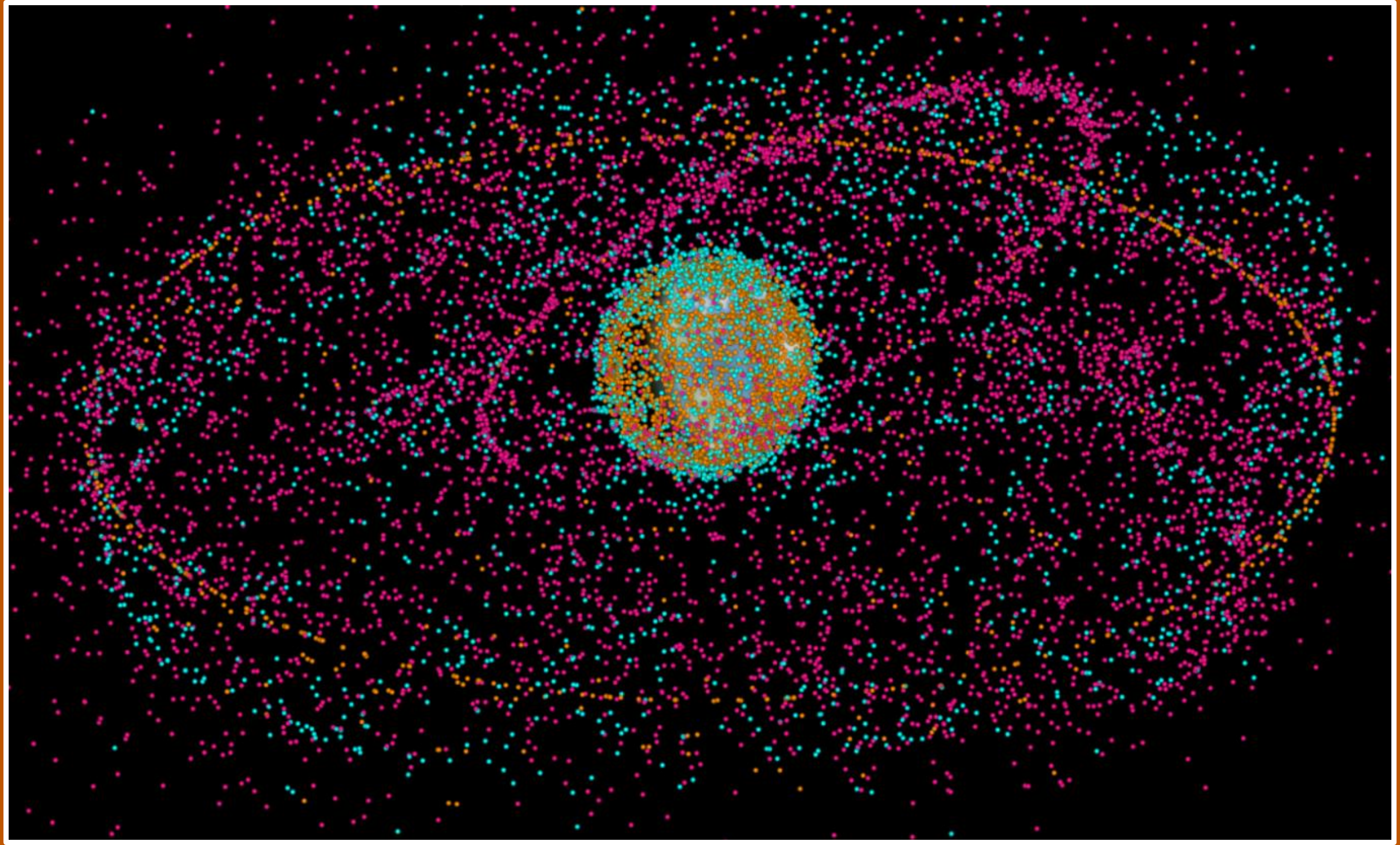
The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

OCTOBER 2022

MONTE CARLO ANALYSIS OF J2 GRAVITY PERTURBATION

ASE374K – Adam Nokes
SmallSat Alliance Orbital Debris Design Competition



https://payloadspace.com/regulating-orbital-debris-part-two/

MADISON VAUGHAN
The University of Texas at Austin

J2 GRAVITY PERTURBATION

J2 is a perturbation derived mathematically from spherical harmonics that relates the oblateness, or "equatorial bulge" of the Earth to a subsequent asymmetrical gravitational field [1]. J2 enacts a small, but noticeable torque on a spacecraft, especially in LEO since gravity (and thus, gravity perturbations) vary with different positional distances. If a spacecraft's orbit does not account for J2, the orbital elements can change over time, which has direct concern for our SmallSat mission that we are designing to operate in LEO for multiple years.

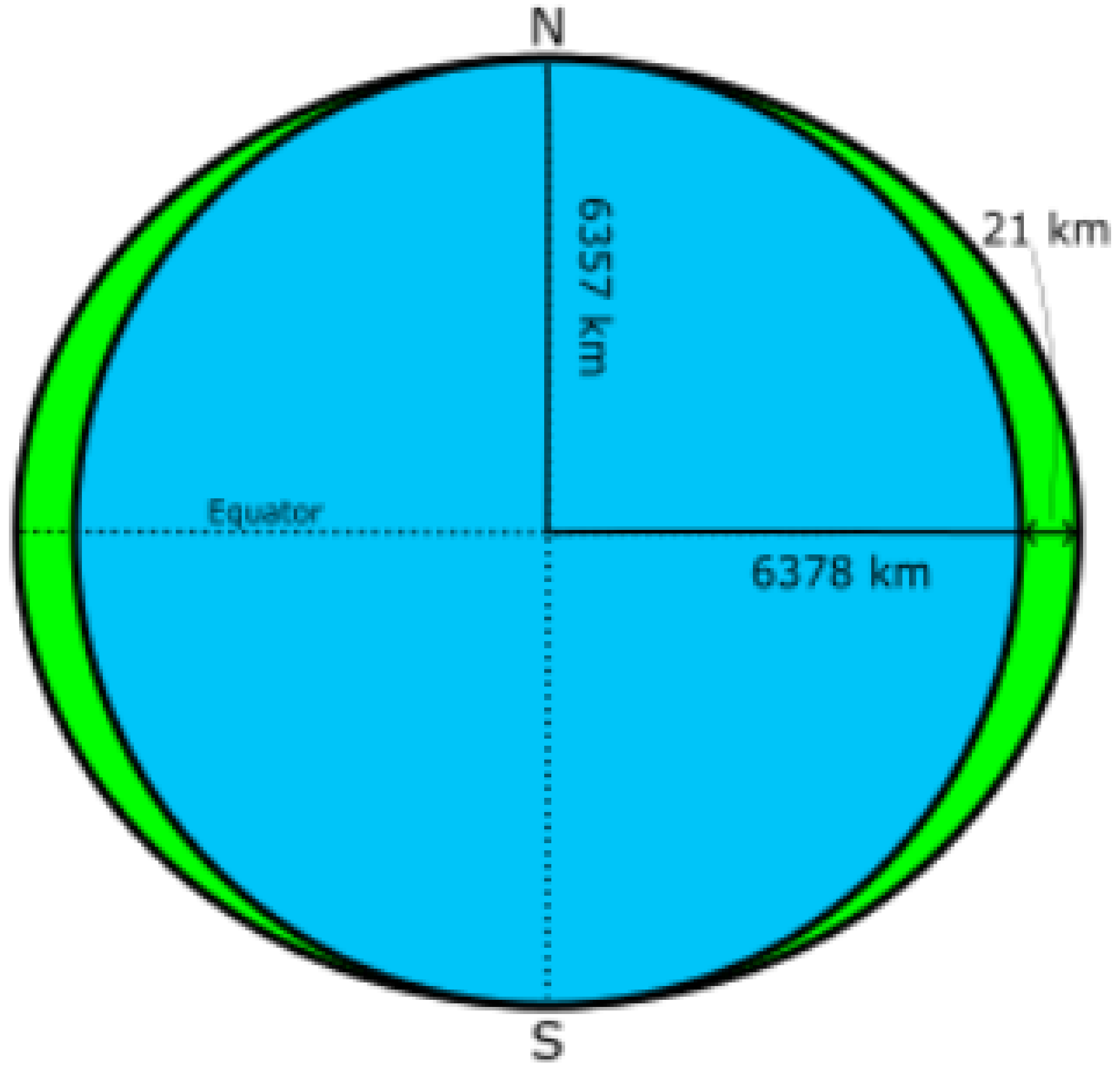


Figure 1: Earth's equatorial bulge [1]

J2 ACCELERATION MODEL FOR ANALYSIS

- The following equation [2] outlines the resulting acceleration perturbation for a spacecraft's motion in all three degrees of freedom (i,j,k)

- There are three varying input parameters
  - Ri, Rj, and Rk are the vector positions of the spacecraft, measured from the center of the Earth
- J2 is the zonal harmonic constant for the Earth
  - J2 = 0.0010826267
- μ is the gravitational parameter for the Earth
  - μ = 398600.4415 km^3/s^2

$$\boldsymbol{a} = -\nabla V = \nabla U = \frac{\partial U}{\partial r_i}\hat{\imath} + \frac{\partial U}{\partial r_j}\hat{\jmath} + \frac{\partial U}{\partial r_k}\hat{k}$$

$$\frac{\partial U}{\partial r_i} = -\frac{\mu r_i}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 1\right]\right)$$

$$\frac{\partial U}{\partial r_j} = -\frac{\mu r_j}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 1\right]\right)$$

$$\frac{\partial U}{\partial r_k} = -\frac{\mu r_k}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 3\right]\right)$$

Figure 2: J2 acceleration perturbation equation [2]

