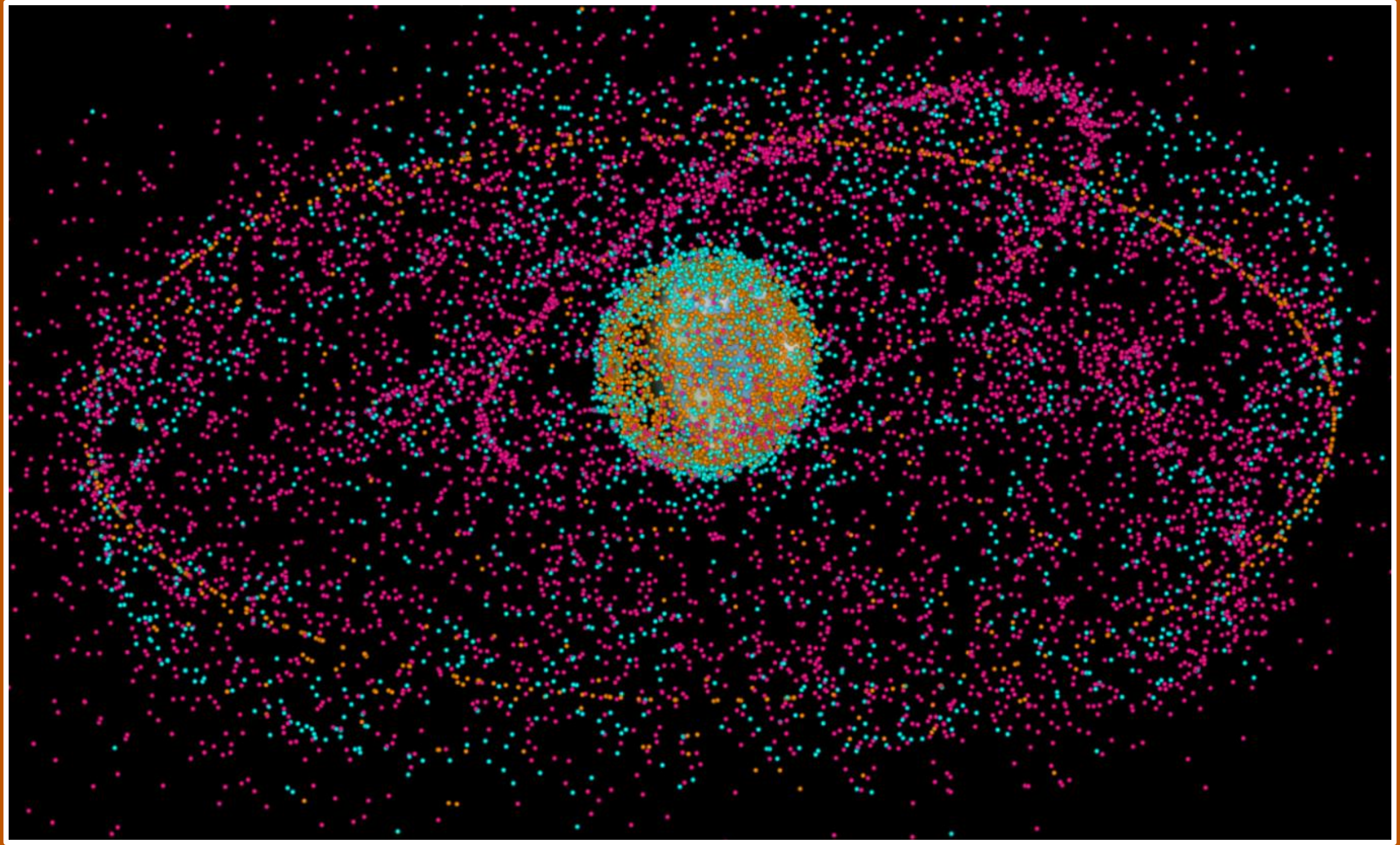
The University of Texas at Austin
Aerospace Engineering and Engineering Mechanics
Cockrell School of Engineering

OCTOBER 2022

ENVIRONMENTAL ANALYSIS ON ORBITAL RELATIVE MOTION AND TARGETING

ASE374K – Adam Nokes
SmallSat Alliance Orbital Debris Design Competition



https://payloadspace.com/regulating-orbital-debris-part-two/

MADISON VAUGHAN
The University of Texas at Austin

HOW DO WE RENDEZVOUS WITH SPACE DEBRIS?

- In order to rendezvous with orbital debris, our satellite must have the capability of monitoring its position and velocity relative to the targeted debris.

- This is in actuality very difficult to do because of 3 reasons:
  - Every piece of debris orbiting in LEO has its own individual orbit with distinct eccentricities, inclinations, velocities, and positions.
  - There are many different types of reference coordinate systems that are used to track and project predicted locations of orbital debris
  - There is no way to measure “how much faster” debris or the spacecraft is moving at when trying to rendezvous – all relative speeds between the two are calculated, which introduces a great possibility of error for a targeting mission

- So how do we model the relative motion between our proposed orbital debris removal satellite and the orbital debris itself?

RELATIVE MOTION

- We start with a target that we want to track and follow (orbital debris in this case)
  - The target has an initial set of orbital elements that we assume to find from the characterization portion of mission – AKA we will assume some initial orbit

- A secondary object called the chaser will be moving relative to the target
  - We do not know its exact orbital elements in reference to the Earth, but we know its initial relative position and velocity with a reference frame centered on the target [1]

- Their relative positions propagated over time can be solved for using two different methods:
  - Inertial Equations of Motion (EOMs)
  - Clohessy-Wiltshire (CW) Equations

TITLE

Using the initial relative position vector and velocity and 2-body dynamics, the following relative acceleration equation can be derived and then used to propagate the relative position over time

$$\ddot{\rho} = -\frac{\mu}{r_T^3}\left(-2\,\rho_R\,\widehat{R} + \rho_S\,\widehat{S} + \rho_W\,\widehat{W}\right)$$ [2]

Alternatively, the Clohessy-Wiltshire equations give us a non-iterative analytic solution to calculate the necessary state vectors at a certain timestep.

$$x(t) = (4 - 3\cos(nt))\,x_0 + \frac{\sin(nt)}{n}\,\dot{x}_0 + \frac{2}{n}(1 - \cos(nt))\,\dot{y}_0$$

$$y(t) = 6(\sin(nt) - nt)\,x_0 + y_0 + \frac{2}{n}(\cos(nt) - 1)\,\dot{x}_0 + \frac{1}{n}(4\sin(nt) - 3nt)\,\dot{y}_0$$

$$z(t) = \cos(nt)\,z_0 + \frac{\sin(nt)}{n}\,\dot{z}_0$$

$$\dot{x}(t) = 3n\sin(nt)\,x_0 + \cos(nt)\,\dot{x}_0 + 2\sin(nt)\,\dot{y}_0$$

$$\dot{y}(t) = 6n(\cos(nt) - 1)\,x_0 - 2\sin(nt)\,\dot{x}_0 + (4\cos(nt) - 3)\,\dot{y}_0$$

