Drexel UNIVERSITY

DEPARTMENT OF PHYSICS

PhD Qualifying Exam — **Classical Physics**

Friday, August 12, 2005 — 9 am - 12 noon

PRINT YOUR NAME________________________

EXAM CODE__________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

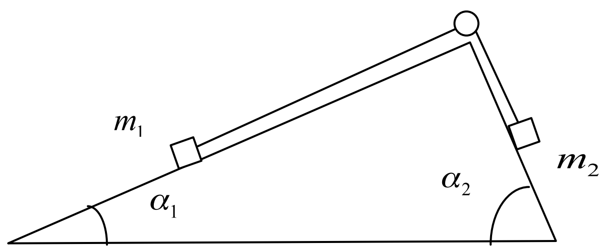
Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

CLASSICAL PHYSICS

PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1. Two masses m_1 and m_2 connected by a massless, inextensible string of length l are placed on two inclined planes as shown. The inclined planes are frictionless, and the pulley is massless and frictionless. Set up Lagrange's equations for the system.



2. An astronaut can high-jump 2 m on Earth. Is he in danger of not returning if he jumps while exploring on the surface of Diemos, one of the moons of Mars? Can he by his own exertions put himself into any orbit? Assume Diemos has roughly the Earth's density and has a diameter of 7 km. (Earth radius: 6371 km).

3. A small permanent magnet M is resting on a disc D of high-temperature superconductor placed inside a cup of Styrofoam. The whole assembly is resting on a spring scale.
To begin the experiment some liquid nitrogen is poured into the cup to cool the superconductor. The scale reads W, the combined weight of the assembly. Once the superconductor is cooled to below its transition temperature the magnetic flux from the superconductor is expelled and the magnet starts to float about 1cm above the superconductor. If the weight of the magnet is W_m, the scale would read (you must justify your choice):

(a) W (b) $W - W_m$ (c) $W+W_m$.

(Note: you can assume that negligible amount of liquid nitrogen is lost due to evaporation. You do not need to know any properties of superconductors to answer this question.)

4. Electric and magnetic fields of the form

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos\left(\vec{k} \cdot \vec{r} - \omega t + \delta\right), \qquad \vec{B}(\vec{r},t) = \vec{B}_0 \cos\left(\vec{k} \cdot \vec{r} - \omega t + \delta\right)$$

obey the respective wave equations, provided that $\omega / k = v$, where v is the velocity of propagation of the waves (more precisely, the phase velocity, but this is not an issue here).

What <u>additional constraints</u> on field directions and amplitudes must be satisfied so that the above fields, $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$, are also solutions of Maxwell's equations?

5. An uncharged conducting sphere is placed in an initially uniform electric field.

(a) Initially, of course, the spherical surface is equipotential. Does it remain equipotential after the new equilibrium configuration is reached a very short time later?

(b) Sketch the shape of the electric field lines in the final equilibrium state.

(c) Sketch the final charge distribution on the surface of the sphere.

Instead of an uncharged conducting sphere, an uncharged dielectric sphere is placed in the initially uniform electric field.

(d) How does the electric field inside the dielectric sphere differ from that inside the conducting sphere of parts a), b), and c)? Explain.

6. Given two iron bars, identical in appearance, one magnetized, the other not. How would you distinguish them without using external magnetic fields or other metal pieces? (You are allowed to measure forces.)

7. A microwave oven is a common household appliance that cooks most food using ***standing*** electromagnetic waves with a frequency of 2.4 Ghz.

(a) Calculate the wavelength of the ***standing*** microwaves and draw its most likely configuration along the length L (about 10 inches) of the cooking chamber. For simplicity, ignore the other 2 dimensions of the oven.

(b) Explain how a microwave oven can be used to cook food that contains water.
(Hint: Water is a polar molecule).

(c) Explain why inside a microwave oven, a compact disk (CD) or aluminum foil heats up rapidly and produce sparks but a spoon will not.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

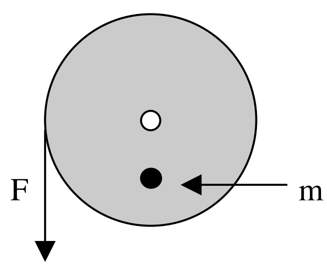
A1.

A light rigid cylinder of radius $2a$ is able to rotate freely about its axis, which is horizontal. A particle of mass m is fixed to the cylinder at a distance a from the axis, and is initially at rest at its lowest point. A light string is wound on the cylinder, and a steady tension F applied to it.

(a) Find the angular acceleration and the angular velocity of the cylinder after it has turned through an angle θ.

(b) Show that there is a limiting tension F_0 such that if $F < F_0$ the motion is oscillatory, but if $F > F_0$ it continues to accelerate.

(c) Estimate the value of F_0 by numerical approximation.



A2.

