Enhanced Sensitivity of Photodetection via Quantum Illumination

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The use of quantum-mechanically entangled light to illuminate objects can provide substantial enhancements over unentangled light for detecting and imaging those objects in the presence of high levels of noise and loss. Each signal sent out is entangled with an ancilla, which is retained. Detection takes place via an entangling measurement on the returning signal together with the ancilla. This paper shows that for photodetection, quantum illumination with m bits of entanglement can in principle increase the effective signal-to-noise ratio by a factor of 2^m, an exponential improvement over unentangled illumination. The enhancement persists even when noise and loss are so great that no entanglement survives at the detector.

The conventional way to detect the presence of an object is to shine light in its direction and to see whether any is reflected back. If the object is far away, only a small percentage of the light will be reflected to a detector. If the object is immersed in noise and thermal radiation, then whatever light is reflected must be distinguished from the noisy background. In the case of quantum bits, it is known that the sensitivity of estimation processes can be enhanced by entangling a signal qubit with an ancilla and by making an entangling measurement on the returning qubit together with that ancilla (*1*–*3*). Entanglement and squeezing are known to enhance various aspects of amplification and imaging (*4*–*6*). This paper shows that entanglement can in principle give an enhancement of sensitivity for photon counting. For example, the photons can be entangled via frequency, as in the output from a spontaneous parametric downconverter. The intuition is that, if the signal is entangled with an ancilla, then it is harder for noise to masquerade as the returning signal. This intuition turns out to be correct, even though noise and loss completely destroy the entanglement between signal and ancilla at the detector. In fact, the entanglement-induced enhancement holds only in the presence of noise and loss. This resistance to noise and loss stands in stark contrast with existing methods that use entanglement to enhance the precision of measurement, where noise and loss rapidly destroy any enhancement [for a review of such methods, see (*7*)]. Moreover, the entanglement-induced enhancement of sensitivity is substantial: It grows exponentially with the number of bits of entanglement between signal and ancilla.

This paper presents the simplest possible mathematical treatment of quantum illumination, where signal and ancilla consist of individual photons. The entangled states used correspond to the single-photon sector of the output of a parametric downconverter, and the measurements required can be performed by upconversion. Two models are analyzed, one a low-noise model and the other with high levels of noise: Both exhibit exponential enhancement of sensitivity. Multiphoton signals and ancillae that exhibit coherence between states with different photon number are likely to surpass the bounds derived here.

Suppose that a single photon is sent at the object to be detected. If the object is not there, the signal received at the detector consists only of thermal and background noise. Even if the object is there, the majority of time the photon is lost, and again only noise is received. Once in a while, however, the photon is reflected back in a perturbed form. The dynamics corresponding to this situation can be modeled as beam splitter with reflectivity, η, that mixes in the vacuum with the signal state, followed by thermalization, which injects noise with average photon number, b, into each optical mode (e.g., frequency or polarization). No object corresponds to a beam splitter with $\eta = 0$. The presence of an object corresponds to a beam splitter with a very small nonzero reflectivity.

To make the analysis tractable, the following simplifying assumptions are made. In the first model analyzed here, the number of noise photons per mode is taken to be small: $b \ll 1$. This scenario corresponds, for example, to single photons in the optical regime directed at a distant target that is bathed in thermal radiation at temperature significantly below optical energies. The detector can distinguish between d modes per detection event: $d = WT$, where W is the bandwidth of the detector and T is the temporal length of the detection window. The time window for detection is sufficiently small that at most one noise photon is detected at a time, i.e., $db \ll 1$. Additional, nonthermal noise can be tolerated as well, as long as fewer than one photon arrives per detection event. These assumptions, small reflectivity and small numbers of noise photons, are made for ease of analysis only. Quantum illumination can also work in the presence of high numbers of noise photons and high detection rates. In the second noise model analyzed below, arbitrarily high amounts of noise can be tolerated.

