

# Course Project: Observing the Effects of Special Perturbations on Orbit Propagation

Madison Vaughan

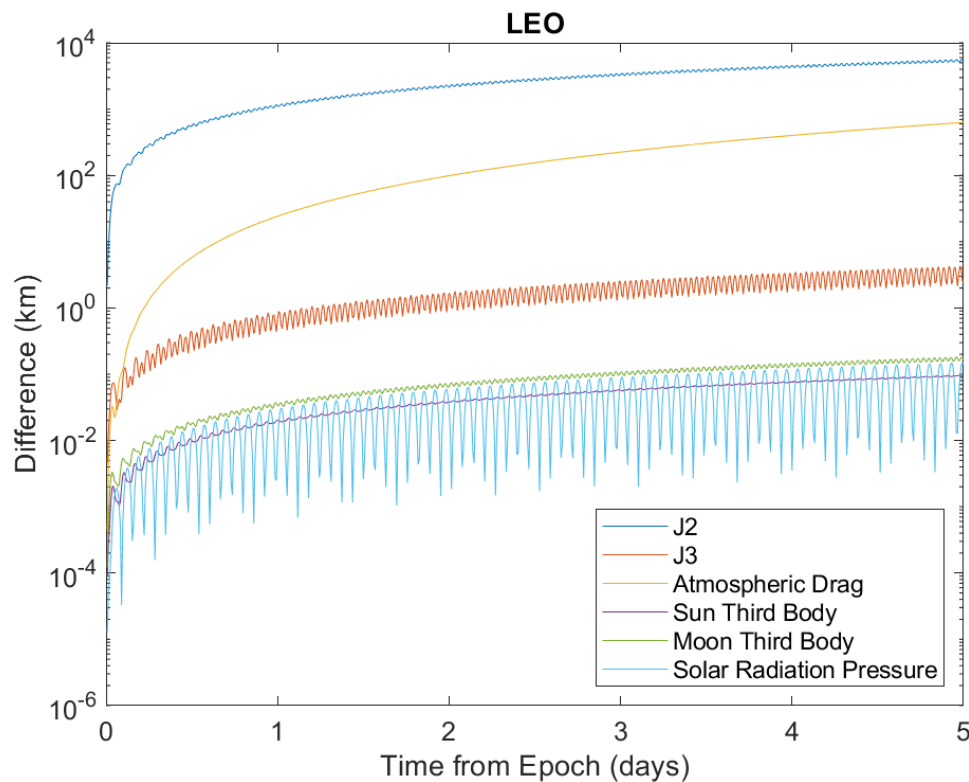
EID: mgv562

Applied Orbital Mechanics

ASE366L

## Task 1

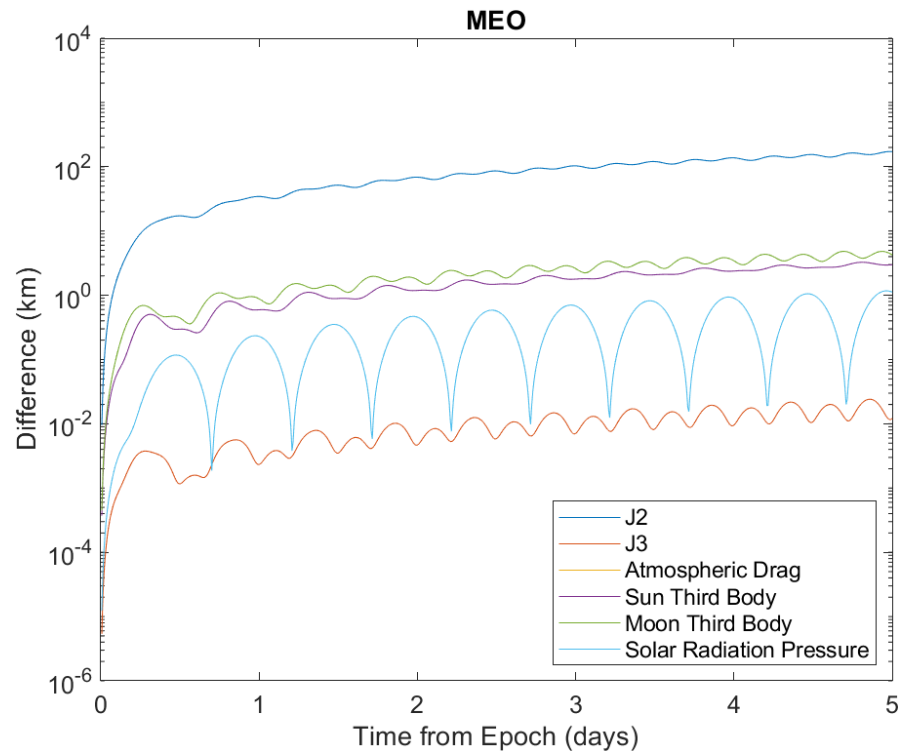
This portion of the project observes how different perturbations will affect the propagation of an orbit. For the orbit types, Low Earth Orbit (LEO), Medium Earth Orbit (MEO), Geostationary Orbit (GEO), and the Molniya spacecraft orbit, a two-body gravitational model orbit will be propagated. Seven more two-body orbits will be propagated including perturbations each from J2, J3, atmospheric drag, the Sun third-body model, the Moon third-body model, and solar radiation pressure (SRP). The difference between the regular two-body and the two-body (including perturbations) position vectors will be taken and the magnitudes will be plotted logarithmically across a timespan of five days. The resulting perturbations will then be ranked according to how much of an effect they have on the spacecraft. The results have been displayed below.