$$\dot{z}(t) = -n\sin(nt)\,z_0 + \cos(nt)\,\dot{z}_0$$

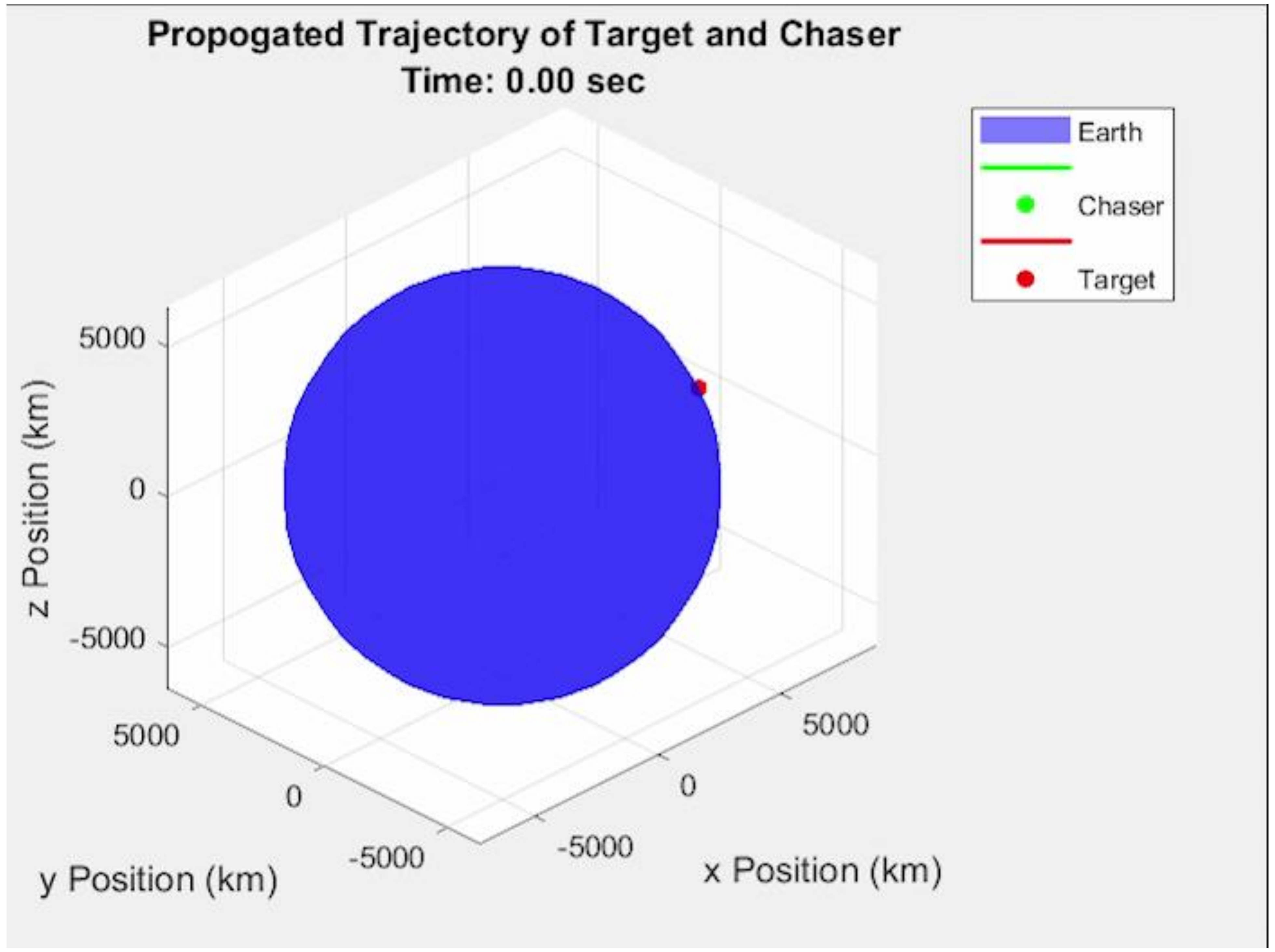
[2], [3]