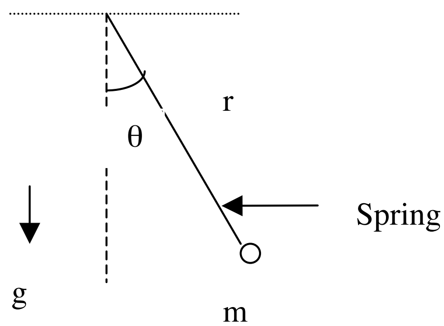
A massless spring of rest length l_0 (with no tension) has a point mass m connected to one end and the other end fixed so that the spring hangs in the gravity field as shown in the figure below. The motion of the system is only in one vertical plane.

(a) Write down the Lagrangian.

(b) Find Lagrange's equations using variables $\theta, \lambda = \dfrac{r - r_0}{r_0}$, where r_0 is the rest length hanging with mass m. Use $\varpi_s^2 = \dfrac{k}{m}, \varpi_p^2 = \dfrac{g}{r_0}$.

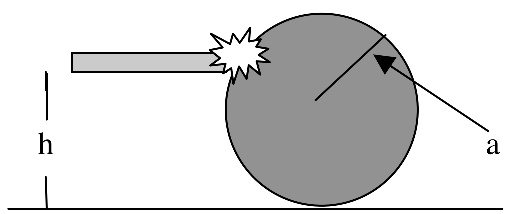
(c) Discuss the lowest-order approximation to the motion, when λ and θ are small with the initial conditions: $\theta = 0, \dfrac{d\lambda}{dt} = 0, \lambda = A, \dfrac{d\theta}{dt} = \varpi_p B$ at t=0. A and B are constants.

(d) Discuss the next-order approximation to the motion. Under what conditions will the λ motion resonate?



A3.

The usefulness of the impulse formulation of the equations of motion is best exemplified by the treatment of the dynamics of billiard shots. Consider the case where the cue stick hits the ball in its vertical median plane in a horizontal direction.



(a) Without explicit knowledge of the forces involved, show that if the ball (of radius *a*) is struck at a height h, then the relation between the spin ω and velocity of the CM of the ball V immediately after the impulse is

$$\omega = \frac{5}{2}\left(\frac{h-a}{a^2}\right)V \ .$$

(b) What should h be if the ball is to roll without slipping after impact?

(c) Discuss the role that friction plays with regard to what happens if the cue stick strikes above and below that critical height.

Hint: The moment of inertia of a sphere is given by $I = (2/5)Ma^2$.

B1. Charged sphere

A sphere of radius R carries a charge density $\rho = kr$, where k is a constant.

(a) With the help of Gauss' law, calculate the electric field inside the sphere ($r < R$)

(b) Calculate the electric field outside the sphere, i.e. for $r > R$.

(c) Find the electrostatic energy of this configuration.

(d) What happens to the electrostatic energy if $R \to 0$? Justify your answer (I was under the impression that the energy of a charged sphere approached infinity when the sphere became a point particle).

B2. Electromagnetic Shielding

Consider electromagnetic waves of frequency ω and wave number k impinging a conductive medium (a metal) with conductivity σ, permeability μ and permitivity ε, The wave equations (for **E** and **B**) can be shown to be as follows:

$$\nabla^2 \mathbf{E} = \mu\varepsilon\, \partial^2\mathbf{E}/\partial t^2 + \mu\sigma\partial\mathbf{E}/\partial t \qquad \text{and} \qquad \nabla^2 \mathbf{B} = \mu\varepsilon\, \partial^2\mathbf{B}/\partial t^2 + \mu\sigma\, \partial\mathbf{B}/\partial t$$

(a) Assuming plane wave solutions of the form:

$$\mathbf{E}(x,t) = \mathbf{E_o}\, e^{\,i(kx-\omega t)} \qquad \text{and} \qquad \mathbf{B}(x,t) = \mathbf{B_o}\, e^{\,i(kx-wt)}$$

show that the wave number k is complex and satisfies the expression:

$$k^2 = \mu\varepsilon\omega^2 + \mathbf{i}(\mu\sigma\omega)$$

(b) Show that upon taking the square root, the wave number k is:

$$k = k_+ + \mathbf{i}\, k_-$$

where $k_\pm = \omega \sqrt{(\varepsilon\mu/2)}\, \{\sqrt{[\, 1 + (\sigma/\varepsilon\omega)^2]} \ \pm 1\}^{(1/2)}$

(c) For the case of a good conductor ($\sigma >> \omega\varepsilon$), show that $k_+ = k_-$ and determine a simple expression for skin depth.

(d) A graduate student wishes to build a ***plastic*** sample box that is shielded from microwave radiation of frequencies greater than 1 Ghz. To do this and to save money (machining metal is more expensive than plastic), he/she machines a plastic box and plates its exterior with a metal with $\sigma = 10^7$ /(ohm-m), $\mu^{= 10\text{-}6}$ N/A^2 and $\varepsilon = 10^{-11}$ N/m^2. How thick should the layer of metal be for this type of shielding ?