DREXEL UNIVERSITY

## DEPARTMENT OF PHYSICS

**PhD Qualifying Exam** **Modern Physics**

Friday, September 20, 2002 1 pm - 4 pm

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper…even the short ones. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. *Circle* the numbers below to indicate which questions you have answered—write nothing on the lines.

| *Short questions* | | *Long Problems* | |
|---|---|---|---|
| *circle* | grade | *circle* | grade |
| 1. | _____ | A1. | _____ |
| 2. | _____ | A2. | _____ |
| 3. | _____ | A3. | _____ |
| 4. | _____ | B1. | _____ |
| 5. | _____ | B2. | _____ |
| 6. | _____ | | |
| 7. | _____ | | |

# MODERN PHYSICS

## *PART I: Short answers (25%)*

**ANSWER 5 OF 7 QUESTIONS**

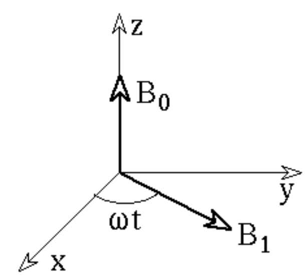
1. Consider a particle in the potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & x > 0 \\ \infty & x \le 0 \end{cases}$$

The only difference between this problem and the standard simple harmonic oscillator is the presence of the infinite barrier, which causes the wave function to vanish at $x = 0$. With the guidance of the solution of the simple harmonic oscillator problem, find the new eigenvalues. Note: No calculations are needed to answer this question.

2. What is the <u>nature</u> and <u>physical origin</u> of the force exerted on a neutral atom by a nearby conducting surface with a zero net charge? What is the essential difference between this force and the Van der Waals force between two neutral atoms?

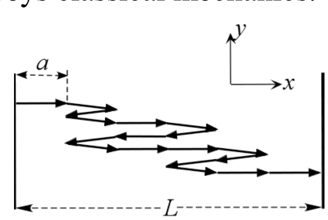
3. A particle is under the action of two magnetic fields of constant magnitude. One ($B_0$) is oriented along the z axis and the second ($B_1$) rotates in the x-y plane at a constant angular frequency $\omega$, as sketched in the diagram to the right.
In the rotating reference system where $B_1$ is stationary give a qualitative description of the evolution of a magnetic moment originally oriented along the positive direction of the z axis.




**4**. A particle is confined in a 2-d rectangular box with infinite potential at the walls, with sides of length L and L/2. What are the energies corresponding to the 4 lowest states?

5. A particle of mass *m* moves in one-dimensional space within the range $0 \le x \le a$, and is reflected (via elastic collision) by walls at $x=0$ and $x=a$. Sketch the trajectory of this particle in the phase space $(x, p)$, where $p$ is the momentum. Assume that the particle obeys classical mechanics.

6. A 1-dimensional chain has $N$ (>1) elements of length $a$, and the angle between adjacent elements can only be 0° or 180°. The joints can turn freely and the two ends of the chain are fixed at a distance $L$. If the entropy of chain is $S$, make a *rough* sketch of the dependence of $S$ on $L$, and *briefly* justify your sketch. No mathematical expressions or details are required. (The elements in the drawing are displaced in the $y$ direction for clarity).




**7.** Suppose a single particle has two possible energy states: -1/2 ε and 1/2 ε. Show that the average energy at temperature T is simply 1/2 ε tanh (ε/2kT)

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

## A1. Found around the quad

A system with angular momentum $l = 1$ is described by the quadrupolar Hamiltonian

$$H = \frac{\omega_0}{\hbar}\left(L_x^2 - L_z^2\right).$$

a) Derive a matrix representation for $H$.
b) Calculate the eigenvalues and the stationary states of this system.
c) At time $t = 0$ the system is prepared in the state

$$|\psi(0)\rangle = |l = 1, m = 1\rangle \equiv |1\rangle.$$

Calculate the state of the system at time $t > 0$.

d) At some time t a measurement of $L_z$ is carried out. What are the possible outcomes and the respective probabilities.

Note: $L_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

## A2. Out of whack

In a particular basis, the spin operators for a spin 1/2 particle (an electron with intrinsic magnetic moment $\mu_B$) may be expressed as the Pauli Spin Matrices:

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_y = i\frac{\hbar}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

At time $t = 0$, a constant magnetic field of amplitude $B$ is induced in the $x$ direction.

(a) What is the matrix representation of the Hamiltonian of the system in this basis?
(b) Assume that the initial state of the electron is expressed as $\psi(0) = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}$

What is the probability of measuring the $S_z$ as $\hbar/2$ at time t?

(c) What is the probability of measuring the $S_x$ as $\hbar/2$ at time t?
(d) Imagine that you measure the x-spin in part c and then immediately measure $S_x$. What then is the probability of measuring $\hbar/2$.

## A3. **Out of state residence?**

Let $|b'\rangle$ and $|b''\rangle$ be the eigenstates of a Hermitian operator B with eigenvalues $b'$ and $b''$, respectively ($b'$ is not equal to $b''$). The Hamiltonian operator is given by

$$H = |b'\rangle C \langle b''| + |b''\rangle C \langle b'| ,$$

where $C$ is just a real number.

(a) Calculate the eigenvalues and eigenvectors of the Hamiltonian.

(b) Suppose the system is known to be in state $|b'\rangle$ at $t$=0, what is the state vector in the Schroedinger picture for $t > 0$?

(c) What is the probability of finding the system in $|b''\rangle$ for $t > 0$, if the system is known to be in state $|b'\rangle$ at $t$=0?

## B1. **Perfect Harmony?**

A quantum harmonic oscillator has energy levels $E_n = \hbar\omega(n+1/2)$ where n = 0, 1, 2…

(a) Show that the probability for finding the oscillator in its n-th quantum state at temperature $T$ is

$$P_n = (1 - e^{-\hbar\omega/kT})e^{-n\hbar\omega/kT}$$

(b) What is the average internal energy of this harmonic oscillator at temperature $T$? What are its limiting values at very low and very high temperatures?

(c) What is its specific heat at temperature $T$? What are its limiting values at very low and very high temperatures?

NOTE: for $x < 1$, $\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}$

## B2. **Heavier and heavier**

An ideal gas is introduced into a "test tube" (actually a sealed cylinder). The gas density is $\rho_o$, and the mass of each molecule is $M$. The test tube is placed into a centrifuge, which then spins the tube at an angular velocity $\omega$. The equilibrium temperature is $T$.

(a) Compute the density of the gas in the test tube as a function of height, $h$, above the bottom. (The bottom of the tube, h = 0, is the point farthest from the rotation axis of the centrifuge).

(b) Describe how you would use a centrifuge to separate the isotope ${}^{235}_{92}U$ from ${}^{238}_{92}U$.

(c) Suppose the naturally occurring ratio of ${}^{235}_{92}U$ / ${}^{238}_{92}U$ is 0.7. Assume the Uranium is gasified by allowing it to react with fluorine (${}^{19}_{9}F$), forming the gas $UF_6$. Assuming that R = 100 cm, T = 300 K, and $\omega$ = 60,000 rpm, estimate the ratio ${}^{235}_{92}U$ / ${}^{238}_{92}U$ at the top of the "test-tube".

$k = 1.38 \times 10^{-23}$ J/K. Mass of the proton = $1.67 \times 10^{-27}$ kg.