MECH 544 - Robotics

Project II

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1 Introduction

In daily life, industry, and the scientific exploration processes, the robots, especially the robotic arms, plays an important role in terms of efficiency and accuracy. It is a relatively rigid truth that the forward and inverse kinematics of a robotic arm can be well identified via using explicitly-structured conventions, such as *Denevit-Hartenberg Convention*. However, there are still ambiguous points which are evolving and being developed each day. One of these point is the subject of motion planning. The motion planning task, in summarized form, can be defined as the process of the determination of the route of the robotic arm end effector for a given start point and end point under a constrained environment. In a process similar to this, there are a lot of factor that should be considered and traded-off. Therefore, in literature, there are various methods that distributes importance to different factors.

In this project, the aim is to plan the motion of a robotic arm end effector under different obstacle conditions and various spatial properties, such as link lengths and start and end points. In Figure 1, the representative image that describes the possible physical configuration and the environmental constraint can be investigated.

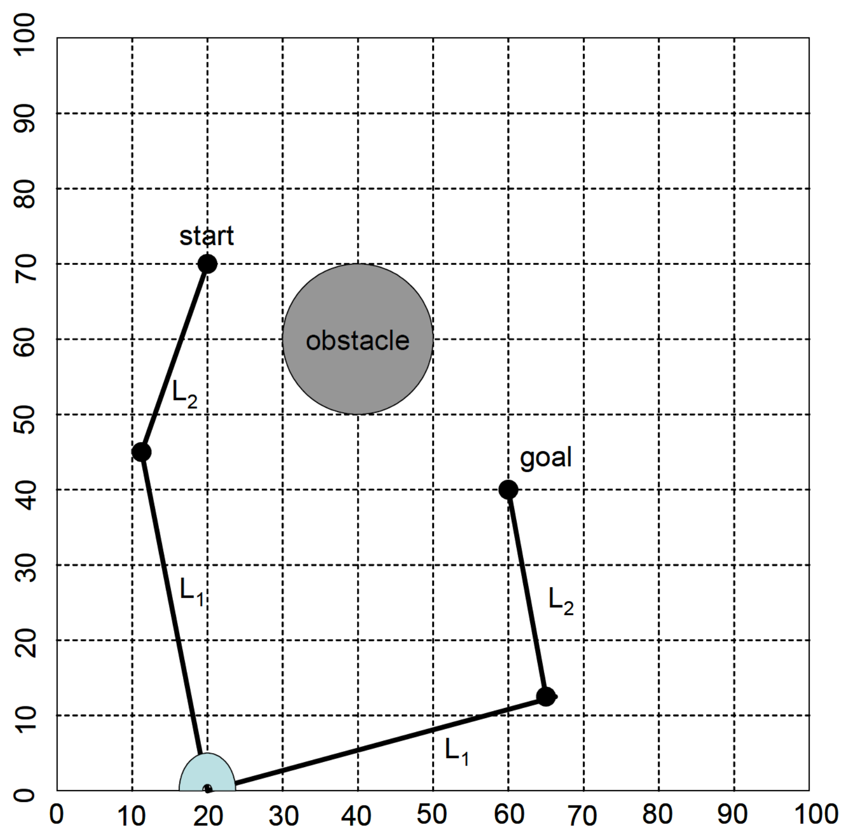

Figure 1: Representative Visualization of the Possible Configuration

2 Configuration Space Construction

In the motion planning problems, the serial manipulators should not be in collision with the obstacles and its components. During changing joint angles, the operator has to calculate the end-effector's and links position using forward kinematics. After the getting position in the Cartesian space, the operator needs to check the positions from forward kinematics, one by one. However, the configuration space provides the way to represents all of the possible collision configurations as a matrix.

As seen in figure 1, the robot has a 2 degree of freedoms, and the environment has a obstacle within the limited workspace. It means that configuration space provides possible configurations without touching the obstacle and without exceeding the boundaries to the path planning algorithm. To be able to construct the configuration space, the forward kinematics of the robot is needed.

2.1 Forward Kinematics

The robot consists of two links, and from joint angles, it is straightforward to obtain x and y positions using a geometric approach. Note that the location of base of the robot is not at the exact origin. In the following equations for forward kinematics analysis, *L1* and *L2* represents the links length.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_{\text{base}} \\ y_{\text{base}} \end{bmatrix} + L_1 \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} + L_2 \begin{bmatrix} \cos(\alpha-\beta) \\ \sin(\alpha-\beta) \end{bmatrix}$$

2.2 Inverse Kinematics

The Inverse Kinematics can be derived using geometrical approach and algebraic approach. In this study, algebraic approach is selected as a inverse kinematics solver. Initially, the offset from the base of the robot is neglected to obtain purer equations.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} L_1\cos(\alpha) + L_2\cos(\alpha-\beta) \\ L_1\sin(\alpha) \; + L_1\sin(\alpha-\beta) \end{bmatrix}$$

If the taking squares and sum of the rows of the equations, the following equation is obtained.

