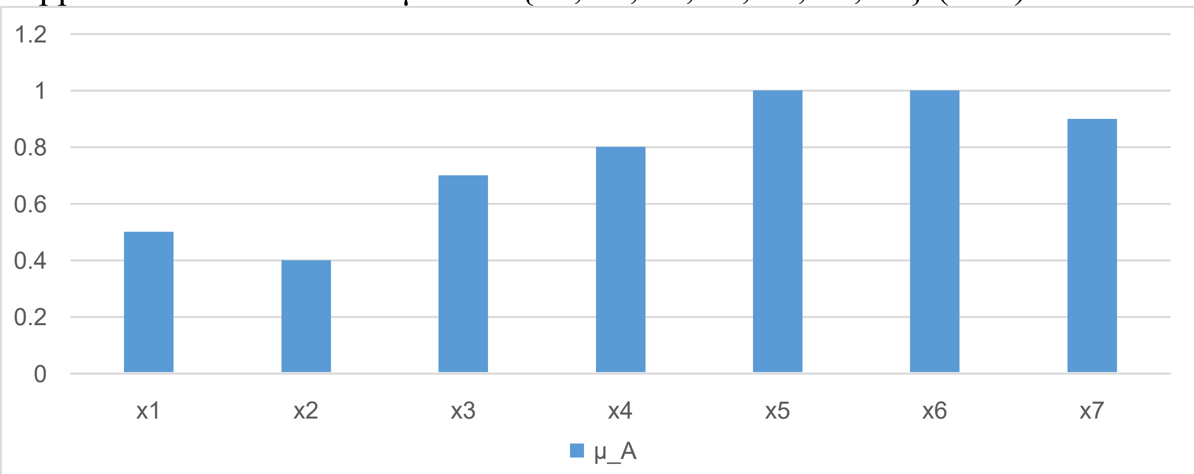
Will these sets be normal? Subnormal? Determine their carriers.

Given fuzzy sets (discrete)

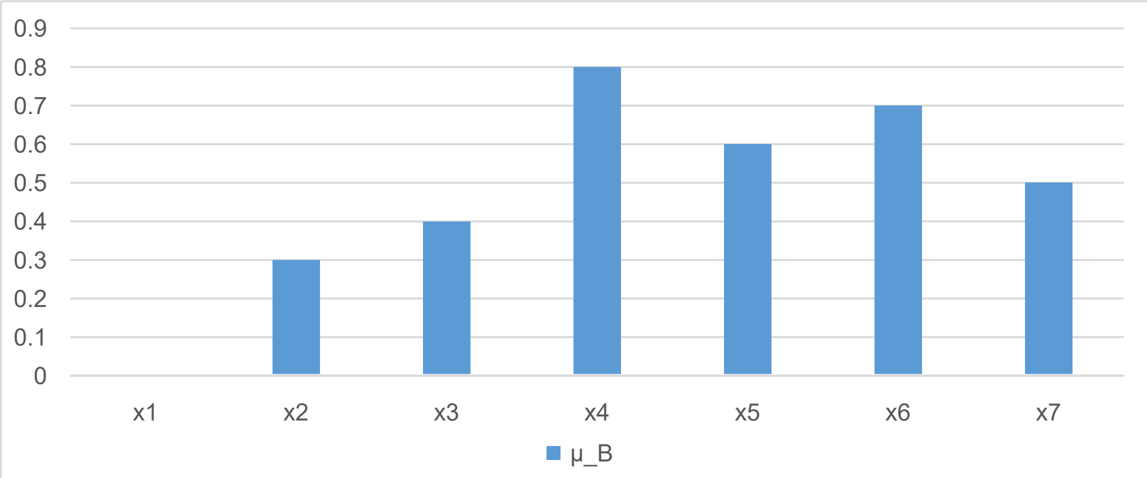
$$A = \begin{array}{c|c|c|c|c|c|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline 0{,}5 & 0{,}4 & 0{,}7 & 0{,}8 & 1 & 1 & 0{,}9 \end{array};$$