First, consider the case of unentangled light. Send a single photon in the state ρ toward the region where the object might be. The two different dynamics corresponding to object there and object not there are as follows (*8*):

Case (0), object not there: $\rho \rightarrow \rho_b \otimes \ldots \otimes \rho_b$, where ρ_b is the thermal state of a mode with b photons on average, and $\otimes$ is the tensor product. Because the average number of photons bd received per detection event is much less than 1, the thermal state can be approximated as

$$\rho_0 = (1 - db)|vac\rangle\langle vac| + b\sum_{k=1}^{d} |k\rangle\langle k| \quad (1a)$$

Here, $|vac\rangle$ is the vacuum state of the modes, and $|k\rangle$ is the state where there is a single photon in mode k and no photons in the other modes. Because $\| \rho_0 - \rho_b \otimes \ldots \rho_b \|_1 = (db)^2 + O[(db)^3]$, the exact thermal state can be safely replaced by ρ_0 as long as we are evaluating the expression to lowest order in db.

Case (1), object there: $\rho \rightarrow (1 - \eta)\rho_0 + \eta\rho_{th}$ where ρ_{th} is the thermalized version of ρ. It is straightforward to verify that $\|\rho - \rho_{th}\|_1 = db + O[(db)^2]$. Accordingly, if we are not interested in terms of $O(\eta db)$, then we can safely approximate $(1-\eta)\rho_0 + \eta\rho_{th}$ as

$$\rho \rightarrow \rho_1 = (1 - \eta)\rho_0 + \eta\rho \quad (1b)$$

Now repeatedly send single photons in the state $|\psi\rangle$ to detect presence or absence of the object. The single-shot minimum probability of error is obtained by projecting onto the positive part of $\rho_1 - \rho_0$ [see (*1*–*3*) for a treatment of discrimination between different dynamics in the case of qubits]. This measurement consists simply of verifying whether a returning photon is in the state $|\psi\rangle$ or not. If the measurement yields a positive result, then we guess that the object is there. If the result is negative, then we guess that the object is not there. The conditional probabilities of the outcomes "yes" and "no" given the presence or absence of the object are

$$p(\text{no}|\text{not there}) = 1 - b$$
$$p(\text{no}|\text{there}) = (1 - b)(1 - \eta)$$
$$p(\text{yes}|\text{not there}) = b$$
$$p(\text{yes}|\text{there}) = b(1 - \eta) + \eta \quad (2)$$

If we iterate the optimal single-shot measurements, Eq. 2 determines the number of trials required to reveal presence or absence of the object. The number of trials required depends on the ratio η/b. If $\eta/b > 1$, then a received photon is more likely to be a signal photon than a noise photon: the signal-to-noise ratio is greater than 1. We'll call this the good regime. In the good regime, the number of trials required to detect the object, if there, goes as $O(1/\eta)$: one simply sends photons

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until one receives one back. If the object is there, then one receives a photon back considerably before one would expect a photon given thermal photons alone.

Similarly, if $\eta/b < 1$, then a received photon is more likely to be a noise photon than a signal photon: The signal-to-noise ratio is less than 1. We'll call this the bad regime. In the bad regime, most of the photons received are noise photons. Here, one must count photons until one can separate the thermal distribution with b photons on average from the distribution when the object is there, which has $b + \eta$ photons on average. By using the usual formulae for sampling the outcomes of Bernoulli trials, one finds that it takes $O(8b/\eta^2)$ photons on average to distinguish between presence or absence of the object in the bad regime.

These estimates of the number of trials required to distinguish between the presence and the absence of the object are just lower bounds because they come from iterating the optimum single-shot measurement. The true asymptotic limit on the number of trials is given by the quantum Chernoff bound (*9*, *10*). The quantum Chernoff bound for photocounting is calculated in (*11*) and confirms that the iteration of the single-shot measurement is indeed optimal.

