Problem Set 5

It's not at all important to get it right the first time.
It's vitally important to get it right the last time.
— Andrew Hunt

- This problem set is due **at 9:00am** on **March 6, 2025** .

- This problem set comprises 3 problems.

- Submit early. Do **not** wait till the last moment.

- Each solution should start on a new page.

- We will give full credit **only** for correct solutions that are described clearly and convincingly.

Problem 5-1.   PPT Question [48 points]

In this assignment, you will define an OCaml module `Polynomial` to represent polynomials. The internal representation of the polynomial is up to you. The module must adhere to the following signature:

```
module type Polynomial = sig

  type t

  (* Convert a list of floats to a polynomial *)
    val of_list : float list -> t

  (* Convert polynomials to strings *)
    val to_string : t -> string

  (* Addition of polynomials *)
    val add : t -> t -> t

  (* Multiplication of polynomials *)
    val mult : t -> t -> t

  (* Division that returns quotient and remainder *)
    val div : t -> t -> t * t

  (* Evaluation of a polynomial *)
    val eval : t -> float -> float

  (* Derivative of a polynomial *)
    val derivative : t -> t

  (* Degree of a polynomial *)
    val degree : t -> int

end
```

Ensure that the following properties are satisfied:

1. For all lists `lst = [c0; c1; c2; ...; ck]`, `of_list lst` returns a representation for the polynomial p such that:

$$p(x) = c_0 + c_1 \cdot x + c_2 \cdot x^2 + \ldots + c_k \cdot x^k = \sum_{i=0}^{k} c_i \cdot x^i$$

2.For all representations of the polynomial p such that $p(x) = \sum_{i=0}^{k} c_i \cdot x^i$, the function `to_string` on input `p` returns `"c0 + c1x + c2x^2 + ... + ckx^k"`. For example, for the polynomial representation of $4x^3 + 2x + 3$, the output should return `"3 + 2x + 4x^3"`.

3.For all polynomials p and q, the value of `add p q` is a polynomial r such that $r(x) = p(x) + q(x)$.

4.For all polynomials p and q, the value of `mult p q` is a polynomial r such that $r(x) = p(x) \cdot q(x)$.

5.For all polynomials m and n, the value of `div n m` is a pair of polynomials (q, r) such that $n(x) = m(x) \cdot q(x) + r(x)$ and the degree of r is less than degree of q.

6.For all polynomials p and floats x, `eval p x` returns the value of $p(x)$.

7.For all polynomials p, `derivative p` returns a polynomail q such that

$$q(x) = \frac{d}{dx}p(x)$$

8.For all polynomials p, `degree p` returns the degree of p, that is, it returns the largest integer d such that the coefficient of x^d in p is non-zero. If p is the zero polynomial, then degree p returns -1.

Problem 5-2. Falling Away From Me [26 points]

Define a function `partitioned: int list -> bool` that returns `true` if there is an element n of the list such that:

- for each element m occurring before n in the list, $m \leq n$, and
- for each element m occurring after n in the list, $m > n$.

```
partitioned [] = false
partitioned [22] = true
partitioned [19;17;18;7] = false
partitioned [7;18;17;19] = true
partitioned [19;13;16;15;19;25;22] = true
partitioned [19;13;16;15;25;19;22] = false
```

Problem 5-3. Ship of Theseus [26 points]

Define a function `connected: string list -> bool` that checks whether the input list of strings is connected. A list of strings is connected iff each string in the list (other than the first) is obtained from the previous one by changing the character in exactly one position, and no string occurs twice in the list. E.g.,

```
connected [] = true
connected ["aa"; "ab";"ba"] = false
connected ["aa";"ab";"bb";"ba"] = true
connected ["aa";"ab";"bb";"ba";"aa"] = false
```