Normal? — Yes. There are elements (x5 and x6) with μ=1.
Subnormal? — No. (because normal)
Support: all elements with μ>0 → {x1, x2, x3, x4, x5, x6, x7} (all 7)



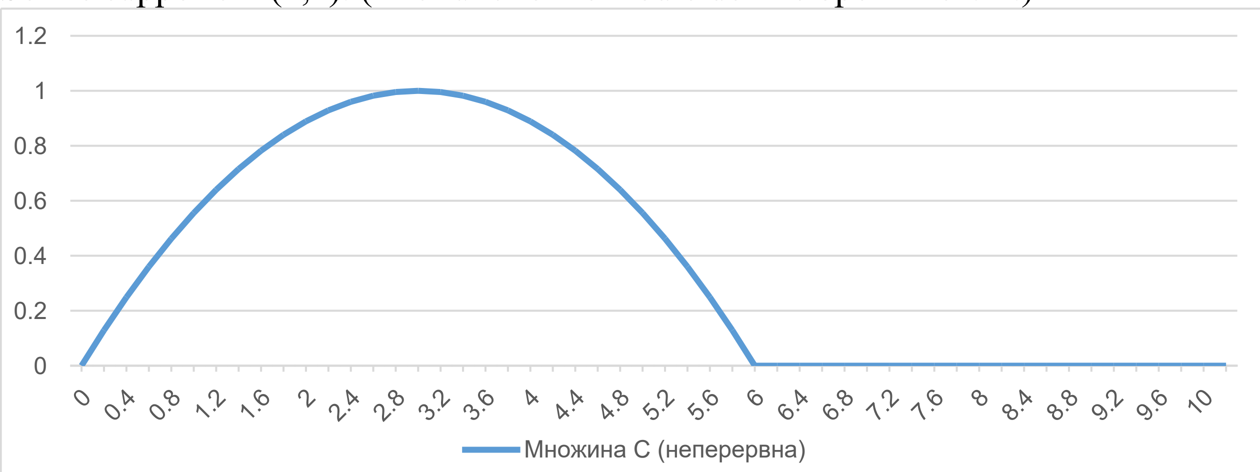
$$B = \begin{array}{c|c|c|c|c|c|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline 0 & 0{,}3 & 0{,}4 & 0{,}8 & 0{,}6 & 0{,}7 & 0{,}5 \end{array};$$

Normal? — No. Maximum = 0.8 < 1.
Subnormal? - Yes.
Support: elements with μ>0 → {x2, x3, x4, x5, x6, x7} (x1 has 0 → not included).



$$C:\quad \mu_C(x)=\begin{cases}-\frac{1}{9}(x-3)^2+1, & \text{якщо } x\in(0;6);\\ 0, & \text{якщо } x\notin(0;6);\end{cases}$$

