Linear Algebra Exercises: Part II

CSCI 567 Machine Learning

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Multiple choice questions may have more than one correct choice.

Q1   Which of the following statements are true? PSD stands for positive semi-definite.

(a) $XX^\top$ is a PSD matrix if and only if X is PSD.

(b) If X and Y are PSD matrices, then so is $\lambda X + \mu Y$ for any $\lambda, \mu \in \mathbb{R}$.

(c) If $X - Y$ and $X + Y$ are PSD matrices, then so are X and Y.

(d) All eigenvalues of a symmetric PSD matrix are non-negative.

Q2   Suppose A and B are two positive definite matrices. Which matrix may NOT be positive definite?

(a) A^{-1}

(b) $A + B$

(c) $AA^\top$

(d) $A - B$

Q3   Consider the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

(a) Compute $x^\top A x$ for $x = (x_1, x_2)^\top$.

(b) Is A positive semidefinite?

(c) Is A positive definite?

(d) Is A invertible?

Q4   Suppose A is a PSD matrix and M is any (not necessarily square) matrix of compatible dimensions. Prove that $M^\top A M$ is PSD.

Q5   Let A be a symmetric matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$.

(a) What are the eigenvalues of $A + \mu I$ for $\mu \in \mathbb{R}$?

(b) For what values of μ is $A + \mu I$ positive semidefinite?