DEPARTMENT OF PHYSICS
Ph.D. Qualifying Exam - Modern Physics
Friday, September 21, 2007
1:00 p.m. - 4:00 p.m.

PRINT YOUR NAME ______________________________

EXAM CODE __________

INSTRUCTIONS

PUT YOUR EXAM CODE, NOT YOUR NAME, ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

You may consult only the references provided or approved in advance by the examiner. You may use a calculator.

In Part I, answer 5 of 7 short answer questions. In Part II, answer 3 of 6 longer problems, including at least one problem from Group A and one from Group B. *Circle the numbers below* to indicate which questions you have answered. Write nothing on the lines below (your grades go there).

Short Problems		Long Problems	
circle	grade	*circle*	grade
1.	________	A1.	________
2.	________	A2.	________
3.	________	A3.	________
4.	________	B1.	________
5.	________	B2.	________
6.	________	B3.	________
7.	________		

PART I: SHORT PROBLEMS (25%)

ANSWER 5 OF 7 QUESTIONS

1. A particle in one dimension is trapped between two rigid walls:

$$V(x) = \begin{cases} 0, & \text{for } 0 < x < L \\ \infty, & \text{for } x < 0,\, x > L \end{cases}$$

At $t = 0$ it is known to be exactly at $x = L/2$. What are the *relative* probabilities for the particle to be found in the different energy eigenstates consistent with this initial position? (You need not worry about absolute normalization, convergence, and other mathematical subtleties.)

2. A neutron, with a kinetic energy of 10 MeV and a rest energy of 940 MeV, encounters an atom with characteristic size $d = 1$ Angstrom. Determine whether the neutron is moving too fast, too slow or just right to probe the atomic structure.

3. An electron is placed in a shallow square well potential: $V = 0$ for $|x| > a, V = -V_0$ for $|x| < a$. The potential supports only three bound states. Sketch the three bound states, paying careful attention to the number of nodes and the decay length of the wavefunction outside the binding region.

4. Suppose you have three particles, and three distinct one-particle states ($\psi_a(x), \psi_b(x), \psi_c(x)$). How many different three-particle states can be constructed if they are

(a) distinguishable particles?

(b) identical bosons?

(c) identical fermions?

To be clear, the particles need not be in different one-particle states. For example, $\psi_a(x_1)\psi_a(x_2)\psi_a(x_3)$ would be one possibility for the three-particle state, if the particles are distinguishable.

5. The entropy S of a system can be expressed as, $S = k \ln \Omega$. Describe briefly what is meant by Ω *and* give a brief argument that shows that the statement $S = k \ln \Omega$ is consistent with the observation that the entropy of an ideal gas increases in an irreversible free expansion.

6. It's very cold outside and you stop at one of the trucks and you ask for a hot cup of half & half (half hot coffee, half cold milk). The proprietor asks you if you want to take the two halves with you, and mix them once you get to class, *or* if you want them mixed there, and then run to class with the whole. What do you answer, in the interests of having the hottest drink after you arrive? Explain quantitatively.

7. How much heat, in Joules, must be added to a system at 300 K in order to increase the number of accessible states by a factor of 1000?

PART II: LONG PROBLEMS (75%)

ANSWER 3 OF 6 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. Quantum Top

A symmetrical top with moments of inertia $I_x = I_y$ and I_z in the body axis frame is described by the hamiltonian

$$\mathcal{H} = \frac{1}{2I_x}(L_x^2 + L_y^2) + \frac{1}{2I_z}L_z^2$$

(a) Compute the eigenvalue spectrum.

(b) Show that l energy eigenvalues are doubly degenerate and one is non degenerate.

(c) Show that L_z commutes with $\mathcal{H}$.

(d) Write down the simultaneous eigenfunctions of $\mathcal{H}$ and L_z.

(e) Compute the expectation value of the operator $L_x + L_y + L_z$ in any eigenstate.

(f) At time $t = 0$ the expectation value of L_z in some eigenstate is 0. Compute $\langle L_z \rangle$ at any later time.

A2. Spin, Spin, Spin

A spin 1 system is in the state u,

$$u = \frac{1}{\sqrt{26}} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

in the basis in which S_z is diagonalized, such that the basis vectors are:

$$|+\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad |0\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad |-\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Note that S_z, the raising operator, $S_+ = S_x + iS_y$, and the lowering operator, $S_- = S_x - iS_y$, have the following representations:

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\hbar, \quad S_+ = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}\hbar, \quad S_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}\hbar,$$

(a) What are the eigenvalues and normalized eigenvectors of S_x?

(b) What is the probability that a measurement of S_x yields +1, when the system is in the state u?

(c) After the measurement of part (b) is made, what is the probability that a measurement of S_z will yield the value of +1?

A3. Bound and Bouncing

Consider a spinless particle of mass m that is moving in a three-dimensional potential

$$V(x,y,z) = \begin{cases} \frac{1}{2}m\omega^2 z^2 & \text{if } 0 < x < a, 0 < y < b \\ \infty & \text{elsewhere} \end{cases}$$

(a) What are the energy eigenvalues of the particle?

(b) What is the total wave function of the particle?

(c) Assuming that $\hbar\omega > 5\pi^2\hbar^2/(ma^2)$, find the energies and degeneracies for the ground state and first excited state.

B1. Oscillators in Heat Bath

(a) Calculate the average energy of N one-dimensional harmonic oscillators placed in a heat bath at temperature T. What is the heat capacity of this system?

(b) Now assume that the potential for the oscillators contains a small quartic "anharmonic term", that is

$$V(x) = \frac{m\omega^2 x^2}{2} + \alpha x^4$$

where $\alpha\langle x^4\rangle \ll kT$. To first order in the parameter α, derive the anharmonic correction to the average energy of the harmonic oscillators.

B2. SM in 2D and 3D

Consider a 2-dimensional membrane, in which N molecules are free to move, and do not interact with each other, except that they are constrained to be on this surface. You may consider the surface to have a small thickness Δz, and molecules span this size (i.e. their diameter is slightly larger than Δz). The membrane and molecules are at fixed temperature T.

(a) Write the Canonical Partition Function for this system.

(b) Using your answer to (a), compute the entropy of the system of the N molecules.

(c) Intuitively and qualitatively, how would your answer to (b) change if the molecules became free to move in 3 dimensions? Be sure to explain the basis for your answer.

(d) Support your answer to (c) by an appropriate calculation.

B3. May I Take Your Disorder, Please?

Consider an ideal gas made of a uniform suspension of non-interacting, randomly oriented electric dipoles, of mass m and dipole moment μ in a uniform electric field of magnitude E.

(a) Write the classical Hamiltonian for each dipole.

(b) Write the canonical partition function for an ideal gas consisting of an ensemble of N such dipoles in a volume V at a temperature T.

(c) Derive an expression for the Helmholtz free energy of the ensemble as a function of field E.

(d) What happens to the entropy S as the field increases?

(e) Explain qualitatively why your answer to part (d) makes sense.