DREXEL
UNIVERSITY

DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam **Classical Physics**

Friday, September 19, 1997 9 am - 12 noon

PRINT YOUR NAME______________________

EXAM CODE________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN.
(This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. By circling the numbers below, indicate which questions you have answered.

Short questions	*Long Problems*
1. _____	A1. _____
2. _____	A2. _____
3. _____	A3. _____
4. _____	B1. _____
5. _____	B2. _____
6. _____	
7. _____	

CLASSICAL PHYSICS

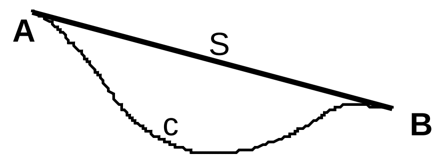
PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. Water exits a hose an hits a stationary vertical wall with velocity which is purely horizontal, of 2 m/s. The water volume flow rate is 0.1 m^3/s. After hitting the wall the water flows down the wall vertically at an average downward velocity of 0.5 m/s. What is the horizontal force and the average vertical force felt by the wall due to the water.

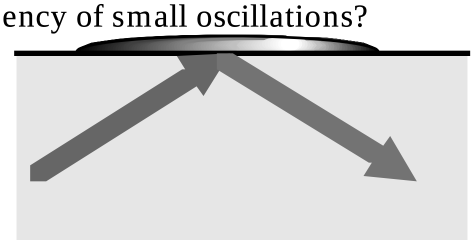
2. A long thin meter stick of mass m is supported at both ends. How great is the force on the remaining support just after one support is suddenly removed?

3. Two tracks connect point A to point B. If balls are started rolling on each track at the same time at A, which one first arrives at point B?



4. A ball of mass m and net charge Q is hung from a pendulum of length L at a distance d above a conducting plane. What is its frequency of small oscillations?

5. A beam of light is incident from within a glass on a glass-liquid interface as shown. It hits the interface at the critical angle. The liquid on the surface contains fluorescent dye. Will the dye fluoresce? Explain.



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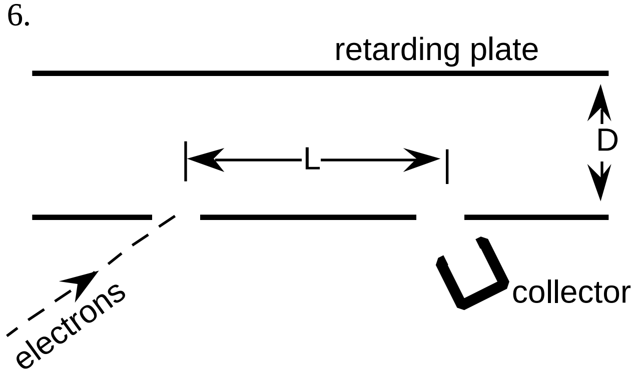


Consider the plane mirror analyzer depicted in the figure. A beam of electrons with kinetic energy E traverses a path through the entrance slit and the exit slit provided that the potential on the retarding (upper) plate is properly selected. (The lower plate is at ground potential.) Thus, sweeping V over time and measuring the electron current I that passes through the exit slit with an electron multiplier will produce an energy spectrum (I vs. V). Considering the electron energy in electron volts, derive the relation E(V) in terms of the electronic charge (e), the electron mass (m_e), the distance between plates (D), the distance between slits (L), and the angle of injection (θ) as needed. What is an obvious difference if you want to detect the kinetic energy of protons?

7. A ball is dropped from a 50 foot ceiling onto a hard floor. The ball bounces with increasing frequency on the same spot until all its energy is dissipated (when the ball comes to rest.) If it has taken 20 sec for the ball to come to rest, how can you make a quick calculation to determine the coefficient of restitution of the ball.

PART II: Long problems (75%)

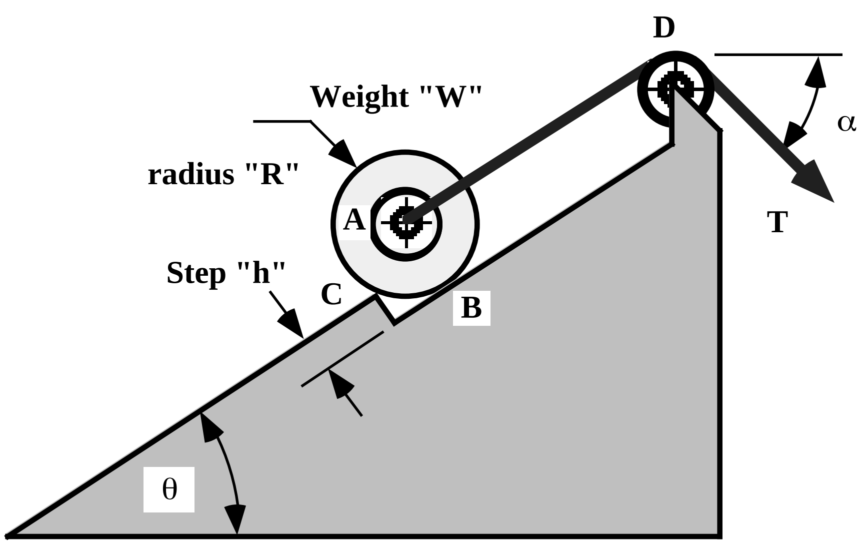
ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1 *High roller*

A heavy roller (radius = R and weight =W) is held on a frictionless incline, against a step **C** that is *h* high, by a rope passing over a smooth light pulley D under a tension of **T**. The angle of the incline is θ, and assuming that the tension is zero, find the limiting value of the angle θ before the roller starts to roll over the step and down the incline.

