Drexel UNIVERSITY

DEPARTMENT OF PHYSICS

PhD Qualifying Exam **Modern Physics**

Friday, September 24, 2004 1 pm - 4 pm

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper…even the short ones. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. *Circle* the numbers below to indicate which questions you have answered—write nothing on the lines.

Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

MODERN PHYSICS

PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1. The radial part of the Schroedinger equation for an arbitrary central potential can be cast into the form

$$-\frac{\hbar^2}{2\mu}\frac{d^2U(r)}{dr^2}+\left[V(r)+\frac{\hbar^2\ell(\ell+1)}{2\mu r^2}\right]U(r)=EU(r),$$

where U(r) is the radial wave function. Consider the special case of the hydrogen atom.
(a) For a state with zero angular momentum, an uncritical look at the shape of the Coulomb potential energy suggests that the most energetically favorable state (i.e. the state of lowest potential energy) would correspond to the electron coming as close as possible to the proton. How does quantum mechanics prevent this collapse?
(b) Consider now the general case of <u>non-zero</u> angular momentum. With the help of the above radial equation, provide a convincing argument (no calculations are needed) in support of the existence of a range of energy values for which bound stationary states can be found.
(c) Again with the help of the above radial equation, argue for the existence of unbound states of the hydrogen atom corresponding to a continuous spectrum of energies.

2. (a) What is the ground state electronic configuration of a lithium atom (Z=3)?
   (b) Give a rough estimate of the energy of the 2p state of lithium.

3. 3 particles are confined to a cubical box with sides of length a, centered at x=y=z=0.
(a) What is the ground state energy if they are 3 bosons?
(b) What is the ground state energy if they are 3 fermions?
(Hint: write down the eigenfunctions and energy levels of the 1D box, generalize to 3D, then populate the energy levels in a way that obeys the exclusion principle, where appropriate.)

4. For a spin 1/2 particle, suppose we measure the sum of the x and z components of the spin **S**. The Pauli matrices are:

$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

(a) What are the possible results of this measurement of S_x+S_z?
(b) Suppose that the measurement yielded the largest possible eigenvalue. Find a spin wave function that describes the state of the system immediately after measuring the sum of the x and z components.

5. Brrrrr! It is KKCCCOOOOOOOOLLLLDDDD outside!! In running from 30th St. Station to Drexel at 8:10 in the morning, you remember that you have the Physics Department's most notorious professor for the most boring class in the world. If you don't stop along the way for a hot cup of coffee you will surely fall asleep even before you sit down in your seat. Soooooo, you stop at one of the trucks and ask for a hot cup of half & half (half hot coffee, half cold milk). The proprietor/ress asks you if you want to take the two halves with you, and mix them once you get to class, or if you want them mixed there, and then run to class with the whole. What do you answer, in the interests of having the hottest drink after you arrive? Explain.

6. Give a physical interpretation of chemical potential. What roles does chemical potential play in diffusive motions, chemical reactions, and phase transitions?

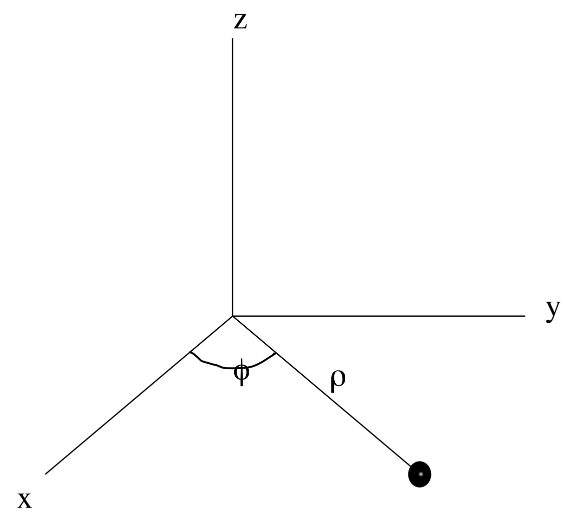
7. A system has two energy levels with an energy gap of 0.50 eV. The upper level is degenerate with 3 states, while the ground state is non-degenerate. What is the probability that any of the excited states is occupied at 25 ºC?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

A particle of mass M is fixed at one end of a rigid rod of negligible mass and length ρ. The other end of the rod holds the mass, and system is free to rotate about the origin about the z-axis. The system is shown in the figure below.



(a) Write an expression for the total energy of the system in terms of its angular momentum **L** and moment of inertia I, and write the time-independent Schroedinger equation using polar coordinates (ρ,φ).

(b) Find the normalized solution, $\Psi(\varphi)$, of the Schroedinger equation. Is there any degeneracy?

