Q1

(a) $G(s) = \dfrac{50}{s(s^2+10s+50)(s+5)}$; $L_d = \dfrac{100}{s}$

$$k = L G^{-1} = \frac{100}{s} \times \frac{s(s^2+10s+50)(s+5)}{50}$$

$$= 2(s^2+10s+50)(s+5)$$

Make a proper controller

$$\Rightarrow \boxed{k = \frac{2(s^2+10s+50)(s+5)}{s^2 \times \left(\frac{s}{10000}+1\right)}}$$

(Adding Integral term & high freq pole)

We know that $G_p = G(1+\alpha\Delta)$ for $\|\Delta\|_\infty < 1$

$$\Rightarrow M = kSGW \quad [W = \alpha]$$

$$= kSG\alpha$$

$$S = \frac{1}{1+GK} = \frac{1}{1+\dfrac{50}{s(s^2+10s+50)(s+5)} \times \dfrac{2\times(s^2+10s+50)(s+5)}{s^2\left(\frac{s}{10000}+1\right)}}$$

$$= \frac{1}{1+\dfrac{100\times10^4}{s^3(s+10^4)}} = \frac{s^3(s+10^4)}{s^3(s+10^4)+10^6}$$

$$M = \frac{2(s^2+10s+50)(s+5)\times10^4}{s^2\times(s+10^4)} \times \frac{s^3(s+10^4)}{s^3(s+10^4)+10^6} \times \frac{50}{s(s^2+10s+50)(s+5)} \alpha$$

$$\Rightarrow M = \frac{10^6 \alpha}{s^3(s+10^4)+10^6}$$

$$\therefore \|M\|_\infty < 1 \Rightarrow \alpha < \frac{1}{\left\| \frac{10^6}{s^3(s+10^4)+10^6} \right\|_\infty} = \frac{1}{Inf}$$

$$\Rightarrow \boxed{\alpha < 0}$$

Maximum value of α is not positive

The system cannot be robustly stable for $\alpha \in R^+$

Q1(b)

```
s = tf('s');
G = 50/(s*(s^2+10*s+50)*(s+5));
%G = prescale(ss(G));
Gd = 100/s;
[K1,CL,gamma,info] = loopsyn(G,Gd);
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  9.310850e-25.
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.288815e-23.
```

```
S = 1/(1+G*K1);
isstable(S)
```

```
ans = logical
   1
```

```
M = K1*S*G;
gam = norm(M,'inf')
```

```
gam = Inf
```

```
alpha = 1/gam
```

```
alpha = 0
```

Q1(c)

```
OPT = balredOptions('StateElimMethod','Truncate');

K2 = (2*(s^2+10*s+50)*(s+5))/((s^2)*(1+ s/1e4));

order(K1)
```

```
ans = 11
```

```
order(K2)
```

```
ans = 3
```

```

K1 = balred(K1,2,OPT); % Loopsyn
K2 = balred(K2,2,OPT); % By-hand

isstable(K1)
```

```
ans = logical
   0
```

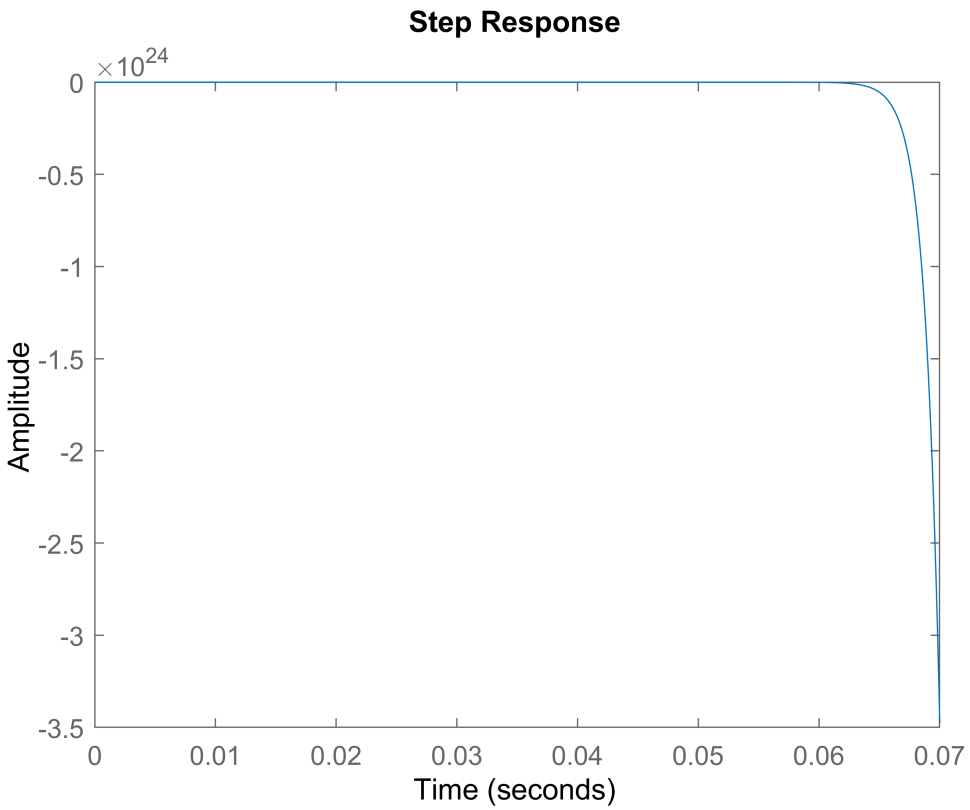
```
isstable(K2)
```

```
ans = logical
   1
```

