

CS 5114 Theory of Algorithms, Spring 2020
Homework 4: Due on 6 April 2020, 11:59pm

I pledge that this test/assignment has been completed in compliance with the Graduate Honor Code and that I have neither given nor received any unauthorized aid on this test/assignment.

Name (Print): _____

Signed: _____

1. **(10%)** Convert the following linear program into standard form:

$$\begin{array}{ll}\text{Minimize} & 2x_1 + 7x_2 + x_3 \\ \text{Subject to} & \\ & x_1 - x_3 = 7 \\ & 3x_1 + x_2 \geq 24 \\ & x_2 \geq 0 \\ & x_3 \leq 0\end{array}$$

2. **(10%)** Convert the following linear program into slack form:

$$\begin{array}{ll}\text{Maximize} & 2x_1 - 6x_3 \\ \text{Subject to} & \\ & x_1 + x_2 - x_3 \leq 7 \\ & 3x_1 - x_2 \geq 8 \\ & -x_1 + 2x_2 + 2x_3 \geq 0 \\ & x_1, x_2, x_3 \geq 0\end{array}$$

What are the basic and nonbasic variables?

3. **(20%)** Solve the following linear program using SIMPLEX:

$$\begin{array}{ll}\text{minimize} & x_1 + x_2 + x_3 \\ \text{subject to} & \\ & 2x_1 + 7.5x_2 + 3x_3 \geq 10000 \\ & 20x_1 + 5x_2 + 10x_3 \geq 30000 \\ & x_1, x_2, x_3 \geq 0.\end{array}$$

(Note: Show all your work. Showing only your answers won't be given any points).

4. **(20%)** Solve the following linear program using SIMPLEX:

$$\begin{array}{ll}\text{Maximize} & x_1 + 3x_2 \\ \text{Subject to} & x_1 - x_2 \leq 8 \\ & -x_1 - x_2 \leq -3 \\ & -x_1 + 4x_2 \leq 2 \\ & x_1, x_2 \geq 0\end{array}$$

(Note: Show all your work. Showing only your answers won't be given any points).

5. **(10%)** Suppose we perform a sequence of n operations on a data structure in which the i -th operation costs i if i is an exact power of 2, and 1 otherwise. Use **aggregate analysis** to determine the amortized cost per operation. Show cost per operation to derive the amortized cost per operation.

6. **(10%)** Suppose we perform a sequence of n operations on a data structure in which the i -th operation costs i if i is an exact power of 2, and 1 otherwise. Use an accounting method of analysis. Show cost, actual cost, and credits per operation to derive the amortized cost for all sequences of n operations.
7. **(20%)** Show the results of inserting the keys F, S, Q, K, C, L, H, T, V, W, M, R, N, P, A, B, X, Y, D, Z, E in order into an empty B-tree with minimum degree 2. Draw only the configurations of the tree just before some node must split, and also draw the final configuration.