MECH 544
Robotics

Instructor: Prof. Cagatay Basdogan

PROJECT 01

In this project, you will derive the forward and inverse kinematics of the PHANToM (Model 1.0) haptic interface (see the details below). The PHANToM haptic device enables 3D touch interactions with virtual objects ($L_1 = 14\ cm,\ L_2 = 14\ cm$).

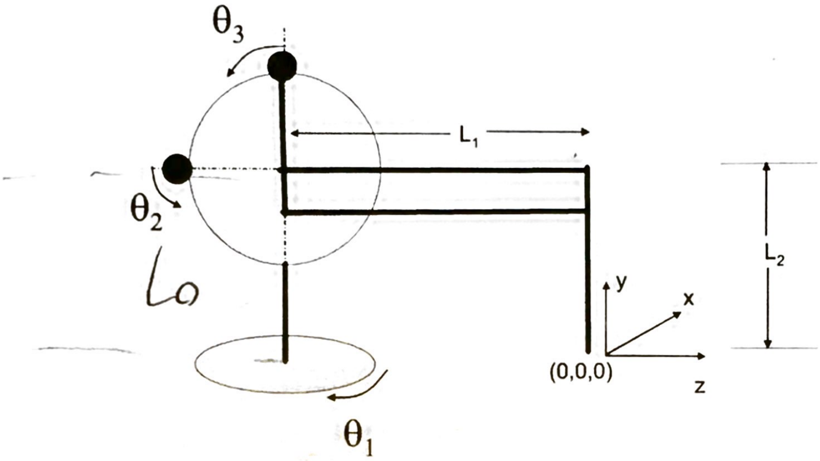
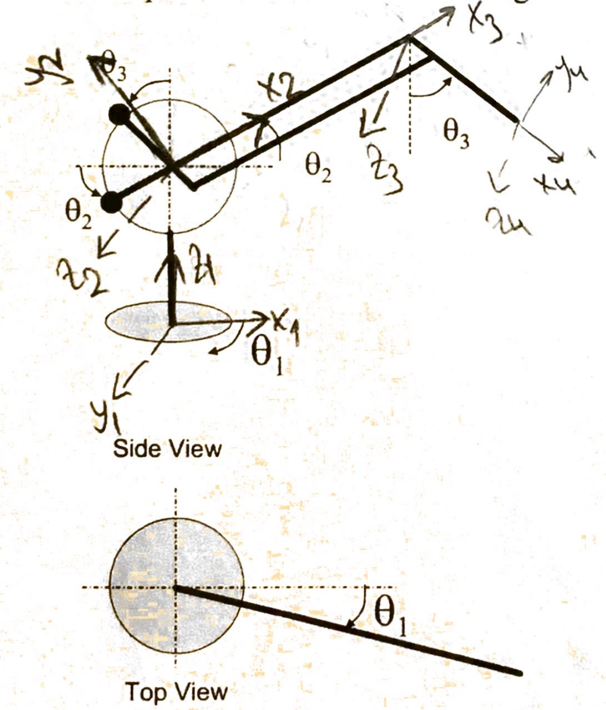


Figure 1. The equilibrium position for PHANToM haptic device. Note that the encoders read end-effector position with respect to the coordinate frame given in the figure.



link	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1^*
2	0	$\pi/2$	L_0	θ_2^*
3	L_1	0	0	θ_3^*
4	L_2	0	0	$-\pi/2$

$$ {}^{i-1}_{\ \ i}T = \begin{matrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{matrix} $$

Figure 2. The top and side views of the PHANToM haptic device.

$$ {}^{0}_{4}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, {}^{3}_{4}T $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & L_0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_1 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} 0 & 1 & 0 & L_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

