

# Probability Exercises

CSCI 567 Machine Learning

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**MULTIPLE-CHOICE QUESTIONS:** one or more correct choices for each question.

**Q1** A bag contains 2 red balls and 3 blue balls. First, Alice draws a ball from the bag randomly (and removes it from the bag). Then, Bob draws a ball randomly too. 1) What is the probability that Alice gets a red ball and Bob gets a blue ball? 2) What is the probability that Alice gets a blue ball given that Bob gets a blue ball?

- (a)  $\frac{3}{10}$  and  $\frac{1}{2}$
- (b)  $\frac{3}{10}$  and  $\frac{2}{5}$
- (c)  $\frac{6}{25}$  and  $\frac{1}{2}$
- (d)  $\frac{6}{25}$  and  $\frac{2}{5}$

**Q2** For events  $A$ ,  $B$  and  $C$ , which of the following identities are correct?

- (a)  $P(A) - P(A \cap B) = P(A \cup B) - P(B)$
- (b)  $P(A \cup B) \leq P(A) + P(B) - P(A)P(B)$
- (c)  $P(A) = P(A \cap C) + P(A \cap \bar{C})$ , where  $\bar{C}$  denotes the complement of event  $C$ .
- (d)  $P(A) = P(A|C) + P(A|\bar{C})$ , where  $\bar{C}$  denotes the complement of event  $C$ .

**Q3** For events  $A$ ,  $B$ ,  $C$  and  $Z_1, \dots, Z_T$ , which of the following identities are correct?

- (a)  $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- (b)  $\frac{P(A|B,C)}{P(A|C)} = \frac{P(B|A,C)}{P(B|C)}$
- (c)  $P(\bigcap_{t=1}^T Z_t) = \prod_{t=1}^T P(Z_t)$
- (d)  $P(\bigcap_{t=1}^T Z_t) = \prod_{t=1}^T P(Z_t|Z_1, \dots, Z_{t-1})$

**Q4** Which of the following statements on the density function of a Gaussian distribution are true?

- (a) The density for a one-dimensional Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$  is  $f(x) \propto \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right)$ .
- (b) The density for a one-dimensional Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$  is  $f(x) \propto \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ .
- (c) The density for a  $d$ -dimensional Gaussian distribution with mean  $\mu \in \mathbb{R}^d$  and covariance matrix  $\Sigma \in \mathbb{R}^{d \times d}$  is  $f(\mathbf{x}) \propto \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^\top \Sigma (\mathbf{x} - \mu)\right)$ .
- (d) The density for a  $d$ -dimensional Gaussian distribution with mean  $\mu \in \mathbb{R}^d$  and covariance matrix  $\Sigma \in \mathbb{R}^{d \times d}$  is  $f(\mathbf{x}) \propto \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^\top \Sigma^{-1} (\mathbf{x} - \mu)\right)$ .

**Q5** Which of the following statements are true?

- (a) Suppose  $X$  and  $Y$  are two jointly Gaussian random variables. Then  $Z = X - 2Y$  is also Gaussian.
- (b) Suppose  $X$  and  $Y$  are two jointly Gaussian random variables. Then the marginal distribution of  $X$  is also Gaussian.
- (c) Suppose  $X$  and  $Y$  are two jointly Gaussian random variables. Then the conditional distribution of  $X$  given  $Y$  is also Gaussian.
- (d) For a random vector  $X \in \mathbb{R}^n$ , its covariance matrix is  $\mathbb{E}[XX^\top] - \mathbb{E}[X]\mathbb{E}[X]^\top$ .

**SHORT-ANSWER QUESTION.**

**Q6** Suppose your spam classification software gives the guarantee that (1) if an email is spam, it will mark it as spam with probability 90%, (1) if an email is not spam, it will only mark it as spam with probability 10%. Suppose you know that 1% of all your emails are spam. If your spam classification software marks a certain email as spam, what is the probability that it is actually spam?