The error probabilities for detecting presence or absence of the object do not depend on the number of signal and detector modes, d. The number of modes doesn't matter because all modes other than the one in which the photon is sent are in a thermal state. These other modes give us no information about the presence or absence of the object. To detect the object, we need only monitor the mode in which we sent the photon to see whether more than the expected number of photons come back.

Now look at the effect of entanglement on our ability to detect the object. Construct the entangled state $|\psi\rangle_{SA} = (1/\sqrt{d})\sum_k |k\rangle_S |k\rangle_A$ for signal photon and ancilla photon and send the signal photon to toward where the object is likely to be. This state can be created, for example, by taking the output of a spontaneous parametric downconverter in the low-photon number regime, matching its time-bandwidth product to the time-bandwidth product of the detector, and selecting out the one signal photon/one idler photon sector [photodetection at the receiver automatically postselects this state (*11*)]. The signal photon is sent off, and the idler photon is retained as the ancilla.

The two different dynamics corresponding to object or no object now take a different form because the state of the ancilla must be included. If the signal photon is lost, the ancilla photon goes to the completely mixed state:

Case (0), object not there:

$$|\psi\rangle_{SA}\langle\psi| \rightarrow \rho_{SA0} = \rho_0 \otimes \frac{I_A}{d} =$$
$$[(1-db)|vac\rangle_S\langle vac| + bI_S] \otimes \frac{I_A}{d} + O(b^2) \quad (3a)$$

Case (1), object there:

$$|\psi\rangle_{SA}\langle\psi| \rightarrow \rho_{SA1} = (1-\eta)\rho_{SA0} + \eta|\psi\rangle_{SA}\langle\psi| + O(b^2, \eta b) \quad (3b)$$

Here I_S and I_A are the identity operators on the single photon Hilbert spaces for the system and ancilla, respectively. As before, the single-shot minimum error probability is obtained by projecting onto the positive part of $(\rho_{SA1} - \rho_{SA0})$, which in turn simply corresponds to determining whether any returning photon is in the state $|\psi\rangle_{SA}$.

Making the optimal single-shot measurement and evaluating the conditional error probabilities for the entangled case yields

$$p_e(\text{no}|\text{not there}) = 1 - \frac{b}{d}$$
$$p_e(\text{no}|\text{there}) = (1 - \frac{b}{d})(1-\eta)$$
$$p_e(\text{yes}|\text{not there}) = \frac{b}{d}$$
$$p_e(\text{yes}|\text{there}) = (1-\eta)\frac{b}{d} + \eta \quad (4)$$

Comparison with the unentangled case, Eq. 2, immediately reveals that the effect of entanglement is to reduce the effective noise from b to b/d. This reduction reflects the fact that in the entangled case a noise photon together with the fully mixed ancilla is d times less likely to be confused for a signal photon entangled with the ancilla than a noise photon is likely to be confused with a signal photon in the unentangled case. Entanglement reduces the effective signal to noise by a factor of d. Again, the single-shot minimum error probability for the entangled case can be shown to coincide with the asymptotic minimum error probability by evaluating the quantum Chernoff bound (*11*).

Comparing the error probabilities for entangled states, as given by Eq. 4, with those for unentangled states, Eq. 2, we see that there are once more two regimes. The good regime now occurs when $\eta d/b > 1$. In the good regime, it once again takes $O(1/\eta)$ trials to determine whether the object is there. Comparing the entangled case to the unentangled case above, we see that the number of trials is the same in the good regime in both cases, but the good regime extends d times further in the entangled case than in the unentangled case, where the good regime occurred for $d/b > 1$.

