DREXEL
UNIVERSITY

DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam | **Classical Physics**

Friday, September 22, 1995 | 9 am - 12 noon

PRINT YOUR NAME______________________

EXAM CODE________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. By circling the numbers below, indicate which questions you have answered.

Short questions	*Long Problems*
1. _____	A1. _____
2. _____	A2. _____
3. _____	A3. _____
4. _____	B1. _____
5. _____	B2. _____
6. _____	
7. _____	

CLASSICAL PHYSICS

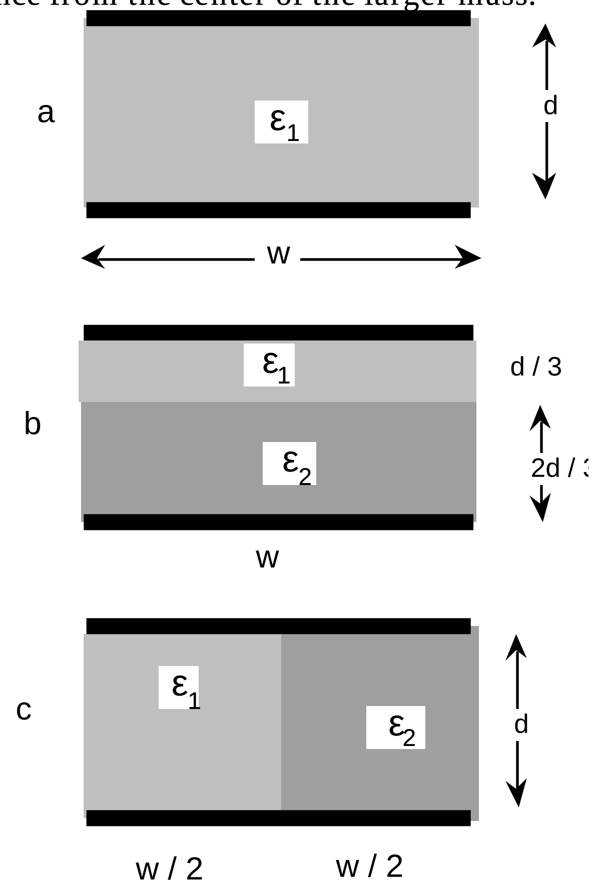
PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. A neutron star in our galaxy has mass = 1.4 M_{sun} and radius = 10 km, where the mass of the sun M_{sun} = 2 x 10^{33} g. There is a small lump on the equator of this star.
(a) If the star is spherical and has uniform rotation, calculate the maximum velocity with which the lump can rotate. (G = 6.67 X 10^{-8} cm^3/gs^2)
(b) Could this star be a millisecond pulsar? Explain.

2. A hole is drilled through the diameter of a large spherical mass M of uniform mass density ρ (mass / volume). There are no other masses around. A small mass m is dropped into the hole. Show that the force on the mass is $F = -4\pi G m \rho r / 3$ where G is the gravitational constant and r is the distance from the center of the larger mass.

3. Calculate the capacitance for each of the three arrangements of parallel plates (separation d) and dielectric constants ε shown at the right.



4. Two infinite conducting planes meet at a right angle. A charge is placed near the corner, a distance h from each plane. What is the force on the charge?

5. A wooden house burns down leaving only a narrow brick chimney standing. A wind later topples the chimney, which pivots around its base as its fall begins. Why is it likely to break, and where does that occur?

6. A partially filled can of Coke™ floats upright with 1 cm of the can out of the water surface. If it is displaced 0.5 cm further into the water and released, with what frequency will it oscillate? (Ignore damping.)

7. Plane polarized light is normally incident on a linear polarizer with its polarization axis oriented 45° to the E field of the incoming beam. A second polarizer is placed following the first with 45° additional rotation (90° to the initial E field direction). What is the intensity of the emergent light (if any) and its state of polarization?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Things that go bump in the night*

As seen in a laboratory reference frame, a particle of known mass m moving with known velocity v undergoes a completely elastic collision with a particle of mass 2m which is initially at rest in the lab frame. The mass m emerges from the collision moving perpendicular to its original direction of motion.

