Drexel
UNIVERSITY

DEPARTMENT OF PHYSICS

PhD Qualifying Exam — **Modern Physics**

Friday, September 22, 2006 — 1 PM – 4 PM

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper…even the short ones. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. *Circle* the numbers below to indicate which questions you have answered—write nothing on the lines.

Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

MODERN PHYSICS

PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1. Consider the following statement: "By definition, the Hamiltonian acting on any allowed state of the system ψ will give the same state back, i.e., $H\psi = E\psi$, where E is the energy of the system." Explain in detail why you agree or disagree with this statement (simply answering agree or disagree will earn zero credit).

2. We all know that electrons obey Fermi statistics, and thus, are spin 1/2. Without going to low temperature or small number, describe an experiment which shows that electrons are fermions and not Bosons.

3. A hydrogen atom in the metastable 2S state can deexcite to the 1S state via several mechanisms. Describe at least two mechanisms, and for each describe how the deexcitation rate depends on: (a) particle density; (b) radiation temperature.

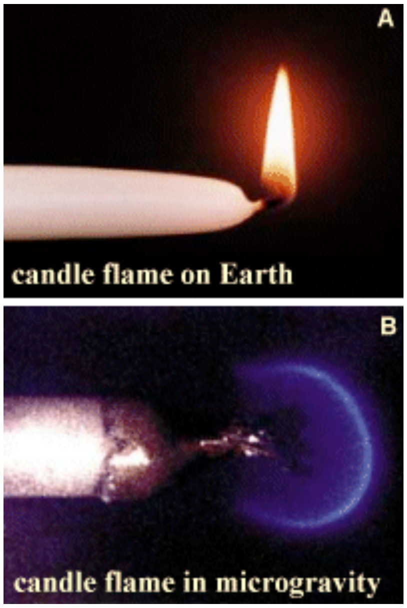
4. If the proton in a hydrogen atom is replaced by a positron, the resulting bound system is an exotic atom called positronium.

a) If, loosely speaking, we use the diameter of the atom as a measure of its size, what is the ratio of the size of an ordinary hydrogen atom to that of the positronium?
b) How does the energy spectrum of the positronium compare with that of a hydrogen atom in the simple case when one considers only Coulomb interaction and no relativistic effects?

5. In the microcanonical ensemble approach to statistical mechanics, the entropy of a thermodynamic system of energy E can be defined in terms of either its phase space volume, $\Gamma(E)$, or its density of states, $W(E) = \dfrac{\partial \Gamma}{\partial E}$. Explain to what extent these two definitions are equivalent.

6. Last August during the heat wave, a Physics graduate student preparing for this Qualifier was driven to distraction by the heat. He decided that he could do something constructive about it, because he was so brilliant. Sooo: He surrounded his apartment with lots and lots of insulation and figured that he could use his refrigerator to cool the apartment.
The refrigerator is rated at 3 amps. It uses a compression cycle that is 40:1 with a working fluid that liquefies at 30 °F. The refrigerator achieves 80% of the Carnot efficiency. The specific heat of the apartment is 0.03KWH/°F.
He is finally ready to cool off at 9 PM, when the temperature is 80 °F. So, he plugs in the refrigerator and opens the refrigerator door.
What is the temperature at midnight?

7. On the surface of the earth, the shape of a candleflame is like that of a yellow teardrop (Figure A). Under the same temperature and pressure conditions but in microgravity conditions (e.g. on the Space Shuttle or in a bottle dropped from a tower on earth's surface), the flame becomes bluish and spherical (See Figure B). Explain the difference in shape and color.



PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

The wavefunction of an electron in a one-dimensional infinite square well of width a, $x \in (0,a)$, at time t=0 is given by

$$\psi = \sqrt{\frac{2}{7}}\,\psi_1(\mathrm{x}) + \sqrt{\frac{5}{7}}\,\psi_2(\mathrm{x})$$

where $\psi_1(\mathrm{x})$ and $\psi_2(\mathrm{x})$ are the ground and first excited states of the system,

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\!\left(\frac{n\pi x}{a}\right)$$

with energy

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}.$$

(a) Write down the wavefunction ψ (x,t) at time t in terms of $\psi_1(\mathrm{x})$ and $\psi_2(\mathrm{x})$.

(b) You measure the energy of an electron at time t = 0. Write down the possible values of the energy and the probability of measuring each.

(c) Calculate the expectation value of the energy in the state ψ (x,t) from part (a).

(d) What is the minimum momentum of this electron, if we measure it?

(e) We add a second electron to the well. Write the properly symmetrized wavefunction (including both spatial and spin wavefunctions) for the *ground* state. Indicate the spin component of the wavefunction using χ_+ and χ_-, where χ_+ means that particle 1 is in a spin up eigenstate. Indicate the spatial part of the wavefunction using $|n_1, n_2\rangle$ where n_1, n_2 refer to the spatial wavefunctions above.

A2.

A two-dimensional harmonic oscillator is placed in a magnetic field of strength, B, perpendicular to the plane of motion.