$$x^2 + y^2 = (L_1)^2 + (L_2)^2 \; + 2L_1L_2\cos(\beta)$$

$$D = \frac{(-L_1^2 - L_2^2 + x^2 + y^2)}{(2L_1L_2)} = \cos(\beta)$$

$$\sin(\beta) = \pm\sqrt{1-D^2}$$

$$\beta = -\text{atan2}\left(\pm\frac{\sqrt{(1-D^2)}}{D}\right)$$

From the configuration of the robot, β angle should be the opposite direction. Also, there is two solutions for the inverse kinematics, one called elbow up and the another one called elbow down solutions. For the obtaining α angle, some of the trigonometric transformations are needed, and finally following equations are obtained.

$$a = L_2 \sin(\beta)$$

$$b = L_1 + L_2 \cos(\beta)$$

$$c = y$$

$$\alpha = \operatorname{atan2}\left(\frac{c}{\sqrt{a^2 + b^2 - c^2}}\right) - \operatorname{atan2}\left(\frac{a}{b}\right)$$

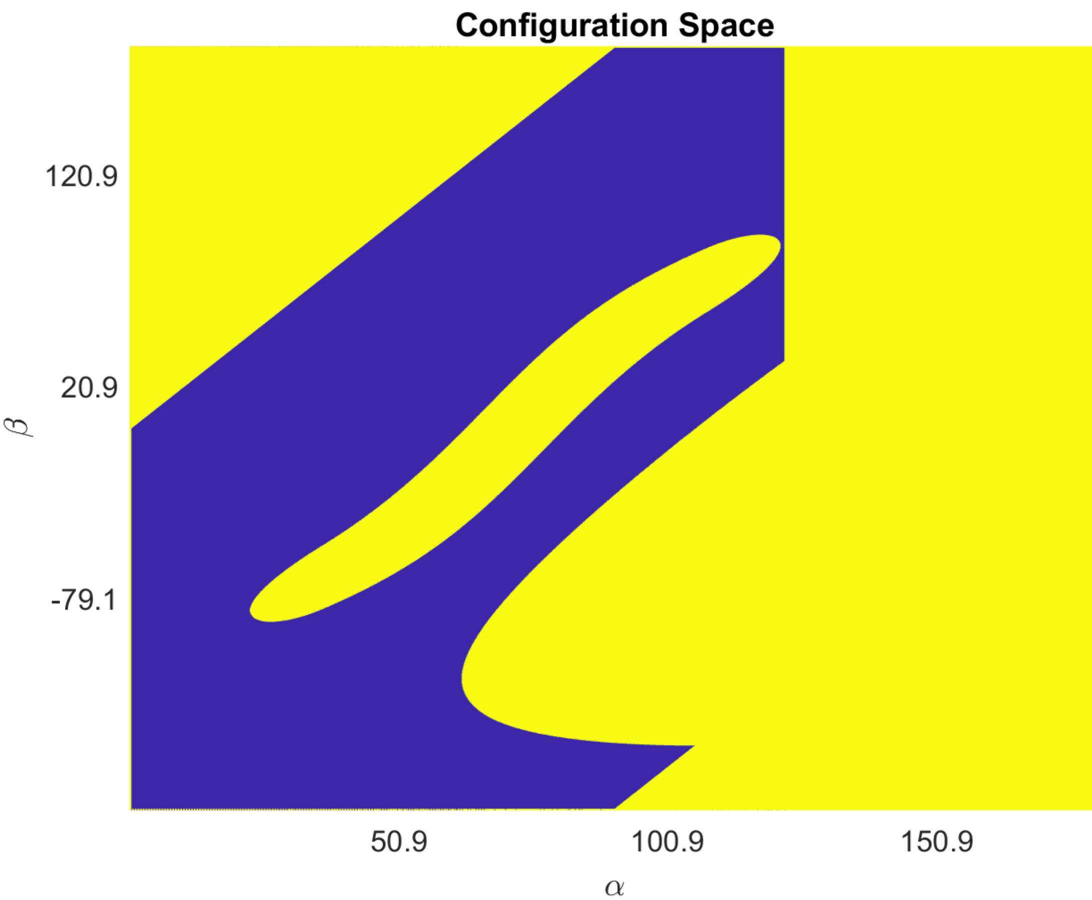

Figure 2: Configuration Space

After the deriving forward and inverse kinematics equations, the configuration space is ready to construct. The statements of the construction of the configuration space are listed as follows:

- The end-effector and the links cannot exceed the boundaries: for x direction [0 - 100], for y direction [0 - 100].

- The end-effector and the links cannot collide with the obstacle.

As seen in figure 2, all of the available configurations were defined into the configuration space. The yellow dots stands for the obstacles, and the purple dots for the available configurations to the path planning. The middle yellow dot cluster represents the possible collisions with the obstacle defined in the assessment description.

3 Potential Field Extraction

As was discussed in Section 1, there are multiple ways of motion planning. In this project, one of the methods used to plan the motion is the *Potential Field Method*. In potential field method, the basic idea is to model the environment such that the obstacles will behave as a repulsive potential and the final configuration will behave as a attractive potential trying to pull the robotic arm end effector to itself. The physical intuition behind this method allows one to construct a force field that will be affected on the end effector of the robotic arm as if it is a point mass that swims in the ocean of the potential field.

