

PHYS 502: Mathematical Physics II

Winter 2015, Homework #1

(Due January 19, 2015)

1. (a) Let $\{\mathcal{R}_j\}$ be a random sequence of real numbers, with \mathcal{R}_j distributed uniformly between -1 and 1 . For the N -point discrete Fourier transform of $\{\mathcal{R}_j\}$:

$$r_k = \sum_{j=0}^{N-1} \mathcal{R}_j e^{2\pi i j k / N},$$

calculate the expectation value and variance of the “periodogram estimate” of the (unnormalized) power spectrum, $P_k = |r_k|^2 + |r_{N-k}|^2$, for $k = 1, \dots, N/2$.

(b) Generate a sequence of random numbers (using, for example, the `random` function in Linux or Python, or `ran2` in Numerical Recipes in C, p. 282) with properties as in part (a), and compute $\{P_k\}$ numerically using a fast Fourier transform with $N = 32768$. Plot first P_k , then $\log_{10} P_k$ against $\log_{10} k$, for $k = 1, \dots, N/2$.

Be sure to include appropriate data windowing, as described in *Numerical Recipes*.

How does your graph compare with analytic expectations? Repeat the calculation, averaging the plotted data over an interval of width 64 centered on each data point.

(c) Repeat the computation in part (b) for a *random walk* $\{w_j\}$ defined by $w_0 = 0$, $w_{j+1} = w_j + \mathcal{R}_j$. Can you account for the differences in appearance between this graph and one you obtained in part (b)?

2. A “corrupted” real-valued dataset may be found in the file `corrupt.dat` on the PHYS 502 Web site. It is a time sequence consisting of two columns of data, j and c_j , for $j = 0, \dots, N-1$. The original data have been convolved with a Gaussian transfer function of the form $g_j \sim \exp(-j^2/a^2)$ (normalized so that $\sum_j g_j = 1$), with $a = 2048$, and are subject to random noise of some sort at some level.

Find a filter to apply to the data, and plot your best-guess reconstruction of the original uncorrupted dataset. Rather than attempting to characterize the noise in detail and applying an optimal filter as in *Numerical Recipes*, it will be sufficient simply to truncate the data in Fourier space at some frequency dictated by the form of the power spectrum you obtain. Turn in (i) your program, (ii) a graph of the untruncated power spectrum, clearly indicating where you chose to truncate (and why—hint: create a log-log plot of the power spectrum), and (iii) a plot of your final “uncorrupted” time sequence.

Can you characterize the type of noise in the data?