FIT 2004 Assignment 2

1) Required To prove:
The number of leaves (e) in a proper binary tree equals number of internal nodes (i) + 1
$e = i + 1$

Proof:

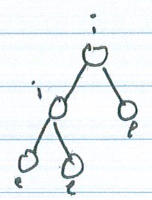
Base case: $i = 0$, only 1 leaf node (root) $\cdot e = 1$
$\therefore \quad e = i + 1$

induction step: assume $e = i + 1$ for $i <= n$

given a Tree T with n internal nodes, we remove 2 sibling leaves, thus new tree T' has i leaves by the induction hypothesis

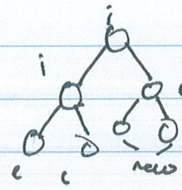
for $i = n$, where n = number of internal nodes, in the given tree, we add two more sibling leaves. it converts the leaf where we ~~added~~ added the sibling leaves to an internal node. thus $e(n+1) = n + 1 + (2 - 1)$ since we are adding 2 leaves and removing $1 = e(n+1) = n + 2$
hence we can say that
$e = i + 1$ for all i in a proper binary tree.

original Tree T



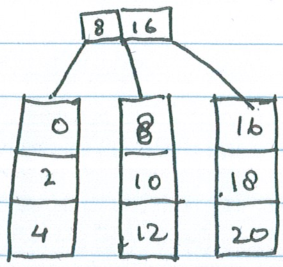
$i = 2$ $e = 3$

add 2 leaves

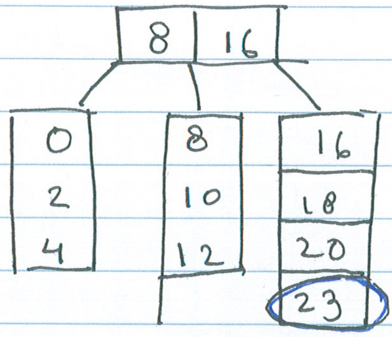


e becomes: $i = 3$
$e = 4$.

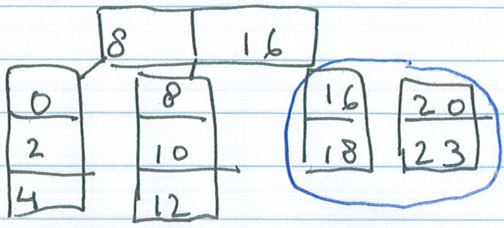
2)



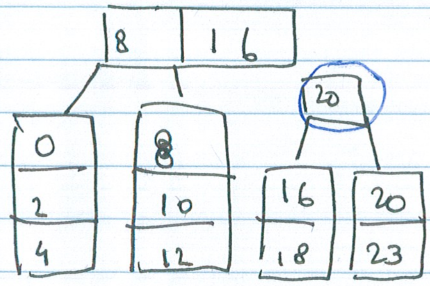
search for 23



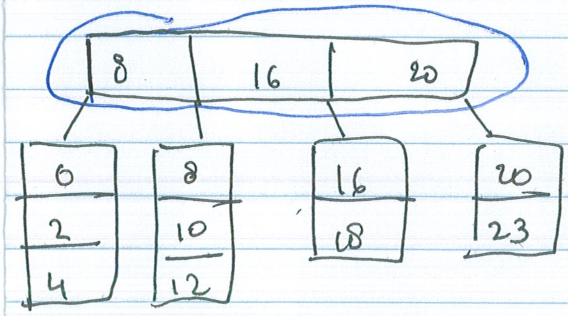
insert 23 B+-tree condition violated.



split leaf and distribute keys



make 20 the parent of new trees



insert 20 into the parent

3) AVL Tree Height

$n(h) = 1 + n(h-1) + n(h-2)$

$n(1) = 1$ and $n(2) = 2$

values : $n(1) = 1$

$n(2) = 2$

$n(3) = 4$

$n(4) = 7$

$n(5) = 12$

Fibonacci Series : $F(n) = F(n-1) + F(n-2)$

$F(0) = 0 \qquad F(1) = 1$

values : $F(1) = 1$

$F(2) = 2$

$F(3) = 3$

$F(4) = 5$

$F(5) = 8$

seeing the relation we can see that $n(h) = \cancel{F(h+2) - 1}\ F(h+1) - 1$

Using induction:

Base case: $n(1) = 1 = 2-1 = F(2) - 1$

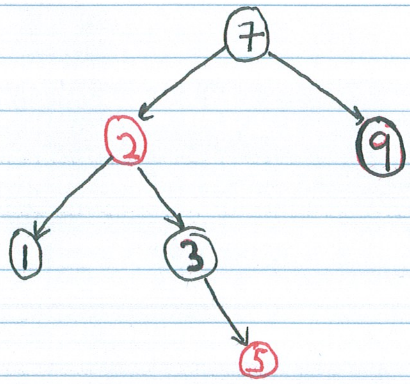
$n(2) = 2 = 3-1 = F(3) - 1$

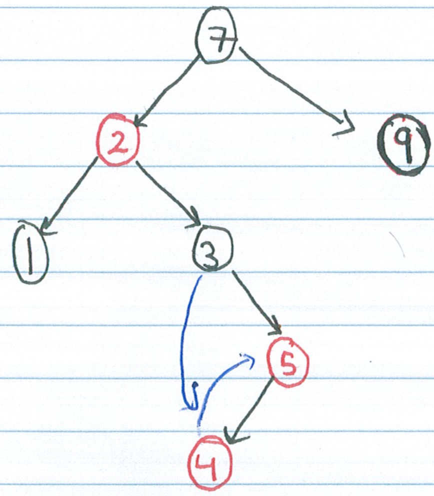
inductive step: $n(h) = F(h+1) - 1$ & $n(h+1) = F(h+2) - 1$

$$n(h+2) = n(h+1) + n(h) + 1$$
$$= F(h+2) - 1 + F(h+1) - 1 + 1$$
$$= F(h+2) + F(h+1) - 1$$
$$= F(h+3) - 1 \qquad \because F(n+2) = F(n+1) + F(n)$$

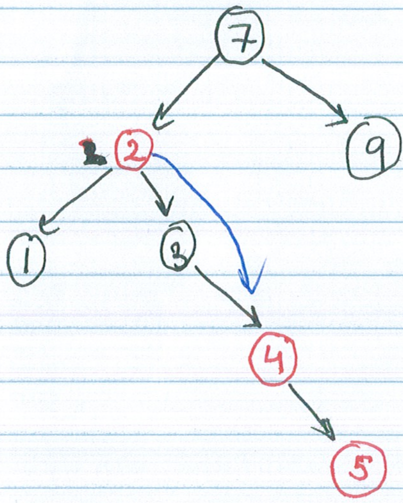
hence we can say that the worst case height of an AVL Tree can be defined by the Fibonacci number.

4) [Red Black Trees]

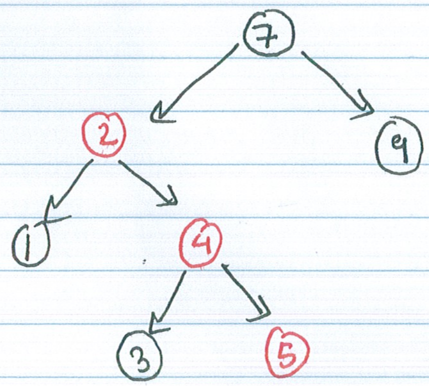




insert 4 in the tree.
node and parent are red node to left of parent, rotate.



node and parent are red, both towards right rotate left



Change colors. make 4 Black. and 3 red.