Home work 1

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Q1.
a). F
b). T
c). F
d). F
e). T
f). F
g). F
h). T
i). T
j). F

Q2.
Using recursive method..,

$$T(n = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$$T(n) = 2\left[2T\left(\frac{n}{4}\right) + \frac{\left(\frac{n}{2}\right)}{\log\left(\frac{n}{2}\right)}\right] + \frac{n}{\log n}$$

$$(T(n) = 4T\left(\frac{n}{4}\right) + \frac{n}{\log\frac{n}{2}} + \frac{n}{\log n}$$

$$T(n) = 4\left[2T\left(\frac{n}{8}\right) + \frac{\left(\frac{n}{4}\right)}{\log\left(\frac{n}{4}\right)}\right] + \frac{n}{\log\frac{n}{2}} + \frac{n}{\log n}$$

$$T(n) = 8T\left(\frac{n}{8}\right) + \frac{n}{log\frac{n}{4}} + \frac{n}{log\frac{n}{2}} + \frac{n}{logn}$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \frac{n}{\log\frac{n}{2^{k-1}}} + \cdots$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} \frac{n}{1log\frac{n}{2^i}}$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} \frac{n}{log^n - i}$$

As this goes on, $\frac{n}{2^k} = 1$

$$n = 2^k$$

$$T(n) = nT(1) + \sum_{i=0}^{log^n - 1} \frac{n}{\log^n - i}$$

$$T(n) = n\left[T(1) + \sum_{i=0}^{\log^n - 1} \frac{n}{\log^n - i}\right]$$

$$T(n) = n[1 + \log(\log(n))]$$

$$T(n) = \Theta(n\log(\log(n))$$

$$A = n\log(\log(n))$$

Q3.

a).

$$T(n) = 3T\left(\frac{n}{2}\right) + n\log n$$

$$a = 3, b = 2, f(n) = n\log n$$

$$n^{\log_b^a} = n^{log_2^3}$$

$$n^{\log_2^3} = n^{1.58} > n\log n$$

Therefore, $T(n) = \Theta\left(n^{\log_b a}\right) = \Theta\left(n^{\log_2^3}\right)$

b).

$$T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$$

$$a = 2, \quad b = 4, \quad f(n) = \sqrt{n}$$

$$n^{\log_b a} = n^{\log_4^2}$$

$$n^{log_4^2} = n^{\frac{1}{2}\log_2^2} = \sqrt{n} = f(n)$$

Therefore, $T(n) = \Theta\left(n^{\log_b a} \log n\right)$

$$T(n) = \Theta(\sqrt{n}\log n)$$

Q4.

a).

Let X be the random variable that the number of bins are empty. Given that n bathe are tossed into n bins and each toss is not dependent to each other.

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right]$$
$$E[X] = E\left[X_1 + X_2 + ... + X_n\right]$$

Where X_i is the variable such that bin i empty. X_i is 1 if bin is empty else 0.

$$E[X_i] = 1 * p(X_i \text{is empty}) + 0 * p(X_i \text{is not empty})$$

$$E[X_i] = p(X_i \text{is empty})$$

$$E[X] = E[X_1] + E[X_2] + E[X_3] + ... + E[X_n]$$

As, n balls goes into $n-1$ bins and keeping the bin X_i empty. The probability is given as

$$p(X) = \left(\frac{n-1}{n}\right)^n$$

$$E[X] = \left(\frac{n-1}{n}\right)^n + \left(\frac{n-1}{n}\right)^n + \ldots \quad ntimes$$

$$E[X] = n\left(\frac{n-1}{n}\right)^n$$

b).

Let X be the random variable with the number of bins having exactly one ball and X_i is the event such that bin i is having exactly one ball.

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right]$$
$$E[X] = E\left[X_1 + X_2 + ... + X_n\right]$$

$$E[X_i] = 1 * p(X_i \text{is exactly having one ball})$$

As exactly one ball is in bin X_i and remaining $n-1$ balls goes into $n-1$ bins. The probability is given by

$$p(X_i) = \frac{(n-1)^{(n-1)} * {}^{n}C_1}{n^n}$$

$$E[X] = \frac{(n-1)^{(n-1)} * {}^{n}C_1}{n^n} + \frac{(n-1)^{(n-1)} * {}^{n}C_1}{n^n} + ...n\text{times}$$

$$E[X] = n * \frac{(n-1)^{(n-1)} * n}{n^n}$$

$$E[X] = \frac{(n-1)^{(n-1)}}{n^{n-2}}$$

Q5.

Min-Heapify and Max-Heapify has the same time complexity of O($\log n$)

Algorithm 1 Min-Heapify

```
procedure MIN-HEAPIFY(A,i)
    l = LEFT(i)
    r = LEFT(i)
    n = A.heap-size
    if l ≤ n and A[l] < A[i] then smallest = l
    else smallest = i
    if r ≤ n and A[r] < A[i] then smallest = r
    if smallest ≠ i then
        Swap A[i] and A[smallest]
        MIN-HEAPIFY(A,i)
=0
```