(a) Sketch the energy level spectrum for B=0. Indicate the degeneracy explicitly.

(b) Describe the energy level splitting in small magnetic field. You can leave the ``g-factor'' expressed as an integral. Write the integral explicitly.

(c) Indicate how the spectrum changes when the field becomes sufficiently large. How large must B be to see the nonlinear effect at the 1% level?

A3.

Consider an electron whose only degrees of freedom are the spin states. The general state is expressed by the spinor wave:

$$\psi = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}.$$

The electron starts off (t = 0) as an eigenstate of the spin oriented in the z direction. At this instant it is placed in a constant t magnetic field B_0 pointing in the z-direction and a smaller oscillating field $B_1 \cos(\omega t)$ in the x-direction. The Hamiltonian of the system is

$$H = - \vec{M} \cdot \vec{B} = \frac{eg\hbar}{4mc} \vec{\sigma} \cdot \vec{B}.$$

(a) Write the time-dependent Schrodinger equation for the system and separate it into coupled differential equations for a(t) and b(t).
(b) By introducing $A(t) = a(t)\, e^{i\omega_0 t}$ and $B(t) = b(t)e^{i\omega_0 t}$, show that these reduce to a second order differential equation for A(t),

$$\frac{d^2 A}{dt^2} - i(2\omega_0 - \omega)\frac{dA}{dt} + \frac{\omega_1^2}{4} A = 0,$$

where m is the electron mass, g the gyromagnetic ratio and

$$B(t) = \frac{2i}{\omega_1}\frac{dA}{dt} e^{i(2\omega_o - \omega)t},$$

$$\omega_0 = \frac{geB_0}{4mc}, \qquad \omega_1 = \frac{geB_1}{4mc}.$$

(c) The general solution is

$$\psi(t) = e^{iwt}\begin{bmatrix} A_+ e^{i\lambda_+ t} + A_- e^{-i\lambda_- t} \\ -\dfrac{2}{\omega_1} e^{-i\omega_0 t}\left(A_+ e^{i\lambda_+ t} + A_- e^{-i\lambda_- t}\right) \end{bmatrix},$$

where

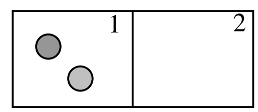
$$\lambda_\pm = \frac{2\omega_0 - \omega \pm \sqrt{(2\omega_0 - \omega)^2 + \omega_1^{\,2}}}{2}.$$

If a(0) = 1 and b(0) = 0, show that the probability that at a time t the spin of the election is in the negative z-direction, $\omega_1 << \omega$, is

$$|b(t)|^2 = (1 - \cos(\omega_1 t))/2$$

Thus, when the frequency of the oscillating field is tuned to $2\omega_0$, the probability of a spin flip is nearly unity.

B1.



A system consists of two distinguishable particles, each of which can be in either of two boxes as shown. The energy of a particle is zero in box 1 and ε in box 2. In addition, there is an interaction term that lowers the energy of the system by Δ when the two particles are in the same box.

(a) List all possible states of the system and the corresponding energy.

(b) Determine the probabilities that the system is in each of the states. Calculate the values of these probabilities for the case of T=300 K, Δ = 0.1 eV and, ε = 0.01 eV, and for the case of T=300 K, Δ = 0.01 eV and, ε = 0.1 eV.

(c) Calculate the average energy of the system as a function of T, Δ, and ε.

(d) Determine the heat capacity of the system for the case of $\Delta <<\varepsilon$ as a function of T, Δ, and ε.

(e) Determine the behaviors of the heat capacity calculated above at high temperatures $\varepsilon << k_B T$, and low temperatures $\varepsilon >> k_B T$, respectively.

B2.

An *ideal gas* occupying volume V_g is in equilibrium with a submonolayer (less than 1 atomic layer) adsorbed film of the same species of atoms. The film can be modeled as a solid surface with N_o adsorption sites, each with a binding energy $-\varepsilon_0$. Assume each site can hold only one adsorbed atom and that the atoms are indistinguishable from each other. Proceed through the following steps to calculate the vapor's pressure as a function of the actual number of atoms adsorbed N_f.

(a) Calculate the partition function for a single particle in the gas phase.

(b) Calculate the total partition function Z_g of the gas. Assume that there are N_g atoms in the gas phase.

(c) Determine the vapor's free energy function $F = - k_B T \ln Z$ and extract its chemical potential μ_g .You may use Stirling's approximation $\ln N! \approx N \ln N - N$.

(d) Using the ideal gas law equation, express your answer in (c) in terms of the vapor pressure Pg.

(e) To determine the film's chemical potential μ_f, consider N_f particles distributed over N_o possible sites ($N_f < N_o$), each with binding energy $- \varepsilon_0$. What is the partition function Z_f of this film ?

(f) Calculate the film's chemical potential μ_f.

(g) Use the expressions of the two chemical potentials μ_g and μ_f, determine the vapor pressure as a function of the coverage of the film N_f/N_o.

.