Also, find the minimum required tension, T_{min}, in order to prevent the roller from rolling down the incline when the angle $\theta = 45^o$?

Do your answers depend on the angle α?

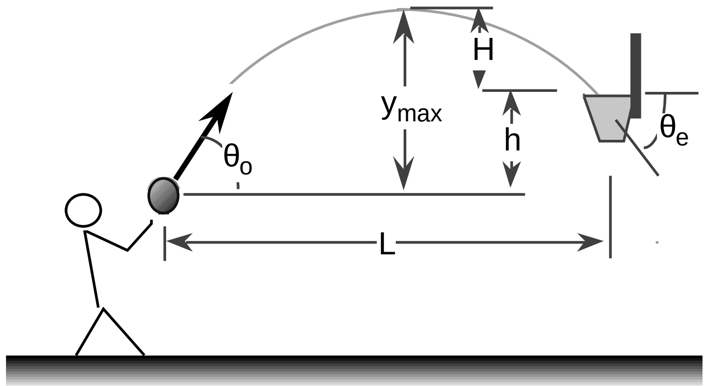


A2. *Nitty Gritty*

A conveyor belt of mass M is moving at a constant horizontal speed v while sand is continuously dropping on the belt from a stationary hopper at a rate *dm/dt.* Show that the power that should be supplied by a force *F* to keep the belt moving at a uniform speed *v* is **twice the rate** at which the kinetic energy is increasing.

A3 *Take your best shot.*

Consider a player shooting a basketball as shown in the figure:



(a) Write the equations of motion for the center of the basketball, *x(t)* and *y(t)* taking the initial position of the basketball as the origin. You may ignore friction and the finite sizes of ball and basket.

(b) Show that the initial speed of the basketball, v_o, is related to the initial angle by the equation

$$v_o^{-2} = (2\cos^2\theta_o / gL)\ (\tan\theta_o - h/L)$$

where g is the acceleration of gravity.

(c) Show that the initial angle for minimum initial speed is given by

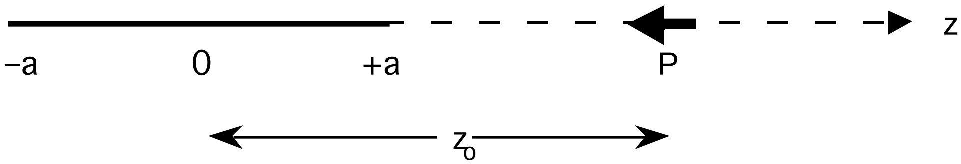
$$\tan\theta_{o,min} = h/L + (1 + h^2/L^2)^{1/2}$$

(d) Show that in general it will be impossible to make a basket unless $\tan\theta_o >$ $2h/L$. Hint: first calculate the angle θ_e.

(e) If the minimum-speed angle $\theta_{o,min}$ is optimal in shooting a basketball, why does a taller player have an advantage in shooting a basket?

B1 *That same old line...*

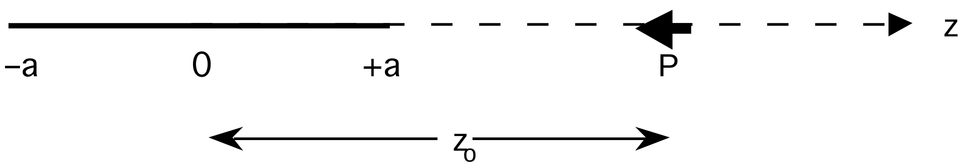
A charge Q is uniformly distributed along the line segment (–a,a) as shown below:



a. Find the electrostatic potential at point P.

b. Calculate the electric field at P.

c. Determine the translational force on an electric dipole at P oriented as shown below, if the dipole moment of magnitude p .



Is the dipole attracted or repelled by the line of charge?

d. In the limit z >>a, write an approximate expression for the electrostatic potential at a point off the z axis.

B2. *The Death of the Classical Atom*

The radiation field of an accelerating, nonrelativistic point charge q is given by

$$\vec{E} = \frac{q}{c^2}\frac{\vec{r}\times(\vec{r}\times\vec{a})}{r^3} \qquad \text{and} \qquad \vec{B} = \vec{r}\times\vec{E}/r$$

where $\vec{a}$ is the instantaneous acceleration of the charge, $\vec{r}$ is the vector from the charge to the field point, c is the speed of light, and the right hand sides are evaluated at the usual retarded time , $t - r/c$.