CASE 1

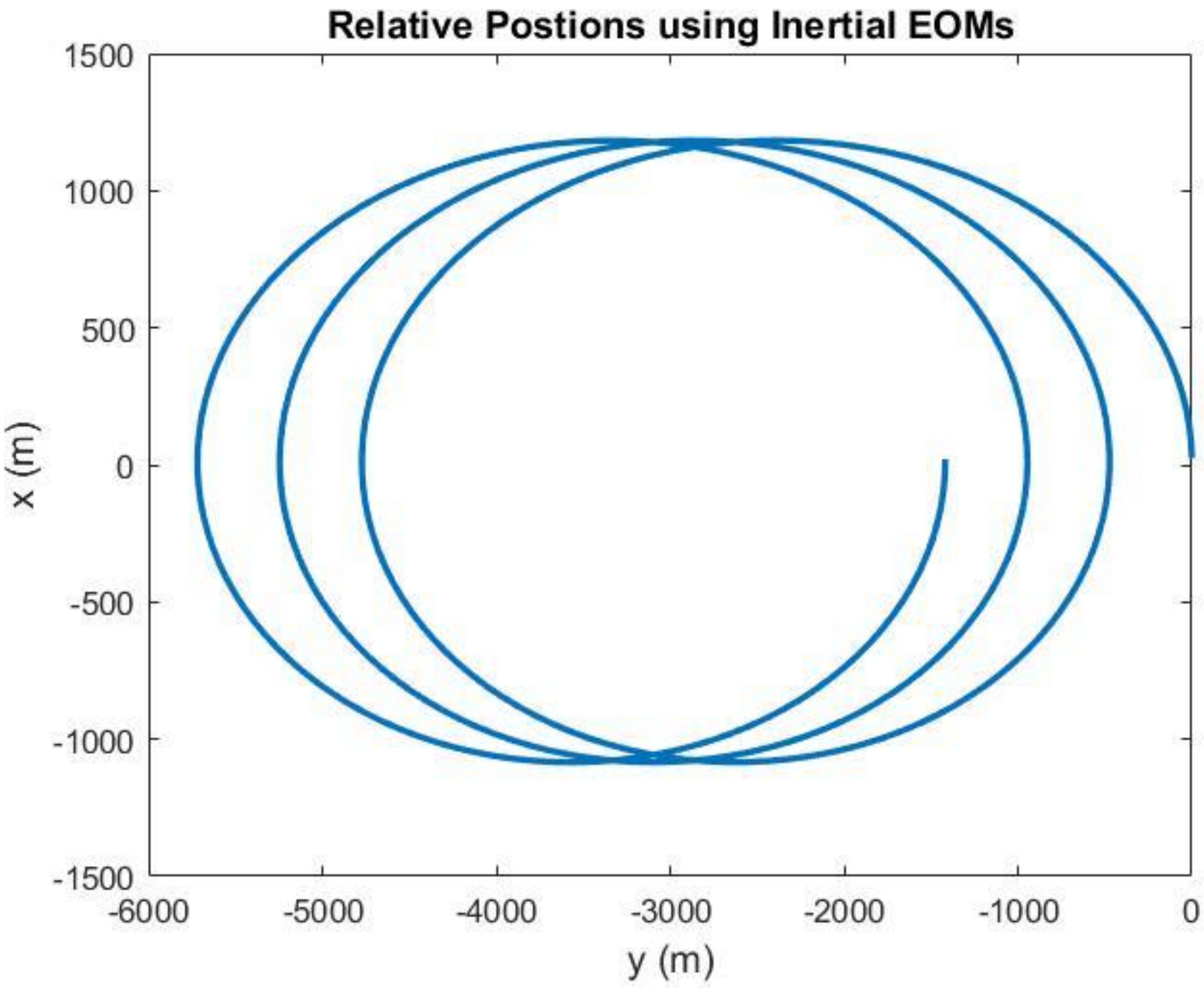
CASE 1 PARAMETERS

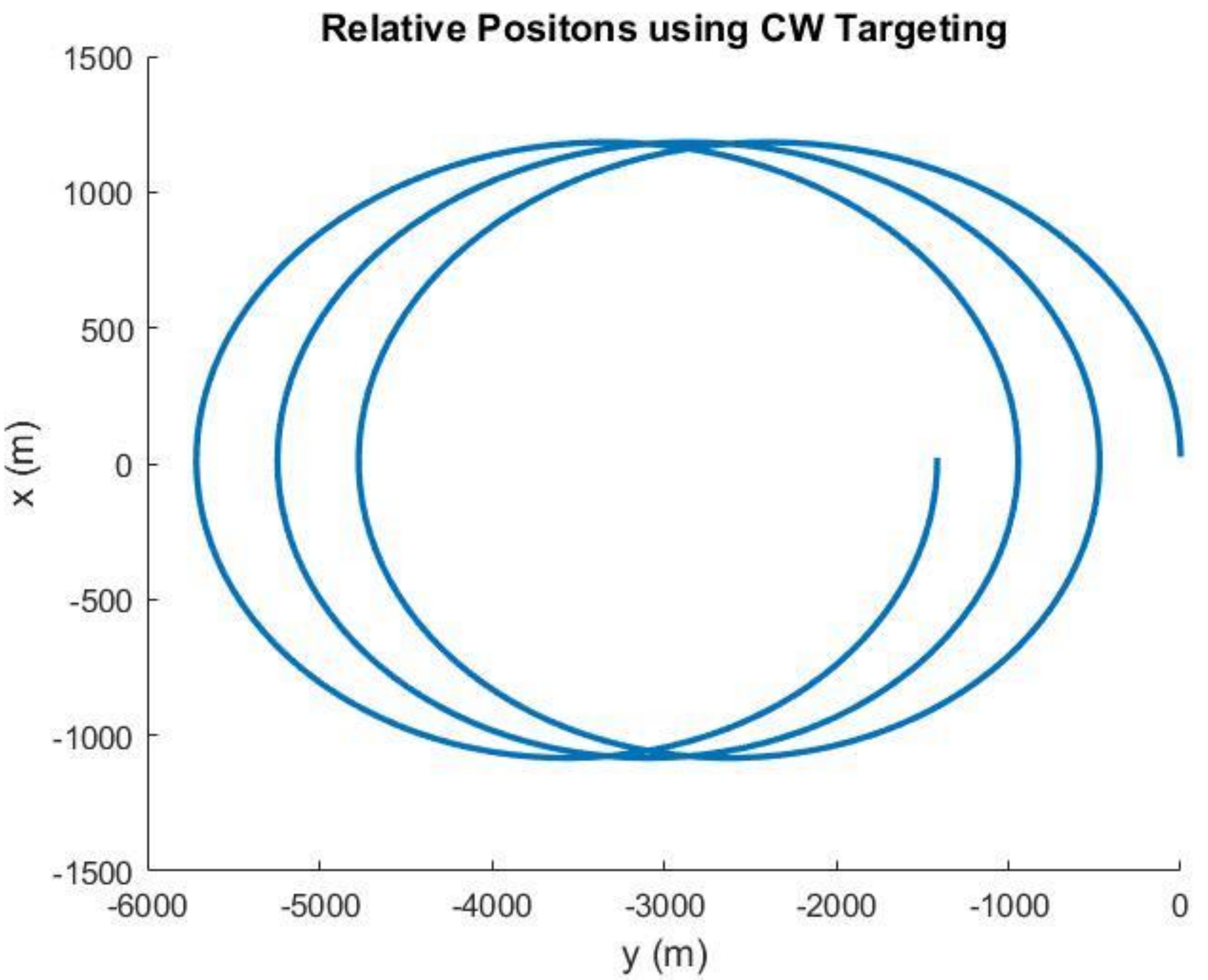
Target Orbital Elements
Semimajor axis = 8000 km
Eccentricity = 0.0
Inclination = 85 degrees
Argument of Periapsis = 0.0 degrees
RAAN = 0.0 degrees
True Anomaly = 0.0 degrees

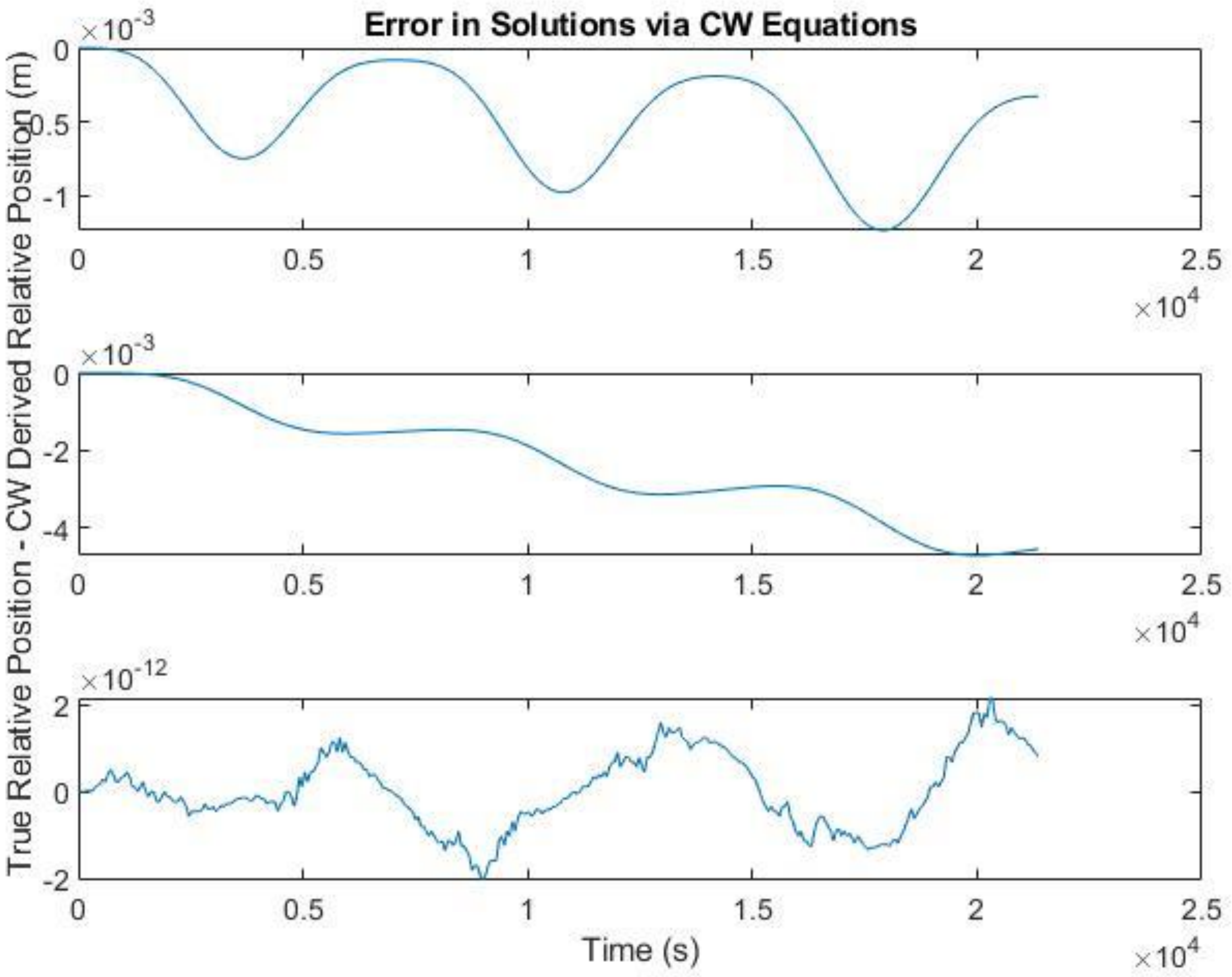
Chaser Relative State
Relative Position = 0.025 km (in R direction)
Relative Velocity = 0.001 km/s (in R direction)



