DEPARTMENT OF PHYSICS

PhD Qualifying Exam **Classical Physics**

Friday, September 19, 2003 9 am - 12 noon

PRINT YOUR NAME________________________

EXAM CODE__________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

CLASSICAL PHYSICS

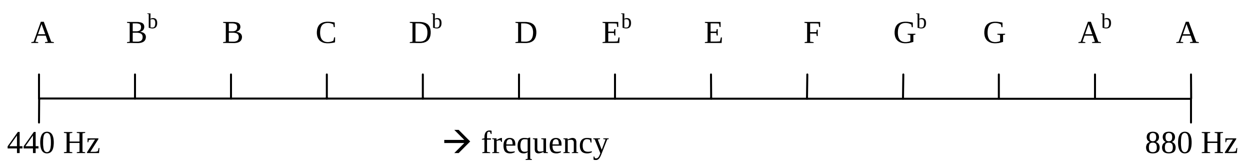
PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1. A wound up yo-yo is attached to a spring scale. The weight of the yo-yo is W, the weight of the string can be neglected.
a) If the yo-yo is allowed to unwind downward, would the scale indicate a reading less than, equal to, or greater than W?
b) After reaching the lowest point, the yo-yo moves upward, would the scale now indicate a reading less than, equal to, or greater than W?

2. A particle moves in a circular orbit of radius r_0 under the influence of an attractive central force of magnitude $F(r)= kr^{-n}$, where k is a constant. Derive a condition on F near radius $r = r_0$ for the orbit to be stable.

3. Astronomers have recently discovered that a very massive black hole in the Perseus galaxy cluster is whistling at a practically pure note. Specifically, it is emitting sound waves (pressure-density waves) in a thin gas in the cluster. The wavelength is λ = 30,000 light years and the period is 10,000,000 years.



a) What is the speed of these waves?
b) What is the frequency?
c) How many octaves below the concert scale (above) is this frequency?
d) What is the pitch of these waves?

4. An athlete can throw a javelin 60 m from a standing position. If he can run 100 m at a constant velocity in 10 s, how far could he hope to throw the javelin from a running start? (Neglect air resistance.)

5. An aluminum sphere of radius r = 10.0 cm is subjected to monochromatic ultraviolet radiation of wavelength λ = 2900 $\mathring{A}$ for a long time.
What is the charge on the sphere?

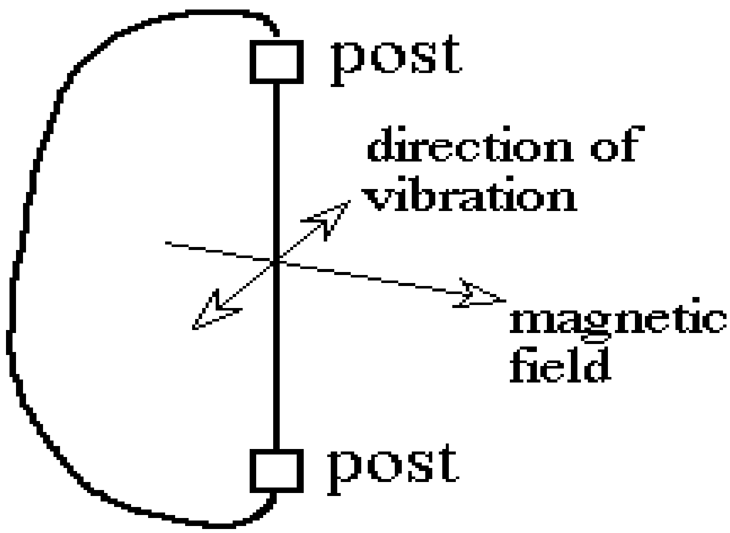
The work function of Al is ϕ = 4.19 eV, corresponding to a wavelength of λ_0 = 2960 $\mathring{A}$.
$h = 6.625 * 10^{-27}$ erg sec
$c = 3*10^{10}$ cm/sec
$hc = 12.41*10^{-5}$ eV-cm
$1\,\mathring{A} = 10^{-8}$ cm

6. As a lecture demonstration, a short cylindrical bar magnet is dropped down a vertical aluminum pipe a few meters long. It takes several seconds for the magnet to emerge at the bottom. An otherwise identical piece of unmagnetized metal makes the trip in a fraction of a second. Provide a *convincing* argument to explain why the magnet falls more slowly.

7. A taut metal wire is clamped between two fixed posts and a circuit loop is completed with some more loose wire, as sketched in the figure. If plucked transversely, the taut part of the wire oscillates, as expected. A horse-shoe magnet is now placed around the wire so that the direction of the magnetic field is roughly perpendicular to the direction of vibration.

a) Will a current run through the wire? Why, or why not?

b) If the wire can oscillate for about one minute before the oscillations are damped out in the absence of the magnet, will the length of the oscillations be affected by the magnetic field? Explain.

