Drexel UNIVERSITY

# DEPARTMENT OF PHYSICS

**PhD Qualifying Exam** **Modern Physics**

Friday, August 12, 2005 1 pm - 4 pm

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper…even the short ones. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. *Circle* the numbers below to indicate which questions you have answered—write nothing on the lines.

| *Short questions* | | *Long Problems* | |
|---|---|---|---|
| *circle* | grade | *circle* | grade |
| 1. | _____ | A1. | _____ |
| 2. | _____ | A2. | _____ |
| 3. | _____ | A3. | _____ |
| 4. | _____ | B1. | _____ |
| 5. | _____ | B2. | _____ |
| 6. | _____ | | |
| 7. | _____ | | |

# MODERN PHYSICS

## *PART I: Short questions (25%)*

**ANSWER 5 OF 7 QUESTIONS**

1. A particle is trapped in a one-dimensional finite square well potential with a barrier height that corresponds to a photon frequency of 5 Ghz. To keep the particle from escaping by thermal activation, how low a temperature should the particle be kept at? (Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K)

2. How much heat, in calories, must be added to a system at 300 K in order to increase the number of accessible states by a factor of 1000? (1cal = 4.19 J)

3. Quantum mechanics was first formulated to explain the particle-wave duality, and was applied to the hydrogen spectrum problem. The Heisenberg uncertainty principle states that the product of uncertainties in measurement of position and momentum is greater that a number of the order of Planck's constant, which is $6.6 \times 10^{-34}$ J.s. However, there are macroscopic quantum mechanical phenomena of great importance. Name two such phenomena with a brief description.

4. The radial wavefunction of an electron in a hydrogen-like atom is $\psi(r)=C\exp(-r/a)$, where $a = a_0/Z$ and $a_0 \approx 0.5$ Angstrom is the Bohr radius, Ze is the nuclear charge, and the atom has only one
electron. Compute the normalization constant C.

Hint: It may be helpful to know the definite integral,

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{\Gamma(n+1)}{\alpha^{n+1}}, \text{ for } n > -1 \text{ and } \alpha > 0.$$

Also note that

$$\Gamma(n+1)=n!, \text{ for } n=0,1,2....$$

5. Give one example of irreversible processes from daily experience. Does it violate the principle of microscopic reversibility in mechanics? How do you explain the phenomenon of the irreversibility of many daily processes?

6. Normally the specific heat is positive, $\frac{\partial U}{\partial T} > 0$. A cloud of hot, self-gravitating gas is observed to radiate away energy in the form of photons. After this, the kinetic energy (Temperature) of that cloud is observed to *increase*. Is this a violation of the laws of thermodynamics? If so, why? If not, explain.

7. You can make a cup of coffee containing 100 gm at 80 deg C
in two ways:

(a) Heat 100 gm of water from 20 deg to 80 deg and pour it into coffee grounds.
(b) Heat 75 gm of water from 20 deg to 100 deg, pour it into coffee grounds, and then add 25 gm of water at 20 deg. (deg in Centigrade)

Compute the excess entropy produced using method (b) rather than (a).

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

## A1.

A spin 1 system is in the state u,

$$u = \frac{1}{\sqrt{2c}}\begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix},$$

in the basis in which $S_z$ is diagonalized, such that the basis vectors are:

$$|+\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, |0\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, |-\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Note that $S_z$, the raising operator, $S_+ = S_x + i\,S_y$, and the lowering operator, $S_- = S_x - i\,S_y$, have the following representations:

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\hbar, S_+ = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}\hbar, S_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}\hbar.$$

(a) What are the eigenvalues and normalized eigenvectors of $S_x$?

(b) What is the probability that a measurement of $S_x$ yields +1, when the system is in the state *u*?

(c) After this measurement of Part (b) is made, what is the probability that a measurement of $S_z$ will yield the value of +1?

## A2.

A particle with mass m is in an infinite square well. The boundary of the well is between 0 and 3. The initial wave function is:

$$\psi(x,0) = 2x(3-x)(2-x).$$

(a) Normalize $\psi(x,0)$.
(b) Draw the wave function graphically. What stationary state does this wave function most resemble?
(c) You should be able to easily derive the generalized formula for the stationary states for the infinite square well. That is starting from the time-independent Schroedinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

and an initial guess $\psi(0)$ = A sin(kx) + Bcos(kx) give the quantization of the energy. Use this formula to ***estimate*** the expectation value for the energy of the system.

(d) Calculate the following expectation values, <x>, <p>, <H>. Compare <H> with the estimation you gave in part (c).

## A3.

The initial state of a harmonic oscillator with a fundamental frequency ω is

$$|\psi(0)\rangle = C_0\,|0\rangle + C_2\,|2\rangle\,,$$

where $C_0$, and $C_2$ are known <u>real</u> coefficients and $|0\rangle$ and $|2\rangle$ are the lowest and 3rd eigenstates of the Hamiltonian.

(a) What is the state of the oscillator at the arbitrary time t?

(b) What is the expectation value <P> of the momentum operator at time t? Make sure that you cast <P> in a form that shows convincingly the harmonic nature of the motion.

(c) What is the expectation value <X> of the coordination operator at time t? What is the classical analog?

Hint: The annihilation operator is given by

$$a = \left(\frac{m\omega}{2\hbar}\right)^{\frac{1}{2}} X + i\left(\frac{1}{2m\omega\hbar}\right)^{\frac{1}{2}} P.$$

## B1.

Consider a system in which a particle may be in one of two energy states: $E_0 = 0$, or $E_1 = \varepsilon$, and in either of two spin states, ↑ , or ↓ . The energy is independent of spin, and thus, the complete eigenstate of a particle may be expressed as:

$$|\,0/\varepsilon;\ \uparrow/\downarrow\,\rangle$$

Imagine that two identical particles are placed within this system.

(a) Find the partition function of the system, if
(i) The particles are classical (thus distinguishable).
(ii) The particles are bosons.
(iii) The particles are fermions.

(b) In the limit of $T \to \infty$, what fraction of systems will have $E_{tot} = 0$ in each of the three types of systems (classical, bosons, fermions)?

(c) What is the expectation energy of each of the three systems?

(d) What is the high temperature limit of the expected energy in all three systems?

**B2.**

(a) Suppose above some temperature, $T_o$, the heat capacity $c_v$ is temperature independent for a particular monatomic substance. Derive an expression for its Helmholtz Free Energy as a function of temperature.

(b) Derive an expression for the Helmholtz Free energy as a function of temperature for a monatomic ideal gas, beginning with the partition function.

(c) Now consider a monatomic solid, with each atom free to oscillate in an identical harmonic potential well about its equilibrium location in the solid. The oscillation frequency is $\omega$. Beginning with the partition function, derive an expression for the Helmholtz free energy as a function of temperature.

(d) Although both (b) and (c) lead to constant heat capacity, the heat capacities are different. Give a qualitative explanation for this fact.

Hint: (i) Recall the difference in heat capacity between monatomic and diatomic gas.
(ii) If you need entropy, S, to derive formulas, recall that $S = -\sum_{v} f_v \ln f_v$, $f_v$ is the ensemble probability of microstate $v$.