

CS-583 Assignment 1

Q1. $x = [5, -3, -1, 2]^T$

1) $\|x\|_2^2 = 25 + 9 + 1 + 4$
 $= 39$

2) $\|x\|_1 = 5 + 3 + 1 + 2$
 $= 11$

3) $a = [4, -2, 6, -1]^T$

$$a^T x = \begin{bmatrix} 4 \\ -2 \\ 6 \\ -1 \end{bmatrix} \cdot [5, -3, -1, 2]$$

$$= 20 + 6 - 6 - 2 = 18$$

Q2. $A = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix}$, $b = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}_{3 \times 1}$

1) $Ab = \begin{bmatrix} -24 + 5 - 4 \\ 20 + 35 + 18 \end{bmatrix}$
 $= \begin{bmatrix} -23 \\ 73 \end{bmatrix}$

$$2.) \quad AA^T = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} 6 & -5 \\ 1 & 7 \\ -2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 36 + 1 + 4 & -30 + 7 - 18 \\ 30 + 7 - 18 & 25 + 49 + 81 \end{bmatrix}$$

$$= \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}$$

Q 3.

$$x = [9, 1, \frac{1}{2}]$$

$$y = \frac{x_1^2}{2} + \log_e x_2 - \frac{x_1}{x_3}$$

$$\frac{dy}{dx} = \left[\frac{dy}{dx_1}, \frac{dy}{dx_2}, \frac{dy}{dx_3} \right]$$

$$\frac{dy}{dx_1} = x_1 + 0 - 2 = 7$$

$$\frac{dy}{dx_2} = \frac{1}{x_2} = 1$$

$$\frac{dy}{dx_3} = \frac{+x_1}{x_3^2} = 9 \times 4 = 36$$

$$\therefore \frac{dy}{dx} = [7, 1, 36]$$

Q4. $f(w) = \|Xw - y\|_2^2 + \lambda \|w\|_2^2$

~~$\frac{\partial f(w)}{\partial w} = 2[X(Xw - y) + \lambda w]$~~

$$\begin{aligned} \frac{\partial f(w)}{\partial w} &= 2 \frac{\partial \|Xw - y\|_2^2}{\partial w} + 2 \frac{\partial \lambda \|w\|_2^2}{\partial w} \\ &= 2(Xw - y) \frac{\partial (Xw - y)}{\partial w} + 2\lambda w \frac{\partial w}{\partial w} \\ &= 2X(Xw - y) + 2\lambda w \\ &= 2[X(Xw - y) + \lambda w] \end{aligned}$$