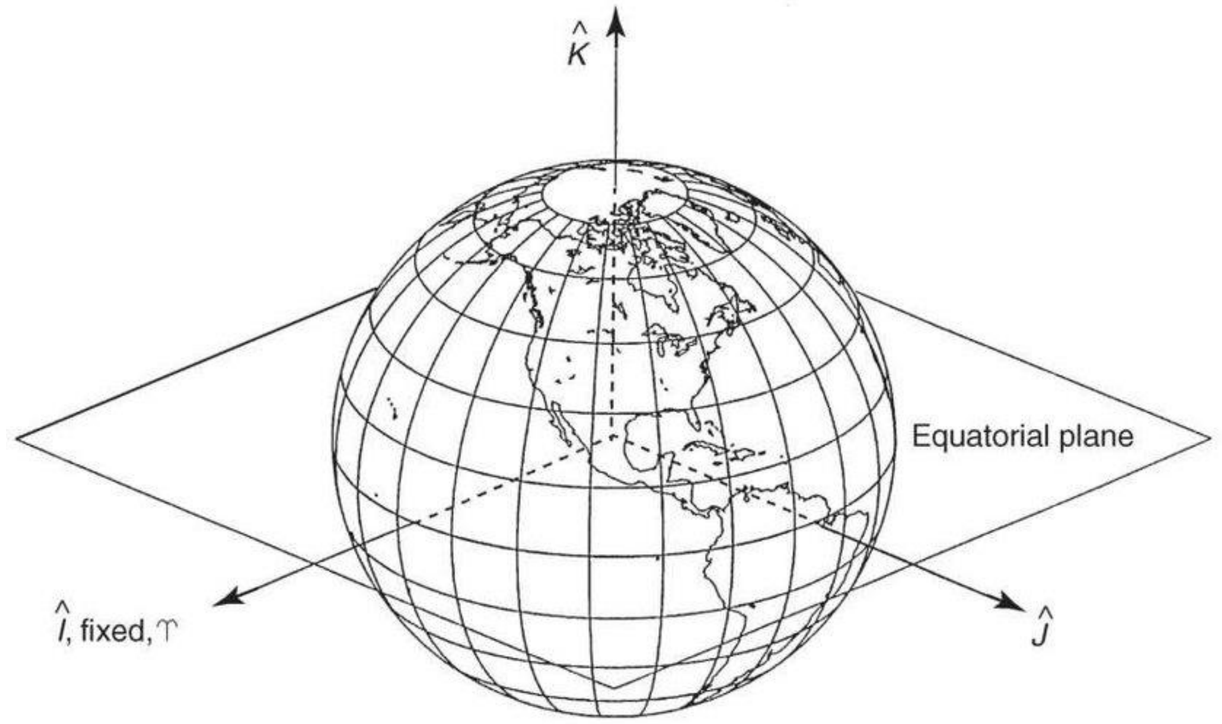
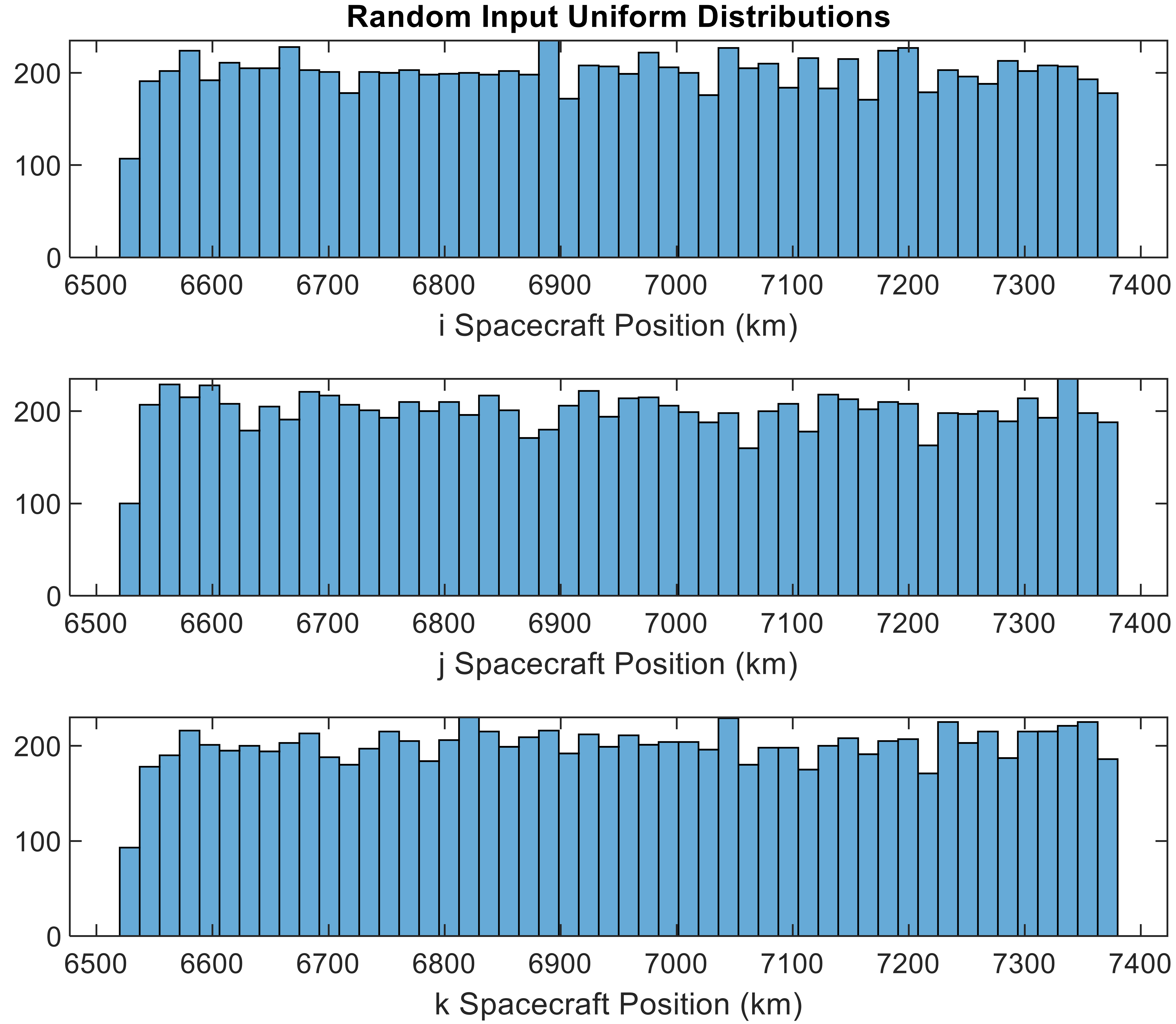


Figure 3: I-J-K Earth Center Fixed Coordinate System [3]

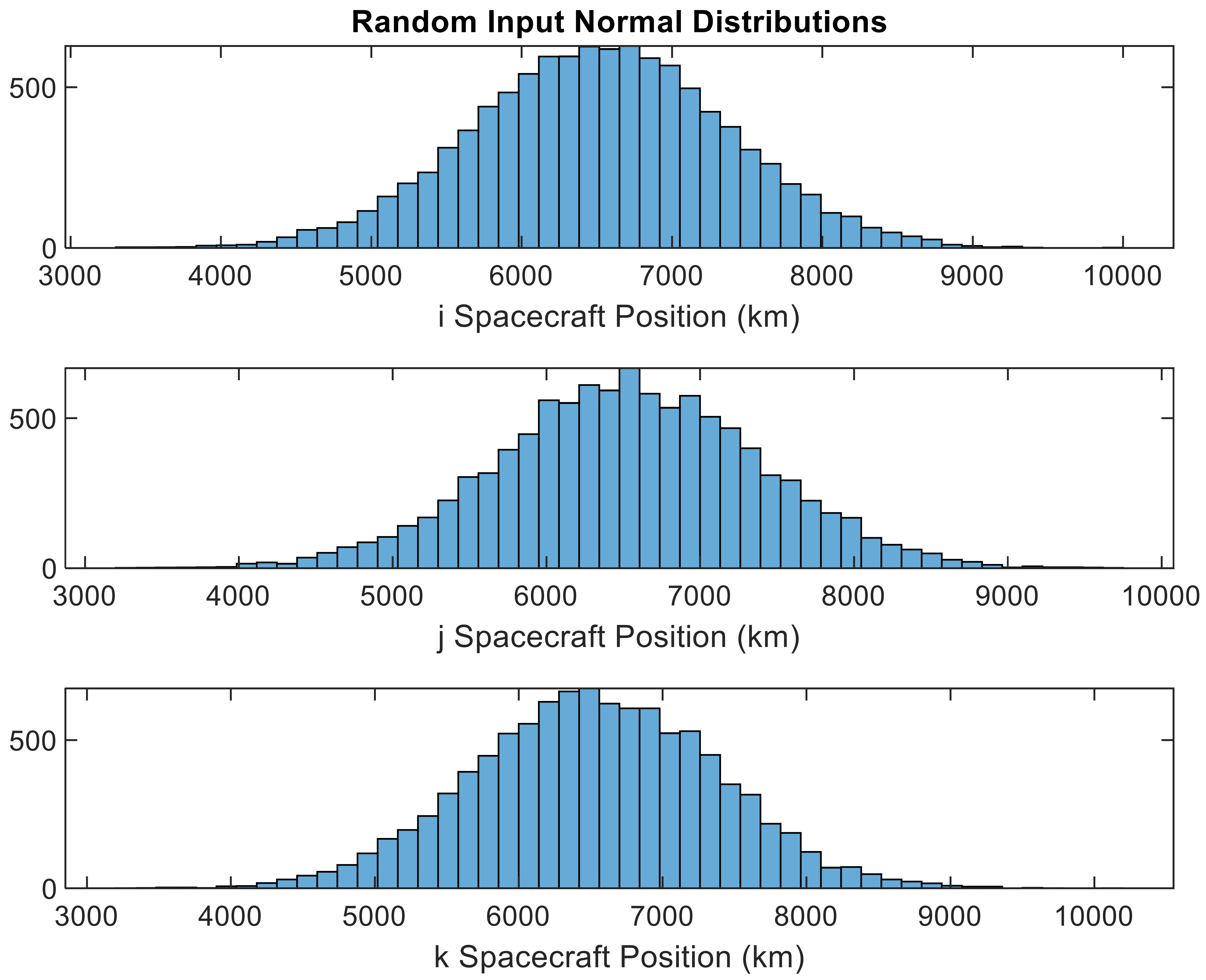
INPUT VARIABLES

- All three input positions were generated uniformly for suitable LEO orbit altitudes (150km to 1000km)
- It is assumed that these positions will be necessary spots for a debris removal satellite to travel to
  - Thus, this also assumes a uniform distribution of the debris field, which is not so accurate, and results should be checked against another distribution



INPUT VARIABLES

- Positions were still generated for suitable LEO orbit altitudes, but now with a normal distribution
  - I expect this to be a more accurate representation of the positions where debris is located because many debris fields are statistically modeled with normal distributions
- I would expect a tighter range of output values in my analysis, but will have to confirm in the results