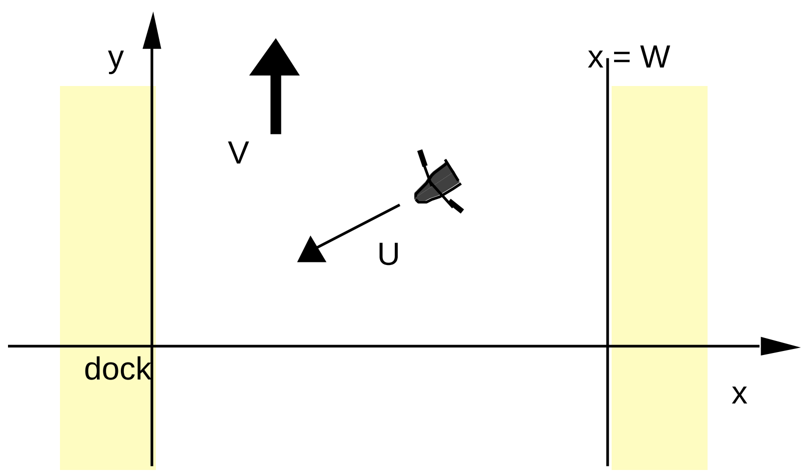
(a) What is the initial speed and the final speed of each particle <u>when viewed from the center of mass (CM) frame?</u>

(b) What is the scattering angle in the CM frame?

(c) What is the final speed of the incident particle?

A2. *Row, row, row your boat...*

You are in a boat and are rowing it across a river of width W. You direct the boat so that it always points towards a dock on the opposite bank. Also, you are rowing so that the boat moves at a constant speed U relative to the water which flows at a constant speed V as shown in the diagram.



(a) Show that the boat's motion is specified by the equations

$$dx/dt = -Ux/r, \qquad dy/dt = V - Uy/r$$

where $r^2 = x^2 + y^2$. The y axis is taken to be one bank and the origin is the dock, while the line $x=W$ is the other bank.

(b) By suitably eliminating the time variable, show that the (xy)-phase curves for the boat are given by

$$r + y = W^{\Delta} x^{(1-\Delta)}$$

where we define $\Delta = V/U$.

(c) Sketch these phase curves for $\Delta < 1$ in the neighborhood of the origin. Explain in some detail what would happen if you should row in such a way that $\Delta > 1$.

A3. *A bouncing ball*

A tennis ball with radius 6.5 cm and mass 0.06 kg is incident without spinning on a rough cement surface with an angle of incidence (relative to the normal) of 45°. The coefficient of restitution, which is the ratio of the normal component of the velocity after impact to the normal component before impact, is 0.75.

(a) Show that the moment of inertia of a hollow tennis ball is $2/3\ MR^2$.

(b) Assuming the ball rolls without skidding during impact, show that the angle of reflection of the tennis ball relative to the normal is 39°.

(c) If the incident speed of the tennis ball is $20\,m/s$, calculate the final angular velocity of the tennis ball and the fraction of energy lost during the collision.

B1. *Discovering your potential.*

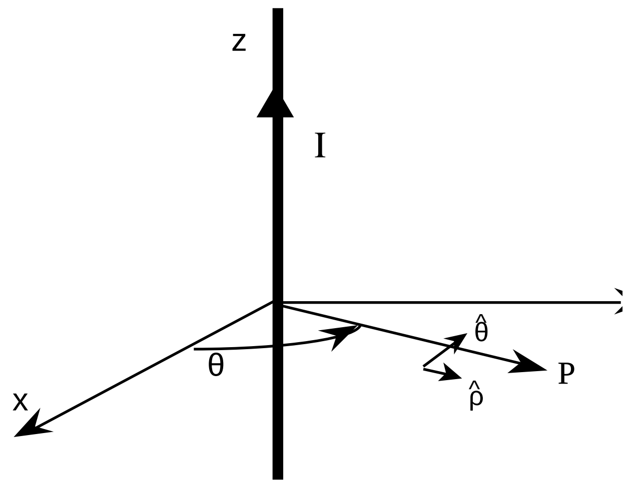
Consider two concentric spherical <u>surfaces</u> (NOT spherical conductors). The inner sphere is of radius a with a potential on its surface given by $\Phi = \Phi_o P_2(\cos\theta)$. The outer sphere is of radius b and is at zero potential.

(a) What is the potential at distance r for $a \le r \le b$?

(b) What is the potential for $r < a$?

B2. *Point of View*

An infinitely long cylinder is placed along the z axis as shown in the figure. It has a linear charge density λ and a current I along the z direction. The x-y plane contains the field point P located by (x,y,z) or (ρ,θ,z).



(a) Find the electric field at the point P. Express your answer in terms of the unit vectors ρ and θ.

(b) Find the magnetic field at P.