https://youtu.be/fMUTEcpOn4E


Relative Postions using Inertial EOMs
x (m)
y (m)
1500
1000
500
0
-500
-1000
-1500
-6000
-5000
-4000
-3000
-2000
-1000
0


Relative Positons using CW Targeting
1500
1000
500
0
-500
-1000
-1500
x (m)
-6000
-5000
-4000
-3000
-2000
-1000
0
y (m)



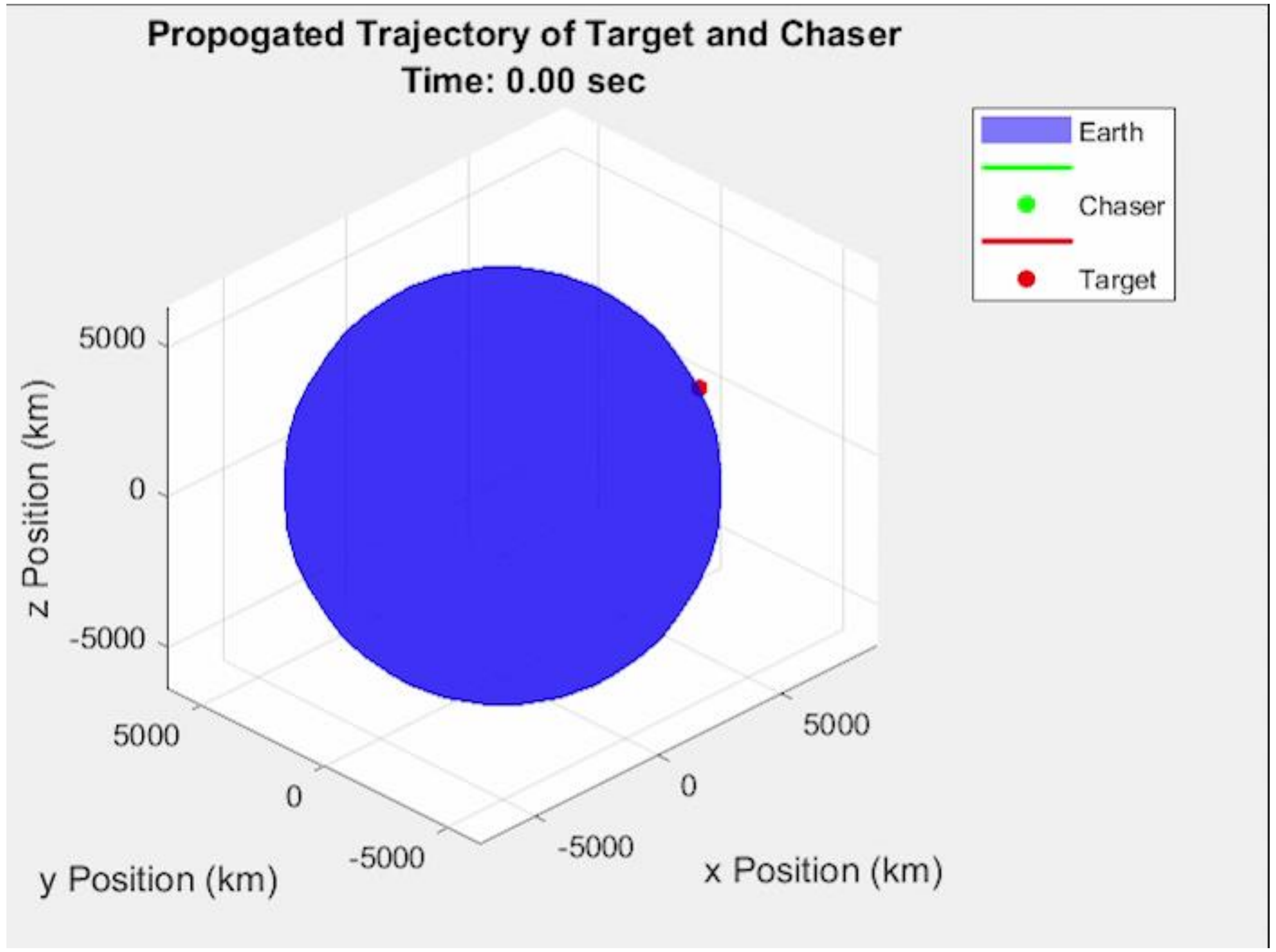
Error in Solutions via CW Equations
True Relative Position - CW Derived Relative Position (m)
×10-3
0
-0.5
-1
0
0.5
1
1.5
2
2.5
×10^4
×10-3
0
-2
-4
×10-12
2
0
-2
Time (s)