As might be guessed, there can be multiple ways of the definition of the potentials. In this project, the conventions, that will be discussed in Section 3.1 and Section 3.2, is used.

3.1 Attractive Field Potential

In attractive field potential, the idea is to model the *final configuration* as a base of the spring, and model the end effector as the free end of the same spring. Due to this analogy, one can write the attractive potential field as the following:

$$P_{attractive} = \frac{1}{2}\zeta\rho^2_{final}(\mathbf{q})$$

where $\mathbf{q}$ represents the instantaneous position of the end effector in the *configuration space*. Then, it can be defined that:

$$\rho_{final}(\mathbf{q}) = \|\mathbf{q} - \mathbf{q_{final}}\|$$

3.2 Repulsive Field Potential

By constructing a similar physics-based analogy, one can emulate the repulsive field as an potential that imposed on the electron that travels through the different energy states. To be able to materialize the analogy, one can start to write the repulsive potential field as:

$$P^k_{repulsive} = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{\rho_k(\mathbf{q})} - \frac{1}{\rho_0}\right)^2 & \rho(\mathbf{q}) < \rho_0 \\ 0 & \rho(\mathbf{q}) > \rho_0 \end{cases}$$

$$\rho_k(\mathbf{q}) = \|\mathbf{q} - \mathbf{q_k}\|$$

where $\mathbf{q_k}$ represent the individual points that is located in the configuration space point cloud and labeled as obstacle or *unpermitted* point. Finally, the ultimate form of the repulsive potential field can be written as the following:

$$P_{repulsive} = \sum_k P^k_{repulsive}$$

The three dimensional representation of the potential field can be investigated in Figure 3. As can be inferred from the Figure 3, the final configuration exposes a decreasing potential, while the obstacles implies an increasing potential.

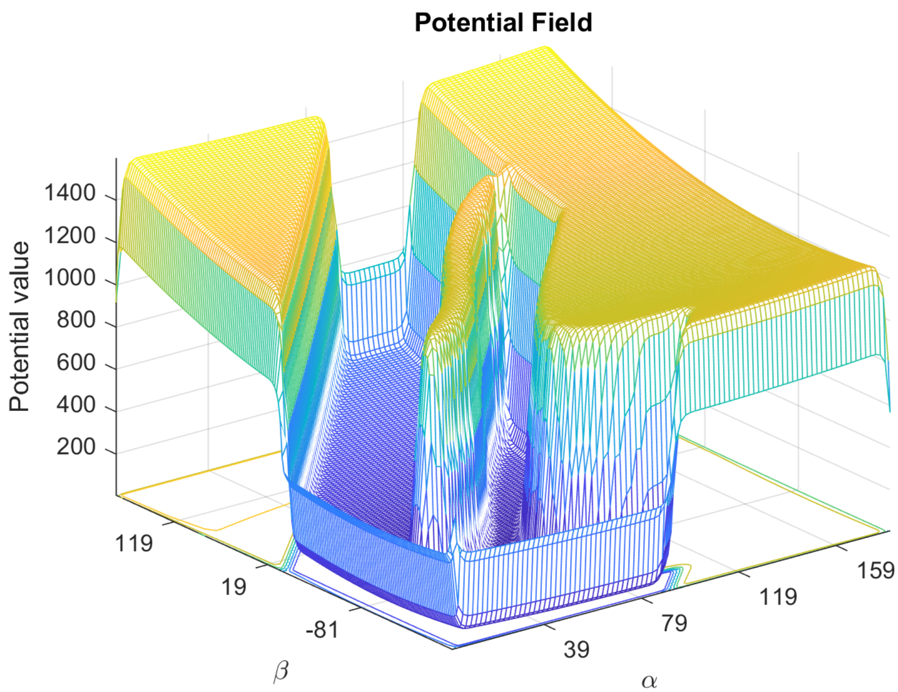

Figure 3: Potential Field

Moreover, the two dimensional version of the Figure 3 can be observed in Figure 4, *contour plot*.

