

minute half-life, after subtraction of a long-lived background ( $\sim 4$  percent) and the 40-minute  $\text{Sn}^{123}$  contribution ( $\sim 5$  percent).

An aluminum absorption curve was run on this activity by making a number of separate identical irradiations of the tin foil and following each decay through a different thickness of aluminum. The absorption curve is shown in Fig. 1. The tin sample used in these irradiations had a thickness of  $80 \text{ mg/cm}^2$ . Analysis of the curve as shown in the figure gave a beta-component with a range of approximately  $670 \text{ mg/cm}^2$  of aluminum corresponding to an energy of  $1.5 \pm 0.2 \text{ Mev}$ , plus a gamma-ray background amounting to approximately 3 percent of the beta-counting rate. The beta-ray range and energy were estimated by a Feather analysis, using as reference standard a sample of  $\text{P}^{32}$  mocked up to resemble the  $\text{In}^{118}$  source in self-absorption.

There was not sufficient intensity for a lead absorption curve, although measurements taken through  $1.0 \text{ g/cm}^2$  and  $4.6 \text{ g/cm}^2$  of lead gave counting rates equal within experimental error to that of the thick absorber background in Fig. 1.

The  $\text{In}^{119}$  was identified by 23-Mev irradiation of a sample of tin foil enriched to 95.4 percent in  $\text{Sn}^{120}$ , followed by dissolution of the foil and precipitation of indium as indium hydroxide from hot 2 M sodium hydroxide solution. The precipitate showed a pure  $17.5 \pm 1$  minute decay. An aluminum absorption curve on a second portion of the irradiated  $\text{Sn}^{120}$  foil is shown in Fig. 2. A Feather analysis carried out as described above

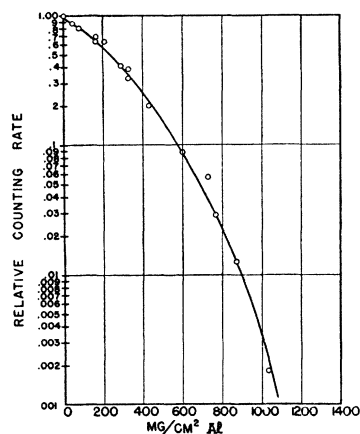


FIG. 2. Aluminum absorption curve of radiation from  $\text{In}^{119}$ .

gave a beta-range of  $1350 \text{ mg/cm}^2$  of aluminum, corresponding to  $2.7 \pm 0.2 \text{ Mev}$ . Gamma-rays, if present at all, had a counting rate less than 0.001 that of the beta-counting rate.

\* Assisted by the joint program of ONR and AEC.

<sup>1</sup> O. Hitzel and W. Waffler, *Helv. Phys. Acta* **20**, 373 (1947).

<sup>2</sup> The enriched  $\text{Sn}^{119}$  and  $\text{Sn}^{120}$  used in this investigation were supplied by Carbide and Carbon Chemicals Corporation, Y-12 Plant, Oak Ridge, Tennessee, and obtained on allocation from the Isotopes Division of the Atomic Energy Commission.

### Note on Dirac's Theory of Magnetic Poles

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IN a note bearing the above heading, Professor H. A. Wilson<sup>1</sup> has described a simple method for finding out the value of Dirac's free magnetic poles. I may point out that this method was described by me nearly thirteen years ago<sup>2</sup> in a paper "On the origin of mass in neutrons and protons." I may just quote the result:

"It was Dirac who first showed that quantum mechanics

demand the existence of free magnetic poles, having the pole strength (or magnetic charge)  $ch/4\pi e = e/2\alpha$ , where  $\alpha$  = Sommerfeld fine-structure constant. Recently, the present author deduced the existence of free magnetic poles from very simple considerations. If we take a point charge  $e$  at  $A$  and a magnetic pole  $\mu$  at  $B$ , classical electrodynamics tells us that  $\frac{A}{B}$  the angular momentum of the system about the line  $AB$  is just  $e\mu/c$ . Hence following the quantum logic, if we put this  $= \frac{1}{2} \cdot h/2\pi$ , the fundamental unit of angular momentum, we have  $\mu = ch/4\pi e = e/2\alpha$  which is just the result obtained by Dirac."

<sup>1</sup> H. A. Wilson, *Phys. Rev.* **75**, 308 (1949).

