Controlling the spectrum of single photons from triply-resonant parametric down-conversion

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* goal: efficienct single photon atom interaction with a stable contrate and Lorentzian line profile
* insdistinguishable photon pairs widely studied for non-resonant systems.
* We show how PDC phase-matching temperature affects the photon spectra.

I. INTRODUCTION

II. SPECTRUM OF THE PARAMETRIC PHOTONS DEPENDING FREQUENCY MISMATCH

The optical resonator coupling can be described by the resonator bandwidth γ:

$$\gamma = \gamma' + \gamma'' \,. \tag{1}$$

The resonator bandwidth is the sum of the external coupling rate γ' and the internal loss rate γ'' and can be interpreted as an overall intensity decay rate of the internal resonator field. This interpretation becomes obvious in ring down spectroscopy, where a switch-off of the external pump at time $t = 0$ leads to an exponential decay of the internal field given by $|\alpha(t)|^2 = |\alpha(t = 0)|^2 \cdot \mathrm{e}^{-2\pi\gamma t}$. The coupling rates γ' and γ'' are directly connected to the mirror transmittance T and the material absorption b [1]:

The resonator frequency response is then given by

$$\mathcal{G}(\nu) = 1/\left(1 - i2\left[\frac{\nu - \nu_0}{\gamma}\right]\right) = \frac{1}{1 - i2\delta} \,. \tag{2}$$

The Hamiltonian $\hat{H}$ for the three interacting fields is:

$$\hat{H} = \sum_{j\in\{\mathrm{p,s,i}\}} \hbar 2\pi\, \nu(\ell_\mathrm{j}, \mathrm{q_j}, \mathrm{p_j}) \left(\hat{a}_\mathrm{j}^\dagger \hat{a}_\mathrm{j} + 1/2\right) + \underbrace{i\hbar\pi \left(g \hat{a}_\mathrm{p} \hat{a}_\mathrm{s}^\dagger \hat{a}_\mathrm{i}^\dagger - \mathrm{h.c.}\right)}_{\equiv \hat{H}_\mathrm{I}} \,. \tag{3}$$

In the following, we consider the case of a monochromatic pump at frequency ν_p. The parametric spectum can be calculated by solving the coupled wave equations. The different frequency components of signal and idler are conveniently summarized by the frequency mismatch Δ, which is the residual mismatch between the pump electric field at frequency ν_p and the parametric resonance frequencies $\nu(\ell_\mathrm{s,i}, \mathrm{q_{s,i}}, \mathrm{p_{s,i}})$ normalized to the average bandwidth γ_si of signal and idler:

$$\Delta = \underbrace{\frac{2}{\gamma_\mathrm{s} + \gamma_\mathrm{i}}}_{=1/\gamma_\mathrm{si}} \cdot \left[\nu_\mathrm{p} - \nu(\ell_\mathrm{s}, \mathrm{q_s}, \mathrm{p_s}) - \nu(\ell_\mathrm{i}, \mathrm{q_i}, \mathrm{p_i})\right] \,. \tag{4}$$

For $\Delta = 0$, energy conservation allows for a maximally efficient conversion from the pump electric field frequency ν_p to the exact parametric resonance frequencies $\nu(\ell_\mathrm{s,i}, \mathrm{q_{s,i}}, \mathrm{p_{s,i}})$. In a typical experiment, the pump laser at frequency ν_p is locked to the resonance frequency of the pump mode ($\delta_\mathrm{p} = 0$) for a high intracavity power. The frequency mismatch Δ is then controlled via the temperature-dependence of the resonance frequencies $\nu(\ell_\mathrm{p,s,i}, \mathrm{q_{p,s,i}}, \mathrm{p_{p,s,i}})$.