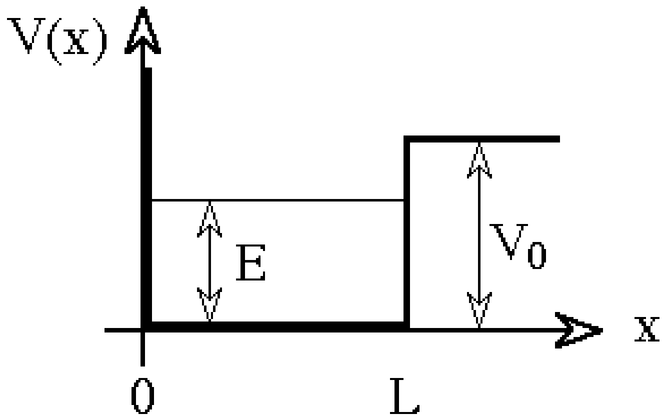
(c) Find the allowed energies for the two-dimensional rotator.

(d) Using the old quantum theory (Wilson-Sommerfeld rules), what are the allowed energies of the two-dimensional rotator? Compare this result to (c).

(e) Consider a particle of mass M fixed at one end of a rigid rod of negligible mass and length ρ, but this time the mass is free to move about the origin in three dimensions. Without proof, what are the allowed energies of this system, and compare this set of values to the two-dimensional rotator. Can this more general case reduce to the simpler case?

A2.

A particle of mass m is located in a one dimensional potential field $V(x)$ whose shape is shown in the figure below, where $V(0) = +\infty$



(a) Find the transcendental equation defining the possible bound state energies of the particle.

(b) Sketch a graphical solution of this equation and identify (of course only qualitatively) the values of the discrete energy spectrum.

(c) Find the minimum value of the quantity $L^2 V_0$ for which the first bound state energy appears for $E < V_0$.

A3.

A particle of mass *m* moves under the action of a two-dimensional field of force whose potential energy is given by

$$V(X,Y) = \frac{m\omega^2}{2}\left[(1+k)^2 X^2 + (2-2k)^2 Y^2\right],$$

where $\omega > 0$ and *k* is a parameter whose range of variation is $0 \le k \le 1$.

(a) Write the Hamiltonian of this system in terms of the creation and annihilation operators $a_x^\dagger$, $a_y^\dagger$, a_x, and a_y. Also, write down the eigenstates and the corresponding eigenvalues.

(b) Plot the ground state energy and the energies of the lowest <u>three</u> excited states as functions of the parameter *k*. Discuss the possible degeneracies.

(c) When $k = 0$, the angular momentum operator $L_z = XP_y - YP_x$, expressed in terms of creation and annihilation operators, takes the form

$$L_z = i\hbar\left(a_x a_y^\dagger - a_x^\dagger a_y\right).$$

If the oscillator is prepared in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle + i|0,1\rangle\right),$$

what is the expectation value of its angular momentum along the z-axis?

Hint: $X = \sqrt{\frac{\hbar}{2m\omega}}\left(a_X^\dagger + a_X\right)$, $\qquad P_X = i\sqrt{\frac{m\hbar\omega}{2}}\left(a_X^\dagger - a_X\right)$.

B1.

N atoms are arranged to lie on a simple cubic crystal lattice. Then M of these atoms moved from their lattice sites to lie at the interstices of the lattice, that is, points which lie centrally between the lattice sites (i.e., centers of the unit cells). Assume that the atoms are placed in the interstices in a way which is completely independent of the positions of the vacancies; therefore a lattice atom does not necessarily go to one of the nearest interstitial sites. Assume also that no two atoms may be placed at the same interstitial site and N and M are much larger than 1.

(a) Show that the number of ways of taking M atoms from the lattice sites and placing them on interstices is

$$\mathrm{W} = \left[\frac{N!}{M!(N-M)!}\right]^2,$$

if there are N interstitial sites where displaced atoms can sit.

(b) Suppose that the energy required to move an atom off its lattice site into an interstitial site is ε. The energy is U = Mε, if there are M interstitial atoms. Show that the fraction of interstitial atoms over the total number of atoms at temperature T is given by

$$\frac{M}{N} = \frac{1}{e^{\frac{\varepsilon}{2k_B T}} + 1}.$$

B2.

Consider an ideal gas made of a uniform suspension of non-interacting, randomly oriented electric dipoles, of mass m and dipole moment μ in a uniform electric field of magnitude E.

(a) Write the classical Hamiltonian for each dipole.
(b) Write the canonical partition function for an ideal gas consisting of an ensemble of N such dipoles in a volume V at a temperature T.
(c) Derive an expression for the Helmholtz free energy of the ensemble as a function of field E.
(d) What happens to the entropy S as the field increases?
(e) Explain *qualitatively* why your answer to part (d) makes sense.