Week 5 - Thermal Properties of Phonons
Comprehension Check
Total points = 25 (scaled by a factor of 1/10 in the system)

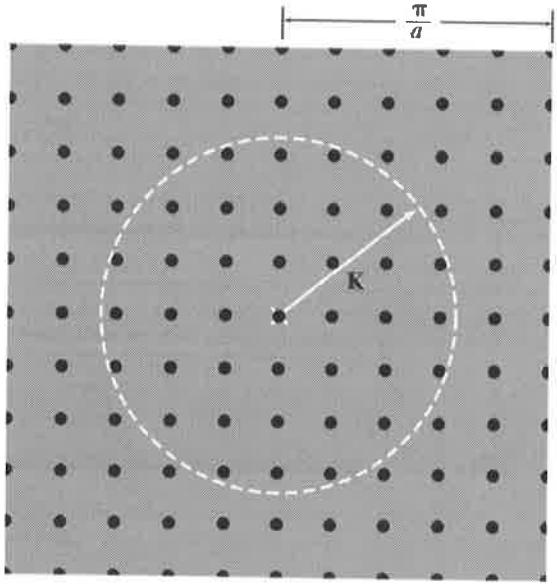
Question 1 (3 points)
The internal energy of a given system is given by $U = 2T^3 + 5T^2 + \text{const.}$
What is the heat capacity of the system?

$$C = \frac{dU}{dT} = 6T^2 + 10T$$

Question 2 (6 points)
State the three assumptions that go into Debye's model for the phonon heat capacity. Be brief.

- at off frequency ω_D
- linear dispersion relation $\omega = vK$
- velocity v independent of polarization

Question 3 (8 points)



The k-space sketch shown here is for a 2D square lattice with lattice constant a and where the periodic boundary condition was applied over a square of side $L=10a$. Hence, the spacing between modes in k-space is $2\pi/L$.
(a) What is the total number of modes N with phonon wavevector less than K?

$$N = \frac{\pi K^2}{(2\pi/L)^2} = \frac{\pi K^2 L^2}{4\pi^2} = \frac{K^2L^2}{4\pi}$$

~~(b) What is the density of states $D(\omega)$? Assume a linear dispersion relation.~~

Question 4 (3 points)
Two nanofilms A and B have the same chemical composition and same dimensions. Nanofilm A is defect free whereas nanofilm B has many structural defects. All other things being equal, what can you say about the relative thermal conductivities of A and B:

I. A is a better heat conductor than B
II. B is a better heat conductor than A
III. Not enough information to judge

in B, mean free path ℓ is limited by scattering off defects. Hence, $\chi_B < \chi_A$
because $\chi = \frac{1}{3} C v \ell$