INPUT VARIABLES

Uniform Distribution

i-Min = 6528.2 km
i-Max = 7378.1 km

j-Min = 6528.3 km
j-Max = 7378.1 km

k-Min = 6582.2 km
k-Max = 7378.1 km

Normal Distribution

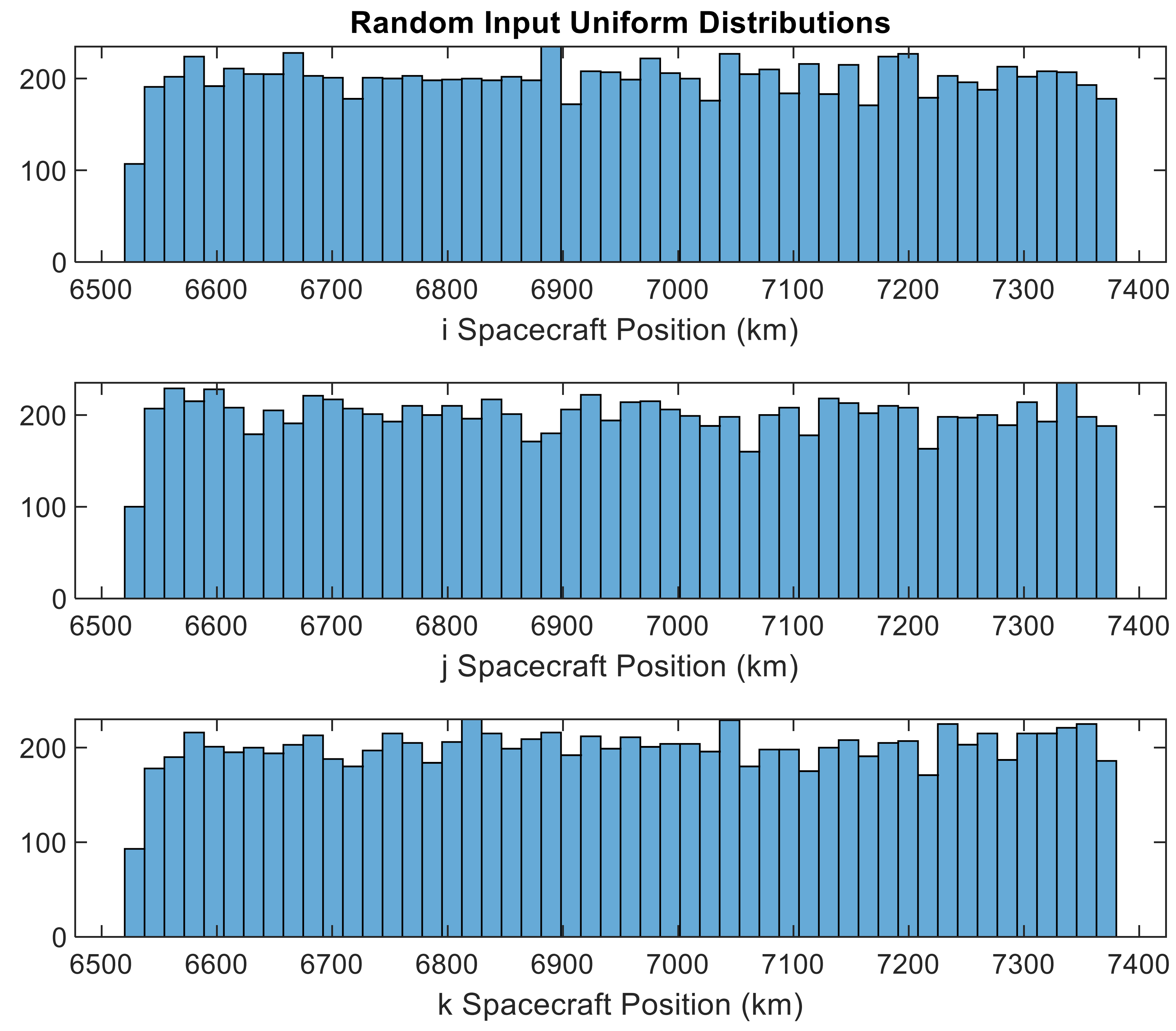
i-Mean = 6521.7 km
i-StDev = 842.7645

j-Mean = 6534.5 km
j-StDev = 852.6896

k-Mean = 6527.1 km
k-StDev = 841.9080

Both distributions were created with 10,000 randomized sample values in order to run the simulations 10,000 times

UNIFORM DISTRIBUTION RESULTS

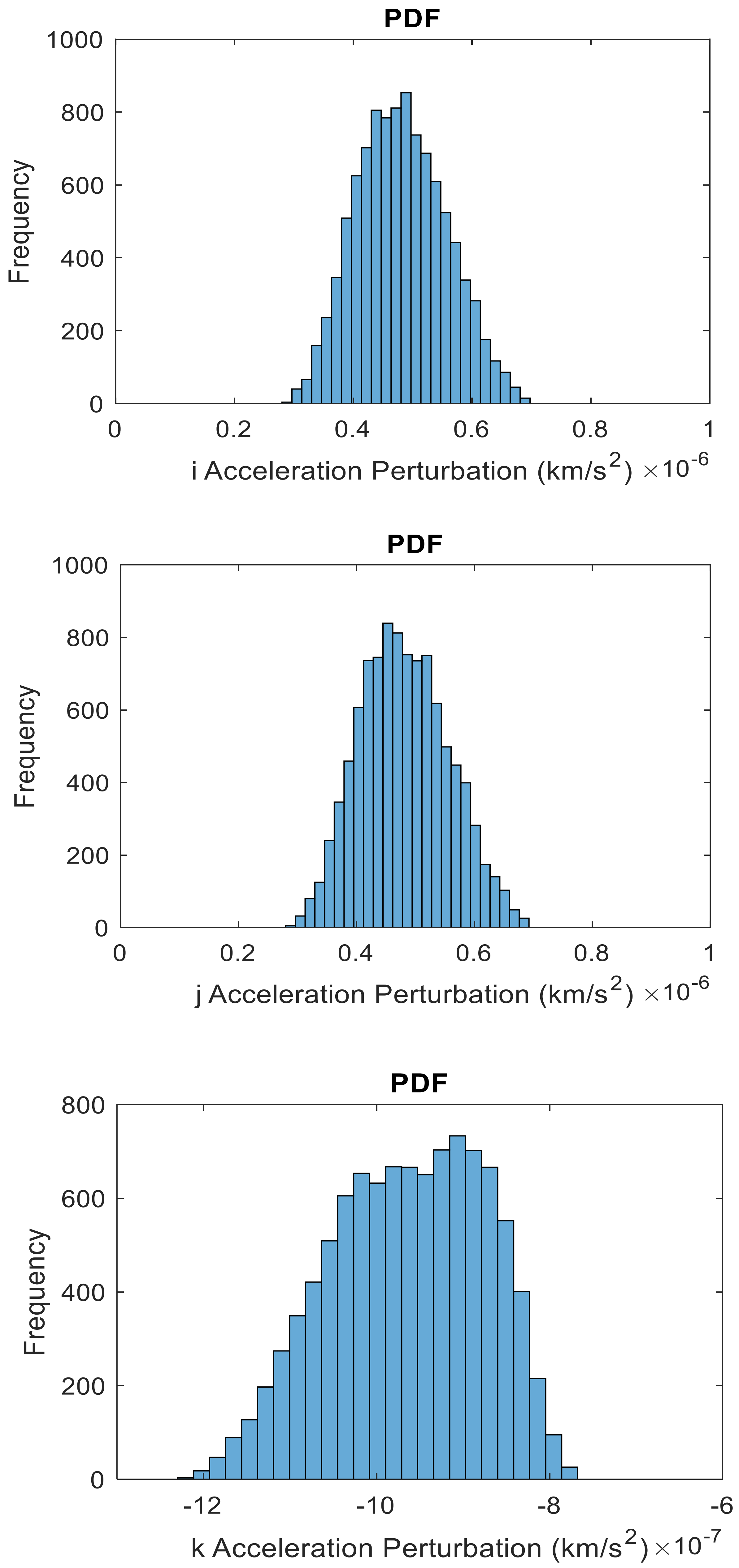


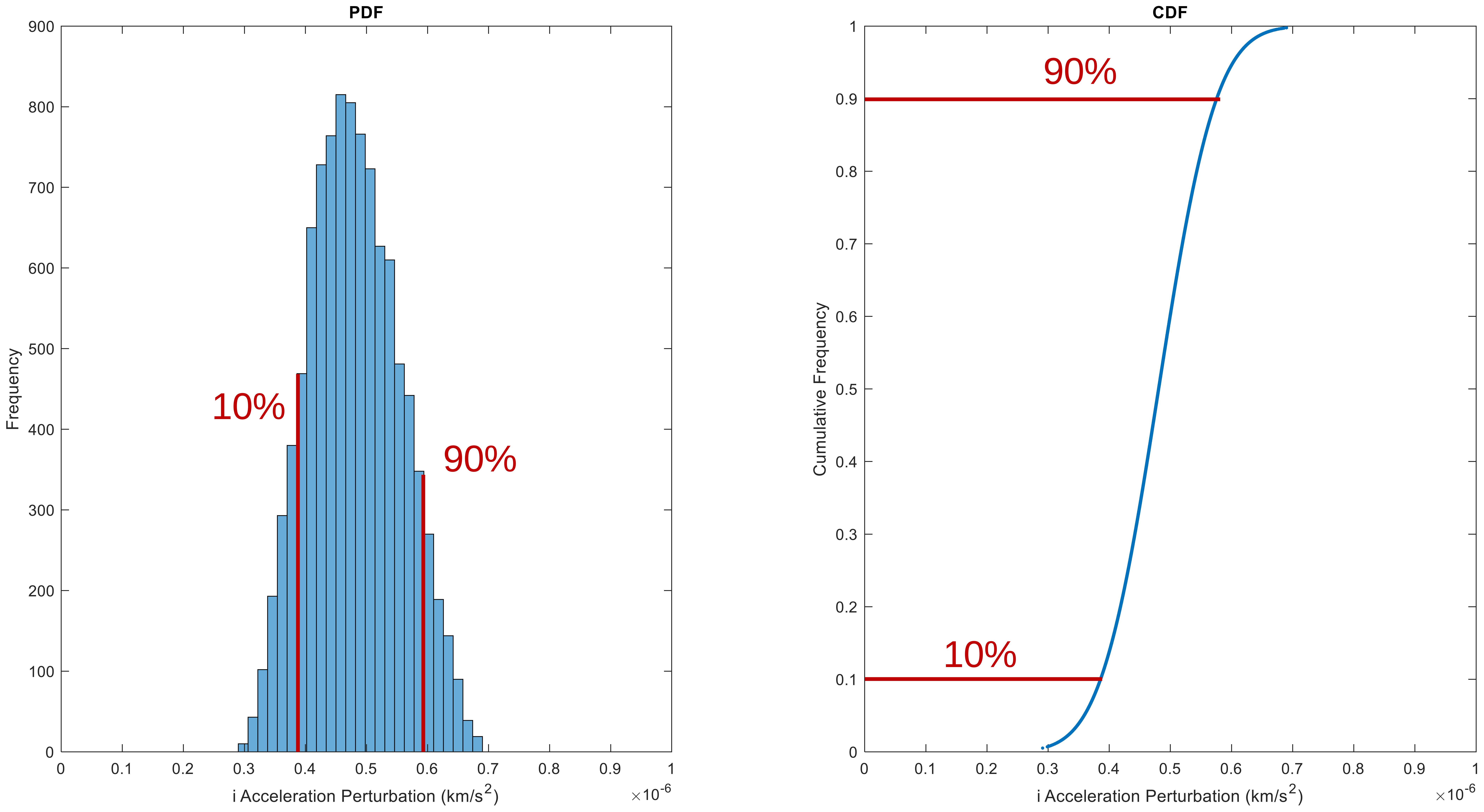
$$\boldsymbol{a} = -\nabla V = \nabla U = \frac{\partial U}{\partial r_i}\hat{i} + \frac{\partial U}{\partial r_j}\hat{j} + \frac{\partial U}{\partial r_k}\hat{k}$$