CASE 2

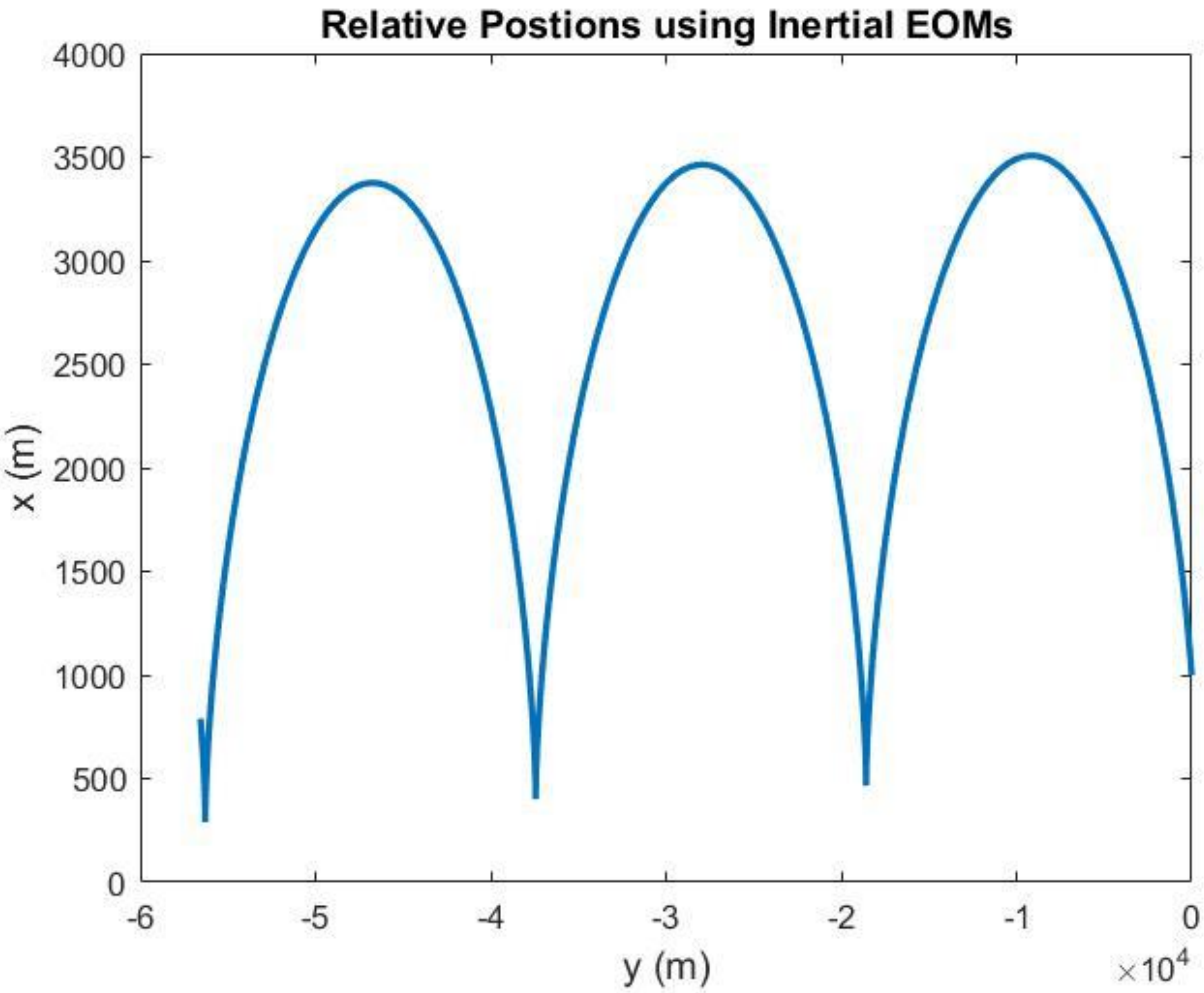
CASE 2 PARAMETERS

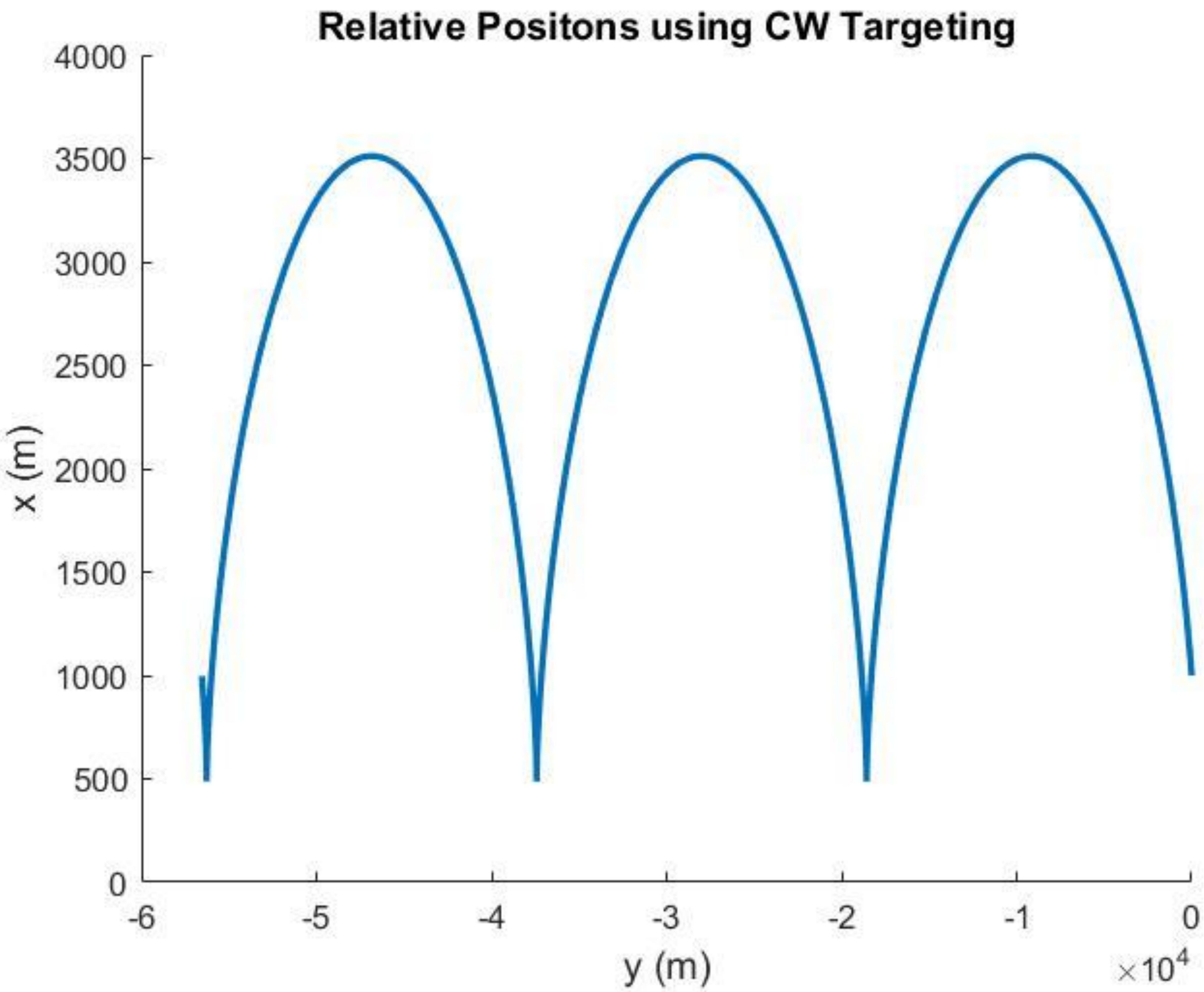
<u>Target Orbital Elements</u>
Semimajor axis = 8000 km
Eccentricity = 0.0
Inclination = 85 degrees
Argument of Periapsis = 0.0 degrees
RAAN = 0.0 degrees
True Anomaly = 0.0 degrees