The extension of the good regime via the use of entangled photons can be understood as follows. As before, the quantum Chernoff bound analysis given in (11) shows that the optimal detection strategy is to measure any incoming photon together with the ancilla to see whether the two photons are in the state $|\psi\rangle_{SA}$. If the photon that returns is the signal photon, then it will pass the test. If the photon that returns is a noise photon, then the ancilla is in the state I_A/d, and the noise photon together with fully mixed ancilla are d times less likely to be found in the state $|\psi\rangle_{SA}$. A noise photon in the entangled case is d times less likely to pass the test and be confused as a signal photon than a noise photon in the unentangled case. In other words, the presence of entanglement makes it d times harder for a noise photon to masquerade as a signal photon. Entanglement effectively enhances the signal-to-noise ratio by a factor of d.

The bad regime for the entangled case occurs for $\eta d/b < 1$. In this case, the number of iterations required to determine whether the object is there or not is $O(8b/\eta^2 d)$. Comparing with the unentangled bad regime, we see that the entangled bound is d times better than the unentangled bound: Quantum illumination reduces the number of trials needed to detect the object by a factor d. Entanglement effectively enhances the signal-to-noise ratio by the degree of entanglement, even in the bad regime.

The fact that entanglement yields an enhancement in the bad regime is particularly interesting because in the bad regime the combination of noise and loss ensures that no entanglement between signal and ancilla survives, an effect that also appears in the qubit case (*1*–*3*). Nonetheless, even though signal and ancilla are unentangled at the detector, a noise photon still finds it d times harder to masquerade as a signal photon entangled with an ancilla photon. Entanglement effectively enhances the signal-to-noise ratio of detection by a factor of d, where d is the number of entangled modes. Measured in terms of m, the number of e-bits of entanglement, the enhancement is $d = 2^m$. The enhancement is exponential in the number of e-bits.

The noise model above assumed small numbers of noise photons per mode and a sufficiently short detection window that at most one photon is detected during that window. These assumptions can be relaxed. Consider, for example, a noise model in which if the object is not there, then the state received at the detector is $\rho_S = I_S/D$, where D is the total number of possible noise states that possess the same energy as the signal. In other words, the noise is described by the microcanonical ensemble, and $D = 2^m$, where m is the entropy of the noise measured in bits. If the object is there, then with a small probability η the signal is reflected back unchanged, whereas with probability $1 - \eta$ the signal is replaced by noise. This model corresponds, for example, to the detection of a reflective, multifaceted object tumbling through space. Most of the time, one receives only background noise, but every once in a while a facet reflects the signal back to the detector.

This "microcanonical" noise model can be analyzed by using the same tools used above, including the quantum Chernoff bound. As shown in (*11*), the model yields similar conclusions to the low-temperature model analyzed above. When an unentangled state $|\psi\rangle$ is sent, then there are two regimes for detection. When $\eta D > 1$, we are in the good regime, with signal-to-noise ratio greater than one, and $O(1/\eta)$ signals must be sent to detect the presence of the object. When $\eta D < 1$, we are in the bad regime, with signal to noise less than one, and $O(8/D\eta^2)$ signals must be sent. By contrast, when half of a fully entangled state is sent out, the good regime is extended to the case $\eta D^2 > 1$: Just as in the low-noise case above, entanglement extends the good regime by a factor D. In the bad regime, where $\eta D^2 < 1$, the number of signals that must be sent to detect the object is $O(8/D^2\eta^2)$: Just as in the low-noise case, initial entanglement reduces the number of signals that needs to be sent by a factor of $D = 2^m$, where m is the number of bits of entanglement. As before, in the bad regime, no entanglement survives at the receiver.

This example elucidates nature of the enhancement afforded by entanglement. For the microcanonical noise model, the sensitivity of the unentangled detection goes as the dimension of the signal space, whereas the sensitivity of the entangled detection goes as the dimension of the signal space squared. This enhancement is similar to that obtained in superdense coding (*12*), in which entanglement allows one to use a single qudit (signal space with dimension D) to send two classical dits (dimension D^2). In the low-noise model, by contrast, the linear nature of the noise model rendered the sensitivity of detection independent of the number of signal modes. For both noise models, adding entanglement enhances the sensitivity by the dimension of the signal space.

