DEPARTMENT OF PHYSICS

PhD Qualifying Exam **Modern Physics**

Friday, September 19, 2003 1 pm - 4 pm

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper…even the short ones. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. *Circle* the numbers below to indicate which questions you have answered—write nothing on the lines.

Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

MODERN PHYSICS

PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1. In two dimensions the parity operator Π is defined by the relation

$$\Pi\psi(x,y) = \psi(-x,-y),$$

where $\psi(x,y)$ is the wave function of the system of interest.

a) What does this definition become if ψ is calculated in polar coordinates, i.e. what is $\Pi\psi(\rho,\varphi)$?

b) What is the parity of the wave function

$$\psi(\rho,\varphi) = \frac{\cos\varphi}{\sqrt{\rho^2 + A^2}},$$

where A is a constant?

2. Consider a diatomic molecule (H_2, N_2, O_2) as a two point molecule with each particle having mass m connected by a massless rod of length r. Suppose the molecule rotates about an axis perpendicular to the rod through its midpoint. Using the Bohr-Sommerfeld quantization hypothesis, derive an expression for the allowed rotational kinetic energy states for this diatomic system.
Hint: Recall that the Bohr-Sommerfeld quantization hypothesis is $\oint pdq$ = nh for integer n.

3. Suppose that the outermost electrons in the Carbon atom are both in the 2p orbit and produce a $^1D_2$ spectroscopic term (the lowest energy is $^3P_0$). What is the degeneracy of this term? In other words, how many different states produce a $^1D_2$ term?
Hint: How many different spin states m_S are allowed? How many different total angular momentum states m_J are allowed?

4. Consider an ideal free-electron gas (N non-interacting electrons). Let ε_1, ε_2,..., ε_N denote the single-particle energies.

a) What is the ground state energy of this system when $T \to 0$, where T denotes the absolute temperature?
b) What do we mean by Fermi energy, E_F?
c) When $T \to 0$, the free-electron gas in a box has a density of states given by

$$\rho(E) = \frac{3}{2} N \frac{E^{1/2}}{E_F^{3/2}}, \qquad E \le E_F.$$

Sketch this function, and show with a second sketch how this function changes qualitatively when T is different from zero but still very small. Explain the physical reason for this behavior.

5. Explain what is meant by the thermal de Broglie wavelength and give an expression for this.

6. Physics principles can be used to understand problems associated with traffic jams. In a simple model, the probability of a car having speed in the range of (v, $v+dv$) is assumed to be $p(v) = Av\exp(-\frac{v}{v_0})dv$, where v_0 is a characteristic speed and A is a normalization constant. Find A and the average speed.

7.

a) You have heard that there is a finite probability of all the air molecules in your room being in the other half, leaving you breathless. How concerned are you about this? Why?

b) Shrink the "room" to a few (say 10 ~ 50) molecules, answer the same questions as in Part (a).

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

The initial state of the hydrogen atom is described by the state vector

$$|\psi(0)\rangle = \frac{1}{\sqrt{6}}\left(|1,0,0\rangle - i\sqrt{2}|2,0,0\rangle + \sqrt{3}|3,2,-2\rangle\right),$$ where |n,l,m> is a hydrogen atom eigenstate.

a) What is the state vector of the atom at time $t > 0$?
b) What is the expectation value of the energy at time t.
c) What are the expectation values of the operators L^2 and L_z?

A2.

A quantum mechanical harmonic oscillator is prepared in the initial state

$$|\psi(0)\rangle = |1\rangle - i\sqrt{2}|2\rangle .$$

a) Check if the state is properly normalized. If not, supply the required normalization constant.
b) Calculate the normalized state vector $|\psi(t)\rangle$ at the arbitrary time $t > 0$.
c) Calculate the expectation value $\langle P\rangle_t \equiv \langle\psi(t)|P|\psi(t)\rangle$ of the momentum operator at time t. Interpret the motion of the oscillator.
d) Calculate the expectation value $\langle H\rangle_t \equiv \langle\psi(t)|H|\psi(t)\rangle$ of the energy of the oscillator at the time t.

Hint: |n> is an eigenvector of the number operator, and $P = i\sqrt{\frac{m\omega\hbar}{2}}\left(a^\dagger - a\right)$.

A3.

Consider two non-interacting particles, both of mass m, in a one-dimensional box with infinite walls at x=0 and x=L.

a) If the particles are distinguishable, the states can be designated $|n_1, n_2\rangle$ where n_1, n_2 are integers. What is the degeneracy (number of states with same energy) of the first *excited* state? (You must show in detail how you computed this number. Simply guessing the answer without justification will earn zero credit.)

b) If the particles are electrons, write the properly symmetrized wavefunctions (including the spin part) for the *ground* and first *excited* states. Indicate the spin component of the wavefunctions using χ_+ and χ_-, where $\chi_+(1)$ means that particle 1 is in a spin-up eigenstate.

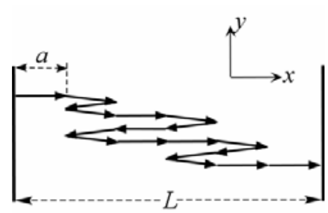
B1.

A monatomic ideal gas with spin ½ atoms in a magnetic field B has, in addition to its kinetic energy, a magnetic energy of $\pm\mu B$ per atom, where μ is the magnetic dipole moment. It is assumed that the gas is so dilute that the interaction of magnetic dipole moments may be neglected.

a) Derive an expression for the canonical partition function of N such atoms.
b) Derive expressions for the Helmholtz free energy and the internal energy of this N-atom system from the partition function.
c) Calculate heat capacity, C_v.

B2.

A 1-dimensional chain has N (>>1) elements of length a, and the angle between adjacent elements can only be 0° or 180°. The joints can turn freely and the two ends of the chain are fixed at a distance L. (The elements in the drawing are displaced in the y direction for clarity).



a) Show that the number of arrangements that can give an overall length of L=2ma, with m>0, is $\omega = \dfrac{N!}{(\frac{N}{2}+m)!(\frac{N}{2}-m)!}$.

b) Show that the entropy of the system is $S \approx k_B\left(N\ln 2 - \dfrac{L^2}{2Na^2}\right)$ under the condition of N >> 1 and L << Na (i.e., m << N).

c) Using the thermodynamic relation for change in internal energy *dU=TdS + fdL*, find the force, f, that is required to maintain the length L at a fixed temperature T under the same condition of N >> 1 and L << Na.

Hint: Useful formulas: $\ln(n!) \cong n\ln(n) - n$, for n>>1;
$\ln(1+x)=x-(1/2)x^2+(1/3)x^3+...+(-1)^n(1/n)x^n+...$ for $-1<x<1$.