The oscillation threshold for the optical pump power $P_\mathrm{th}(\delta_\mathrm{p}, \Delta)$ is then given by

$$P_\mathrm{th}(\delta_\mathrm{p}, \Delta) = \hbar 2\pi\nu_\mathrm{p} |\alpha_\mathrm{p}^\mathrm{in}|^2 = P_0 \cdot \left(1 + 4\delta_\mathrm{p}^2\right) \cdot \left(1 + \Delta^2\right) \,. \tag{5}$$

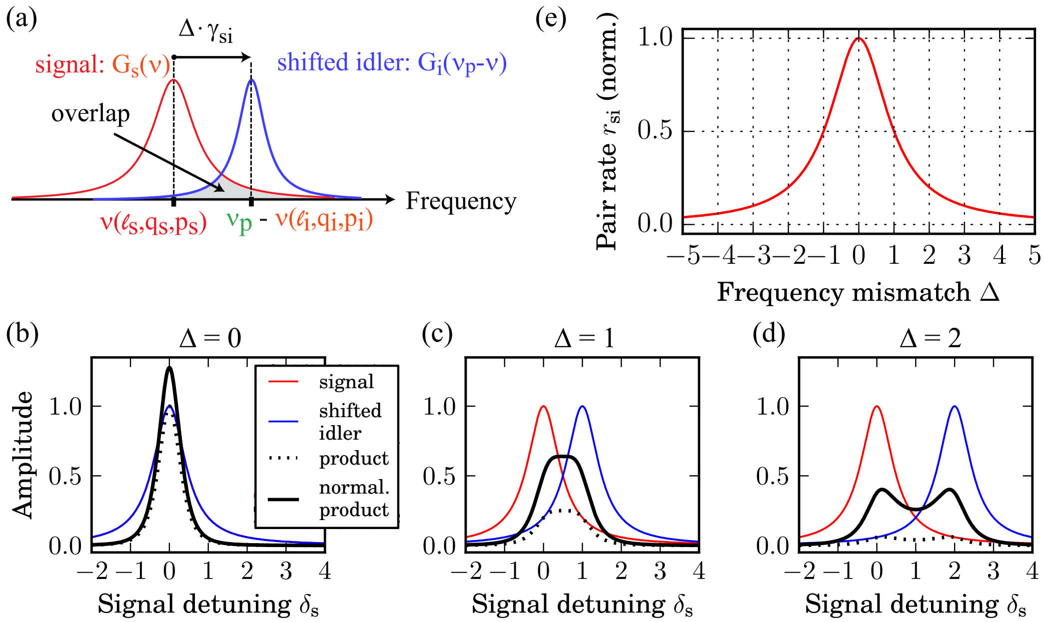

FIG. 1. Frequency spectrum of parametric down-conversion below threshold. (a) The overlap integral of the parametric response functions $\mathrm{G}_{\mathrm{s,i}}(\nu) = |\mathcal{G}_{\mathrm{s,i}}(\nu)|^2$ in frequency space (see Eq. ??) determines the parametric photon numbers $N_{\mathrm{s,i}}$ according to Eq. 7. (b-d) The product of the signal $\mathrm{G}_{\mathrm{s}}(\nu)$ and the shifted idler $\mathrm{G}_{\mathrm{i}}(\gamma_{\mathrm{si}}\Delta + \nu(\ell_{\mathrm{s}}, \mathrm{q}_{\mathrm{s}}, \mathrm{p}_{\mathrm{s}}) + \nu(\ell_{\mathrm{i}}, \mathrm{q}_{\mathrm{i}}, \mathrm{p}_{\mathrm{i}}) - \nu) = \mathrm{G}_{\mathrm{i}}(\nu_{\mathrm{p}} - \nu)$ response functions ($\gamma_{\mathrm{s}} = \gamma_{\mathrm{i}} = \gamma_{\mathrm{si}}$ in this example) are plotted for various frequency mismatches Δ (see Eq. 4). The black line shows the product normalized to a unity area. (e) The pair production rate $r_{\mathrm{si}} \propto \left(1 + \Delta^2\right)^{-1}$ (see Eq. 8) as a function on the frequency mismatch Δ is Lorentzian.

In the limit of low gain, i.e far below threshold, we can iteratively solve the coupled wave equation for pump, signal, and idler with first-order pertubation theory for $|\alpha_{\mathrm{p}} g|/\gamma_{\mathrm{s,i}} \ll 1$. The noise power $S_{\mathrm{s,i}}(\nu)$ of the parametric photons at frequency ν in the resonator follows from

$$\begin{aligned}\left\langle \hat{a}^{\dagger}_{\mathrm{s,i}}\left(\nu'\right) \cdot \hat{a}_{\mathrm{s,i}}\left(\nu\right) \right\rangle &= S_{\mathrm{s,i}}(\nu) \cdot \delta(\nu - \nu') \\ &= \frac{2}{\pi \gamma_{\mathrm{s,i}}} \frac{\mathrm{N_p}}{\mathrm{N_{th}}} G_{\mathrm{s,i}}\left(\nu + \nu(\ell_{\mathrm{s,i}}, \mathrm{q}_{\mathrm{s,i}}, \mathrm{p}_{\mathrm{s,i}})\right) G_{\mathrm{i,s}}\left(\gamma_{\mathrm{si}}\Delta + \nu(\ell_{\mathrm{i,s}}, \mathrm{q}_{\mathrm{i,s}}, \mathrm{p}_{\mathrm{i,s}}) - \nu\right). \qquad (6)\end{aligned}$$

