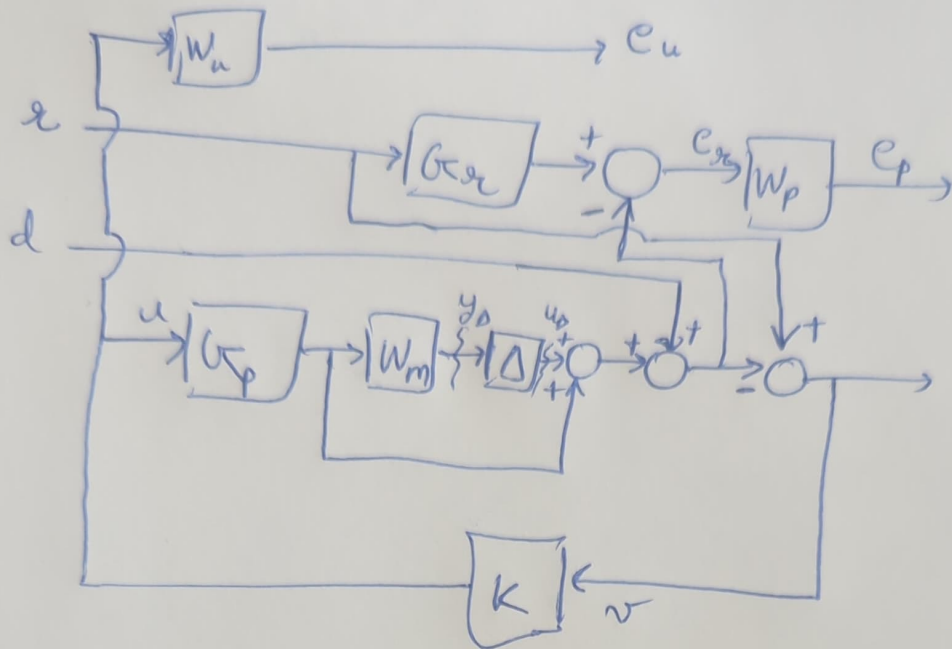


Q3



$$y_d = W_m G_p u$$

$$W = \begin{bmatrix} x \\ d \end{bmatrix}$$

$$z = \begin{bmatrix} e_u \\ e_p \end{bmatrix} \Rightarrow e_u = W_u u$$

$$e_p = W_p (G_x x - (d + u_d + G_p u))$$

$$= W_p (G_x x - d - u_d - G_p u)$$

$$= W_p G_x x - W_p d - W_p u_d - W_p G_p u$$

$$v = x - (d + u_d + G_p u)$$

$$= x - d - u_d - G_p u$$

$$\begin{bmatrix} y_d \\ z \\ v \end{bmatrix} = \begin{bmatrix} y_d \\ e_u \\ e_p \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -W_p & W_p G_x & -W_p \\ -I & I & -I \end{bmatrix}}_P \begin{bmatrix} W_m G_p \\ W_u \\ -W_p G_p \\ -G_p \end{bmatrix} \begin{bmatrix} u_d \\ x \\ d \\ u \end{bmatrix}$$

$$N = F_d(P, k) = P_{11} + P_{12} k (I - P_{22} k)^{-1} P_{21}$$

$$\Rightarrow N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -w_p & w_p k r & -w_p \end{bmatrix} + \begin{bmatrix} w_m k r_p \\ w_u \\ -w_p k r_p \end{bmatrix} k \left(I - (-k r_p) \right)^{-1} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -w_p & w_p k r & -w_p \end{bmatrix} + \begin{bmatrix} w_m k r_p k \\ w_u k \\ -w_p k r_p k \end{bmatrix} \left(1 + k r_p k \right)^{-1} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -w_p & w_p k r & -w_p \end{bmatrix} + \begin{bmatrix} w_m k r_p k / (1 + k r_p k) \\ w_u k / (1 + k r_p k) \\ -w_p k r_p k / (1 + k r_p k) \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

Let $T = k r_p / (1 + k r_p k)$, $T = 1 - S \Rightarrow S = 1 - T$
 $\Rightarrow S = \frac{1}{1 + k r_p k}$

$$\Rightarrow N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -w_p & w_p k r & -w_p \end{bmatrix} + \begin{bmatrix} w_m T \\ w_u k S \\ -w_p T \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -w_p & w_p k r & -w_p \end{bmatrix} + \begin{bmatrix} -w_m T & w_m T & -w_m T \\ -w_u k S & w_u k S & -w_u k S \\ w_p T & -w_p T & w_p T \end{bmatrix}$$

$$N = \begin{bmatrix} -w_m T & w_m T & -w_m T \\ -w_u k S & w_u k S & -w_u k S \\ -w_p(1-T) & w_p k r - w_p T & -w_p(1-T) \end{bmatrix} = \begin{bmatrix} -w_m T & w_m T & -w_m T \\ -w_u k S & w_u k S & -w_u k S \\ -w_p S & w_p(k r - T) & -w_p S \end{bmatrix}$$