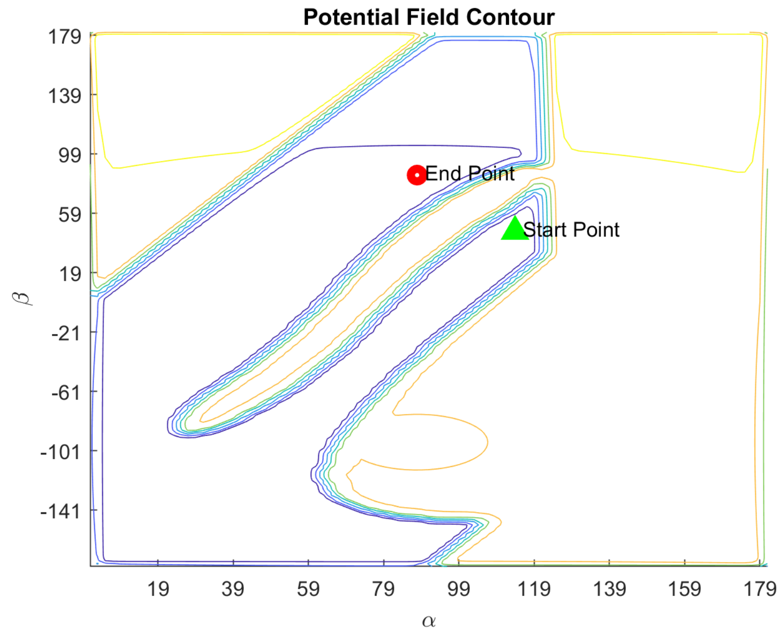

Figure 4: Potential Field Contours

4 Motion Planning

4.1 Potential Field Method

In Section 3, the methodology to construct the potential field is discussed, however, the use case that enables one to construct a path to plan the motion is not discussed. Actually, in this point, the virtue of the *potential field* can be acknowledged in two ways. The first way is to use directly the physical fact behind the method. That is, if there exists a potential in the environment, it can be proved that the mass located in this potential will be subjected to a force, which can be quantified as following:

$$\mathbf{F} = -\nabla P = \begin{bmatrix} -\frac{\partial P}{\partial x} \\ \\ -\frac{\partial P}{\partial y} \end{bmatrix}$$

where P represents the sum of the $P_{attractive}$ and $P_{repulsive}$.

The second way to make use of the constructed potential field is to consider this potential field as a objective function and solve an optimization problem under these constraints. That is, if the potential field, *objective function*, is determined beforehand, one can easily incorporate a basic *Gradient Descent Algorithm* on it and construct a route/path.

In this project, the harmony of the two aforementioned phenomenons are used. From a force field point of view, one can write the attractive and repulsive force as in the following manner:

$$\mathbf{F_{attractive}} = -\zeta\left(\mathbf{q} - \mathbf{q_{final}}\right)$$

$$\mathbf{F^{k}_{repulsive}} = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{\rho_k(\mathbf{q})} - \frac{1}{\rho_0}\right)\frac{1}{\rho_k^2(\mathbf{q})}\nabla\rho_k(\mathbf{q}) & \rho(\mathbf{q}) < \rho_0 \\ 0 & \rho(\mathbf{q}) > \rho_0 \end{cases}$$

where

$$\nabla\rho_k(\mathbf{q}) = \frac{\mathbf{q} - \mathbf{q_k}}{\|\mathbf{q} - \mathbf{q_k}\|}$$

Finally, for this project, the $\mathbf{F_{repulsive}}$ and $\mathbf{F_{attractive}}$ can be written as:

$$\mathbf{F_{attractive}} = -\zeta\left(\mathbf{q} - \mathbf{q_{final}}\right)$$

$$\mathbf{F_{attractive}} = \max_k(\mathbf{F^{k}_{repulsive}})$$

In the same way, from a optimization problem point of view, one can write the iterative scheme as the following:

$$\mathbf{x_{t+1}} = \mathbf{x_t} + \Delta t \frac{\mathbf{F_{total}}}{\|\mathbf{F_{total}}\|} \tag{1}$$

Now, it should be pointed out that although the physical intuition behind the potential field method is very strong, it has also a few drawbacks. One of these drawbacks is the fact that it can include local minimum points in the overall potential field/objective function. Since, the mission of the iterative scheme in (1) is to minimize the potential field such that the end effector of the robotic arm will move from start configuration to final configuration, these undesired local minimum points can halt optimization process and result in unacceptable outcomes. For example, in Figure 5, the situation where the solution is stuck at the local minimum point can be observed.