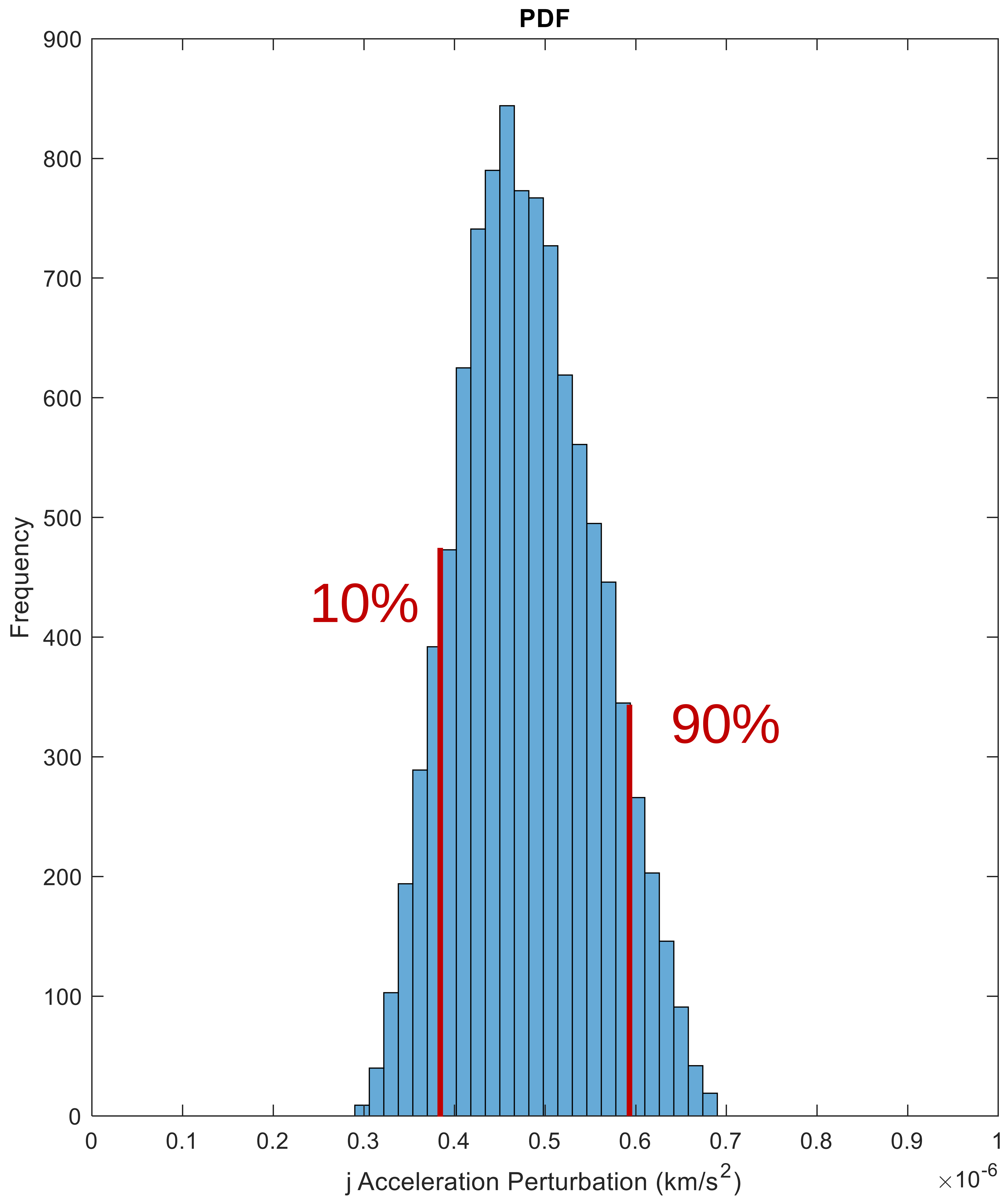
$$\frac{\partial U}{\partial r_i} = -\frac{\mu r_i}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 1\right]\right)$$

$$\frac{\partial U}{\partial r_j} = -\frac{\mu r_j}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 1\right]\right)$$

$$\frac{\partial U}{\partial r_k} = -\frac{\mu r_k}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 3\right]\right)$$