(c) It is possible to view the cylinder from an inertial frame x' y' z' moving with a velocity **v** along the z axis relative to x,y,z and see a pure *magnetic* field. Recalling the transformation equations

$$E'_{\rho}\,\hat{\rho} = \gamma\,(E_{\rho}\,\hat{\rho} + \mathbf{v} \times \mathbf{B})$$

$$B'_{\theta}\,\hat{\theta} = \gamma\,(B_{\theta}\,\hat{\theta} - \frac{\mathbf{v} \times \mathbf{E}}{c^2})$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

find an expression for the speed v.

(d) A particle of charge q at rest in the frame x' y' z' will experience zero force. Show <u>with a calculation</u> what force this charge experiences in the x y z frame.

(e) Suppose the charge q is at rest in the x y z frame of reference. What is the force **F'** that it experiences in the x' y' z' frame. The primed frame moves with the speed calculated in part c.

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MODERN PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. Give a qualitative argument based on the concept of resonance in quantum mechanics that the simplest molecule, H_2^+, is stable against dissociation.

2. A one dimensional harmonic oscillator has the unperturbed Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2} x^2$$

Find the change in the eigenvalues if it is perturbed by the potential $V = bx^2$.

3. Consider a quantum mechanical system (such as a diatomic molecule) whose energy levels vary gradually as a function of a parameter R (such as the atomic separation). Assume the Hamiltonian can be written as a sum of a zeroth order Hamiltonian H_o and the perturbation V, i.e., $H(x,R) = H_o(x,R) + V(x,R)$ where x is some internal coordinate. We consider two eigenstates of H_o, φ_o and φ_1, with the eigenvalues $E_1(R)$ and $E_2(R)$, which cross each other at a value $R=R_x$. In the basis of φ_o and φ_1, $H(x,R)$ can be represented in the following matrix form:

$$H = \begin{bmatrix} E_1(R) & V_{12}(R) \\ V_{21}(R) & E_2(R) \end{bmatrix}$$

where $V_{ij}(R) = \int \varphi_i^* V \varphi_j \, dx$

Assume both H_o and V possess a certain geometrical symmetry. For the zeroth-order level crossing to become an avoided crossing, must φ_o and φ_1 belong to the same symmetry class or a different symmetry class?

4. Explain why the room temperature specific heat is so similar for most solids. (No calculations, please!)

5. Give an equation for the entropy of a 2 dimensional n x n square lattice (periodic boundary conditions) in which a given site can interact with one and only one nearest neighbor, regardless of which neighbor the other site interacts with.

6. What is the shape of the curve in phase space which is drawn by a classical harmonic oscillator?

7. A certain table of physical constants only lists C_p (heat capacity at constant pressure) for solids and liquids, but C_v is required for a practical calculation. What is the difference between the two in terms of other thermodynamic quantities, and why is the error in using C_p for C_v likely to be small?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Have you got a moment?*

An atom with an electron in a J = 3/2 multiplet is immersed in a crystalline electric field described by the Hamiltonian

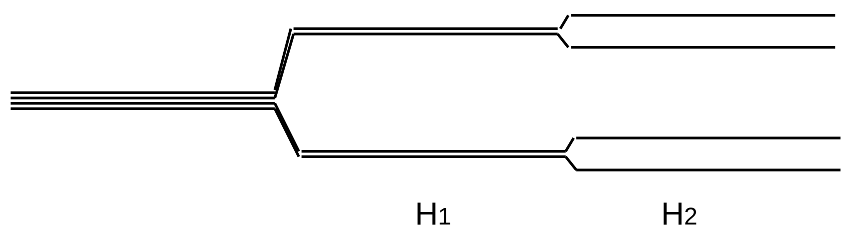
$$H_1 = Q\,(J_x^2 - J_y^2)$$

(a) Compute the eigenvalues and eigenfunctions of this electron.

An additional small perturbation of the form

$$H_2 = \varepsilon\ J_z \qquad (\varepsilon << Q)$$

is introduced to split the degeneracies.



(b) Compute the eigenvalues and eigenfunctions now.

An external probe field of the form $J_x \cos \omega t$ is now applied to the atom

(c) State the selection rules for transitions from the ground state to the the two excited states.

(d) For the more strongly allowed transitions, which polarization state (J_+ or J_-) is allowed and which is forbidden?

