CS 5114 Theory of Algorithms, Spring 2020
Homework 1: Due on 11 Feb. 2020, 1pm

I pledge that this test/assignment has been completed in compliance with the Graduate Honor Code and that I have neither given nor received any unauthorized aid on this test/assignment.

Name (Print): _____

Signed: _____

1. **(30%)** Check if $B =$ (or $\neq$) $O(A), \Omega(A), \Theta(A), o(A)$, or $\omega(A)$ where $k \geq 1, \epsilon > 0$, and $c > 1$. Say T or F in the following questions.
   (a) (3%) Given $A = \lg^k n, B = n^\epsilon$, $B = \Theta(A)$. (T, F)
   (b) (3%) Given $A = \lg^k n, B = n^\epsilon$, $B \neq \Omega(A)$ for $\epsilon \to 0$; $B = \Omega(A)$ otherwise. (T, F)
   (c) (3%) Given $A = n^k, B = c^n$, $B = \Theta(A)$. (T, F)
   (d) (3%) Given $A = n^k, B = c^n$, $B \neq \Omega(A)$. (T, F)
   (e) (3%) Given $A = \sqrt{n}, B = n^{\sin n}$, $B = \Omega(A)$. (T, F)
   (f) (3%) Given $A = \sqrt{n}, B = n^{\sin n}$, $B \neq \Theta(A)$. (T, F)
   (g) (3%) Given $A = 2^n, B = n^{n/2}$, $B = \Theta(A)$. (T, F)
   (h) (3%) Given $A = 2^n, B = n^{n/2}$, $B \neq o(A)$. (T, F)
   (i) (3%) Given $A = n^{\lg c}, B = c^{\lg n}$, $B = \Theta(A)$. (T, F)
   (j) (3%) Given $A = n^{\lg c}, B = c^{\lg n}$, $B = o(A)$. (T, F)
2. **(20%)** Solve $T(n) = 2T(n/2) + \frac{n}{\lg n}$ and present $T(n)$ in Θ notation (i.e., $T(n) = \Theta(A)$; find A).
3. **(10%)** Solve the following problems using Master Theorem.
   (a) **(5%)** $T(n) = 3T(n/2) + n \lg n$
   (b) **(5%)** $T(n) = 2T(n/4) + \sqrt{n}$
4. **(20%)** Suppose that n balls are tossed into n bins, where each toss is independent and the ball is equally likely to end up in any bin. (a) (10%) What is the expected number of empty bins? (b) (10%) What is the expected number of bins with exactly one ball?
5. **(20%)** Starting with the procedure MAX-HEAPIFY, write pseudocode for the procedure MIN-HEAPIFY (A, i), which performs the corresponding manipulation on a minheap. How does the running time of MIN-HEAPIFY compare to that of MAX-HEAPIFY?