PDF
Frequency
10%
90%
i Acceleration Perturbation (km/s²)
×10⁻⁶
CDF
Cumulative Frequency
90%
10%
i Acceleration Perturbation (km/s²)
×10⁻⁶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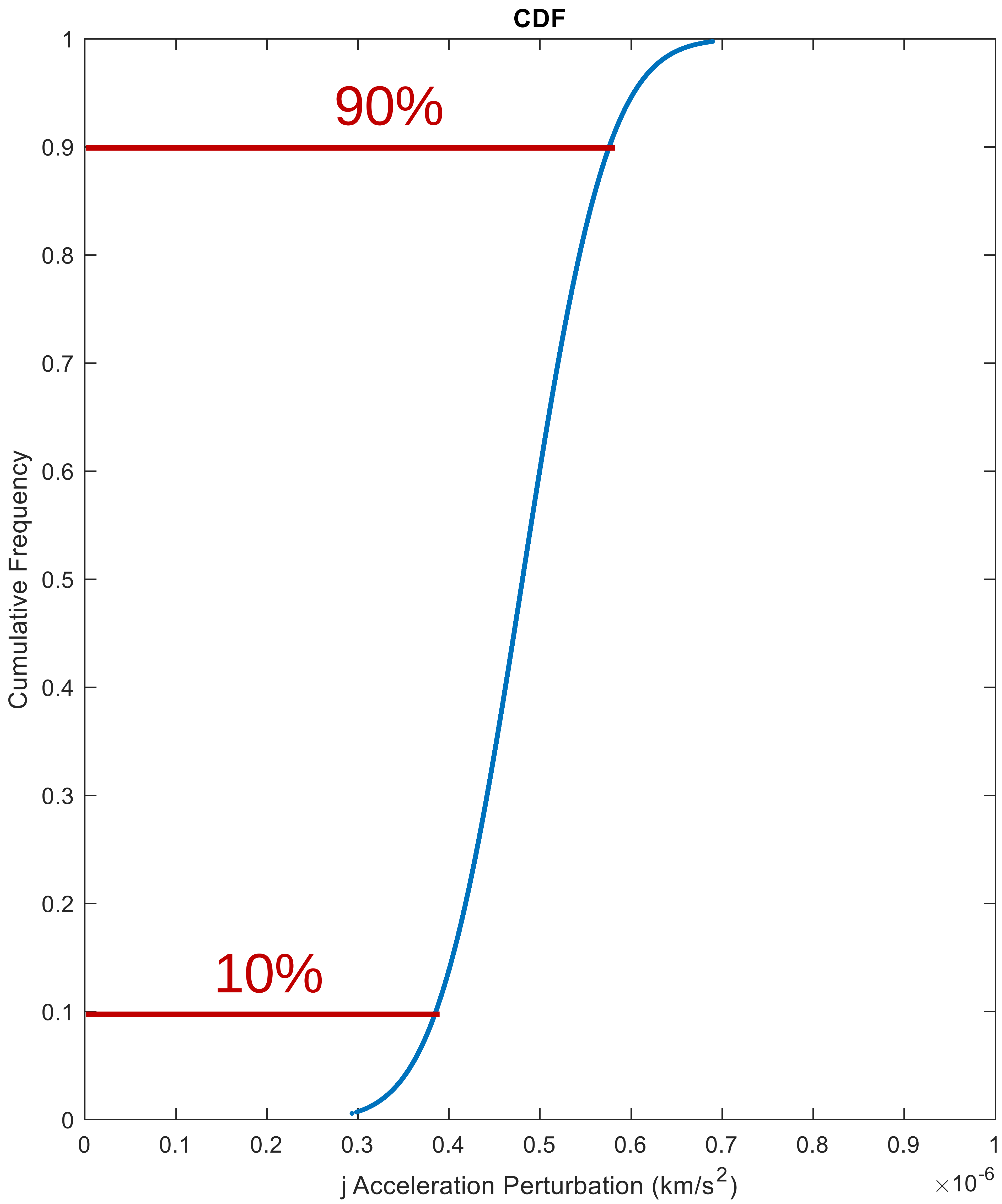
10%
90%
j Acceleration Perturbation (km/s^2)
$\times 10^{-6}$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Cumulative Frequency
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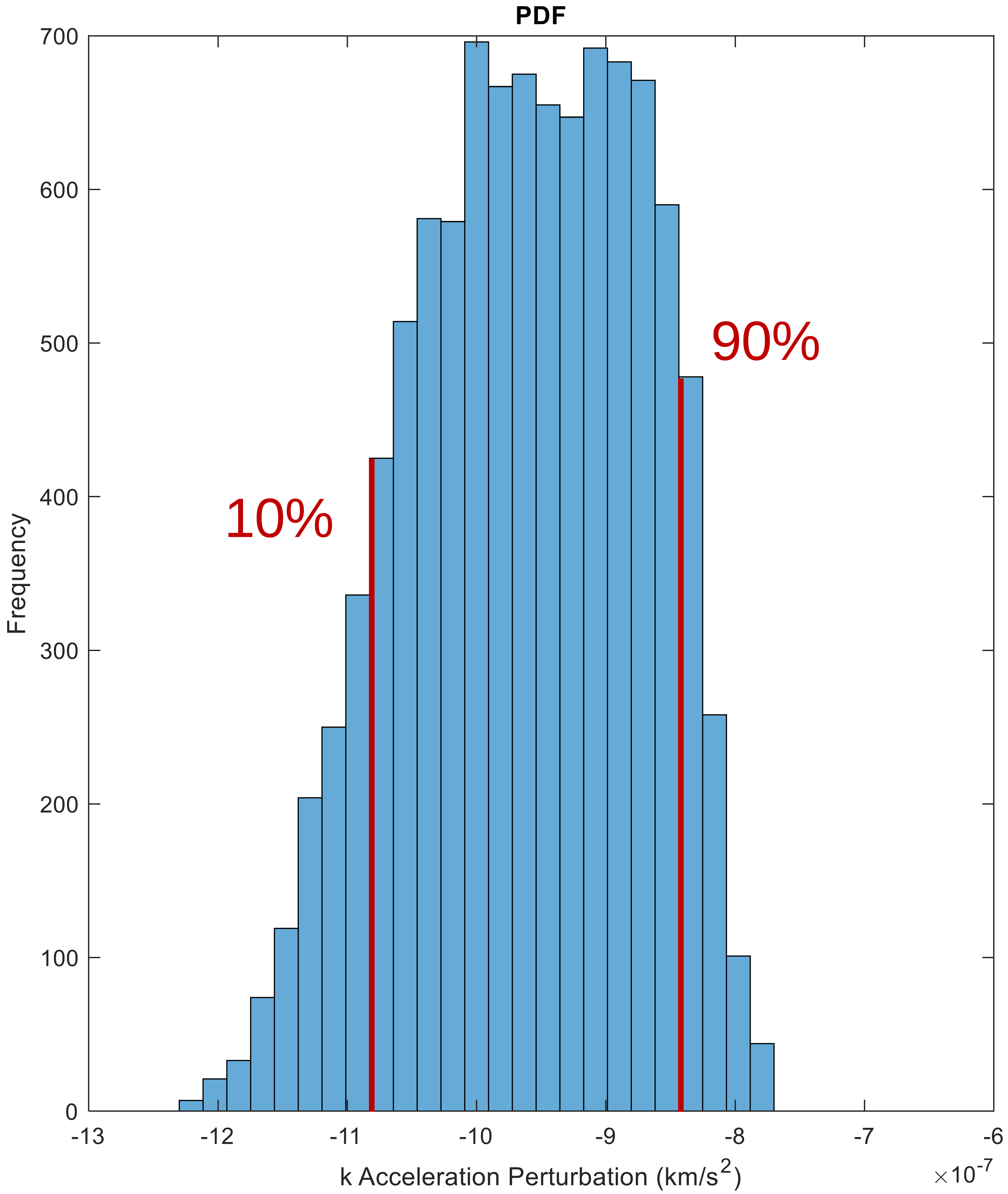
1
90%
10%
j Acceleration Perturbation (km/s^2)
$\times 10^{-6}$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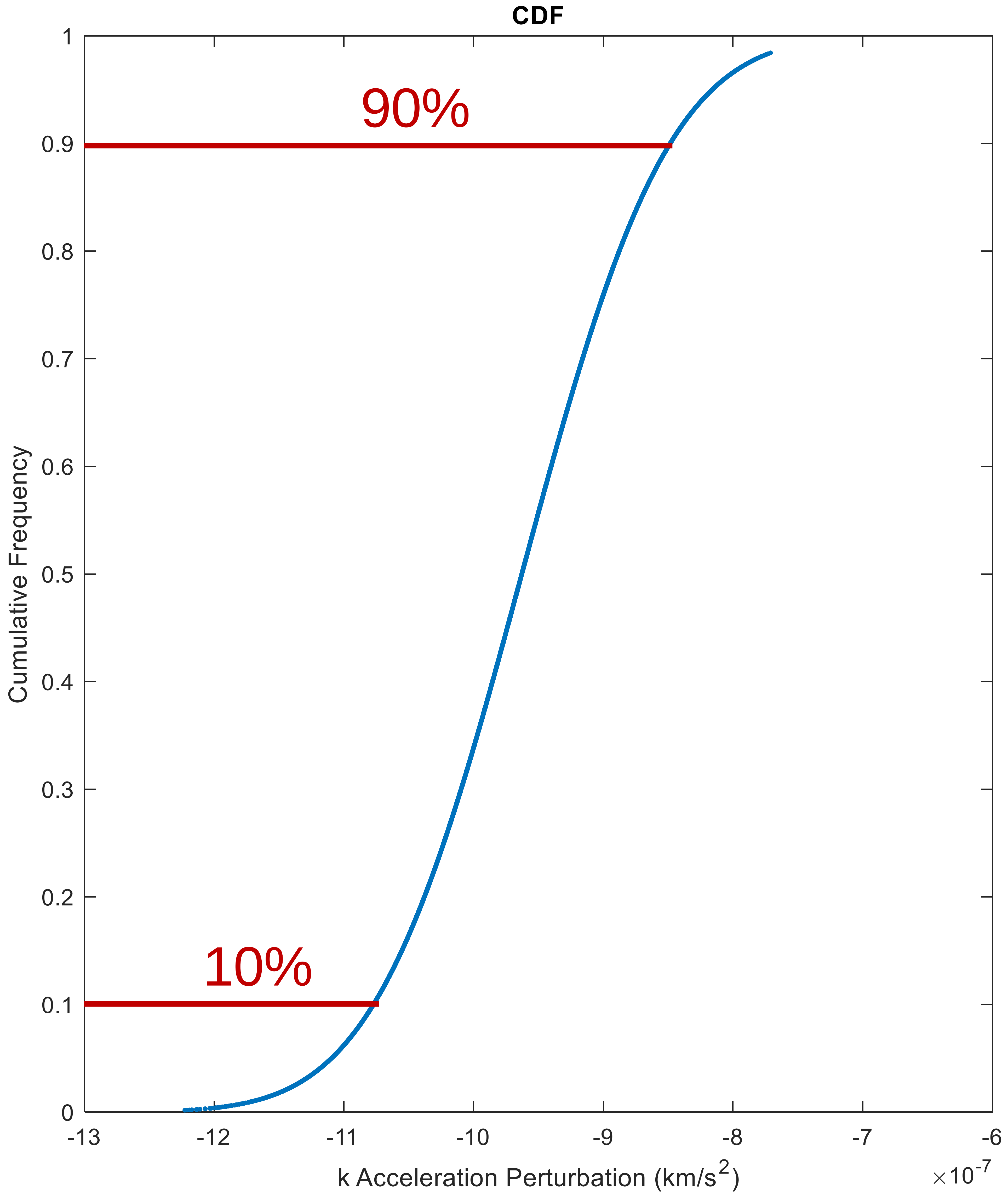
-6
k Acceleration Perturbation (km/s2)
×10-7
10%
90%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Cumulative Frequency
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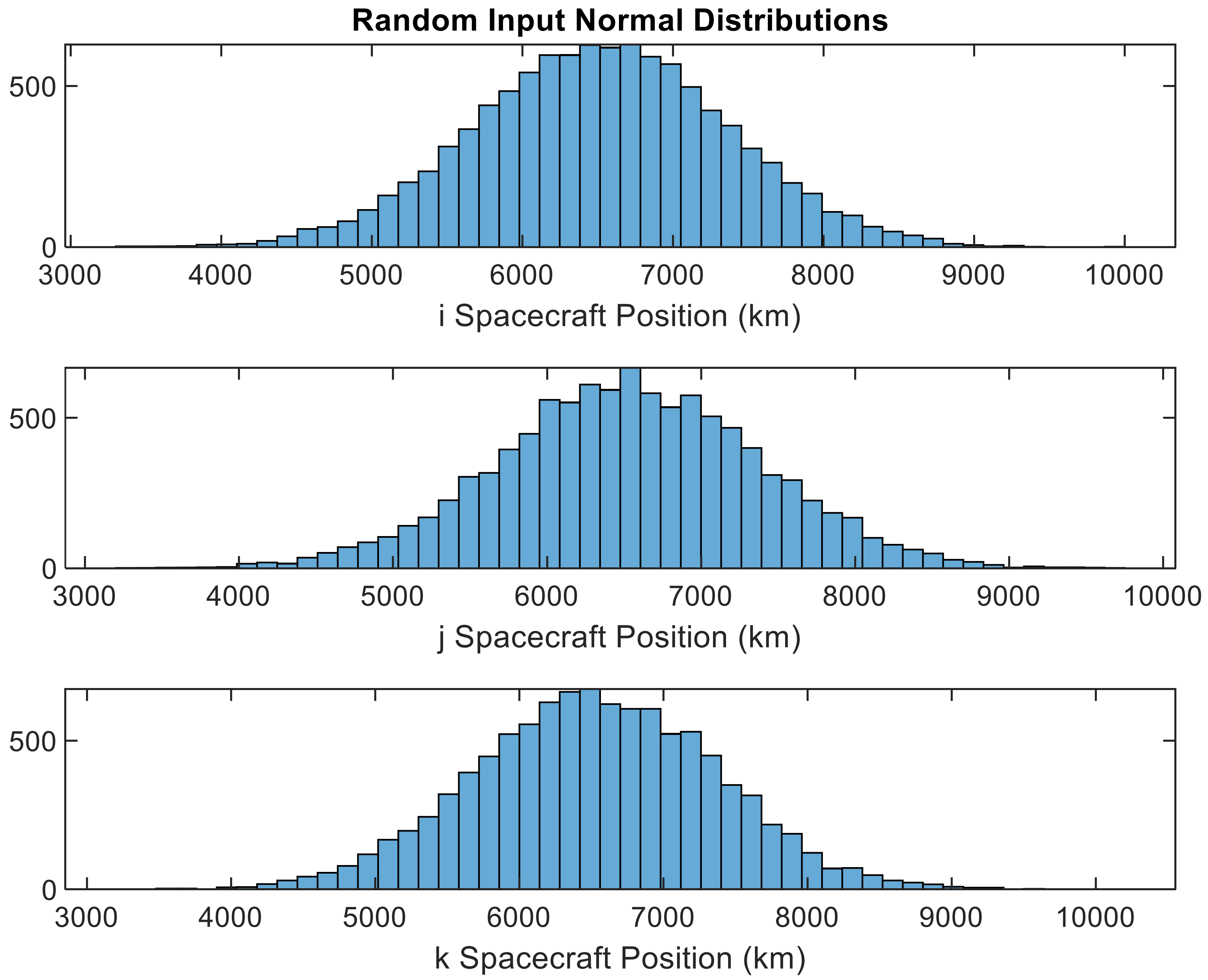
0
-13
-12
-11
-10
-9
-8
-7
-6
k Acceleration Perturbation (km/s2)
×10-7
90%
10%