A2. *Spending Time out in the Field*

The purpose of this problem is to study the quantum mechanical motion of a charged particle of mass m and charge q in a static magnetic field $\mathbf{B}_o$ The uniform magnetic field is described by the vector potential

$$\mathbf{A}_o(\mathbf{r}) = \frac{1}{2}[\mathbf{B}_o \times \mathbf{r}]$$

and is parallel to the z axis of the chosen reference frame. The Hamiltonian for the particle (assumed to have no spin) is

$$H = \frac{1}{2m}[\,\mathbf{p} - q\,\mathbf{A}_o(\mathbf{r})\,]^2$$

(a) Derive the expressions for the three components v_x, v_y, and v_z of the velocity operator as functions of the canonical operators **p** and **r**, whose commutation relations are

$$[r_m, p_n] = i\hbar\,\delta_{mn} \qquad (m,n = x, y, z)$$

(b) Calculate the commutation relations of the operators v_x, v_y, and v_z.

(c) Write the Heisenberg equations

$$i\hbar\, dO/dt = [O, H]$$

for the velocity components. Deduce from these equations the character of the particle motion and derive an expression for the cyclotron frequency.

A3. *Electrons shaken, not stirred*

The purpose of this problem is to establish a connection between the classical picture of an oscillating dipole and that of the corresponding quantum mechanical system. The dipole operator for a one-electron atom is given by **d** = – e **r.**

(a) Show or argue convincingly that the expectation value of the dipole operator vanishes when a hydrogen atom is in a stationary state.

(b) If a perturbation (for example, an electromagnetic field) is turned on, the state of the atom will no longer be stationary, of course. Suppose that at some time t the atom is in the state

$$\psi(\mathbf{r},t) = (1+\gamma)^{-1/2}\left[\varphi_{100}(\mathbf{r})e^{-iE_1 t/\hbar} + \gamma\,\varphi_{210}(\mathbf{r})\,e^{-E_2 t/\hbar}\right]$$

where γ is a fixed real constant, $\varphi_{100}(\mathbf{r})$ and $\varphi_{210}(\mathbf{r})$ are the stationary states of the hydrogen atom corresponding to the indicated quantum numbers, and E_1 and E_2 are the respective eigenenergies. Verify that the state $\psi(\mathbf{r},t)$ is correctly normalized. Also, find the expectation values of the energy, E, the square of the angular momentum, L^2, and the projection of the angular momentum, L_z, along the z-axis.

(c) Calculate the expectation value of the dipole moment operator for the state $\psi(\mathbf{r},t)$ and show that, on the average, the atom behaves as an oscillating electric dipole. What is the frequency of oscillation?

(d) What would be the answer of part (c) if the state $\varphi_{210}(\mathbf{r})$ was replaced by $\varphi_{200}(\mathbf{r})$?

Note: Explicit calculations of overlap integrals are not required. For this reason you are not given the explicit and detailed form of the hydrogen wave functions.

B1. *Spin Counts*

A solid contains N mutually noninteracting nuclei of spin 1. Each nucleus can therefore be in any of three quantum states labelled by the quantum number m, where m = 1, 0 or –1. Because of electric interactions with internal fields in the solid, a nucleus in the state m = 1 or in the state m = –1 has the same energy $\varepsilon > 0$, while its energy in the state m = 0 is zero.

(a) What is the ratio of the probability of a nucleus being in the state m = 1 to the probability of its being found in the state m = 0 ?

(b) Find an expression, as a function of temperature, of the nuclear contribution to the internal energy of the solid.

(c) Calculate the nuclear contribution to the heat capacity of the solid. Make a quantitative graph showing the temperature dependence of this heat capacity. What is its explicit temperature dependence for large values of T?

(d) Find an expression, as a function of T, of the nuclear contribution to the entropy of the solid. What are the limiting values of the entropy as $T \to 0$ and $T \to \infty$?

B2. *Close enough*

The grand canonical partition function $\Xi(\mu,V,T)$ for an ideal Bose-Einstein or Fermi-Dirac gas can be written as

$$\Xi(\mu,V,T) = \prod_p \left(1 \pm e^{-(\varepsilon_p-\mu)/kT}\right)^{\pm 1}$$

where the product is over momentum states p with energy ε. μ is the chemical potential.

(a) Which sign (+ or –) applies for spin zero particles?

(b) Derive the equation of state for both Bose Einstein and Fermi Dirac cases.

(c) Derive the occupation number for each momentum state p for both cases.

(d) In terms of occupation number, explain the unique effects that happen for spin zero particles at suitably low temperatures and high densities.

(e) How does your answer to (d) compare with a typical phase transition? Give similarities and differences.