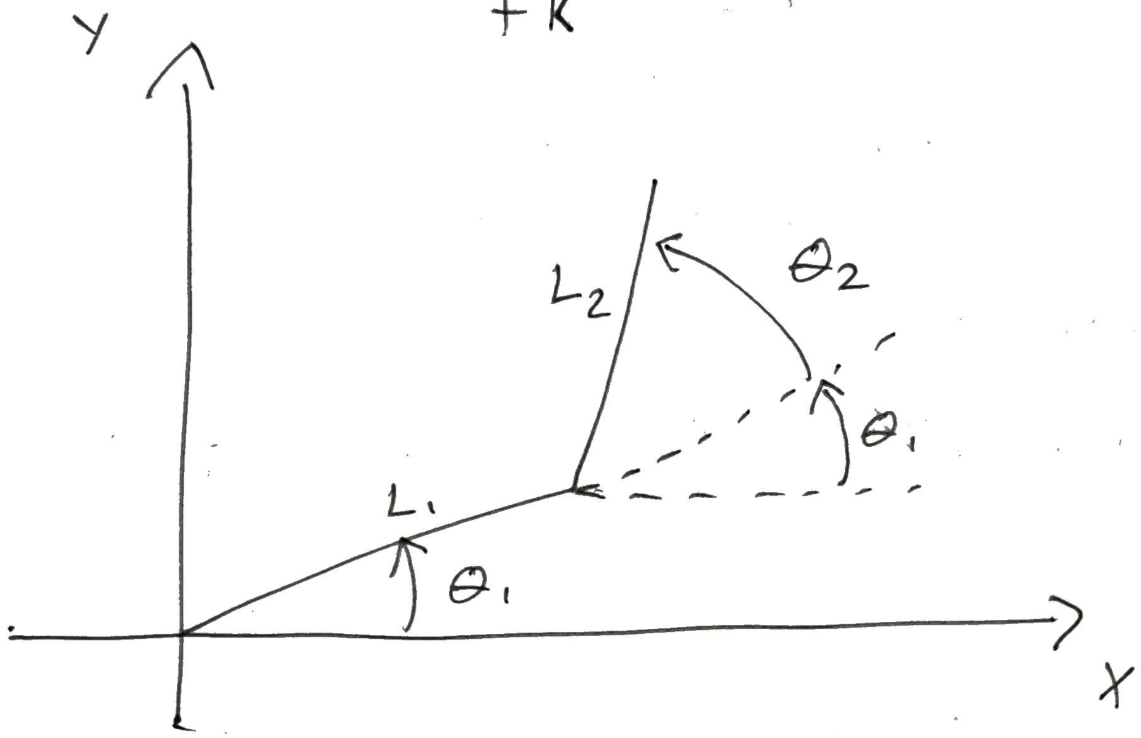
FK

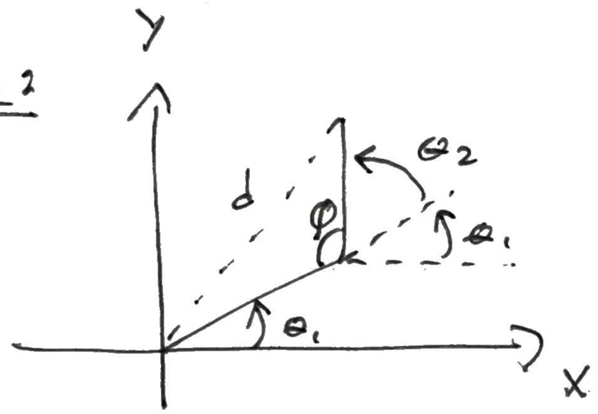


$$X = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$

$$Y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

IK:

θ_2:



cos satsen:

$$d^2 = L_1^2 + L_2^2 - 2L_1L_2\cos(\varphi) \quad (1)$$

$$d = \sqrt{x^2+y^2} \quad , \quad \varphi = 180 - \theta_2$$

$$(1) \quad \cos(\varphi) = \frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2}$$

$$\varphi = \arccos\left(\frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2}\right)$$

$$180^\circ - \theta_2 = \arccos\left(\frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2}\right)$$

$$\theta_2 = 180^\circ - \arccos\left(\frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2}\right)$$

Actually $\pm$, there are 2 solutions

IK:

θ_1: from FK: $X = L_1 c_1 + L_2 c_{1+2}$
$Y = L_1 s_1 + L_2 s_{1+2}$

Trig identities:

$$X = L_1 c_1 + L_2 c_1 c_2 - L_2 s_1 s_2 = c_1 (L_1 + L_2 c_2) - L_2 s_1 s_2$$

known from before

$$Y = L_1 s_1 + L_2 s_1 c_2 + L_2 s_2 c_1 = s_1 (L_1 + L_2 c_2) + L_2 s_2 c_1$$

with $a = L_1 + L_2 c_2$, $b = L_2 s_2$:

$X = c_1 a - b s_1$ (1)
$Y = c_1 b + a s_1$ (2)

(1) $c_1 = \dfrac{X + b s_1}{a}$ (3)

(2) & (3) $b\left(\dfrac{X + b s_1}{a}\right) + a s_1 = \dfrac{bX + b^2 s_1}{a} + a s_1 = Y$

$\Rightarrow s_1 = \dfrac{Ya - bX}{b^2 + a^2}$ (4)

(3) & (4) $c_1 = \dfrac{X + b\left(\dfrac{Ya - bX}{b^2 + a^2}\right)}{a} = \cdots = \dfrac{Xa + bY}{b^2 + a^2}$

$\tan(\theta_1) = \dfrac{s_1}{c_1} = \dfrac{Ya - bX}{Xa + bY} \Rightarrow \theta_1 = \operatorname{atan2}\left(\dfrac{Ya - bX}{Xa + bY}\right)$