DREXEL UNIVERSITY

DEPARTMENT OF PHYSICS

PhD Qualifying Exam **Modern Physics**

Friday, September 22, 2006 1 – 4 PM

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper…even the short ones. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. *Circle* the numbers below to indicate which questions you have answered—write nothing on the lines.

Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

MODERN PHYSICS

PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1.
(a) What is the physical origin of the hyperfine structure in a hydrogen atom?

(b) What is the main observable consequence of the hydrogen hyperfine structure? (Think astrophysics)

2.
(a) A classical magnetic moment $\vec{\mu}$ is oriented initially along the z axis. A uniform magnetic field is turned on along the y axis. Sketch the subsequent evolution of the classical magnetic moment.

(b) The quantum mechanical state of a particle having a magnetic moment $\vec{\mu}$ evolves according to the time evolution equation

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}Ht\right)|\psi(0)\rangle,$$

where $H = -\vec{\mu}\cdot\vec{B}$ and $|\psi(0)\rangle$ is the state of the system at time $t = 0$. In what sense does the quantum particle evolve just as the classical particle does? There seems to be little obvious connection between the quantum evolution equation and the behavior of a classical compass needle. Explain.

3. The specific energy density per unit volume of photons is given by the blackbody radiation equation,

$$\frac{du}{d\nu} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1},$$

which is known as a blackbody spectrum.

Consider only the regime for which k_B T >> $m_\mu c^2$ (or, in other words, assume a massless neutrino), and explain how the low frequency limit of blackbody neutrino radiation would differ from that of photon radiation. Which is a steeper function of frequency?

4. Why is the 2S $\rightarrow$ 1S Hydrogen transition forbidden? Please indicate not simply the selection rule, but why the selection rule is necessary.

5. In a statistical mechanical description of systems, we sometimes refer to a system to be at negative temperature. What does it mean by negative temperature? Give an example of physical systems or devices operating at negative temperature.

6. A physical system X is made of two subsystems A and B, with the number of microstates Ω_A and Ω_B, respectively. (a) What are the entropies of A and B? (b) What is the number of microstates of X? (c) What is the entropy of X?

7. An ice cube of mass 100 g at -20°C is dropped into a coffee cup containing 500 g water at 60°C.
(a) What is the final temperature?
(b) What is the change in entropy?

Given: The specific heat for water is 1 cal/g and that for ice is 0.5 cal/g. The latent heat of ice is 80 cal/g.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

A beam of particles of spin $\frac{\hbar}{2}$ is sent through a Stern-Gerlach apparatus, which contains a uniform magnetic field $\mathbf{B} = B_0 \hat{z}$. The device divides the incident beam into two spatially separated components depending on the quantum numbers m_s of the particles. One of the resulting beams is removed and the other beam is sent through a similar apparatus, the magnetic field of which is inclined by an angle θ with respect to the first. Again, the incoming beam is split into two components.
You should solve the following problems using the Pauli spin formalism,

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$$

(a) What is the Hamiltonian of the interaction of the particle spin and magnetic field in the first apparatus?

(b) What are the spin eigenvectors of this interaction?

(c) The second apparatus is rotated in the x-z plane by an angle θ with respect to the first apparatus. What is the Hamiltonian of the interaction of the particles with the second apparatus?

(d) What are the spin eigenvectors of the second interaction?

(e) One of the beams coming out of the first apparatus is blocked. The other beam is sent on to the second apparatus. Write the spin wavefunction of the beam that approaches the second apparatus as a superposition of eigenvectors of the Hamiltonian of the second apparatus.

A2.

Consider a particle of mass m subject to the potential

$$V(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & elsewhere \end{cases}.$$

$|\psi_n\rangle$ are the eigenstates of the Hamiltonian H of the system, and their eigenvalues are

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

The state of the particle at time $t = 0$ is

$$|\psi(t=0)\rangle = A_1|\psi_1\rangle + A_2|\psi_2\rangle + A_3|\psi_3\rangle + A_4|\psi_4\rangle$$

where the complex numbers $A_1, \ldots A_4$ are known.

(a) If the energy of the particle in the state $|\psi(t=0)\rangle$ is measured, what is the probability of finding a value smaller than $\dfrac{3\pi^2\hbar^2}{mL^2}$?

(b) What is the mean value of the energy of the particle in the state $|\psi(t=0)\rangle$

(c) When the energy is measured, the result $\dfrac{8\pi^2\hbar^2}{mL^2}$ is found. After the measurement, what is the state of the system? What is the result if the energy is measured again?

A3.

Take the spin Hamiltonian for the hydrogen atom in an external magnetic field B_0 in the z direction to be

$$H = \frac{2A}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2 + \omega_0 S_{1z},$$

where A is a positive, real number, $\omega_0 = geB_0/(2mc)$ in Gaussian units, m is the mass of the electron, $\vec{S}_1$ is the spin operator of the electron and $\vec{S}_2$ is the spin operator of the proton. The contribution $-\vec{\mu}_2 \cdot \vec{B}_0$ of the proton has been neglected because the mass of the proton is about 2000 times larger than that of the electron.

(a) Construct the matrix representation of the spin Hamiltonian in the basis of the vectors $|\pm,\pm\rangle$.
(b) Find the energies of this system.
(c) Examine your results in the limiting case $A << \hbar\omega_0$.

Hint: $2\vec{S}_1 \cdot \vec{S}_2 = S_1^+ S_2^- + S_1^- S_2^+ + 2S_{1z}S_{2z}$. Also note that the matrix representative of H is block-diagonal.

B1.

Consider a cubic lattice in 3-dimensions with each volume V containing N x N x N points. Each lattice has a molecule which will bond with any one of its 6 nearest neighbors, but not with more than one. Point a may bond with b, irrespective of whether b already made a bond with a. Let each bond energy be denoted ε. ($\varepsilon < 0$) The lattice is effectively infinite (equivalently, the boundary conditions are periodic.)

(a) Write an expression for the entropy of a volume V for this lattice.
(b) Write an expression for the Helmholtz free energy in volume V, incorporating your result from (a).

Now let there be a single impurity in the lattice volume V that only the impurity molecule cannot bond at all.

(c) How does your answer to (a) change?
(d) How does your answer to (a) change?

Now suppose the impurities in 2 adjacent volumes were adjacent to each other. (That is, you have 2 adjacent impurities in total volume 2V.)

(e) Now write an expression for the Helmholtz free energy of volume 2V in which two impurities are adjacent. Compare your answer with the Helmholtz free energy of 2 volumes with a single isolated impurity (i.e. twice what you did in (d)). Is the system more or less stable with the two impurities adjacent than separated?

B2.

Consider a real, non-ideal gas with N atoms in a volume V. The interatomic potential between any two atoms is given by

$$u(r) = \varepsilon \exp(-\alpha r^2),$$

where r is the interatomic distance between the two atoms. In the following questions, assume that ε is small so that the partition function may be expanded in a series in ε.

(a) Calculate the partition function of this real gas and find the correction to the ideal gas behavior to terms of order ε.

(b) Show that the equation of state to order ε for this gas is given by

$$P = \frac{Nk_B T}{V} + \left(\frac{N}{V}\right)^2 \frac{\varepsilon}{2} \left(\frac{\pi}{\alpha}\right)^{\frac{3}{2}}.$$