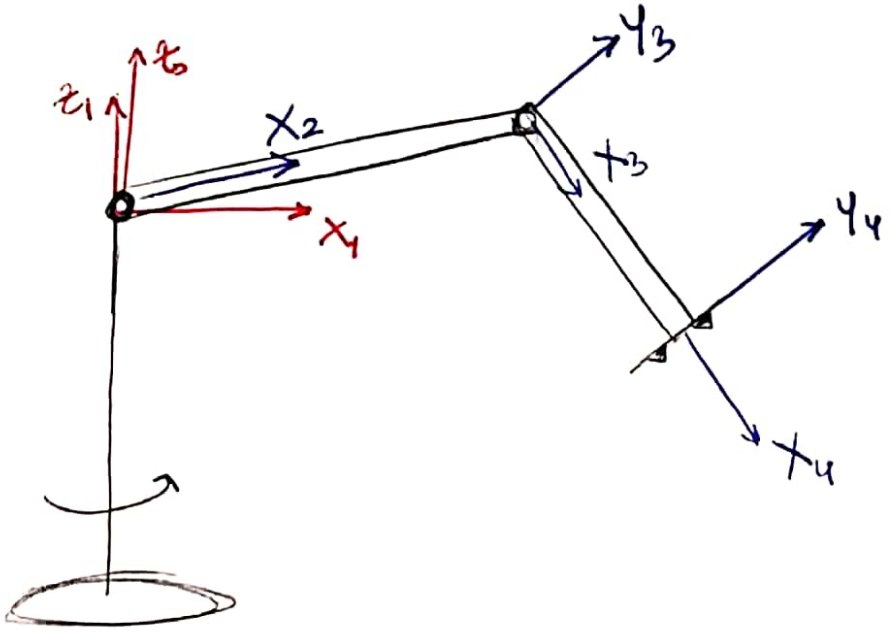
Forward Kinematics



DH Table

	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	$\pi/2$	0	θ_2	0
3	0	L_1	θ_3	0
4	0	L_2	0	0

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$$ {}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & L_1 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{4}T = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1s_2c_3 & s_1 & (c_1c_2c_3 - c_1s_2s_3)L_2 + c_1c_2L_1 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1s_2c_3 & -c_1 & (s_1c_2c_3 - s_1s_2s_3)L_2 + s_1c_2L_1 \\ s_2c_3 + c_2s_3 & -s_2s_3 + c_2c_3 & 0 & (s_2c_3 + c_2s_3)L_2 + s_2L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{4}T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & \underbrace{c_1c_{23}L_2 + c_1c_2L_1}_{x} \\ s_1c_{23} & -s_1s_{23} & -c_1 & \underbrace{s_1c_{23}L_2 + s_1c_2L_1}_{y} \\ s_{23} & c_{23} & 0 & \underbrace{s_{23}L_2 + s_2L_1}_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (*) $$

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Inverse Kinematics (Mathematical Solution)

From (1st), we know that:

$$x = c_1 c_{23} L_2 + c_1 c_2 L_1$$

$$y = s_1 c_{23} L_2 + s_1 c_2 L_1$$

Take square of both sides:

$$x^2 + y^2 = \boxed{c_1^2 c_{23}^2 L_2^2 + s_1^2 c_{23}^2 L_2^2} + \boxed{c_1^2 c_2^2 L_1^2 + s_1^2 c_2^2 L_1^2} + 2 c_1 c_{23} L_2 c_1 c_2 L_1 + 2 s_1 c_{23} L_2 s_1 c_2 L_1$$

$$\boxed{c_{23}^2 L_2^2} \qquad \boxed{c_2^2 L_1^2}$$

$2 c_1^2 c_{23} c_2 L_1 L_2$

$2 s_1^2 c_{23} c_2 L_1 L_2$

$$x^2 + y^2 = \boxed{c_{23}^2 L_2^2} + \boxed{c_2^2 L_1^2} + 2 c_2 c_{23} L_1 L_2$$

$$z^2 = \boxed{s_{23}^2 L_2^2} + \boxed{s_2^2 L_1^2} + 2 s_2 s_{23} L_1 L_2$$

$$\boxed{L_2^2} \qquad \boxed{L_1^2}$$

$$x^2 + y^2 + z^2 = L_1^2 + L_2^2 + 2 L_1 L_2 \cos(\theta_2 + \theta_3 - \theta_2)$$

$$\cos\theta_3 = \frac{x^2 + y^2 + z^2 - L_1^2 - L_2^2}{2 L_1 L_2} = K$$

$$\sin\theta_3 = \pm\sqrt{1 - K^2}$$

$$\boxed{\theta_3 = \text{Atan2}\left(-\sqrt{1-K^2},\ K\right)}$$

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$$z = (s_2 c_3 + c_2 s_3) L_2 + s_2 L_1$$

$$z = s_2 (c_3 L_2 + L_1) + c_2 (s_3 L_2)$$

c , b , a

$$\theta_2 = \text{Atan2}(b, a) \mp \text{Atan2}(\sqrt{a^2 + b^2 - c^2}, c)$$

$$\theta_2 = \text{Atan2}(kL_2 + L_1, -\sqrt{1-k^2}\, L_2) + \text{Atan2}(\sqrt{L_1^2 + L_2^2 + 2kL_1L_2}, \pm)$$

Finally,

$$\cos\theta_1 = \frac{x}{c_{23} L_2 + c_2 L_1} \qquad \sin\theta_1 = \frac{y}{c_{23} L_2 + c_2 L_1}$$

$$\theta_1 = \text{Atan2}(y, x)$$

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