<sup>2</sup> *Ind. J. Phys.* **10**, 145 (1936).

### Note on Proposed Schemes for Nuclear Shell Models\*

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THE two papers by the present writers<sup>1,2</sup> on nuclear shell structure, cover very similar ground, such as assignment of orbital configurations on basis of spins and magnetic moments, statistics of isomerism, and the character of  $\beta$ -transitions. Both papers suggest level schemes to account for the empirically found regularities in nuclear structure. The two schemes are, however, not identical, and even a third proposal has been made by Maria G. Mayer,<sup>3</sup> on basis of the data collected in references 1 and 2. It may thus be of value to explain the relations between these papers.

The basis of all the considerations on shell structure is the observation that the level schemes in a simple potential well give a good account of the regularities of nuclear structure for neutron and proton numbers below 20. Such regularities persist also for heavier nuclei, though they do not correlate with the simple well scheme. These facts suggest, however, that a rearrangement of levels may be successful.

TABLE I. Proposed schemes for nuclear shells.

No. of particles in nucleus	8	20	50	82
No. of particles in shell	2+6	12	30	32
Feenberg and Hammack	$(1s)^2(2p)^6$ $(1s)^2(2p)^6$	$(2s)^2(3d)^{10}$ $(3d)^{10}$	$(4f)^{14}(5g)^{18}$	$(6h)^{22}(4d)^{10}$
Nordheim	$(1s)^2(2p)^6$	$(2s)^2(3d)^{10}$	$(4f)^{14}(3p)^6(4d)^{10}$	$(5g)^{18}(5f)^{14}$
Mayer	$(1s)^2(2p)^6$	$(2s)^2(3d)^{10}$	$(4f)^{14}(3p)^6(5g)^{18}$	$(5g)^{18}(4d)^{10}(3s)^2(6h)^{12}$
Order of levels in potential well	1s, 2p, 3d, 2s, 4f, 3p, 5g, 4d, 3s, 6h, 5f, 4p, 7i			

In the scheme of Feenberg and Hammack, the rearrangement consists in a pushing up of orbits with radial nodes, such as  $2s$ ,  $3p$ ,  $4d$ , which progresses more and more for heavier nuclei. Thus, the level scheme is somewhat different for light and heavy nuclei. A qualitative explanation for this tendency is given by the repulsive action of the Coulomb forces on protons, which will cause a decrease in density of nuclear matter at the center of heavy nuclei.

In Nordheim's scheme, the rearrangement is in the opposite sense; that is, radial nodes are not penalized as much as in a potential well. This may also be described as a discrimination against high orbital momentum states. The latter may be caused by the strong interaction between the nuclear particles,

since a rigid or liquid nucleus as a whole would have no orbital momentum in its lowest state.

The scheme proposed by Mayer follows exactly the order in a potential well. It achieves the breaks at the correct places by the assumption of a very strong spin-orbit coupling at high angular momentum values.

A summary of the three schemes is given in Table I. All three schemes give, of course, the empirical shell numbers and a statistical correlation with observed spins and moments. A decision between the schemes may be hoped for through discussion of new data which may tend to tip the scales in a definite direction, or by more theoretical work. Among the latter would be a refined calculation of the effects of the Coulomb forces on the density distribution in a nucleus, improved treatment of the many body problem, and better understanding of the spin-orbit coupling in nuclei.

It should be emphasized that the existence and the characteristics of nuclear shell structure have become now much more clearly established than formerly in spite of the ambiguities in their interpretation. Particularly there is a definite correlation between spin and shell structure. This does not mean necessarily that the individual particle model is better than hitherto assumed. The shell structure in nuclei, is, however, so pronounced an effect that one may hope to obtain an interpretation even on basis of such a crude approximation as the individual particle model.

\* This letter has been written on request by the editor of the *Physica Review*, who received the papers, reference 1 and 2, by the same mail.

<sup>1</sup> Eugene Feenberg and Kenyon C. Hammack, *Phys. Rev.* **75**, 1877 (1949).

<sup>2</sup> L. W. Nordheim, *Phys. Rev.* **75**, 1894 (1949).

<sup>3</sup> Maria G. Mayer, *Phys. Rev.* **75**, 1969 (1949).