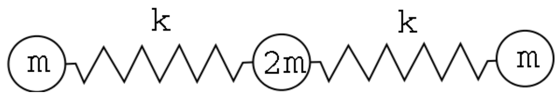


PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

Three particles, of masses m, 2m, and m, respectively, are connected by two springs, each of spring constant k, and are free to move in one dimension without friction, as sketched below.

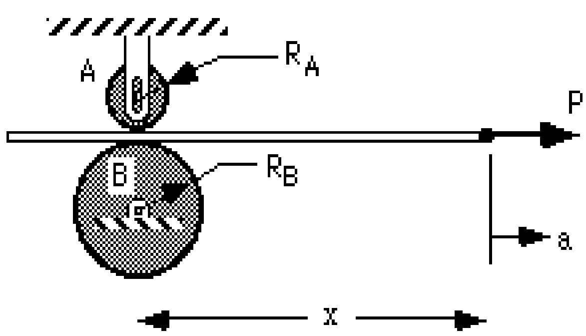


Determine the normal modes of oscillation of the system, and calculate their frequencies.

A2.

A belt of mass 0.8 kg passes between two solid cylinders and is pulled to the right by a constant force "**P**". It has acceleration "a" to the right, as shown below. The top cylinder "**A**" (radius R_A = 75 mm, mass 8.0 kg and moment of inertia $I_A = 2.25 \times 10^{-2}$ kg m^2) is free to move **vertically**, as well as rotate, in the bearing blocks that support its shaft. Cylinder "**B**" (radius R_B = 150 mm, mass 24.0 kg and moment of inertia $I_B = 27 \times 10^{-2}$ kg m^2) is held in its bearings so it only rotates. Coefficients of friction between the belt and the cylinders are **unknown**, but it is known that **only rolling** (without any slipping between the belt and the cylinders) occurs. Neglect the friction in the bearings on the shafts of the cylinders.

a) Draw separate **free-body force diagrams** for the cylinders and the belt and show all the forces on each one of them.



b) If the acceleration of the belt is a = 1.5 m/s^2 to the right, determine the **angular accelerations** α_A and α_B of the cylinders A and B. Indicate the directions of these.

c) What is the **static frictional force** between the belt and the cylinder **B** such that no slipping occurs?

d) What is the **static frictional force** between the belt and the cylinder **A** such that no slipping occurs?

e) What is the magnitude of the required force **P** to perform this task?

f) If the **maximum acceleration of the belt** is a_{max} = 2.0 m/s^2 before any slipping occurs, find the coefficient of static friction, μ_S, between cylinder **B** and the belt.

A3.

Write down the Lagrangian of a system, undergoing small oscillations along a vertical axis, consisting of a mass *M* attached to **a spring of mass *m***, length *L*, and spring constant *k*. By solving the Lagrange's equation(s), show that the frequency of oscillation of mass M is given by, $\omega = [3k/(3M+m)]^{1/2}$.
(Note: Assume that the amplitude of oscillations is much smaller than the length of the spring, and the speed of any part of the spring is linearly proportional to its distance from the rigid support.)

B1.

A long circular cylinder of radius R carries a magnetization

$$\vec{M} = Cr^3\hat{a}_\theta, \qquad r \le R$$

where C is a constant and r is the distance from the axis (of course, the magnetization is zero outside the cylinder). There are no free currents anywhere within our Galaxy.

a) Find the magnetic field $\vec{B}$ <u>inside</u> and <u>outside</u> the cylinder with the help of Ampere's law

$$\oint_\gamma \vec{H} \cdot d\vec{\ell} = I_f^{encl},$$

where γ is a closed Amperian path of your choice.

b) Show that the boundary condition

$$B_1^\perp = B_2^\perp$$

is satisfied at the interface between the inside of the cylinder and the rest of the world outside. The subscript 1 stands for "inside" and 2 stands for "outside".

c) Calculate the bound current density $\vec{J}_b$ in the region $r < R$, and the bound surface current density $\vec{K}_b$ at the surface of the cylinder (i.e., for $r = R$).

d) Calculate the magnetic field $\vec{B}$ <u>inside</u> the cylinder using the alternative form of Ampere's law

$$\oint_{\gamma} \vec{B} \cdot d\vec{\ell} = \mu_0 I^{encl},$$

where γ is a closed Amperian path that you must select properly.

B2.

Two charges $+q$ and $-q$ are separated by a fixed distance d and rotate with a uniform angular velocity ω about an axis that passes mid-way between the two charges and which is perpendicular to the line joining the two charges. Calculate the time-averaged power radiated per unit solid angle $< \frac{dP}{d\Omega} >$ and the total power $< P >$ radiated by this system.

[Given: In the radiation zone the B-field due to a dipole of moment $\vec{p}$ is $B_{rad} = \frac{1}{c^2 r}\left[\ddot{\vec{p}}\right] \times \hat{n}$, where $\hat{n}$ is the unit vector in the direction of observation]