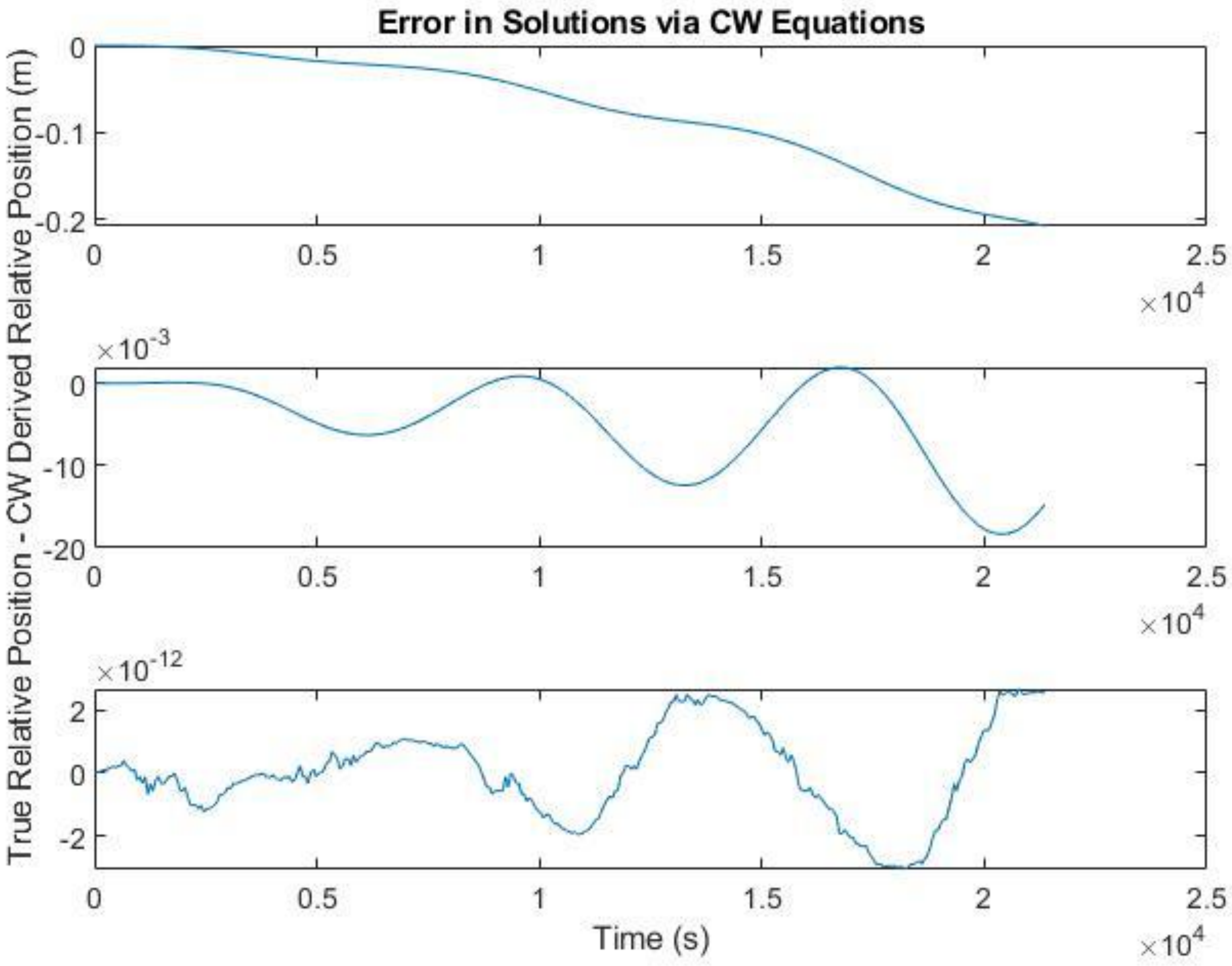
<u>Chaser Relative State</u>
Relative Position = 1 km (in R direction)
Relative Velocity = 0.001 km/s (in R direction)



https://youtu.be/Soaw0xHkwhw


Relative Postions using Inertial EOMs
x (m)
y (m)
4000
3500
3000
2500
2000
1500
1000
500
0
-6
-5
-4
-3
-2
-1
0
×10^4


Relative Positons using CW Targeting
x (m)
4000
3500
3000
2500
2000
1500
1000
500
0
-6
-5
-4
-3
-2
-1
0
y (m)
×10^4

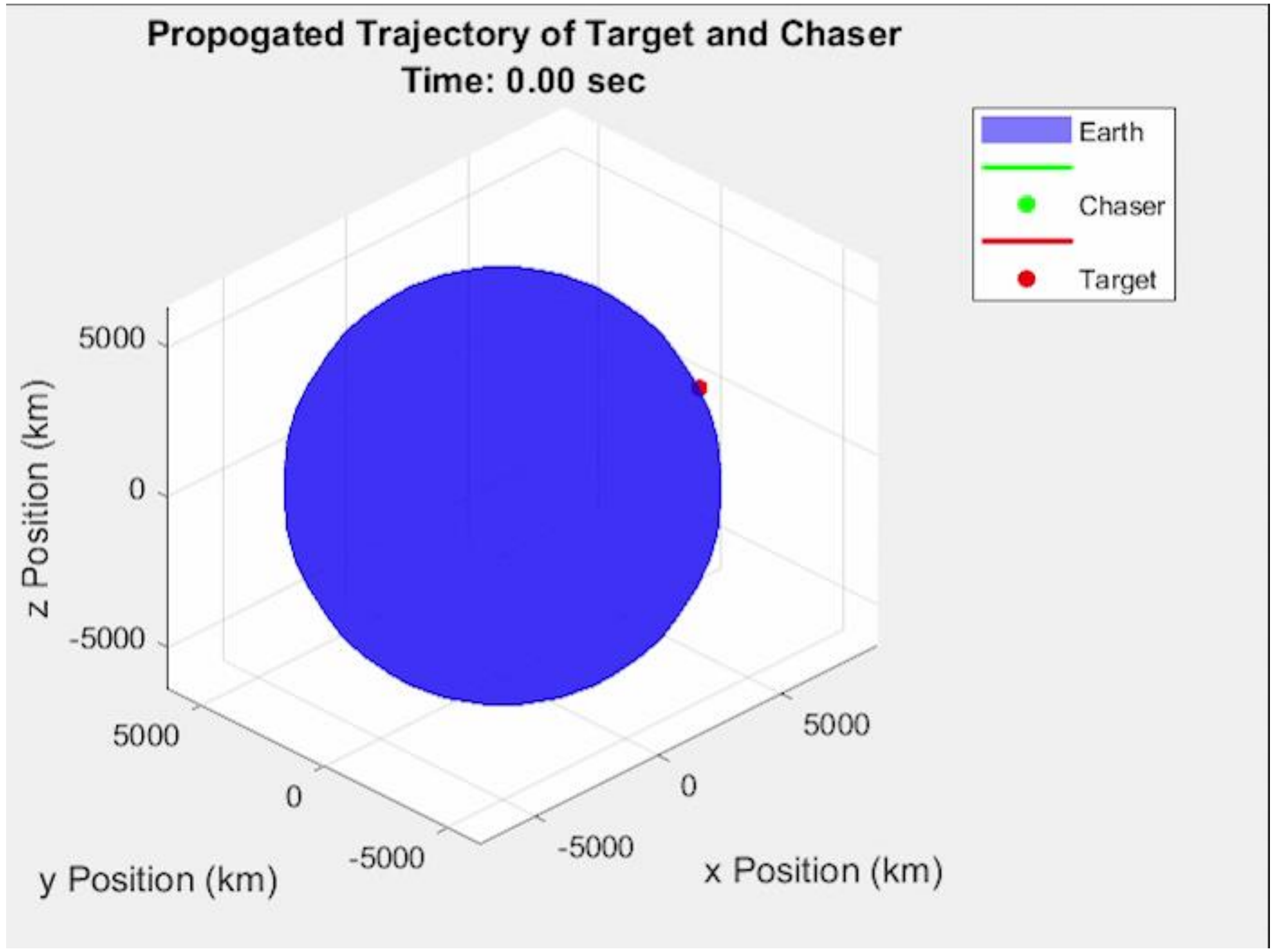

Error in Solutions via CW Equations
True Relative Position - CW Derived Relative Position (m)
0
-0.1
-0.2
0
0.5
1
1.5
2
2.5
$\times 10^4$
$\times 10^{-3}$
0
-10
-20
$\times 10^{-12}$
2
0
-2
Time (s)