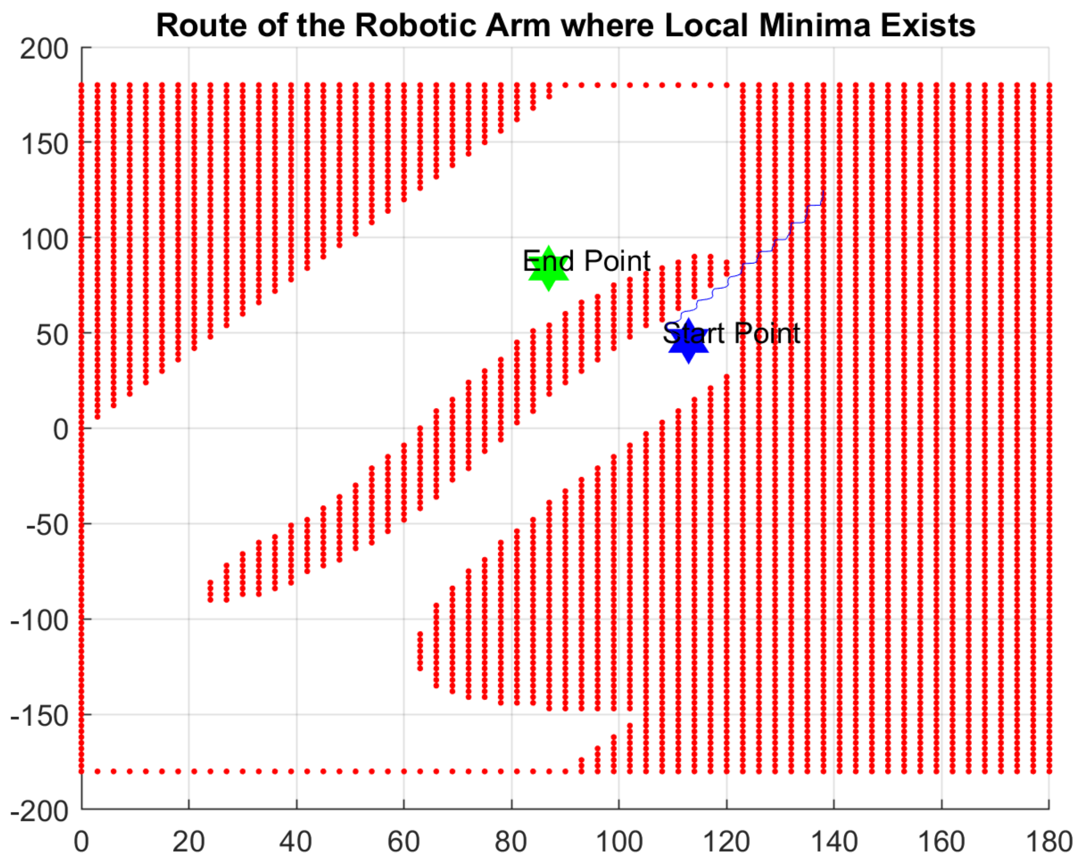

Figure 5: Local Minimum Situation

The reason for the existence of the local minimum incarceration is the fact that the current physical configuration of the robotic arm makes it very difficult to yield a viable solution. To be able to eliminate this problem, one can alter the physical parameters that is present in the robotic setup, such as link lengths, and start and final configurations. In Figure 6, Figure 7, and Figure 8, the planned motions under the altered configurations can be observed.

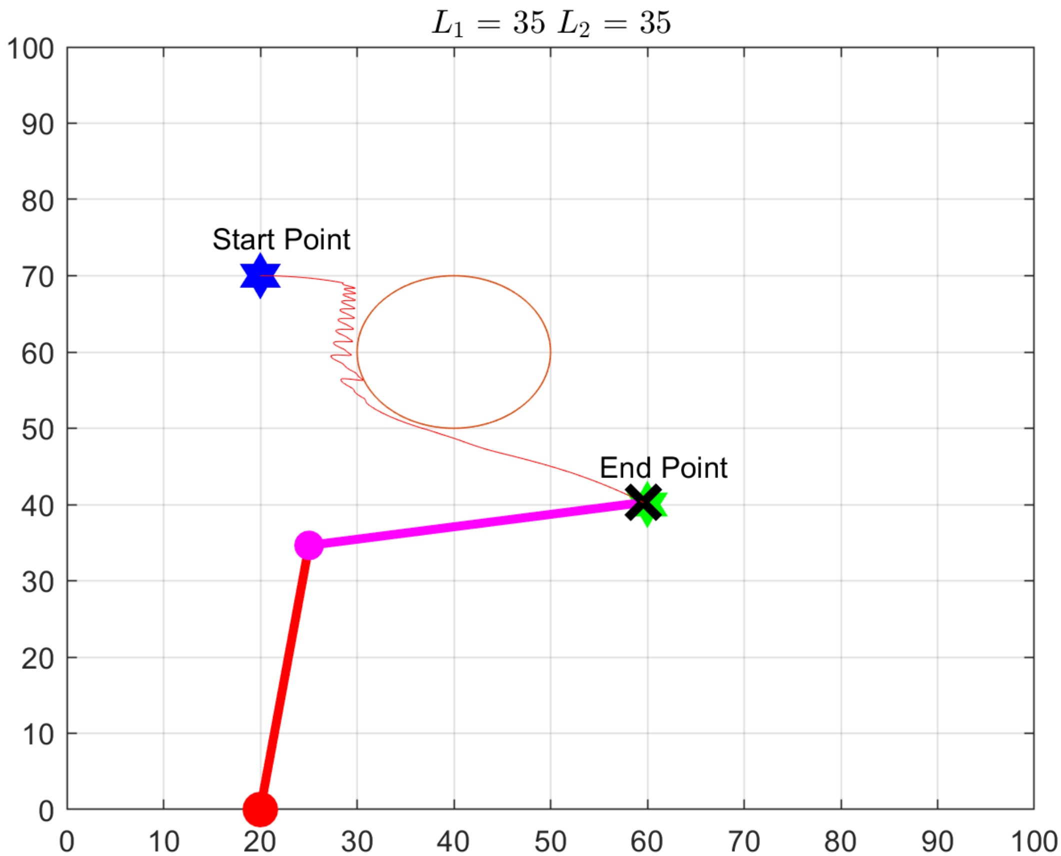

Figure 6: Generated Path 1

The physical configuration for Figure 6:

$$L_1 = 35$$

$$L_2 = 35$$

$$\mathbf{q_{start}} = \begin{bmatrix} 20 \\ 70 \end{bmatrix}$$

$$\mathbf{q_{final}} = \begin{bmatrix} 60 \\ 40 \end{bmatrix}$$

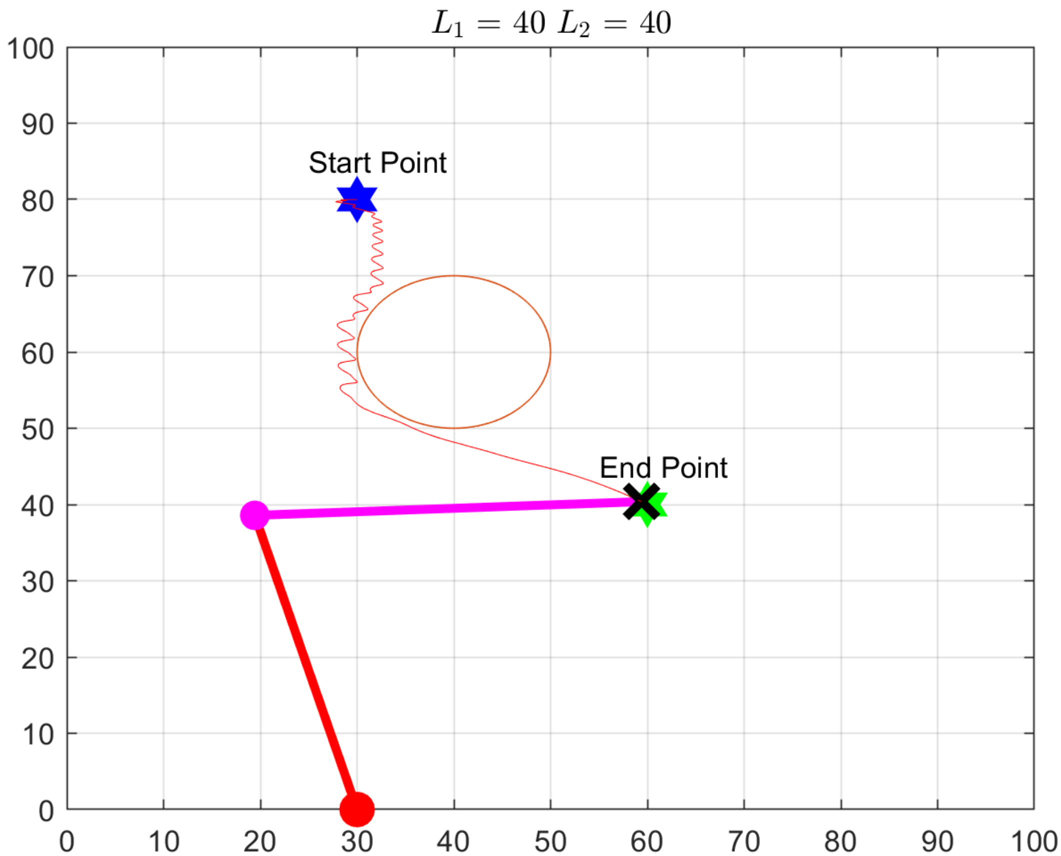

Figure 7: Generated Path 2

The physical configuration for Figure 7:

$$L_1 = 40$$
$$L_2 = 40$$

$$\mathbf{q_{start}} = \begin{bmatrix} 30 \\ 80 \end{bmatrix}$$

$$\mathbf{q_{final}} = \begin{bmatrix} 60 \\ 40 \end{bmatrix}$$

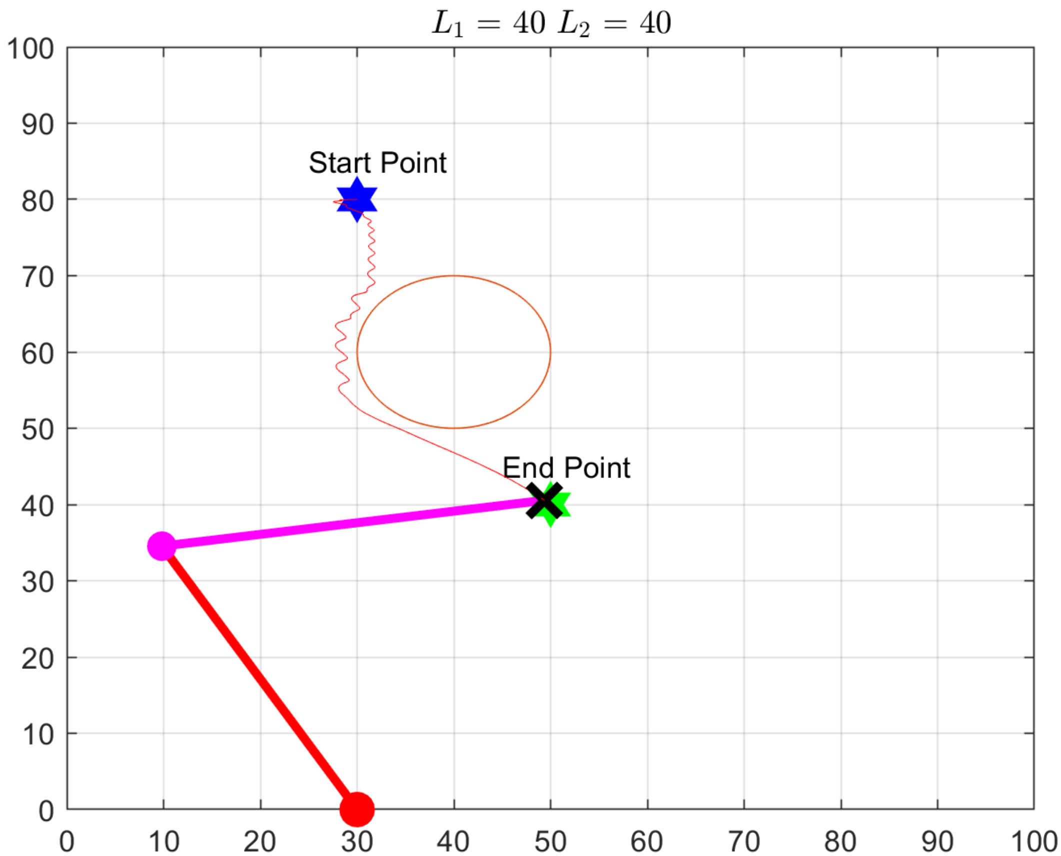

Figure 8: Generated Path 3

The physical configuration for Figure 8:

$$L_1 = 40$$

$$L_2 = 40$$

$$\mathbf{q_{start}} = \begin{bmatrix} 30 \\ 80 \end{bmatrix}$$

$$\mathbf{q_{final}} = \begin{bmatrix} 50 \\ 40 \end{bmatrix}$$

4.2 Wave-Front Method

The Potential Field Method can be stuck in the local minimal. For that reason, the wave-front method was chosen to avoid local minimal problems. The wave-front algorithm uses the configuration space as a grid. Configuration space consists of 0 (free space) and 1 (for obstacles). The algorithm finds a path from start point to goal point inside of the grid. In the grid, every cells are assigned with the integer number and it starts from the goal point as a 2. Each step the value of the squares increase one by one. Moore Neighbor Tracing algorithm can be used to find the neighbours of the cell. Values of the each cells should be equal to minimum value of the its Moore neighbours.

When the assignation the value of the each free cell are done, the C-space map looks like in figure 9. The solid purple color stands for the obstacle, and the gradient colors specify the values of the empty cells. To find the path from the start point to the end point, this grid map is an essential.