**Figure 1:** Difference between perturbed and two-body LEO trajectories

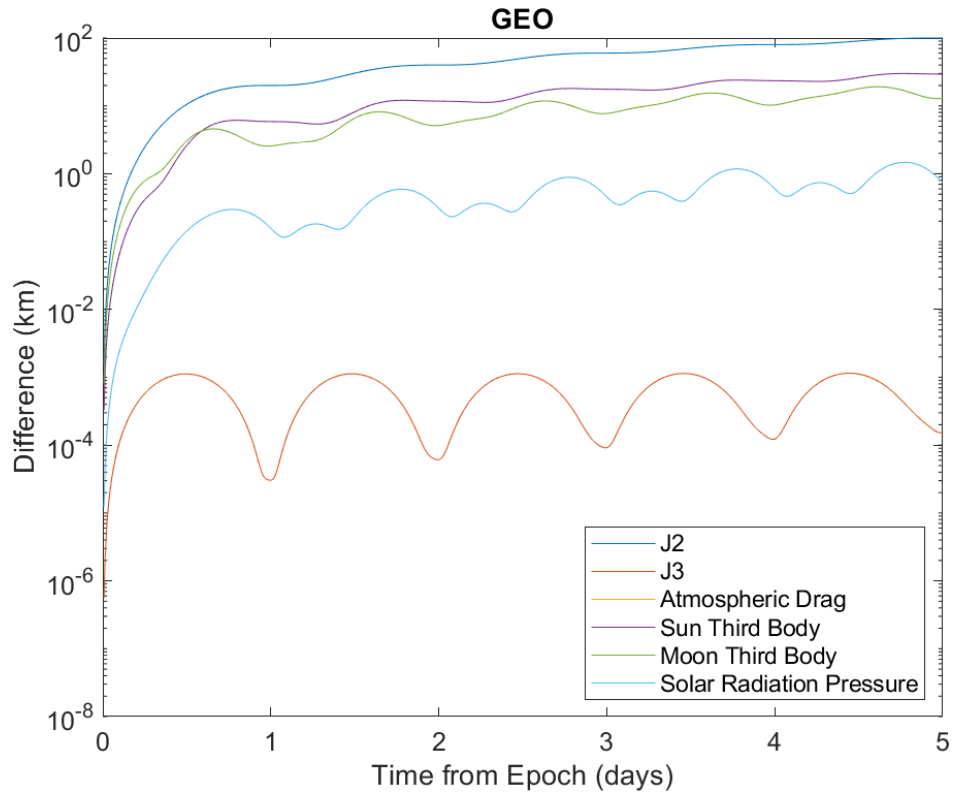
In Figure 1, the six different perturbation types and their effects on the LEO trajectory can be seen. The J2 perturbation seems to have the largest effect on the trajectory of the spacecraft, as the orbital radius becomes shifted from a range of 100 km to nearly 10,000 km. Atmospheric drag has the second largest effect, which is to be assumed due to the nature of a Low Earth Orbit having a lower altitude. The J3 perturbation has the third largest effect, which seems to also be oscillating with a relatively high frequency. The Moon as a third body has smaller effect on the spacecraft, still slightly higher than that of the Sun. The effect of SRP seems to oscillate around the values of

the Moon and Sun third body effects, so it is hard to say which is definitively greater or lesser. It should be noted that the last three perturbations mentioned have a very small magnitude of an effect on the spacecraft when compared to the first three.