Quantum illumination is a potentially powerful technique for performing detection, in which signal is entangled with an ancilla and entangling measurements are made at the detector. Entanglement enhances the effective signal-to-noise ratio because a noise photon has a harder time masquerading as an entangled signal photon compared with a noise photon masquerading as an unentangled signal photon. The enhancement of sensitivity and effective signal-to-noise ratio that quantum illumination provides is exponential in the number of bits of initial entanglement and persists even in the presence of large amounts of noise and loss, when no entanglement survives at the receiver. This entanglement-induced enhancement for detection is reminiscent of entanglement-assisted communication capacity (*12*, *13*), where large enhancements in the presence of noise also occur.

Quantum illumination can be used for ranging and imaging: The effective signal-to-noise enhancement persists as we sweep the detection window in time to determine the distance to the object and as we sweep the signal beam in space to image features of the object. Many practical questions remain; notably, can the requisite entangled measurements be performed efficiently? Detection of entangled photons is a well-studied topic [see, e.g. (*14*)], and the requisite upconversion of entangled photons has been performed experimentally (*15*) [for a more detailed discussion of detection issues see (*11*)]. Does the enhancement persist at higher noise temperatures and for larger numbers of photons in the signal? What are the maximum enhancements obtainable via quantum illumination over multiphoton input states, including Gaussian states? These questions and many others must be answered before quantum illumination can prove itself useful in practice.

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reCAPTCHA: Human-Based Character Recognition via Web Security Measures

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CAPTCHAs (Completely Automated Public Turing test to tell Computers and Humans Apart) are widespread security measures on the World Wide Web that prevent automated programs from abusing online services. They do so by asking humans to perform a task that computers cannot yet perform, such as deciphering distorted characters. Our research explored whether such human effort can be channeled into a useful purpose: helping to digitize old printed material by asking users to decipher scanned words from books that computerized optical character recognition failed to recognize. We showed that this method can transcribe text with a word accuracy exceeding 99%, matching the guarantee of professional human transcribers. Our apparatus is deployed in more than 40,000 Web sites and has transcribed over 440 million words.

A CAPTCHA (*1*, *2*) is a challenge-response test used on the World Wide Web to determine whether a user is a human or a computer. The acronym stands for Completely Automated Public Turing test to tell Computers and Humans Apart. A typical CAPTCHA is an image containing several distorted characters that appears at the bottom of Web registration forms. Users are asked to type the wavy characters to "prove" they are human. Current computer programs cannot read distorted text as well as humans can (*3*), so CAPTCHAs act as sentries against automated programs that attempt to abuse online services. Owing to their effectiveness as a security measure, CAPTCHAs are used to protect many types of Web sites, including free–e-mail providers, ticket sellers, social networks, wikis, and blogs. For example, CAPTCHAs prevent ticket scalpers from using computer programs to buy large numbers of concert tickets, only to resell them at an inflated price. Sites such as Gmail and Yahoo Mail use CAPTCHAs to stop spammers from obtaining millions of free e-mail accounts, which they would use to send spam e-mail.

According to our estimates, humans around the world type more than 100 million CAPTCHAs every day (see supporting online text), in each case spending a few seconds typing the distorted characters. In aggregate, this amounts to hundreds of thousands of human hours per day. We report on an experiment that attempts to make positive use of the time spent by humans solving CAPTCHAs. Although CAPTCHAs are effective at preventing large-scale abuse of online services, the mental effort each person spends solving them is otherwise wasted. This mental effort is invaluable, because deciphering CAPTCHAs requires people to perform a task that computers cannot.

We show how it is possible to use CAPTCHAs to help digitize typeset texts in nondigital form by enlisting humans to decipher the words that computers cannot recognize. Physical books and other texts written before the computer age are currently being digitized en masse (e.g., by the Google Books Project and the nonprofit Internet

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