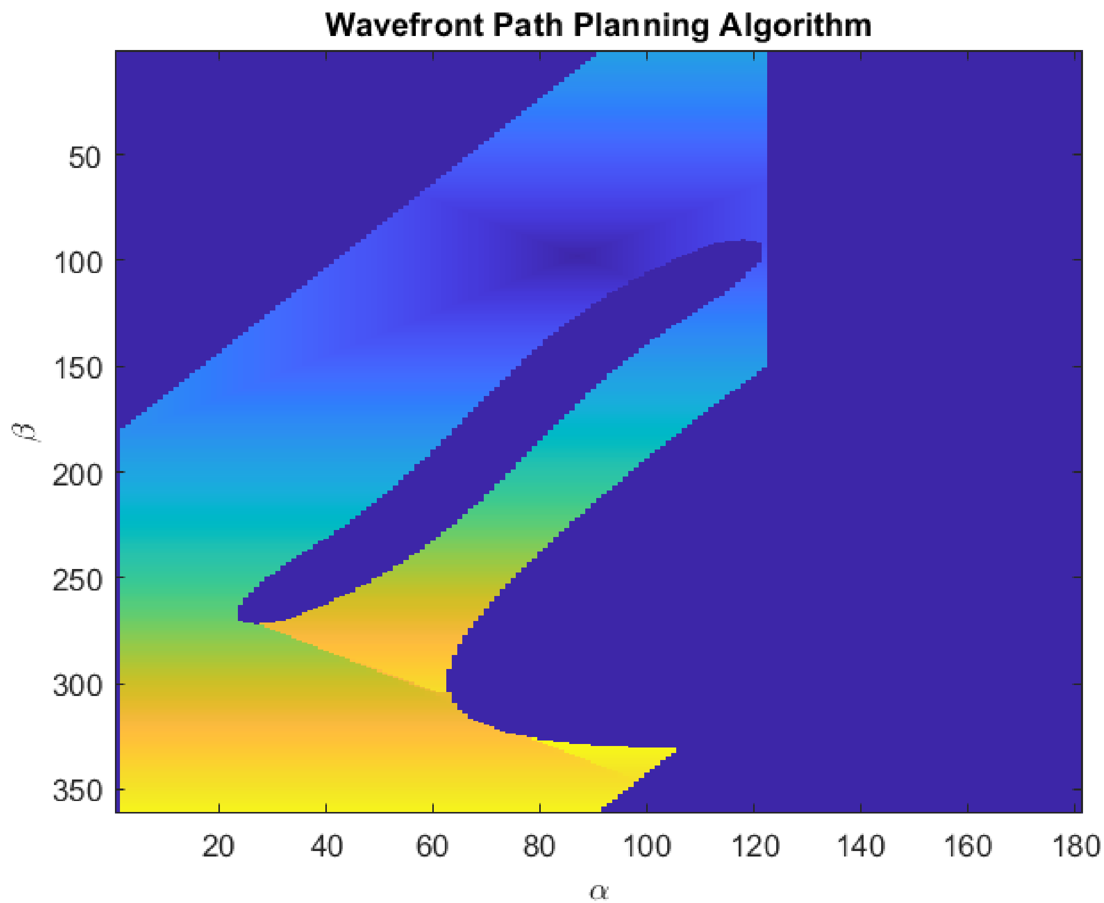

Figure 9: Wave-Front Grid Map

In the grid map, the target point is reached by following the decreasing cell values starting from the starting point. As seen in figure 9, there is two possible way for start point to goal point. However, Between the two, the total number of traversed cells in a path is worth less. If the intersecting angles on this path are defined as a path, the path in the figure 10 is obtained for the Cartesian coordinate plane. The algorithm works well and it is significantly time efficient way to motion planning but it may suggest a longer way which is not energy efficient.

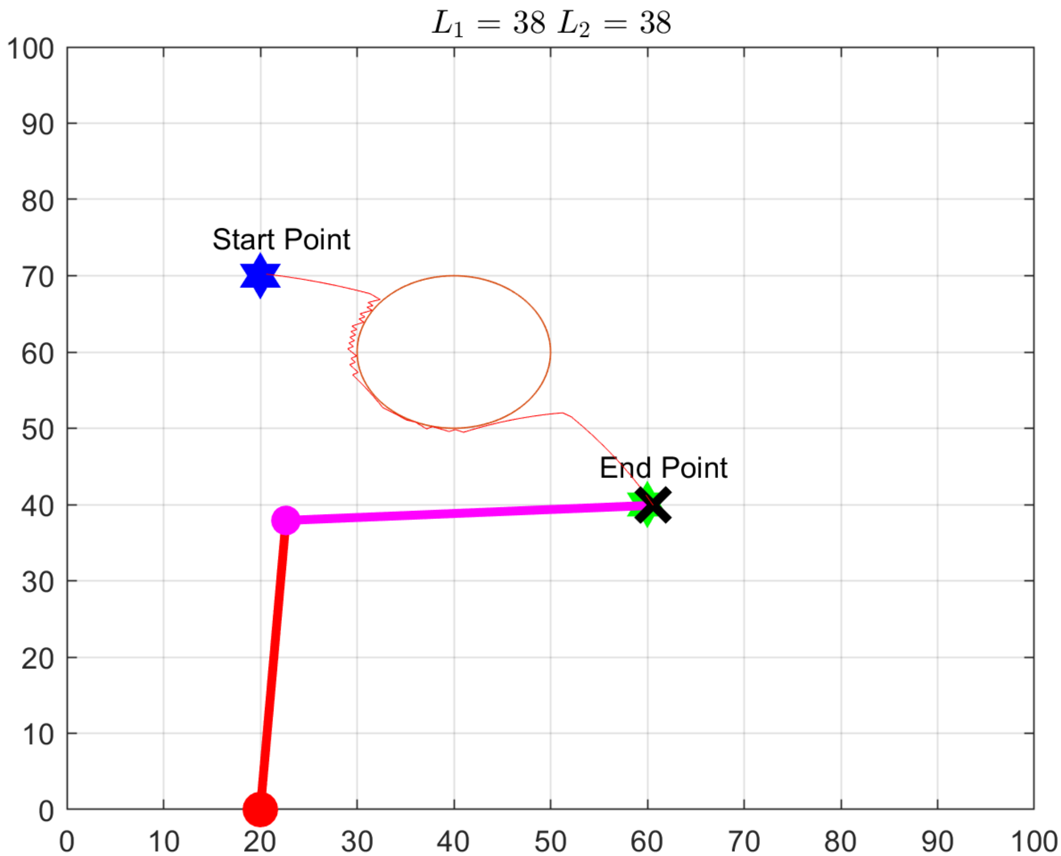

Figure 10: Generated Path via Wave-Front Method

5 Conclusion

All in all, in this project, the motion planning of the 2 degrees of freedom planar serial manipulator was investigated. Firstly, to be able to planning to motion,the Configuration Space was constructed. For the constructing the configuration space, the forward kinematics equations were derived. To find the relevant angles for start and end goal position, the inverse kinematics solutions also obtained. Then, using the configuration space, two motion planning algorithm were tested. Firstly, potential field method has applied into configuration space. The potential field method consists of attractive and repulsive potentials. Taking the derivative of the potentials, attractive and repulsive forces are obtained. Attractive forces is related to the goal position, repulsive forces determining from the obstacles. After the getting the forces, path will be constructing with the gradient descent algorithm. However, the potential field method has not any recover algorithm for the local minima. For the avoid of the local minima, the wavefront path planning algorithm has been implemented. Utilizing the wavefront path planning algorithm, multiple paths are obtained in a fastest way.