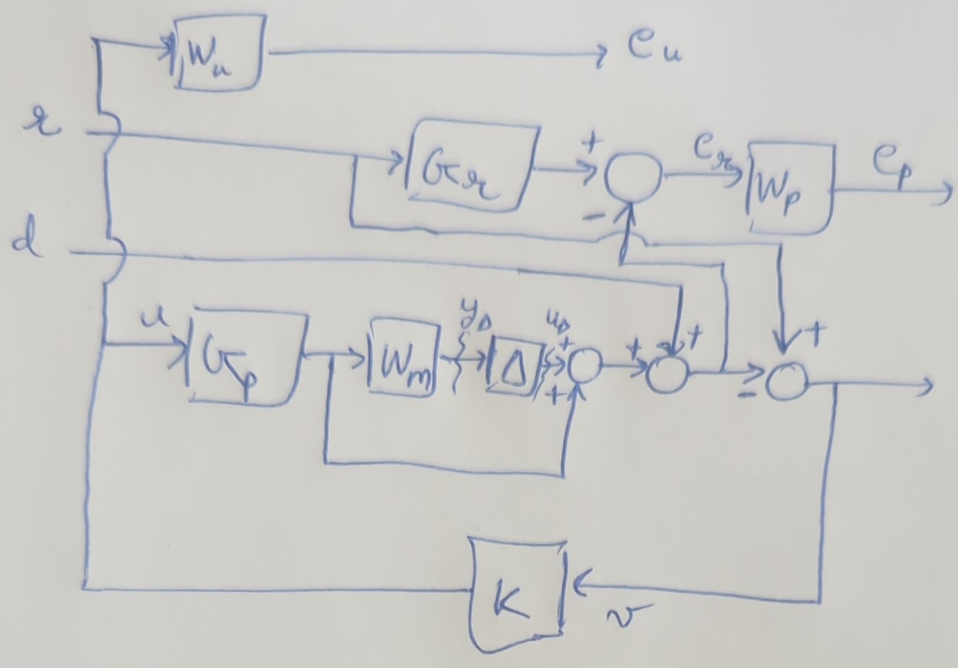
Q3



$$y_\Delta = W_m G_p u \qquad w = \begin{pmatrix} r \\ d \end{pmatrix}$$

$$z = \begin{bmatrix} e_u \\ e_p \end{bmatrix} \Rightarrow e_u = W_u u$$

$$e_p = W_p \left(G_r r - (d + u_\Delta + G_p u) \right)$$

$$= W_p (G_r r - d - u_\Delta - G_p u)$$

$$= W_p G_r r - W_p d - W_p u_\Delta - W_p G_p u$$

$$v = r - (d + u_\Delta + G_p u)$$

$$= r - d - u_\Delta - G_p u$$

$$\therefore \begin{bmatrix} y_\Delta \\ z \\ v \end{bmatrix} = \begin{bmatrix} y_\Delta \\ e_u \\ e_p \\ v \end{bmatrix} = \underbrace{\left[\begin{array}{ccc|c} 0 & 0 & 0 & W_m G_p \\ 0 & 0 & 0 & W_u \\ -W_p & W_p G_r & -W_p & -W_p G_p \\ \hline -I & I & -I & -G_p \end{array}\right]}_{P} \begin{bmatrix} u_\Delta \\ r \\ d \\ u \end{bmatrix}$$

$$N = F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

$$\Rightarrow N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -W_p & W_p G_{r} & -W_p \end{bmatrix} + \begin{bmatrix} W_m G_p \\ W_u \\ -W_p G_p \end{bmatrix} K \left(1 - (-G_p)K\right)^{-1} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -W_p & W_p G_{r} & -W_p \end{bmatrix} + \begin{bmatrix} W_m G_p K \\ W_u K \\ -W_p G_p K \end{bmatrix} \left[(1 + G_p K)^{-1} \right] \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -W_p & W_p G_{r} & -W_p \end{bmatrix} + \begin{bmatrix} W_m G_p K/(1 + G_p K) \\ W_u K/(1 + G_p K) \\ -W_p G_p K/(1 + G_p K) \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

Let $T = G_p K/(1 + G_p K)$, $T = 1 - S \Rightarrow S = 1 - T$

$$\Rightarrow S = \frac{1}{1 + G_p K}$$

$$\Rightarrow N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -W_p & W_p G_{r} & -W_p \end{bmatrix} + \begin{bmatrix} W_m T \\ W_u K S \\ -W_p T \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -W_p & W_p G_{r} & -W_p \end{bmatrix} + \begin{bmatrix} -W_m T & W_m T & -W_m T \\ -W_u K S & W_u K S & -W_u K S \\ W_p T & -W_p T & W_p T \end{bmatrix}$$

$$\boxed{N = \begin{bmatrix} -W_m T & W_m T & -W_m T \\ -W_u K S & W_u K S & -W_u K S \\ -W_p(1-T) & W_p G_{r} - W_p T & -W_p(1-T) \end{bmatrix} = \begin{bmatrix} -W_m T & W_m T & -W_m T \\ -W_u K S & W_u K S & -W_u K S \\ -W_p S & W_p(G_{r} - T) & -W_p S \end{bmatrix}}$$

Q3(b)

```
s = tf('s');
Gp = -25.9/(s^3+24.2*s^2-356*s-8620);
Wp = 5/(0.1*s+0.001);
Wm = (2*s+1.73)/(s+173.2);
Gr = 1e15/(s^3+1.75*1e5*s^2+2.15*1e10*s+1e15);
Wu = 1e-5;

P = [0 0 0 Wm*Gp;
     0 0 0 Wu;
     -Wp Wp*Gr -Wp -Wp*Gp;
     -1 1 -1 -Gp];

[K,CL,gamma] = hinfsyn(P,1,1)
```

```
K =

     []


CL =

     []
gamma = Inf
```

```
delta = ultidyn('del',1);
Gunc_p = (1+Wm*delta)*Gp;
% T = Gunc_p*K/(1+Gunc_p*K);
% S = 1-T;
% N = [-Wm*T Wm*T -Wm*T;
%       -Wu*K*S Wu*K*S -Wu*K*S;
%       -Wp*S Wp*(Gr-T) -Wp*S];
P_hat = [0 0 Wu;
         Wp*Gr -Wp -Wp*Gunc_p;
         1 -1 -Gunc_p];
N = lft(P_hat,K)
```

```
N =

  Uncertain continuous-time state-space model with 3 outputs, 3 inputs, 14 states.
  The model uncertainty consists of the following blocks:
    del: Uncertain 1x1 LTI, peak gain = 1, 2 occurrences

Type "N.NominalValue" to see the nominal value, "get(N)" to see all properties, and "N.Uncertainty" to interact with
```

```
stabmarg = robuststab(N)
```

```
stabmarg = struct with fields:
                LowerBound: 0
                UpperBound: 0
    DestabilizingFrequency: 0
```

```
mu = 1/stabmarg.LowerBound
```

```
mu = Inf
```

```
perfmarg = robustperf(N)
```

```
perfmarg = struct with fields:
           LowerBound: 0
           UpperBound: 0
    CriticalFrequency: 0
```

```
mu = 1/perfmarg.LowerBound
```

```
mu = Inf
```