NORMAL DISTRIBUTION RESULTS



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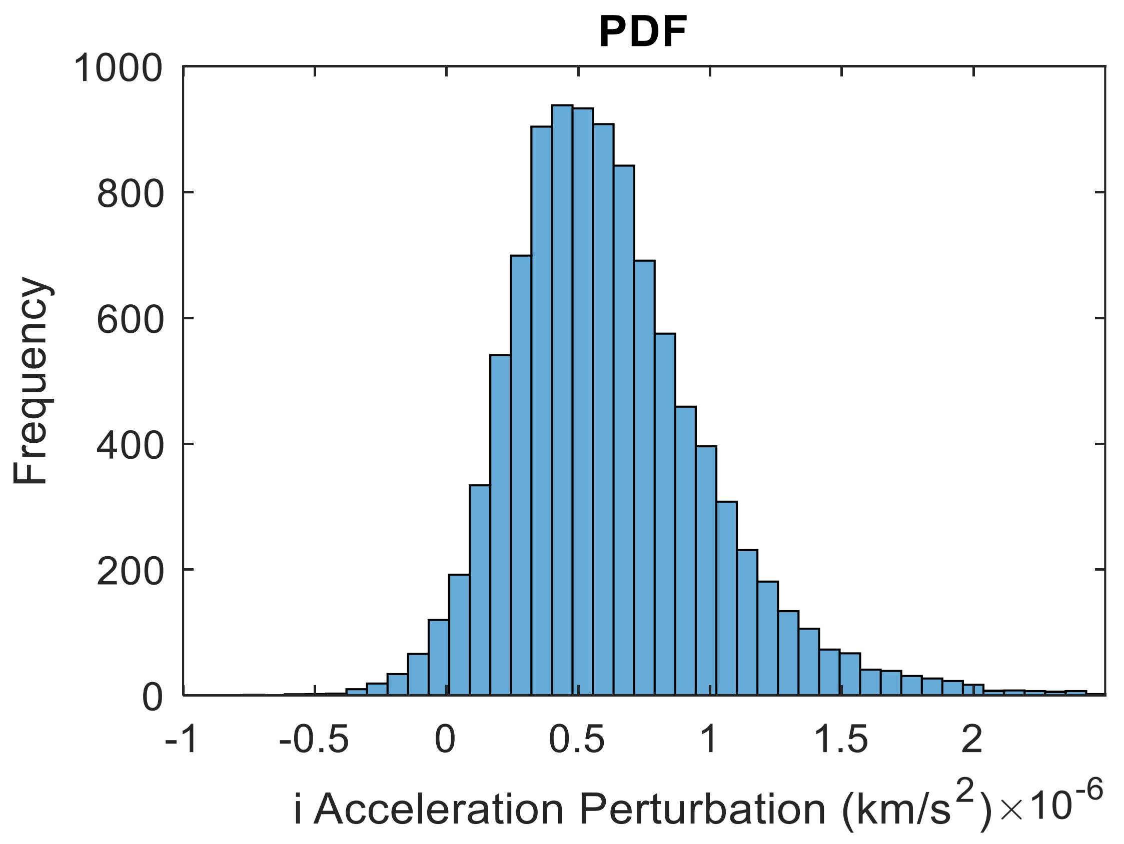
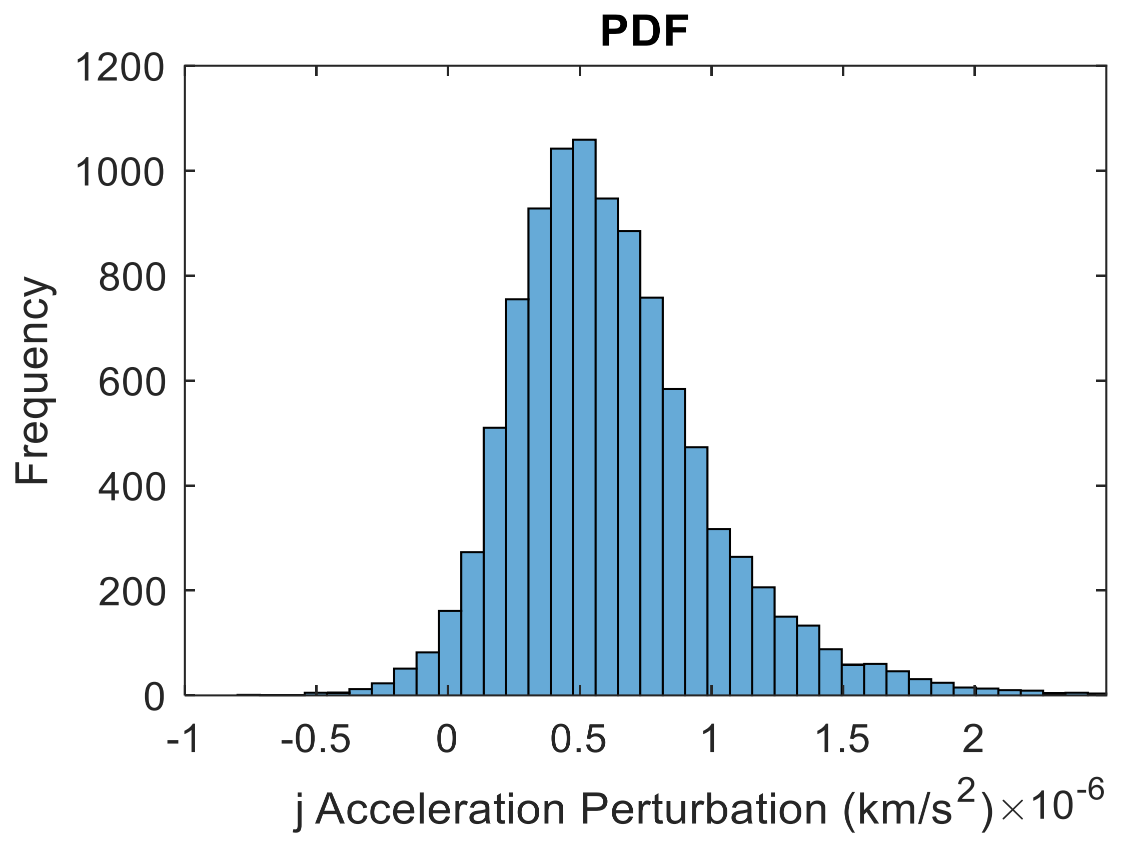
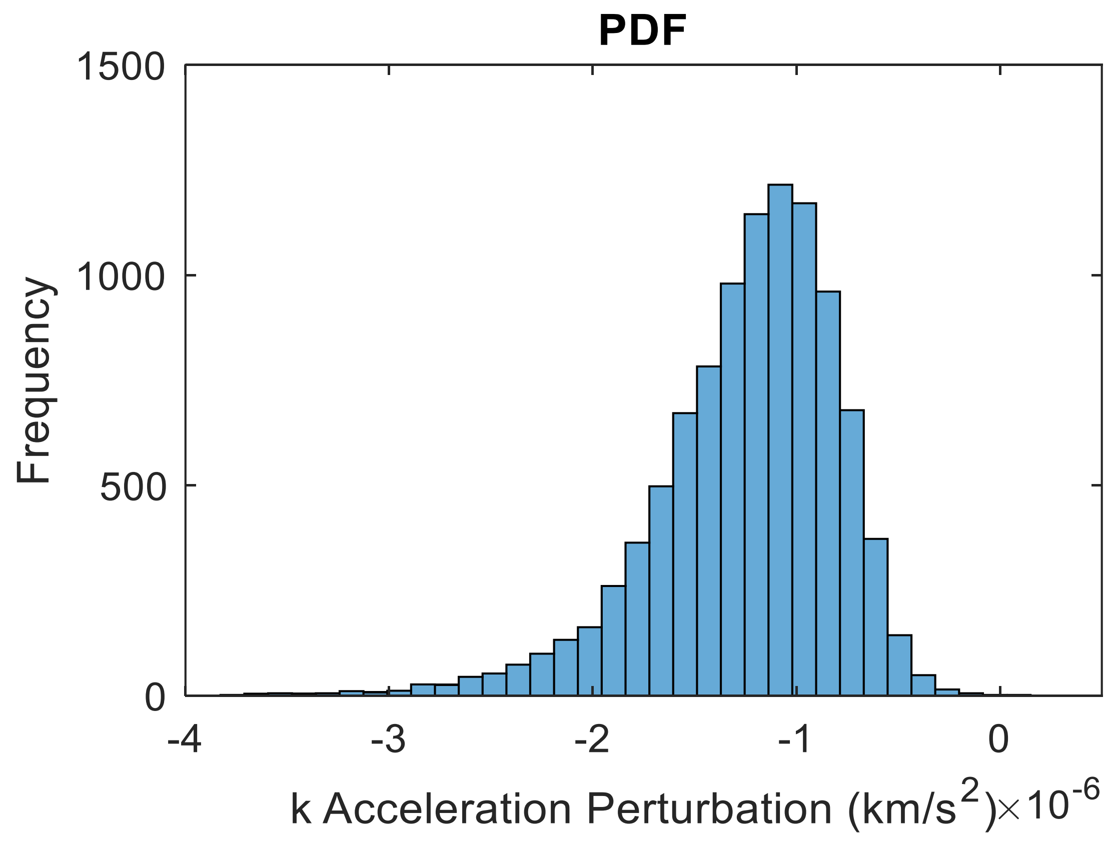
$$\frac{\partial U}{\partial r_i} = -\frac{\mu r_i}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 1\right]\right)$$

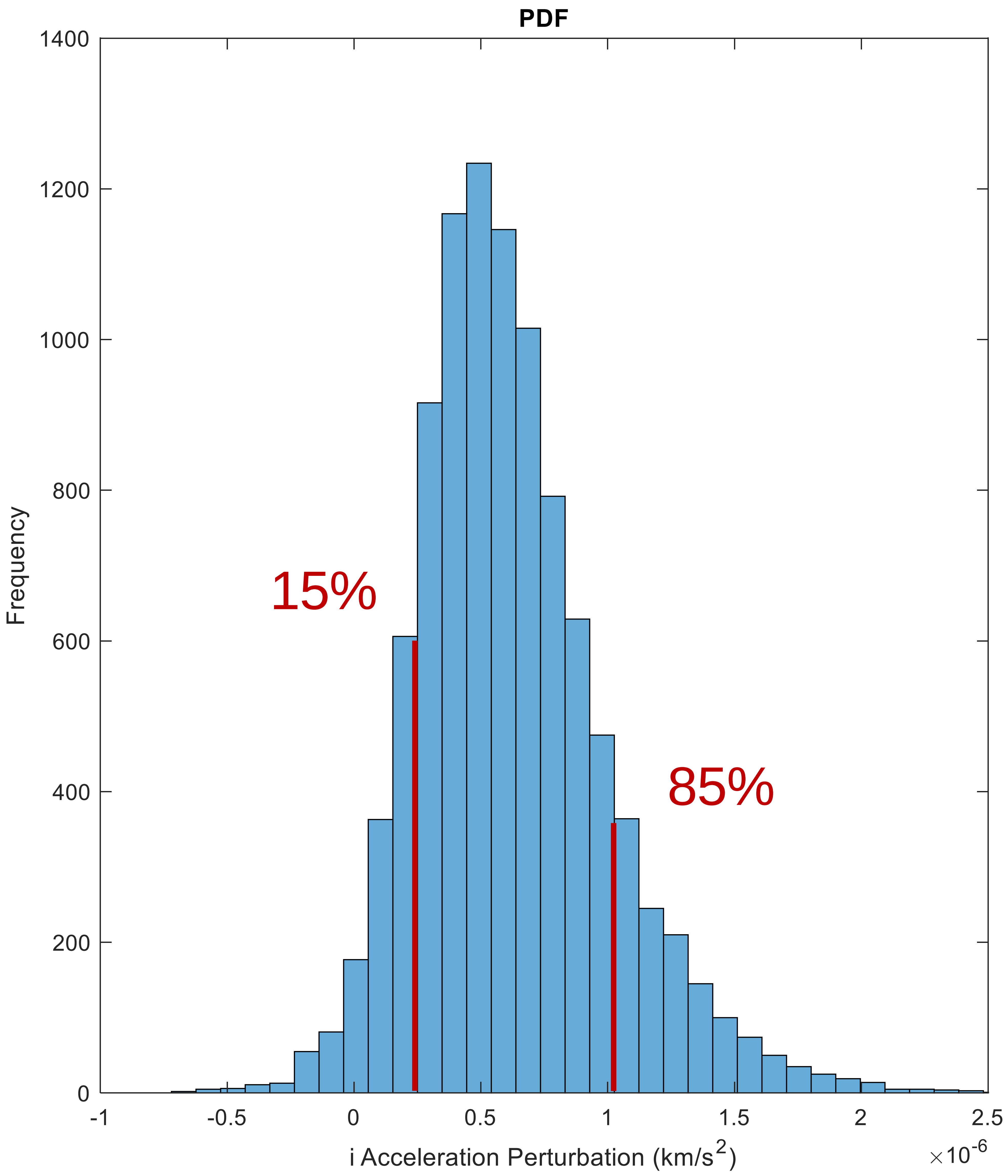
$$\frac{\partial U}{\partial r_j} = -\frac{\mu r_j}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 1\right]\right)$$

$$\frac{\partial U}{\partial r_k} = -\frac{\mu r_k}{r^3}\left(1 - J_2\frac{3}{2}\left(\frac{R_{\text{Earth}}}{r}\right)^2\left[5\left(\frac{r_k}{r}\right)^2 - 3\right]\right)$$