CASE 3

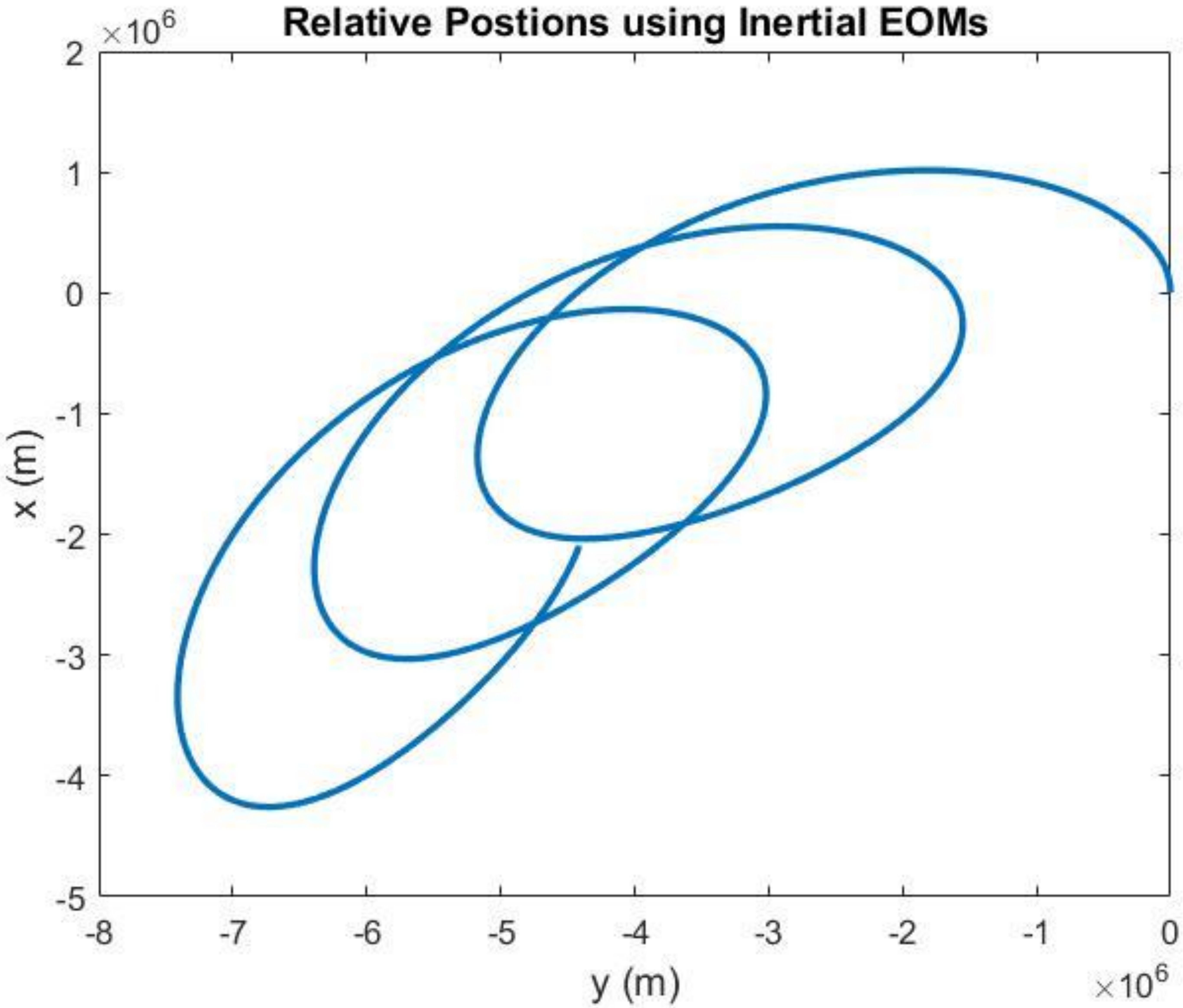
CASE 3 PARAMETERS

<u>Target Orbital Elements</u>
Semimajor axis = 8000 km
Eccentricity = 0.0
Inclination = 85 degrees
Argument of Periapsis = 0.0 degrees
RAAN = 0.0 degrees
True Anomaly = 0.0 degrees

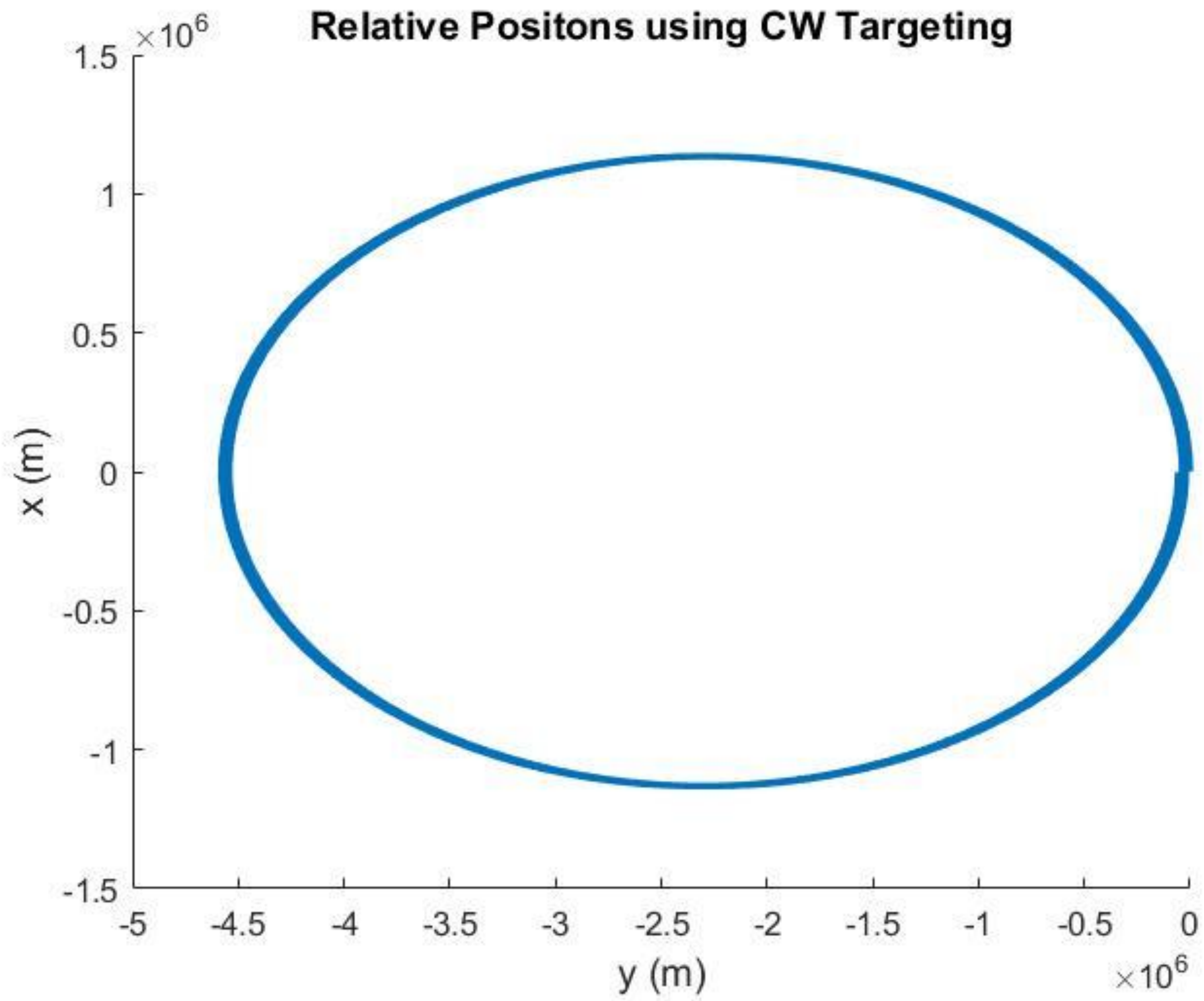
<u>Chaser Relative State</u>
Relative Position = 1 km (in R direction)
Relative Velocity = 1 km/s (in R direction)

