Quantum II HW2

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1 Problem 1

Electronic configurations of
a. He(Z=2) $1s^2$
b. Ne(Z=10) $1s^2 2s^2 2p^6$
c. Ar(Z=18) $1s^2 2s^2 2p^6 3s^2 3p^6$
d. O (Z=8) $1s^2 2s^2 2p^4$
e. Fe (Z=26) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6$

2 Problem 2

Nuclear configuration of:
a) ${}^{4}_{2}He$: Protons(2):$1s^{(2)}_{1/2}$ Neutrons(2): $1s^{(2)}_{1/2}$
b) ${}^{16}_{8}O$: Protons(8): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(2)}_{1/2}$ Neutrons(8): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(2)}_{1/2}$
c) ${}^{40}_{20}Ca$ Protons (20): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(2)}_{1/2}$ $1d^{(6)}_{5/2}$ $2s^{(2)}_{1/2}$ $1d^{(4)}_{3/2}$
Neutrons (20):$1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(2)}_{1/2}$ $1d^{(6)}_{5/2}$ $2s^{(2)}_{1/2}$ $1d^{(4)}_{3/2}$
d) ${}^{56}_{26}Fe$ Protons (26): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(2)}_{1/2}$ $1d^{(6)}_{5/2}$ $2s^{(2)}_{1/2}$ $1d^{(4)}_{3/2}$ $1f^{(6)}_{7/2}$
Neutrons (30): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(2)}_{1/2}$ $1d^{(6)}_{5/2}$ $2s^{(2)}_{1/2}$ $1d^{(4)}_{3/2}$ $1f^{(8)}_{7/2}$ $2p^{(2)}_{3/2}$
e) ${}^{13}_{6}Ca$ Protons (6): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ Neutrons (7): $1s^{(2)}_{1/2}$ $1p^{(4)}_{3/2}$ $1p^{(1)}_{1/2}$

3 Problem 3

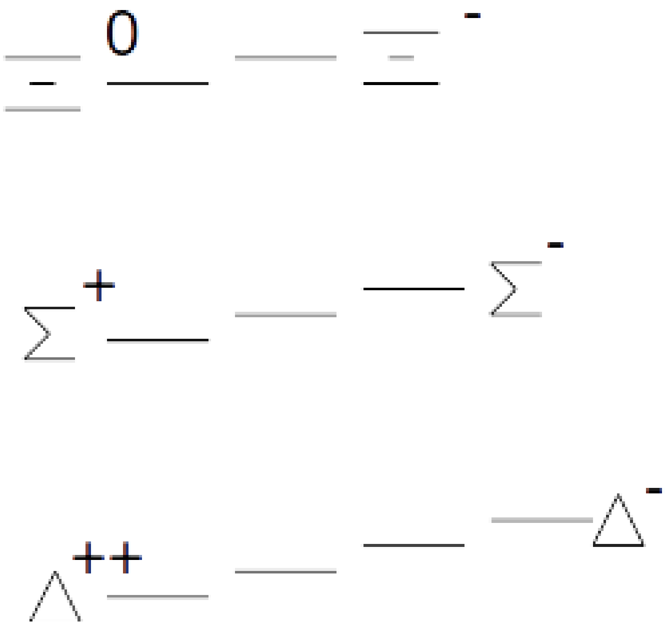
The ground state configurations for the three atoms are:
a) ${}^{57}_{26}Fe$: Protons (26) valence $1f^{(6)}_{7/2}$ Neutrons (31): valence $2p^{(3)}_{3/2}$

b) ${}^{57}_{27}Co$ Protons (27) valence $1f^{(7)}_{7/2}$ Neutrons (30): valence $2p^{(2)}_{3/2}$
c) ${}^{57}_{28}Ni$ Protons (28) "valence" $1f^{(8)}_{7/2}$(full) Neutrons (29): valence $2p^{(1)}_{3/2}$

So ${}^{57}_{26}Fe$ has a nuclear spin of $\frac{3}{2}$ and a parity of $(-1)^{\ell} = -1$ determined by the highest odd nucleon (the last neutron).
${}^{57}_{27}Co$ has a nuclear spin of $\frac{7}{2}$ and a parity of $(-1)^{\ell} = -1$ determined by the highest odd nucleaon (the last proton).
${}^{57}_{28}Ni$ has a nuclear spin of $\frac{3}{2}$ and a parity of $(-1)^{\ell} = -1$, determined entirely by the single valence neutron.

4 Problem 4

a. A "sensible" plot of the energy levels of the lightest nine particles:



b. Since the observed particles split into three groups with an average energy difference of 150 between them, I would guess the energy of the 10th particle to be 1685.
c. We look for a 3D harmonic oscillator model with N=3 and approximately the right energy levels. We break the symmetry of the 3D harmonic oscillator by $\omega_1 = \alpha$, $\omega_2 = \omega_3 = \beta$. Setting $\alpha - \beta = 150$ will create the desired separation between the energy levels. To find the value of β we calculate the lowest energy level $\langle 003 \rangle$ as $\frac{\beta+150}{2} + \frac{1}{2}\beta + \frac{7}{2}\beta = 1233$ so $\beta = 257.33$.
d. The Hamiltonian for the 3D harmonic oscillator can be written using

annihilation and creation operators:

$$H = (a_1^\dagger a_1 + \frac{1}{2})\hbar\omega_1 + (a_2^\dagger a_2 + a_3^\dagger a_3 + 1)\hbar\omega_{23} \tag{4.1}$$

e. We find the best fit energies by varying our basic parameters and minimizing the mean-squared error (see p4.m). With one million trials we find the best fit (in an MMSE sense) to be $\omega_1 = 408.07,\ \omega_2 = 258.78,\ \omega_3 = 256.27$. With these parameters our prediction for the 10th particle is 1686.
f. We test the quality of the MMSE fit with a chi-squared test, incorporating the variance from the measurements. We find a chi-squared value of 0.178 with 9 degrees of freedom, so we can reject the null hypothesis with confidence greater than 99.5