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15%
85%
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1.5
2
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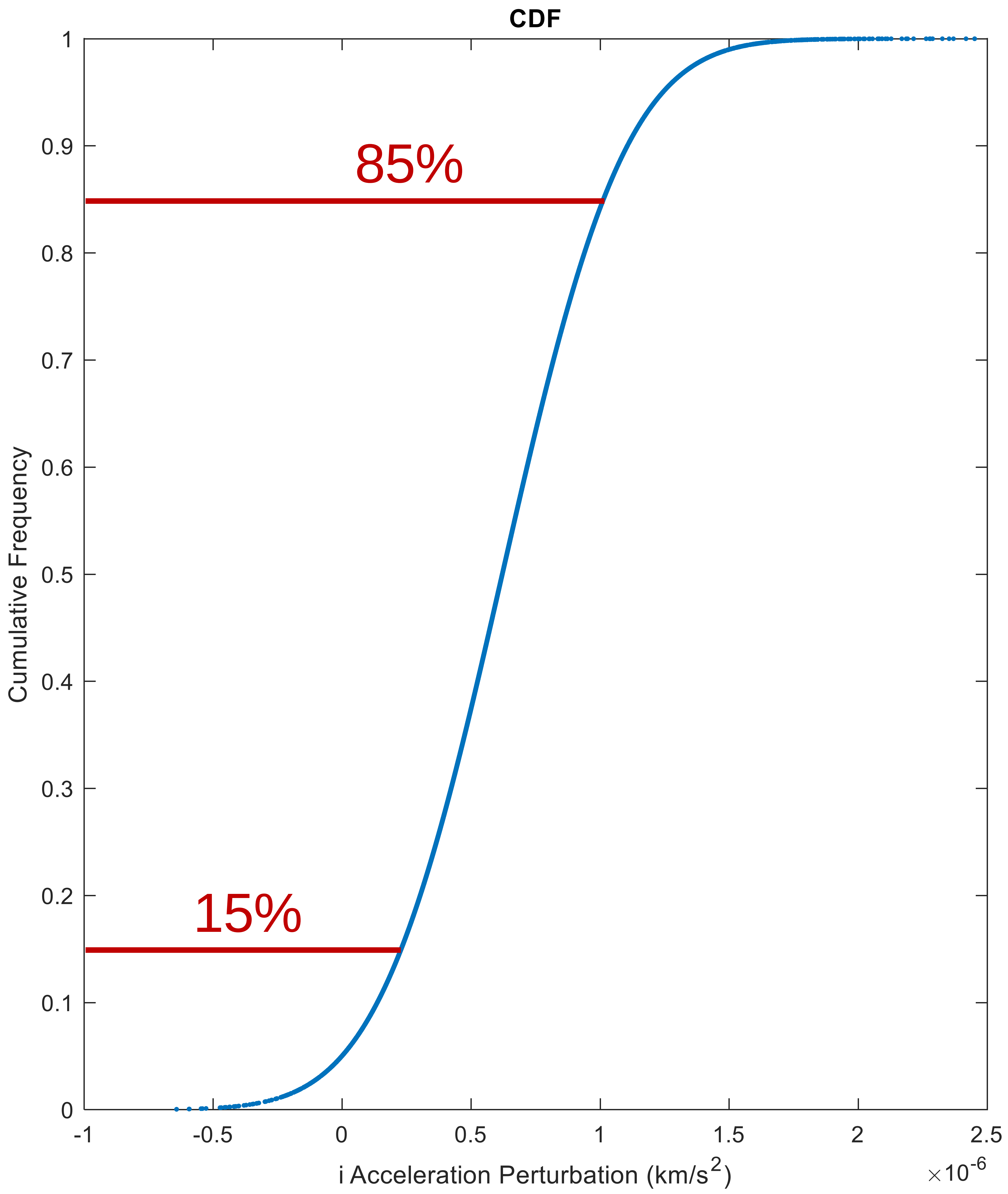
$\times 10^{-6}$
i Acceleration Perturbation (km/s^2)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Cumulative Frequency
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15%
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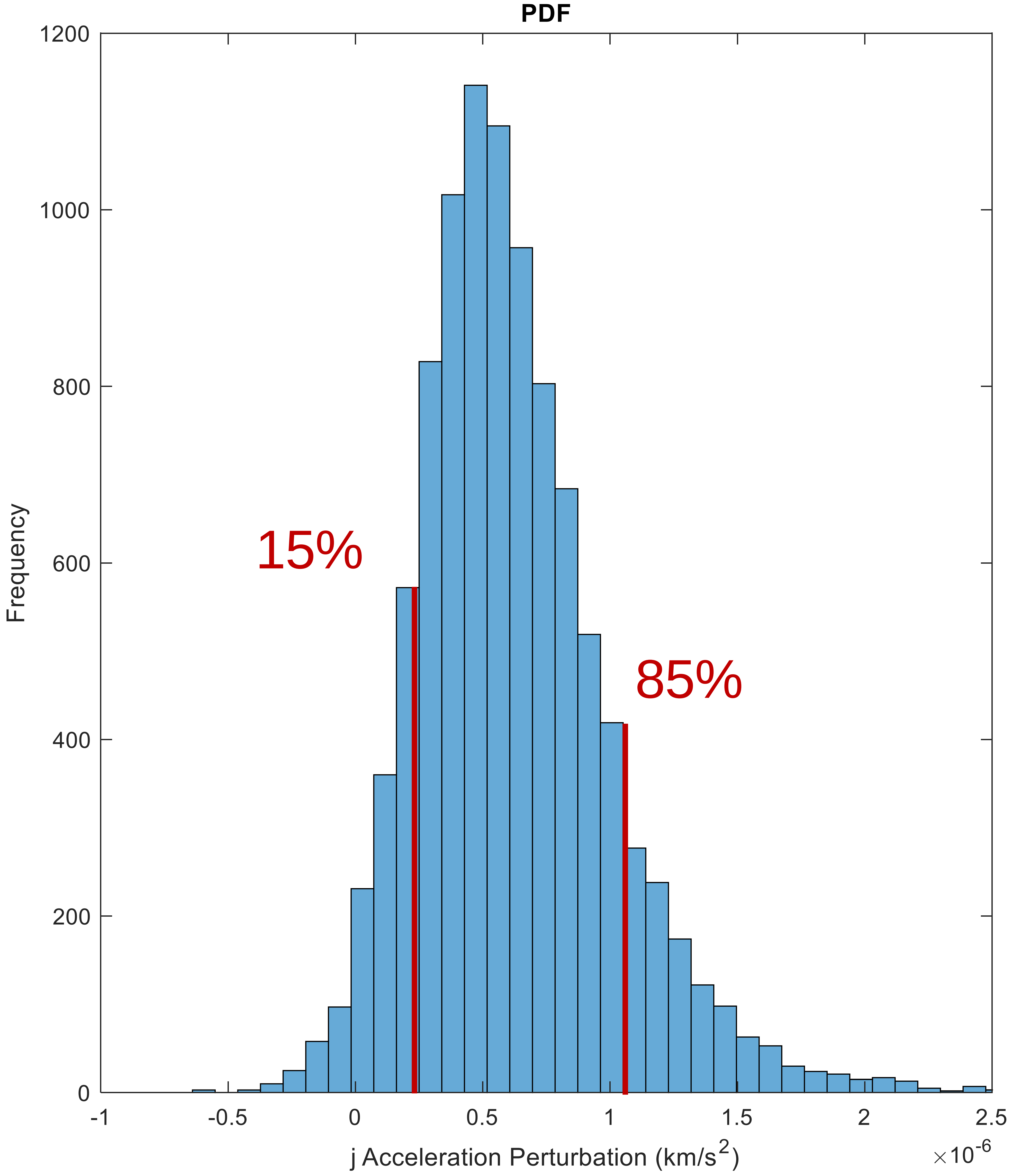
0
0.5
1
1.5
2
2.5
$\times 10^{-6}$
i Acceleration Perturbation (km/s^2)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Frequency
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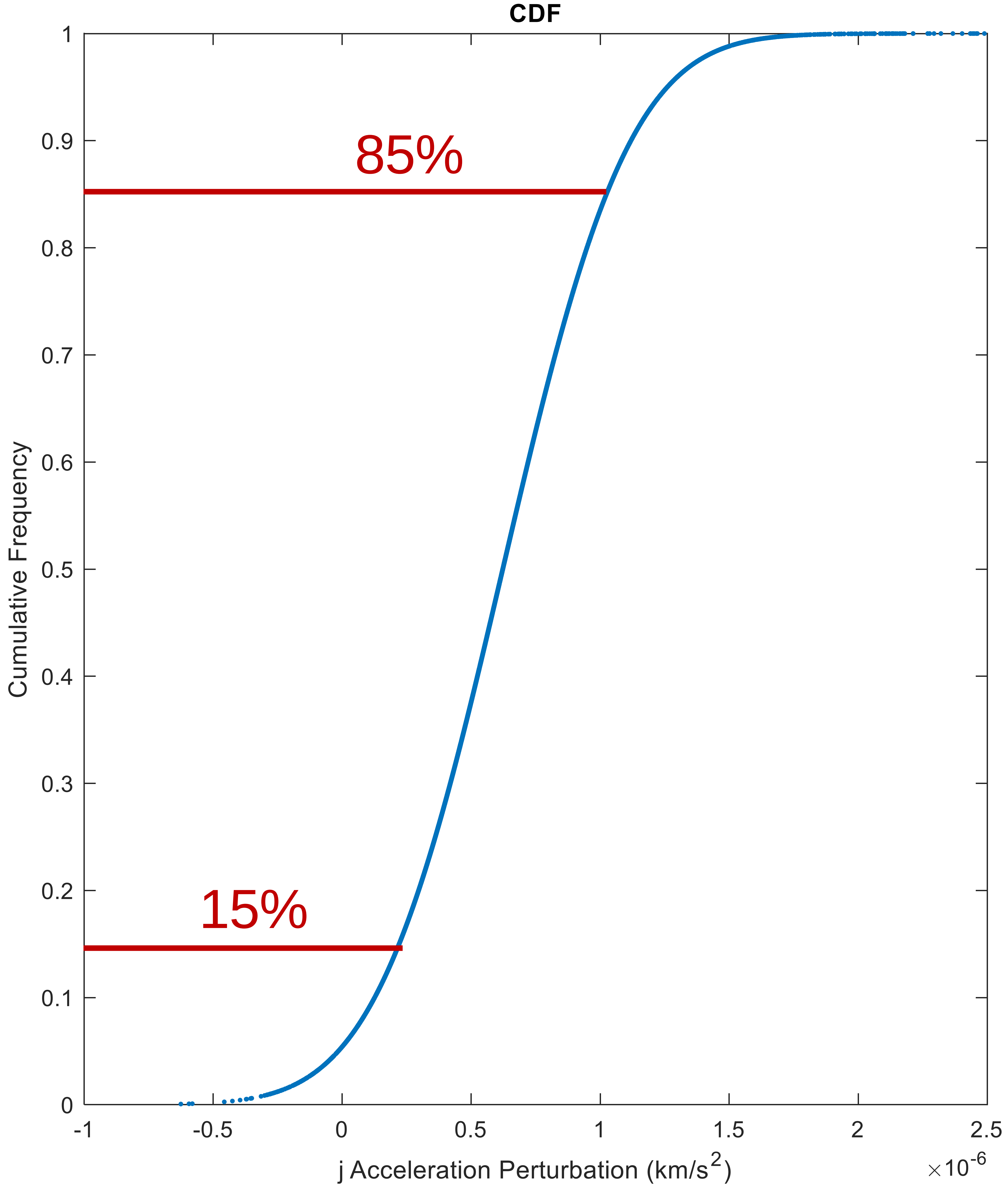
2.5
15%
85%
j Acceleration Perturbation (km/s^2)
×10^{-6}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Cumulative Frequency
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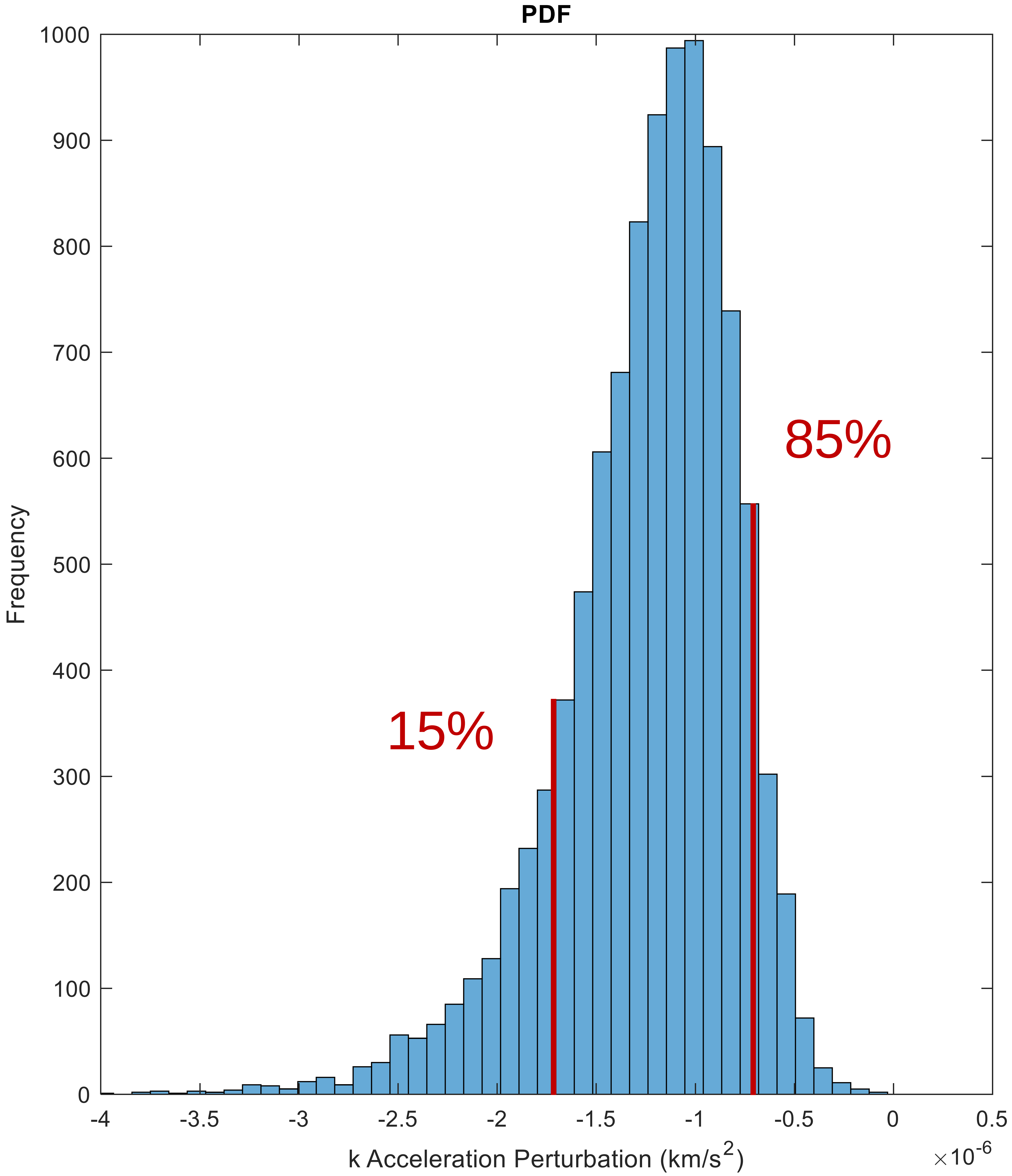
2.5
85%
15%
j Acceleration Perturbation (km/s^2)
×10^{-6}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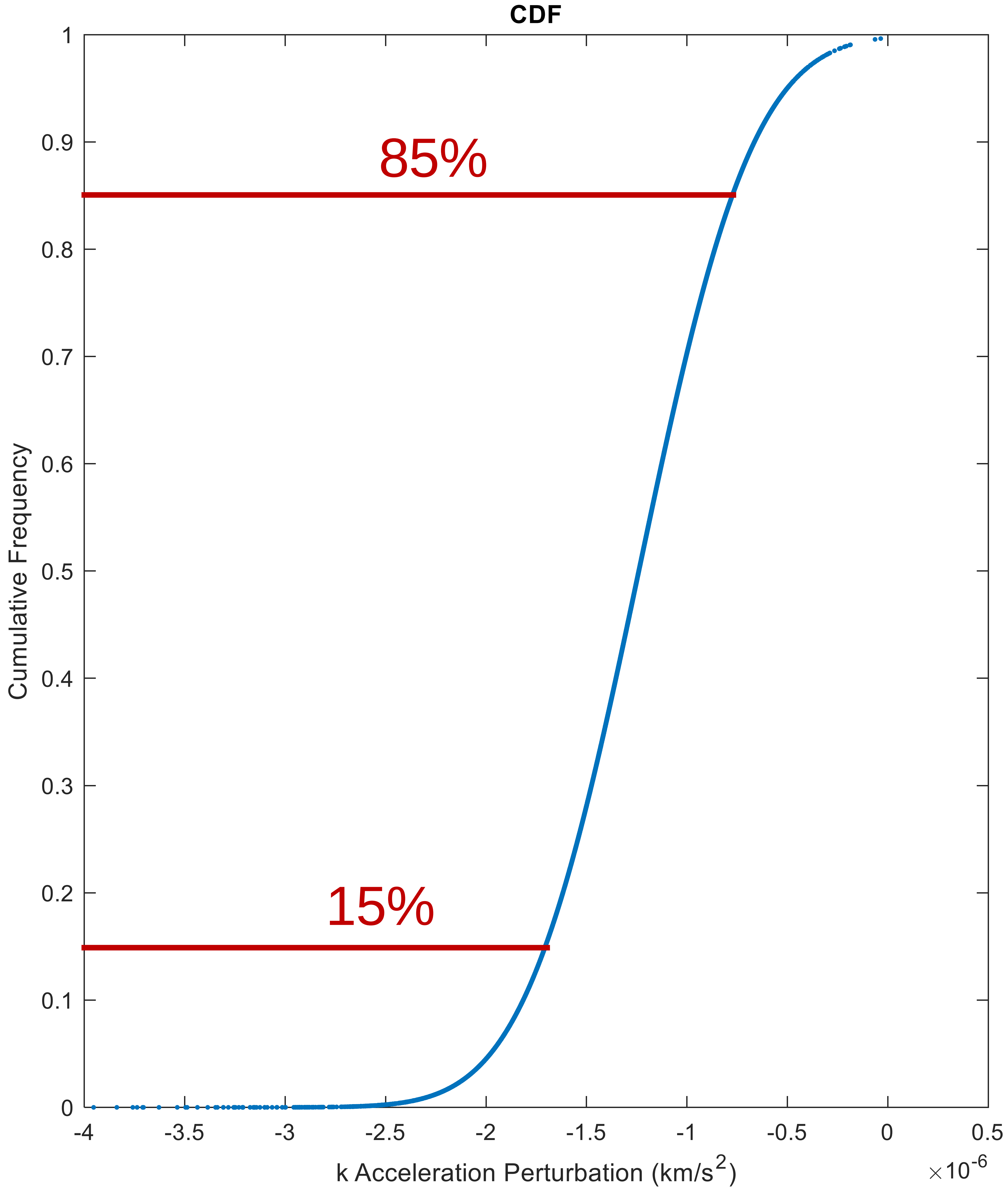
-1
-0.5
0
0.5
k Acceleration Perturbation (km/s2)
×10-6
85%
15%