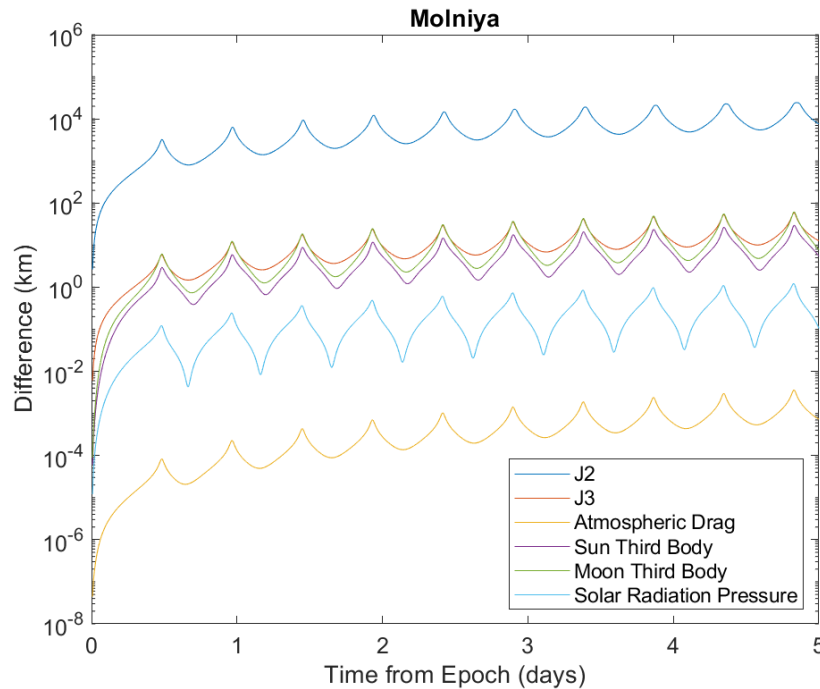
**Figure 2:** Difference between perturbed and two-body MEO trajectories

In Figure 2, J2 still provides the dominant effect on the spacecraft in MEO, but to a lower degree of around 100 km when compared to the LEO J2 effect. Since MEO is at a higher altitude above Earth, one would naturally expect the effect of J2 to decrease. The Moon has the second greatest effect on the spacecraft, which steadily oscillates at a low frequency around 10 km. This is consistent with the understanding of third-body dynamics, where the gravity of a body has a larger effect as an object becomes close and closer to it. Following closely to the Moon, the Sun provides a slightly lower magnitude difference, but still noticeable. SRP provides an interesting effect, jumping up to a higher magnitude before lowering every half-day. The smallest effect seems to be J3, which oscillates with a similar frequency to the Moon perturbation, but at a very small degree compared to the others. Interestingly, there is no atmospheric drag to be seen in the plot. It could be assumed that due to the very high altitude of MEO, atmospheric drag becomes negligible to the spacecraft.



**Figure 3:** Difference between perturbed and two-body GEO trajectories

In Figure 3, J2 still remains the dominant perturbation for GEO, and like in MEO, the magnitude is only up to 10 km. However, the Sun perturbation has become the second largest perturbation on the spacecraft, with the Moon perturbation following closely behind. SRP remains the fourth largest perturbation at a slightly larger magnitude than that in MEO. J3 now has the least effect on the spacecraft, oscillating slowly once each day. The higher peaks of the J3 perturbation could likely be due to a more southern position of the spacecraft, feeling a greater effect due to the North-South asymmetry of the Earth's gravitation. Similar to MEO, the atmospheric drag has become negligible at GEO, which in fact has the highest altitude of all cases.



**Figure 4:** Difference between perturbed and two-body Molniya trajectories

In Figure 4, the Molniya trajectory introduces some interesting shape to the perturbation effects. Still reigning as the dominant perturbation, J2 creates a jagged-sort of oscillation for the spacecraft at a very high magnitude. It is known that the Molniya orbit is highly eccentric, which could explain the shape of most of these perturbations. J3 becomes the second largest perturbation, being closely followed by the Moon and subsequently, the Sun. These three perturbation differences oscillate around a magnitude of 10km, which is still slightly considerable on the spacecraft. The effect of SRP is lower than the aforementioned perturbations, and atmospheric drag reappears as the perturbation which has the smallest effect on the Molniya orbit.