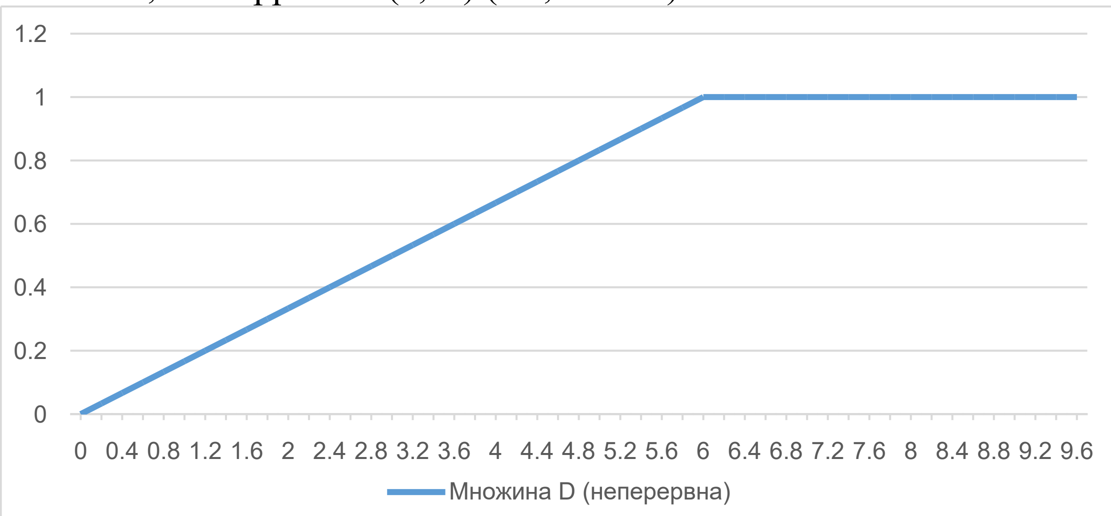
$\mu_C(x) = -\left(\frac{1}{9}\right)(x-3)^2 + 1$for $x \in (0,6)$, and 0 outside this interval.
This is a parabola with vertex at x=3: $\mu_C(3)=1$.
Normal? — Yes. It has the value 1 at x=3.
Subnormal? - No.
Support: all x, where $\mu_C(x)>0$. Solution:$\frac{1}{9}(x-3)^2 + 1 > 0 \Leftrightarrow (x-3)^2 < 9 \Leftrightarrow x \in (0,6)$
So the supports = (0,6). (The function is 0 outside this open interval.)



$$D:\quad \mu_D(x)=\begin{cases}0, & \text{якщо } x\le 0;\\ \frac{1}{6}x, & \text{якщо } x\notin(0;6);\\ 1, & \text{якщо } x\ge 6.\end{cases}$$

$\mu_D(x) = 0$ for $x \le 0$;
$\mu_D(x) = x/6$ for $x \in (0,6)$; $\mu_D(x) = 1$ for $x \ge 6$.
The maximum $\mu_D = 1$ (for all $x \ge 6$).
Normal? — Yes. (has a value of 1 at x=6 and onwards)

Subnormal? - No.
Support: all x with $\mu_D(x) > 0$. Here it is:
for (0,6): $x/6 > 0 \rightarrow$ all $x \in (0,6)$;
for $x \geq 6$: $\mu=1>0 \rightarrow$ all $x \geq 6$.
Therefore, the supports = $(0, \infty)$ (i.e., all x>0).



Determine the intersection and union of the sets: a) A and B, b) C and D from problem 1 (using three definitions).

Definition 1.6. The union of fuzzy subsets A and B is called the fuzzy subset AUB , the membership function of which has the following form:

$$\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\}, x\in E$$

By merging fuzzy subsets$\mu_{A\cup B} = \sup \mu_{A_y}(x, y), x\in E$

Definition 1.6, a. The union of fuzzy subsets A and B can be also be determined using a finite sum of their membership functions, and exactly: $\mu_{A\cup B}(x) = \begin{cases} 1, \text{якщо } \mu_A(\text{x}) + \mu_B(\text{y}) \geq 1 \\ \mu_A(\text{x}) + \mu_B(\text{y}) \text{ в інших випадках} \end{cases}$

By merging fuzzy subsets$\mu_{A\cup B} = \min\{1, \mu_A(\text{x}) + \mu_B(\text{x})\}$

Definition 1.6, b. The union of fuzzy sets can be found also through their algebraic sum, i.e. the union of the fuzzy sets A and B is a fuzzy set with the following membership function:
$\mu_{A\cup B}(x) = \mu_A(\text{x}) + \mu_B(x)\text{-}\mu_A(x) * \mu_B(x)$

Definition 1.7. The intersection of fuzzy subsets A and B of the universal set E is called a fuzzy subset with the function belonging to the following form:$\mu_{A\cap B}(x) = min\{\mu_A(x), \mu_B(x)\}, x\in E$
Intersection of fuzzy subsets $\mu_{A\cap B} = \inf \mu_{A_y}(x, y), x\in E$

