



DEPARTMENT OF PHYSICS

PhD Qualifying Exam

Friday, January 13, 2006

Classical Physics

9 am - 12 noon

PRINT YOUR NAME _____

EXAM CODE _____

PUT YOUR EXAM CODE, **NOT YOUR NAME**, ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

Short questions

circle *grade*

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

Long Problems

circle *grade*

A1. _____

A2. _____

A3. _____

B1. _____

B2. _____

CLASSICAL PHYSICS

PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

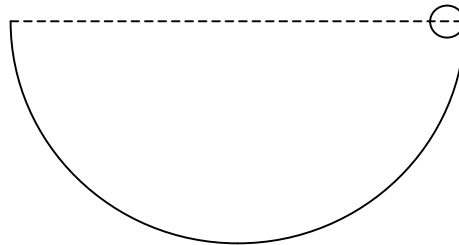
1. Given the Yukawa potential

$$U(r) = -K \frac{e^{-\alpha r}}{r},$$

where $K > 0$ and $\alpha > 0$, determine the Yukawa force. Is it repulsive or attractive?

2. A bullet traveling horizontally pierces in succession three thin screens placed at equal distances D apart. If the retardation at the screens is assumed constant, show that you can compute it from D , and the traversal times T_{12} and T_{23} , from the first screen to the second, and from the second to the third, respectively.

3. A hemispherical thin glass goblet of radius $R = 5$ cm will withstand a perpendicular force up to 2 N. If a 100 g steel ball is released from rest at the tip of the goblet and allowed to slide down the inside, at which point on the goblet will the ball break through? Neglect the radius of the ball and friction.



4. A particle moves in a circular orbit of radius r_0 under the influence of an attractive central force of magnitude $F(r)$. Derive a condition on $d \ln F / d \ln r$ near $r = r_0$ for the orbit to be stable.

5. The equation $\dot{\nabla} \times \dot{\mathbf{E}} = 0$ is true for a class of electromagnetic situations, but not universally valid.

(a) Under what condition is this equation true?

(b) When it is valid, what is its main consequence?

Now consider the first of Maxwell's equations, $\dot{\nabla} \cdot \dot{\mathbf{E}} = \rho / \epsilon_0$ (also known as Gauss' law).

(c) What are the two physical principles (or laws, if you prefer) hiding behind this equation?

6. Earnshaw theorem states that a charged particle cannot be held in stable equilibrium by electrostatic forces alone. Provide a short proof/argument in support of this theorem (no equations are needed).

7. When a charged particle is inserted in a uniform magnetic field with a velocity \mathbf{v} perpendicular to the magnetic field, it undergoes circular motion as it experiences a radial acceleration due to the magnetic force. What modification would you make to the magnetic field so that the particle can also experience a tangential acceleration? Explain your answer.

PART II: Long problems (75%)

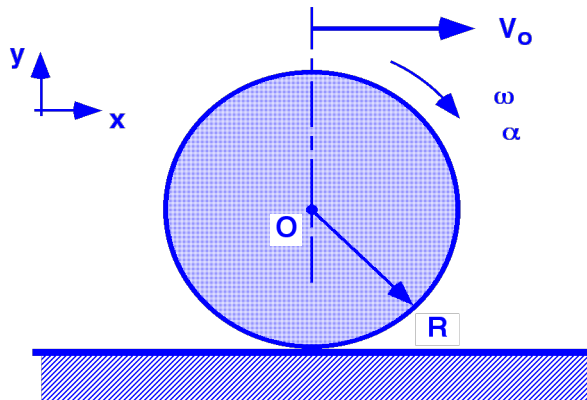
ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

A uniform sphere of mass $m = 2 \text{ kg}$ and radius $R = 0.3 \text{ m}$ is projected along a **rough** horizontal surface with an initial velocity $V_0 = 30 \text{ m/s}$ and **no angular velocity**. The coefficient of kinetic friction between the sphere and the surface is $\mu_k = 0.80$.

The moment of inertia of a sphere is $(2/5)mR^2$. Determine:

- (a) The sphere's **linear acceleration** while it initially slides and starts to roll on the surface.
- (b) The sphere's **angular acceleration** while it initially slides and starts to roll on the surface.
- (c) The amount of time, t , required by the sphere measured from the instant it initially contacts the surface until it **stops sliding** and begins rolling.
- (d) The sphere's **angular velocity** immediately after it stops sliding and starts to roll on the surface.



A2.

A test particle of negligible mass moves under the influence of an attractive central potential $\phi(r)$ which goes to zero as $r \rightarrow \infty$. The particle is unbound and moves on an orbit with velocity at infinity, V , and impact parameter, b .

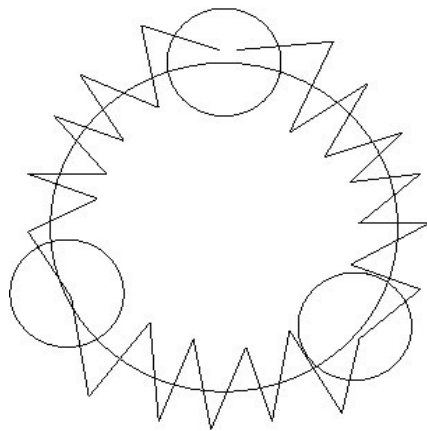
- (a) Write down an (implicit) equation relating the particle's distance of closest approach r_{\min} to the attracting center to V , b , and ϕ .
- (b) Now specialize to the case $\phi(r) = -GM/r$, corresponding to motion under the gravitational field of a body of mass M moving with velocity V through a field of test particles. If the radius

of the body is R and the number density of test particles is n , calculate the rate at which particles strike the body.

A3.

Consider 3 masses of mass m attached by springs with spring constant k around a cylinder as shown in figure below. Let x_1, x_2, x_3 be the displacement of the masses from equilibrium. As the displacements are small you can treat the system as being essentially linear with periodic boundary conditions.

- Write down the equations of motion for the system.
- Find the eigenvalues by solving the secular equation be careful with your signs!
- Find the normal modes by inspection.



B1.

A sphere of radius a has a uniform charge distribution σ on its surface and is rotating with constant angular velocity ω . What are the magnetic field and the vector potential at the center of the sphere?

B2.

A pair of parallel wires of radii a and b are separated by a distance d (center to center) where $d \gg a+b$. A current flows down one wire and back along the other (opposite directions), and is uniformly distributed within the cross-section of the wires. Calculate the self-inductance per unit length.