## **Task 2**

2.1.) Using an iterative solver derived from the theory of T.H. Skopinski and K.G. Johnson's "Determination of Azimuth Angle at Burnout for Placing a Satellite Over a Selected Earth Position", the following orbital elements were determined:

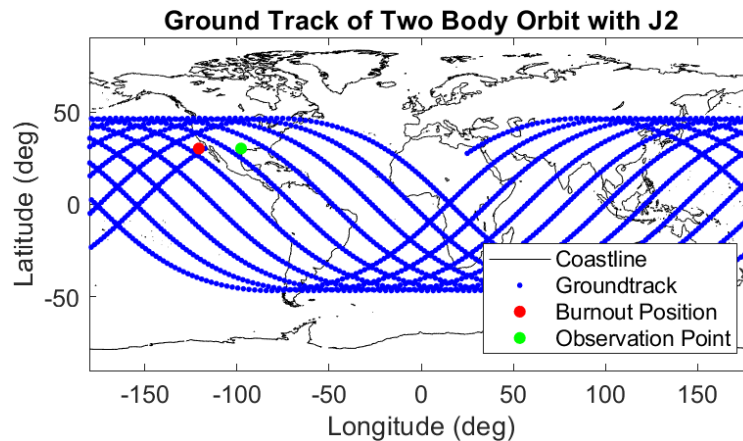
$a = 6,500 \text{ km}$ ,  $i = 133.26 \text{ deg}$ ,  $\Omega = 272.76 \text{ deg}$ ,  $\omega = 23.85 \text{ deg}$ ,  $\theta = 20.00 \text{ deg}$

Using these values, the burnout position and velocity were determined to be:

$$\mathbf{R}_{\text{burnout}} = -2571.21 \mathbf{i} - 4990.80 \mathbf{j} + 3280.67 \mathbf{k} \text{ (km)}$$

$$\mathbf{V}_{\text{burnout}} = -6.6267 \mathbf{i} + 0.7047 \mathbf{j} - 4.1072 \mathbf{k} \text{ (km/s)}$$

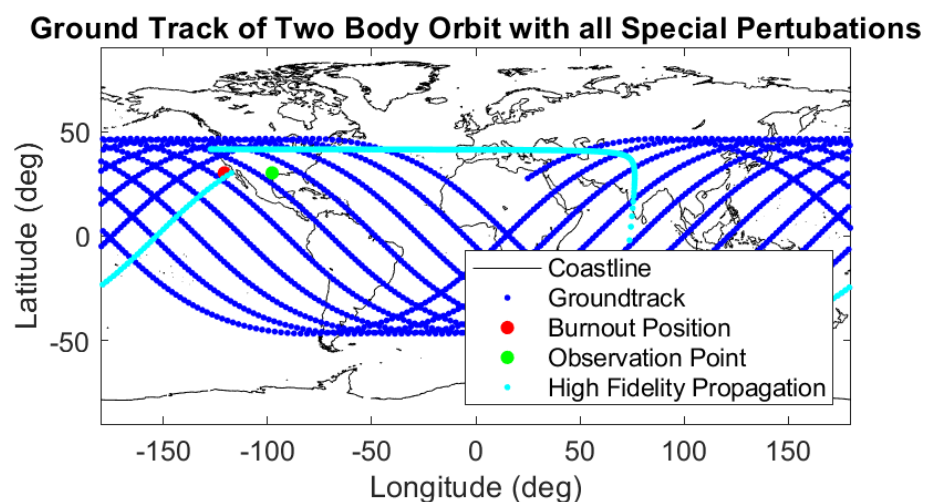
2.2) The above solution was propagated for two-body including J2 dynamics and plotted as a groundtrack.



**Figure 5:** Groundtrack from Propagated Burnout Position and Velocity

From Figure 5, it can be assumed the solver to be very accurate, as the burnout position dot begins at the proper latitude and longitude and moves to the west as the orbit is propagated. I believe my implementation and propagation to not be accurate however, because my groundtrack does not end where it should at the observation point.

2.3) From Figure 6, ignoring the strange error from -100 to 75 deg longitude, it seems like the high fidelity propagator works similarly to the previous propagation, only changing the amount of orbits the spacecraft completes. This lets us assume that the spacecraft crashed at some point within its first orbit, indicating that the calculated burnout velocity was not enough to combat the atmospheric drag and other perturbations that were taken into account with the high fidelity propagator.



**Figure 6:** Groundtrack from High Fidelity Propagated Burnout Position and Velocity