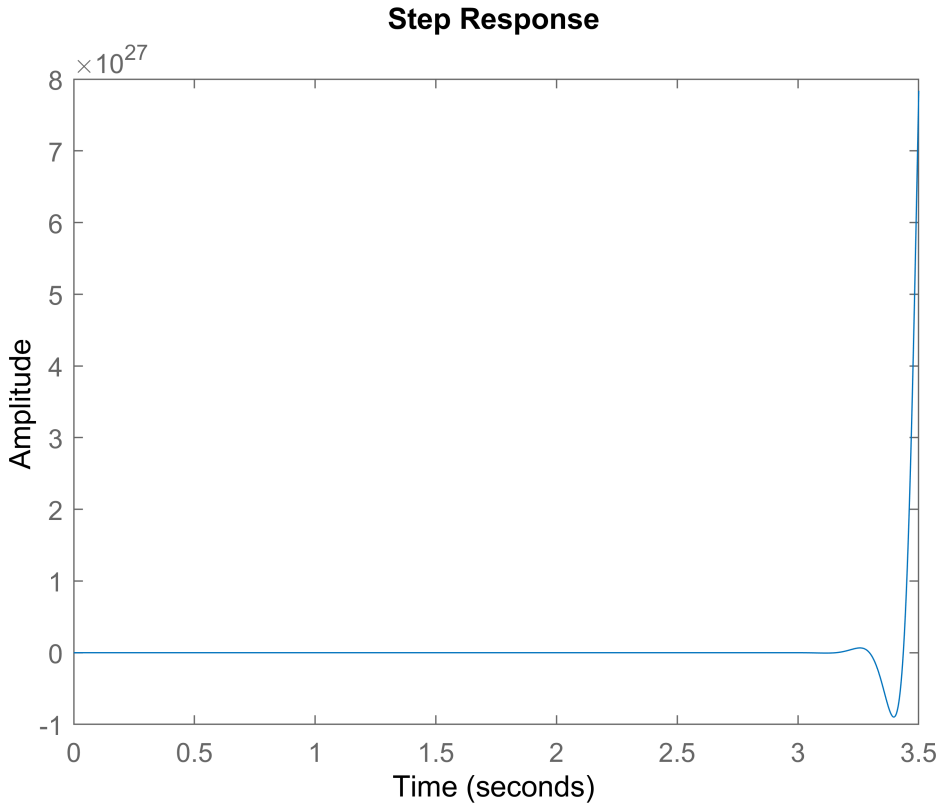
```

T1 = G*K1/(1 + G*K1);
T2 = G*K2/(1 + G*K2);
```

```
step(T1)
```




```
step(T2)
```



```
% Loopsyn controller
S = 1/(1+G*K1);
isstable(S)
```

```
ans = logical
   0
```

```
M = K1*S*G;
gam = norm(M,'inf')
```

```
gam = Inf
```

```
alpha = 1/gam
```

```
alpha = 0
```

```
% By-hand controller
S = 1/(1+G*K2);
isstable(S)
```

```
ans = logical
   0
```

```
M = K2*S*G;
gam = norm(M,'inf')
```

```
gam = 1.0025
```

```
alpha = 1/gam
```

```
alpha = 0.9975
```