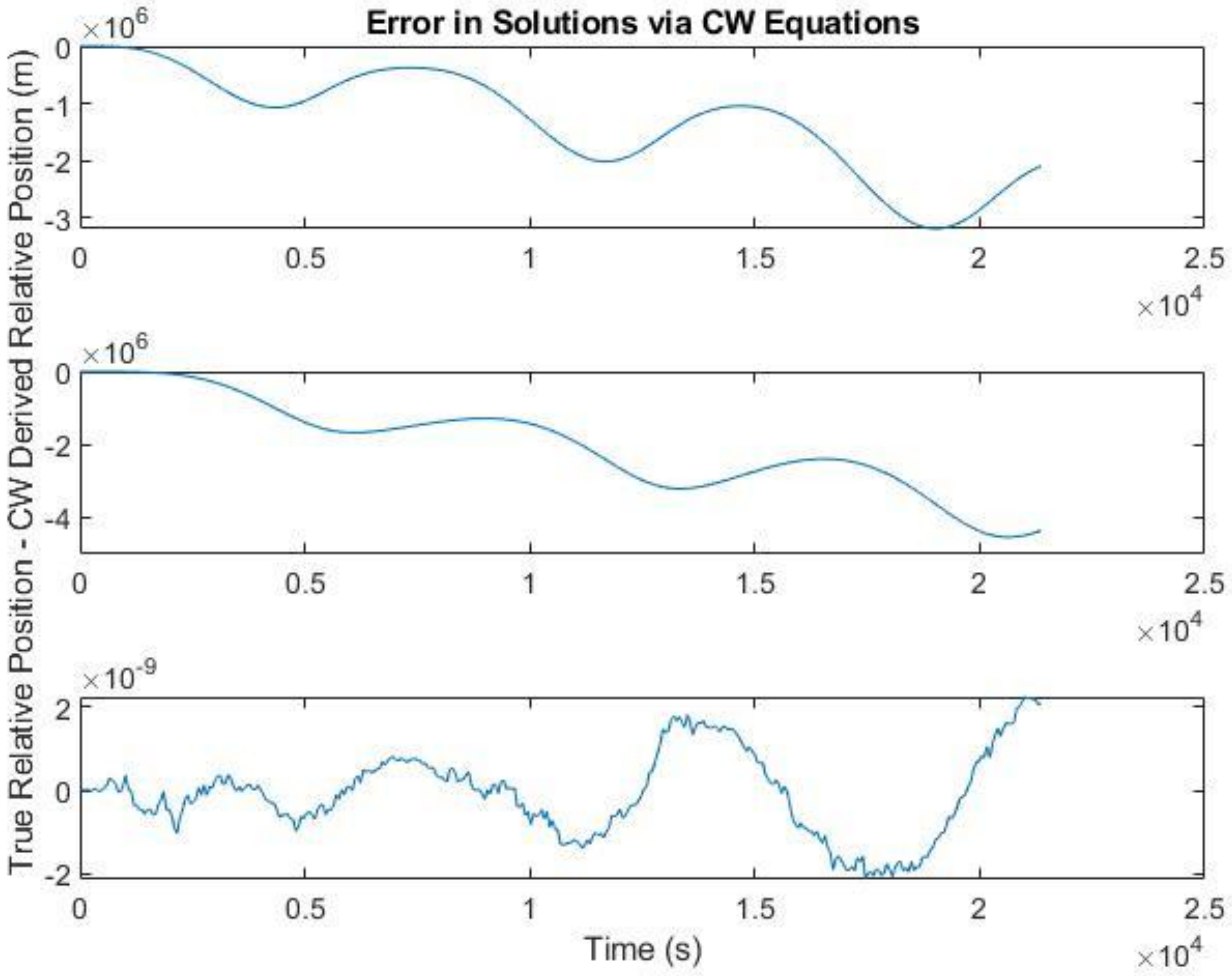


https://youtu.be/OMQvVgcNEMU



Relative Postions using Inertial EOMs
×10^6
2
1
0
-1
-2
-3
-4
-5
x (m)
-8
-7
-6
-5
-4
-3
-2
-1
0
y (m)
×10^6


Relative Positons using CW Targeting
×10^6
1.5
1
0.5
0
-0.5
-1
-1.5
x (m)
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
y (m)
×10^6


Error in Solutions via CW Equations
True Relative Position - CW Derived Relative Position (m)
×10^6
0
-1
-2
-3
×10^6
0
-2
-4
×10^-9
2
0
-2
0
0.5
1
1.5
2
2.5
×10^4
Time (s)

CONCLUSION

For Case 3, it is shown that there is a considerable large error for the X and Y (technically i and k directions in an ijk centered frame) compared to Case 1 and 2. Even the graphs between the EOM-derived positions and the CW-derived positions look visibly different for Case 3 compared to Case 1 and 2.

This error proves that for large magnitude differences in relative position and velocity (to 1 km and 1km/s^2 magnitudes) the Clohessy-Wiltshire equations are not very accurate, and for our mission we should instead try to use the inertial EOM methods for initial tracking operations. The Clohessy-Wiltshire equations will only be applicable to our mission after we are confident that our spacecraft will be within a smaller distance (less than 100 m) with the targeted debris.

FUTURE WORK

Future simulations could target actual debris that we know to exist in the LEO debris field and consider a myriad of different initial relative velocities and positions. Future simulations could also consider the affect of perturbations such as J2 or atmospheric drag on using Inertial EOMs for this propagation.

Further simulation could attempt to model that actual rendezvous with the desired orbital debris using different orbit transfer maneuver methods and integrate the use of perturbations within that simulation.

REFERENCES

[1] Jones, Brandon. “Introduction to Relative Motion”, (Lecture 16, ASE 366L, The University of Texas at Austin, Texas, March 24, 2022

[2] Jones, Brandon. “Formulating Relative Motion Equations”, (Lecture 17, ASE 366L, The University of Texas at Austin, Texas, March 29, 2022

[3] Jones, Brandon. “Clohessy-Wiltshire Equations”, (Lecture 18, ASE 366L, The University of Texas at Austin, Texas, March 31, 2022

The MATLAB script and functions used in this analysis were written by me.

THANK YOU.

- Madison Vaughan, SmallSat Alliance Orbital Debris

cockrell.utexas.edu

The University of Texas at Austin
Aerospace Engineering and Engineering Mechanics
Cockrell School of Engineering