CDF
Cumulative Frequency
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-2
-1.5
-1
-0.5
0
0.5
k Acceleration Perturbation (km/s2)
×10-6
85%
15%

RESULTS

The confidence range of 10% to 90% was chosen for the uniform distribution simulations because there was a relatively tight range of output accelerations, so I assume that most of the values can be estimated here with good confidence. A range of 15% to 85% was chosen for the normal distribution simulations. This is because it seems that the normal distribution simulation resulted in an actual wider distribution of acceleration outputs compared to the uniform distribution simulation, contradicting my initial expectation, so I chose a comparatively smaller confidence range for these outputs. Overall, both simulations resulted in acceleration perturbations in the 10^-6 km/s^2 (or 10^-3 m/s^2) magnitude, which could cause a considerable amount of torque on the spacecraft over time. For both input distributions, the output range seems to be the smallest for the k-direction, so I can conclude that the k-direction acceleration perturbation will be relatively predictable despite a wide range of k-positions. This means that our spacecraft will need a more robust design to account for J2 in the i-j directions if we want to be able to maneuver between a wide range of LEO altitudes to remove targeted debris for a long-term mission.

REFERENCES

[1] J2 Perturbation. [Online]. Available: https://ai-solutions.com/_freeflyeruniversityguide/j2_perturbation.htm. [Accessed: 20-Oct-2022]. https://ai-solutions.com/_freeflyeruniversityguide/j2_perturbation.htm

[2] Jones, Brandon. “Accelerations from J2 and Drag”, (Lecture 12, ASE 366L, The University of Texas at Austin, Texas, March 1, 2022

[3] Ogundele, Ayansola. (2017). Nonlinear Dynamics and Control of Spacecraft Relative Motion. 10.13140/RG.2.2.35073.48484.

The MATLAB script and functions used in this analysis were written by me.

THANK YOU.

- Madison Vaughan, SmallSat Alliance Orbital Debris

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