## On Closed Shells in Nuclei. II

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THE spins and magnetic moments of the even-odd nuclei have been used by Feenberg<sup>1,2</sup> and Nordheim<sup>3</sup> to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin-orbit coupling, increasing with angular momentum, is assumed, a level assignment different from either Feenberg or Nordheim is obtained. This assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

Table I contains in column two, in order of decreasing binding energy, the levels of the square well potential. The quantum number gives the number of radial nodes. Two levels of the same quantum number cannot cross for any type of potential well, except due to spin-orbit splitting. No evidence of any crossing is found. Column three contains the usual spectroscopic designation of the levels, as used by Nordheim and Feenberg. Column one groups together those levels which are degenerate for a three-dimensional isotropic oscillator potential. A well with rounded corners will have a behavior in between these two potentials. The shell grouping is given in column five, with the numbers of particles per shell and the total number of particles up to and including each shell in column six and seven, respectively.

Within each shell the levels may be expected to be close in energy, and not necessarily in the order of the table, although the order of levels of the same orbital angular momentum and different spin should be maintained. Two exceptions,  ${}_{11}\text{Na}^{23}$

with spin 3/2 in stead of the expected  $d_{5/2}$ , and  ${}_{25}\text{Mn}^{55}$  with 5/2 instead of the expected  $f_{7/2}$ , are the only violations.

Table II lists the known spins and orbital assignments from magnetic moments<sup>4</sup> when these are known and unambiguous, for the even-odd nuclei up to 83. Beyond 83 the data is limited and no exceptions to the assignment appear.

Up to  $Z$  or  $N=20$ , the assignment is the same as that of Feenberg and Nordheim. At the beginning of the next shell,  $f_{7/2}$  levels occur at 21 and 23, as they should. At 28 the  $f_{7/2}$  levels should be filled, and no spins of 7/2 are encountered any more in this shell. This subshell may contribute to the stability of  $\text{Ca}^{48}$ . If the  $g_{9/2}$  level did not cross the  $p_{1/2}$  or  $f_{5/2}$

TABLE I.

Osc. No.	Square well	Spect. term	Spin term	No. of states	Shells	Total No.
0	1s	1s	1s <sub>1/2</sub>	2	2	2
1	1p	2p	1p <sub>1/2</sub> 1p <sub>3/2</sub>	4 2	6	8
2	1d 2s	3d 2s	1d <sub>5/2</sub> 1d <sub>3/2</sub> 2s <sub>1/2</sub>	6 4 2	12	20
3	1f 2p	4f 3p	1f <sub>7/2</sub> 1f <sub>5/2</sub> 2p <sub>3/2</sub> 2p <sub>1/2</sub> 1g <sub>9/2</sub>	8 6 4 2 10	8? 22	28? 50
4	1g 2d 3s	5g 4d 3s	1g <sub>7/2</sub> 2d <sub>5/2</sub> 2d <sub>3/2</sub> 3s <sub>1/2</sub> 1h <sub>11/2</sub>	8 6 4 2 12	32	82
5	1h 2 3p	6h 5f 4p	1h <sub>9/2</sub> 2f <sub>7/2</sub> 2f <sub>5/2</sub> 3p <sub>3/2</sub> 3p <sub>1/2</sub> 1i <sub>13/2</sub>	10 8 6 4 2 14	44	126
6	1i 2g 3d 4s	7i 6g 5d 4s	1i <sub>11/2</sub>			

levels, the first spin of 9/2 should occur at 41, which is indeed the case. Three nuclei with  $N$  or  $Z=49$  have  $g_{9/2}$  orbits. No  $s$  or  $d$  levels should occur in this shell and there is no evidence for any.

The only exception to the proposed assignment in this shell is the spin 5/2 instead of 7/2 for  $\text{Mn}^{55}$ , and the fact that the magnetic moment of  ${}_{27}\text{Co}^{59}$  indicates a  $g_{7/2}$  orbit instead of the expected  $f_{7/2}$ .

In the next shell two exceptions to the assignment occur. The spin of 1/2 for  $\text{Mo}^{95}$  with 53 would be a violation, but is experimentally doubtful. The magnetic moment of  $\text{Eu}^{153}$  indicates  $f_{5/2}$  instead of the predicted  $d_{5/2}$ . No  $h_{11/2}$  levels appear. It seems that these levels are filled in pairs only,