Assignment II PHYS 521 (due April 24, 2015)

Problem 1

Show that for a given N, with $\sum_{i}^{N} p_i = 1$, the uncertainty function $S(\{p_i\})$, takes its maximum value when $p_i = 1/N$ for all i, that is, $S(\{p_i\}) = A(N)$.

Problem 2

Find the extremum value of $Ax^2 + By^2$ under the constraint, $ax + by = c$. Do it in two ways, first by direct substitution, then by Lagrange's multiplier method.

Problem #3

Consider the urn problem discussed in class: An urn is filled with balls, each numbered n = 0, 1, or 2. The average value of n is $<n> = 2/7$. Calculate the probabilities, p_0, p_1, p_2, which yield the maximum uncertainty. Find the expectation value, based these probabilities, of $<n^3> - 2<n>$.

Problem #4

Assuming that the entropy, S, and the number of microstates, Ω, of a physical system are related through an arbitrary functional form, $S = f(\Omega)$, show that the additive character of S (an extensive parameter) and the multiplicative character of Ω (meaning, $\Omega = \Omega_1 * \Omega_2$, Ω_i is the number of microscopic states for a subsystem) necessarily require that the function $f(\Omega)$ be of the form of

$$S = k \ln \Omega,$$

where k is a (universal) constant. This form was first written down by Max Planck.

Problem #5

Show that $\ln x \le x - 1$ for all real x. The equality holds for x = 1.
Hint: It may be easier to prove it by a graphical method. Also you can show it is equivalent to proving the inequality, $e^x \ge 1 + x$, and notice that e^x is a monotonically increasing function of x as is its derivative.

Problem #6

Prove that $\log_2 X = \log X / \log 2$.