



	$\alpha_{i-1}$	$a_{i-1}$	$\theta_i$	$d_i$
1	0	0	$\theta_1$	0
2	$\pi/2$	0	$\theta_2$	0
3	0	$L_1$	$\theta_3$	0
4	0	$L_2$	0	0

$${}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & L_1 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4T = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_4T = \begin{bmatrix} c_1 c_2 c_3 - c_1 s_2 s_3 & -c_1 c_2 s_3 - c_1 s_2 c_3 & s_1 & (c_1 c_2 c_3 - c_1 s_2 s_3) L_2 + c_1 c_2 L_1 \\ s_1 c_2 c_3 - s_1 s_2 s_3 & -s_1 c_2 s_3 - s_1 s_2 c_3 & -c_1 & (s_1 c_2 c_3 - s_1 s_2 s_3) L_2 + s_1 c_2 L_1 \\ s_2 c_3 + c_2 s_3 & -s_2 s_3 + c_2 c_3 & 0 & (s_2 c_3 + c_2 s_3) L_2 + s_2 L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_4 T = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & \underbrace{c_1 c_{23} L_2 + c_1 c_2 L_1}_{x} \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & \underbrace{s_1 c_{23} L_2 + s_1 c_2 L_1}_{y} \\ s_{23} & c_{23} & 0 & \underbrace{s_{23} L_2 + s_2 L_1}_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = c_1 c_{23} L_2 + c_1 c_2 L_1$$

$$y = s_1 c_{23} L_2 + s_1 c_2 L_1$$

$$x^2 + y^2 = \underbrace{(c_1^2 c_{23}^2 L_2^2 + s_1^2 c_{23}^2 L_2^2)}_{c_{23}^2 L_2^2} + \underbrace{(c_1^2 c_2^2 L_1^2 + s_1^2 c_2^2 L_1^2)}_{c_2^2 L_1^2} + 2 c_1 c_{23} L_2 c_1 c_2 L_1 + 2 s_1 c_{23} L_2 s_1 c_2 L_1$$

$\rightarrow 2 c_1^2 c_{23} c_2 L_1 L_2$   
 $\rightarrow 2 s_1^2 c_{23} c_2 L_1 L_2$

$$x^2 + y^2 = c_{23}^2 L_2^2 + c_2^2 L_1^2 + 2 c_2 c_{23} L_1 L_2$$

$$z^2 = s_{23}^2 L_2^2 + s_2^2 L_1^2 + 2 s_2 s_{23} L_1 L_2$$

$$x^2 + y^2 + z^2 = L_1^2 + L_2^2 + 2 L_1 L_2 (\cos(\theta_2 + \theta_3 - \theta_2))$$

$$\cos \theta_3 = \frac{x^2 + y^2 + z^2 - L_1^2 - L_2^2}{2 L_1 L_2} = K$$

$$\sin \theta_3 = \pm \sqrt{1 - K^2}$$

$$\theta_3 = \text{Atan2}(\pm \sqrt{1 - K^2}, K)$$

→ 2 solutions

$$z = (s_2 c_3 + c_2 s_3) L_2 + s_2 L_1$$

$$\underbrace{z}_c = s_2 (\underbrace{c_3 L_2 + L_1}_b) + c_2 (\underbrace{s_3 L_2}_a)$$

$$\theta_2 = \text{Atan2}(b, a) \pm \text{Atan2}(\sqrt{a^2 + b^2 - c^2}, c)$$

$$\theta_2 = \text{Atan2}(KL_2 + L_1, \pm \sqrt{1-K^2} L_2) \pm \text{Atan2}(\sqrt{L_1^2 + L_2^2 + 2KL_1L_2}, z)$$

→ 2 solutions (maybe more!)

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$$\cos \theta_1 = \frac{x}{c_{23} L_2 + c_2 L_1}$$

$$\sin \theta_1 = \frac{y}{c_{23} L_2 + c_2 L_1}$$

$$\theta_1 = \text{Atan2}(y, x)$$

→ 1 solution