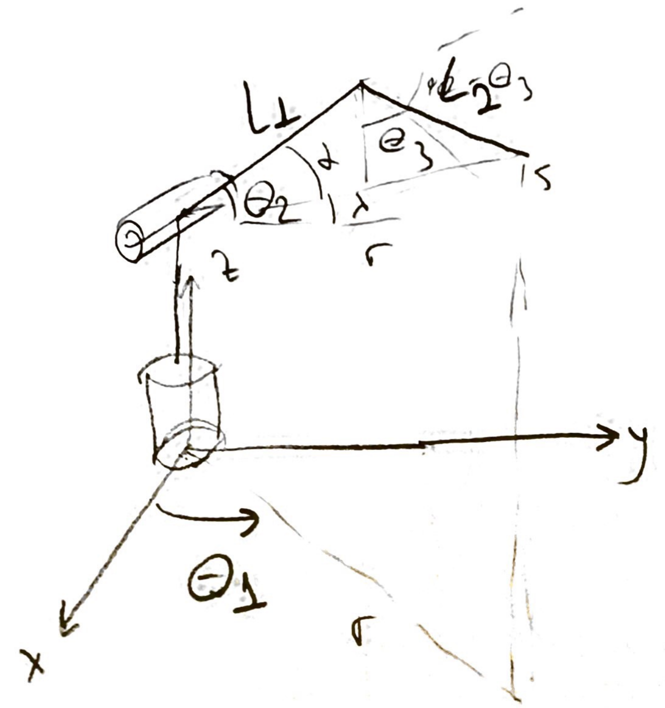
$$
{}^{0}_{1}T\,{}^{1}_{2}T = \begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 & L_0 s\theta_1 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 & -L_0 c\theta_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T\,{}^{3}_{4}T = \begin{bmatrix} 0 & c\theta_3 & 0 & L_2 c\theta_3 + L_1 \\ 0 & s\theta_3 & 0 & L_1 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{4}T = \begin{bmatrix} 0 & c\theta_1 c\theta_2 c\theta_3 - c\theta_1 s\theta_2 s\theta_3 & s\theta_1 & \begin{pmatrix} L_1 c\theta_1 c\theta_2 + L_2 c\theta_1 c\theta_2 c\theta_3 \\ - L_1 c\theta_1 s\theta_2 s\theta_3 + L_0 s\theta_1 \end{pmatrix} \\ 0 & s\theta_1 c\theta_2 c\theta_3 - s\theta_1 s\theta_2 s\theta_3 & -c\theta_1 & \begin{pmatrix} L_1 s\theta_1 c\theta_2 + L_2 c\theta_3 s\theta_1 c\theta_2 \\ - L_1 s\theta_3 s\theta_2 s\theta_3 - L_0 c\theta_1 \end{pmatrix} \\ 0 & s\theta_2 c\theta_3 + c\theta_2 s\theta_3 & 0 & L_1 s\theta_2 + L_2 c\theta_3 s\theta_2 + L_1 s\theta_3 c\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{4}T = \begin{bmatrix} 0 & c\theta_1 (c\theta_1 \theta_2) & 0 & c\theta_1 (L_1 c\theta_2 - L_1 s\theta_2 s\theta_3 + L_2 c\theta_2 c\theta_3) \\ & & & + L_0 s\theta_1 \\ 0 & s\theta_1 (c\theta_1 \theta_2) & 0 & s\theta_1 (L_1 c\theta_2 - L_1 s\theta_2 s\theta_3 + L_2 c\theta_2 c\theta_3) \\ & & & - L_0 c\theta_1 \\ 0 & s\theta_1 \theta_2 & 0 & L_1 s\theta_2 + L_1 c\theta_2 s\theta_3 + L_2 s\theta_2 c\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Inverse Kinematic



$$\theta_1 = atan2(y_c, x_c)$$

geriden gelemiyor diye düşünüyorum

$$r = \sqrt{x_c^2 + y_c^2} \qquad s = z_c - L_0$$

$$r^2 + s^2 = L_1^2 + L_2^2 + 2L_1 L_2 \cos(90 - \theta_3)$$

$$\sin\theta_3 = \frac{r^2 + s^2 - L_1^2 - L_2^2}{2.L_1 L_2} = D$$

$$\cos\theta_3 = -\sqrt{1 - D^2}$$

Sadece elbow up yapısından dolayı

$$\theta_3 = atan2(D, -\sqrt{1-D^2})$$

$$\theta_2 = \alpha + \lambda$$

$\lambda \rightarrow atan2(s, r)$

$\alpha \rightarrow atan2(L_2.\cos\theta_3, L_1 + L_2.\sin\theta_3)$

$$\theta_2 = atan2(s, r) + atan2(L_2.\cos\theta_3, L_1 + L_2.\sin\theta_3)$$