Vacuum fluctuations[2] $\hat{\mathrm{f}}_{\mathrm{i}}\left(\nu\right)$ of the idler mode drive the signal noise power, and visa verse. By integrating over the full frequency space (see Fig. ??), we get the number $N_{\mathrm{s,i}}$ of signal and idler photons in the resonator:

$$N_{\mathrm{s,i}} = \int S_{\mathrm{s,i}}(\nu)\,\mathrm{d}\nu = \frac{1}{\gamma_{\mathrm{s,i}}} \frac{\mathrm{N_p}}{\mathrm{N_{th}}} \frac{\gamma_{\mathrm{s}}\gamma_{\mathrm{i}}}{\gamma_{\mathrm{si}}} \frac{1}{1 + \Delta^2}\,. \qquad (7)$$

In parametric down-conversion, signal and idler are produced in pairs. Hence, we can define the rate of photon pair production r_{si} in the resonator:

$$r_{\mathrm{si}} = 2\pi \frac{\gamma_{\mathrm{s}}\gamma_{\mathrm{i}}}{\gamma_{\mathrm{s}} + \gamma_{\mathrm{i}}} \frac{P_{\mathrm{p}}^{\mathrm{in}}}{P_{\mathrm{th}}\left(\delta_{\mathrm{p}}, \Delta\right)}\,, \quad \left(P_{\mathrm{p}}^{\mathrm{in}} \ll P_{\mathrm{th}}\right). \qquad (8)$$

The simultaneous generation of signal and idler favors a description of parametric down-conversion in terms of temporal correlation functions [3–10]. The time-dependent annihilation and creation operators stem from the Fourier back-transformation $\int \mathrm{d}\nu\, \hat{a}(\nu)\, \mathrm{e}^{i2\pi\nu t} \to \hat{a}(t)$ of Eq. ?? and the input-output relation given by Eq. ??. The normalized cross-correlation function $\mathrm{g}^{(2)}_{\mathrm{si}}(\tau)$ between signal and idler in terms of the detection time difference $\tau = t_{\mathrm{s}} - t_{\mathrm{i}}$ is given by

$$
\begin{aligned}
\mathrm{g}_{\mathrm{si}}^{(2)}(\tau) &= \frac{\left\langle \hat{a}_{\mathrm{s}}^{\dagger\mathrm{out}}(t)\,\hat{a}_{\mathrm{i}}^{\dagger\mathrm{out}}(t+\tau)\,\hat{a}_{\mathrm{s}}^{\mathrm{out}}(t)\,\hat{a}_{\mathrm{s}}^{\mathrm{out}}(t+\tau)\right\rangle}{\left\langle \hat{a}_{\mathrm{s}}^{\dagger\mathrm{out}}(t)\,\hat{a}_{\mathrm{s}}^{\mathrm{out}}(t)\right\rangle \left\langle \hat{a}_{\mathrm{i}}^{\mathrm{out}}(t)\,\hat{a}_{\mathrm{i}}^{\dagger\mathrm{out}}(t)\right\rangle} \\
&= 1 + \frac{P_{\mathrm{th}}(\delta_{\mathrm{p}},\Delta)}{P_{\mathrm{p}}^{\mathrm{in}}} \cdot \begin{cases} \exp(2\pi\gamma_{\mathrm{i}}\tau) & \text{for } \tau < 0 \\ \exp(-2\pi\gamma_{\mathrm{s}}\tau) & \text{for } \tau > 0 \end{cases} .
\end{aligned} \tag{9}
$$

The cross-correlation is usually measured by direct detection of signal and idler.

III. PARAMETRIC DOWN-CONVERSION DEPENDING ON THE PHASE-MATCHING TEMPERATURE

IV. LOCKING THE PHASE-MATCHING TEMPERATURE

V. REFERENCES

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