



## DEPARTMENT OF PHYSICS

### PhD Qualifying Exam

Friday, September 22, 2006

### Classical Physics

9 am - 12 noon

PRINT YOUR NAME \_\_\_\_\_

EXAM CODE \_\_\_\_\_

PUT YOUR EXAM CODE, **NOT YOUR NAME**, ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

#### *Short questions*

*circle*      *grade*

1.      \_\_\_\_\_

2.      \_\_\_\_\_

3.      \_\_\_\_\_

4.      \_\_\_\_\_

5.      \_\_\_\_\_

6.      \_\_\_\_\_

7.      \_\_\_\_\_

#### *Long Problems*

*circle*      *grade*

A1.      \_\_\_\_\_

A2.      \_\_\_\_\_

A3.      \_\_\_\_\_

B1.      \_\_\_\_\_

B2.      \_\_\_\_\_

# CLASSICAL PHYSICS

## PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

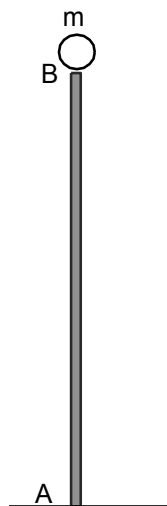
1. You are giving a demonstration in a freshman Physics class. You are sitting on a (frictionless) turntable holding a rapidly spinning bicycle wheel (also frictionless, with handles attached to the hub). The rotation axis of the wheel is vertical and aligned with the axis of the turntable; the angular velocity vector directed upward. You slowly turn the axis of the wheel, keeping it always in the same vertical plane, so that it is eventually spinning in the opposite direction about the vertical, still aligned with the turntable axis. The wheel's rotation rate stays constant. Describe qualitatively what happens as the wheel's orientation changes, and why. What torque must you apply to keep the axis in the plane? If your moment of inertia about the axis is 20 times that of the wheel, describe your own state of motion at the end of the demonstration.

2. A small spherical ball of mass  $m$  rests on the top of a vertically positioned stick  $AB$  of length  $L$ . The stick is of uniform mass density and the diameter of the ball is negligible compared to  $L$ . The stick starts falling (the end A does not slip) and lands on the floor. At the instant B hits the floor, its speed is  $V_B$

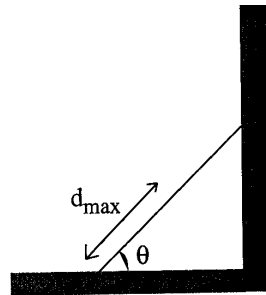
At the instant the stick starts to fall, the ball slips off the end of the stick and falls straight down and hits the floor with speed  $V_m$

Which of the following is true? Please provide a brief explanation for your choice.

- (a)  $V_B > V_m$
- (b)  $V_B < V_m$
- (c)  $V_B = V_m$



3. A ladder of length  $L$  with negligible mass leans against a wall. The ladder forms an angle of  $\theta$  with the floor. The static coefficient between the ladder and the floor and between the ladder and the wall is  $\mu$ . A child of mass  $m$  walks up the ladder. What is the maximum distance  $d_{\max}$  that the child can walk up the ladder before the ladder starts to slip?



4. A rocket of mass  $M_0$  starts off with velocity  $u_0$  (relative to a stationary observer) at time  $t_0$  with constant exhaust velocity  $v$ . After some point in time, the rocket now has mass  $M$  and is going at velocity  $u$ . Using the conservation of momentum, give an equation for the final velocity  $u$  of the rocket (You may assume  $u \ll v$ ).
5. An infinite charged plate sits on the surface of the earth. What charge density is required to exactly levitate a proton above the plate?
6. Describe one *classical* experiment demonstrating that light has particle properties.
7. An experimental package is cooled to 10 mK deep inside an evacuated low temperature metal cryostat. In his hurry, a graduate student mistakenly mounts the sample directly in the path of a long cylindrical metal tube of cross sectional area  $1 \text{ cm}^2$  that terminates in a closed port at room temperature.
- (a) Estimate the **rate** at which heat is delivered to the sample through the room temperature port? For simplicity, assume all permittivity  $\epsilon = 1$ . Neglect any thermal conductivity losses.
- (b) If the sample's heat capacity is  $100 \text{ mJ/K}$ , what is the resulting rate of increase in its temperature?

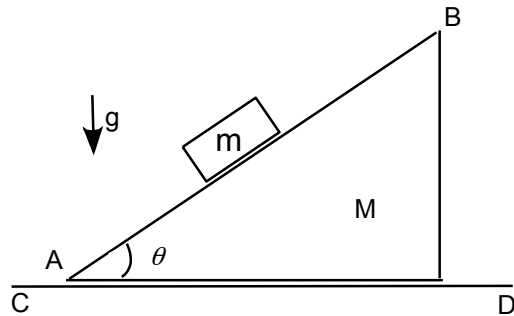
## PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

### A1.

A block of mass  $m$  is released from B on a frictionless surface of an incline of mass  $M$ . The incline itself is free to move on a frictionless horizontal surface CD.

- (a) Write down an expression for the Lagrangian of the system comprising  $m$  and  $M$ .
- (b) Obtain expressions for the accelerations of  $m$  and  $M$ .



### A2.

Suppose I have a solid box  $20 \times 15 \times 5$  cm, which is now spinning in space. Use the coordinate system of the box, such that Axis 1 is left to right along the 15 cm edge, Axis 2 is down to up along the 20 cm edge and Axis 3 is back to front along the 5 cm edge.

- (a) Find the ratios of the two largest principal moments of inertia relative to the smallest one.
- (b) What are the equations of motion for the rotational velocity  $\Omega$  in the box's coordinate system?
- (c) The box gradually loses kinetic energy until it has the smallest possible energy for its angular momentum. What  $\Omega$  is compatible with the condition of least kinetic energy? The box is rotating about what axis?

### A3.

- (a) A dark matter particle is initially at rest on Earth's surface. It interacts with the normal matter in Earth's interior only via the force of gravity. Approximating Earth as a homogeneous sphere of radius  $R = 6400$  km and surface gravity  $g = 9.80$  m/s<sup>2</sup>, calculate the time taken for the particle to return to its starting point. Compare this time to the period of a circular orbit at radius  $R$ .
- (b) Earth's interior matter is not uniformly distributed—the density increases as we approach the center. Calculate the time taken for the particle to return in the limit in which all of Earth's matter is concentrated at the center (without forming a black hole!). Why is this a lower limit on the actual return time?

**B1.**

An uncharged conducting sphere of radius  $R$  is located at the origin, with a point charge  $Q$  located at  $x = D$  and a point charge  $-Q$  located at  $x = -D$ , both located on the  $x$ -axis. Using the method of images, find the induced dipole moment of the sphere.

**B2.**

Two parallel, infinitely thin wires are separated by a distance,  $d$ , and each carry a (positive) linear charge density,  $\lambda$ , and a mass density,  $\mu$ . The wires are instantaneously at rest.

You may also find it useful to be reminded that the Lorentz factor is:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

- (a) What is the electrical acceleration on the wires?
- (b) You have a friend (his observations are in the “primed” frame), who is moving parallel to the wires at a speed of  $0.6c$ . What is the charge density in the primed frame?
- (c) What is the current in the wires in the primed frame?
- (d) What is the *total* acceleration between the wires in the primed frame?
- (e) How can you account for the discrepancy between your answers in parts (a) & (d)?