Definition 1.7, a. The intersection of fuzzy subsets A and B is the bounded product of their membership functions, i.e.

$$\mu_{A\cap B}(x) = min\{0,\ \mu_B(x) + \mu_A(x) - 1\}, x \in E$$

Definition 1.7, b. The intersection of fuzzy sets A and B is called a fuzzy set whose membership function is equal to the algebraic product membership functions of given sets, i.e. $\mu_{A\cap B}(x) = \mu_A(x) * \mu_B(x)$

x	**μA**	**μB**	**1.7(inter) min**	**1.6(union) max**	**1.7 a (inter)**	**1.6 a (union)**	**1.7 b (inter)**	**1.6 b (union)**
x1	0.5	0.0	0.0	0.5	0.0	0.5	0.00	0.50
x2	0.4	0.3	0.3	0.4	0.0	0.7	0.12	0.58
x3	0.7	0.4	0.4	0.7	0.1	1.0	0.28	0.82
x4	0.8	0.8	0.8	0.8	0.6	1.0	0.64	0.96
x5	1.0	0.6	0.6	1.0	0.6	1.0	0.60	1.00
x6	1.0	0.7	0.7	1.0	0.7	1.0	0.70	1.00
x7	0.9	0.5	0.5	0.9	0.4	1.0	0.45	0.95

Decompose the fuzzy sets A and B (from task 1) into level sets.

Level decomposition of the set A

α	**$A_a = \{ x_i : \mu\_A(x_i) \geq \alpha \}$**
1.0	$\{ x_5, x_6 \}$
0.9	$\{ x_5, x_6, x_7 \}$
0.8	$\{ x_4, x_5, x_6, x_7 \}$
0.7	$\{ x_3, x_4, x_5, x_6, x_7 \}$
0.5	$\{ x_1, x_3, x_4, x_5, x_6, x_7 \}$
0.4	$\{ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \}$

Decomposition of the set B into levels

α	**$B_a = \{ x_i : \mu\_B(x_i) \geq \alpha \}$**
0.8	$\{ x_4 \}$
0.7	$\{ x_4, x_6 \}$
0.6	$\{ x_4, x_5, x_6 \}$
0.5	$\{ x_4, x_5, x_6, x_7 \}$
0.4	$\{ x_3, x_4, x_5, x_6, x_7 \}$
0.3	$\{ x_2, x_3, x_4, x_5, x_6, x_7 \}$

Example based on the examples given on page 23

A manufacturing enterprise that has an old ventilation system in the workshop. Management must decide whether the equipment needs to be replaced urgently or whether it can continue to operate for some time without risk to the health of employees. There are two types of information for decision making: probabilistic estimates (obtained from rare measurements of the concentration of harmful substances) and fuzzy expert estimates (formed on the basis of the experience of occupational safety engineers). Over the past 5 years, single measurements have been carried out, and according to statistics: In 80% of cases, the level of air pollution was below the permissible norm, and in 20% it was exceeded. The risk is there, but small. However, this data is very limited - almost no measurements were made. Engineers who work on the shop floor regularly assess the situation more accurately. They believe that: "generally safe" conditions with a degree of membership of 0.7, conditions "require modernization in the near future" - 0.9, "Critical and dangerous" conditions are only 0.1 . Experts are not sure that everything is bad, but they clearly feel that the system is working at the limit of its capabilities.

Since fuzzy sets allow us to take into account the fuzziness of human assessments, a membership function is formed for the fuzzy set “needs replacement.” It gradually increases from 0.5 for weak arguments (minor noise, old equipment) to 1 for situations where the system clearly fails (employee complaints, odors, local exceedances).

So, probability theory is unreliable here due to lack of data. The fuzzy approach allows you to express expert knowledge and shows that the situation is almost certainly approaching a problem. This also leads to the recommendation to carry out a planned replacement of the ventilation system, although statistics do not indicate an urgent danger. The fuzzy data reflects the real state: the system is not yet in an emergency, but is already clearly outdated.