(a) Determine the Poynting vector and hence show that the charge's instantaneous power emission is

$$P = \left(\frac{2q^2}{3c^3}\right)a^2$$

(b) Based on your result, make an estimate of the lifetime of a classical hydrogen atom against radiation losses. Assume that the atom consists of an electron orbiting a proton at a distance of 0.05 nm, corresponding to a classical binding energy of 13.6 eV.
1 eV = 1.6×10^{-12} erg = 1.6×10^{-19} J. q = 4.8×10^{-10} esu.

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DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam **Modern Physics**

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MODERN PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. The ionization potential of hydrogen ($H \rightarrow H^+$) is 13.6 eV. The first ionization potential of helium, E_I, ($He \rightarrow He^+$) is 24.5 eV while the second ionization potential of helium, E_{II} ($He \rightarrow He^{2+}$) is 54.4 eV. Using simple estimates, account for the fact that $E_{II} \neq 2\,E_I$.

2. Using the Heisenberg Uncertainty Principle and an expression for the energy of the Hydrogen atom, derive an expression for the diameter of the hydrogen atom.

3. In the hydrogen atom, the energy of the $2s_{1/2}$ state is about 4.372×10^{-6} eV higher than that of the $2p_{1/2}$ state. Give a brief qualitative explanation of this effect. Can you name the physicist who described and then measured the effect?

4. What is meant by an "atom laser" ? Suggest an experiment in which atom lasers may be useful.

5. Consider a physical system of N identical particles confined to a container of volume V. In the limit $N \rightarrow \infty$ and $V \rightarrow \infty$, maintaining a finite ratio N / V, one can describe the system in the thermodynamic limit. For such a system, what is meant by an *extensive* property. Suppose A(N,V,T) is such an extensive property, T being the temperature of the system. Show that

$$A = \left(\frac{\partial A}{\partial N}\right)_{V,T} N + \left(\frac{\partial A}{\partial V}\right)_{N,T} V$$

6. Explain the physical origin of the Van der Waals force between two electrically neutral atoms. Without getting into mathematical details, explain why the Van der Waals force can lead to bound states between two neutral atoms

7 a) What are the main predictions of the <u>non-relativistic Schroedinger theory</u> with regard to the observable spectroscopic features of a hydrogen atom in vacuum, in the absence of <u>external</u> electric and magnetic fields? Which <u>observed</u> features cannot be explained by the Schroedinger theory?

b) Consider now a hydrogen atom in the presence of an external magnetic field. According to the <u>nonrelativistic Schroedinger theory</u> what are the observable consequences of the external magnetic field? Do these theoretical predictions fit the observed spectral lines in the presence of a magnetic field? Why or why not?

c)What essential theoretical modifications were introduced by Pauli to resolve the problems observed in part (b)? Which experimental problems are resolved by the Pauli correction? Which other main problems are not resolved?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Magnet Opus*

Consider a system of angular momentum $\vec{J}$. We confine ourselves to the three dimensional subspace spanned by the kets $|+1\rangle$, $|0\rangle$, $|-1\rangle$, common eigenstates of J^2 (eigenvalue $2\hbar^2$) and J_z (eigenvalues $+\hbar$, 0, $-\hbar$). The Hamiltonian H_0 of the system is

$$H_0 = aJ_z + \frac{b}{\hbar} J_z^2$$

where a and b are two positive constants, whose dimensions are those of an angular frequency.

a) What are the energy levels of the system? For what values of the ratio b/a is there a degeneracy?

b) A static field $\vec{B}_0$ is applied in the direction parallel to the x-axis. The interaction of the magnetic moment of the system, $\vec{M} = \gamma\vec{J}$ (γ is the gyromagnetic ratio, assumed to be negative), with the external magnetic field is described by the Hamiltonian

$$W = \omega_0 J_x$$

where $\omega_0 = -\gamma\left|\vec{B}_0\right|$ is the Larmor angular frequency in the field $\vec{B}_0$, and J_x is the component of $\vec{J}$ in the x-direction. Assume that $b=a$ and that $\omega_0 << a$. Calculate the energies and eigenstates of the system to first order in ω_0 for the energies and to zeroth order for the eigenstates.

Note: In the Hilbert space of relevance to this problem, we have

$$J_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A2. *What did we know and when did we know it?*

A system is described by the wave function

$$\psi(x,y,z) = N(x+y+z)\exp(-\frac{r^2}{\alpha^2})$$

where α is real and N is a <u>known</u> normalization constant.

a) We wish to measure <u>simultaneously</u> the observables L_z and L^2 . Can we do so, and why?

b) List the possible results of a simultaneous measurement of these quantities and calculate the corresponding probabilities. The following identities may came handy

$$Y_1^0(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^{\pm 1}(\theta,\varphi) = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

A3. *A drop in the well*

(a) Write the energy eigenvalues and normalized wavefunctions for a particle of mass *m* in a one dimensional square well of distance 2a (from –a to a) with infinite potential walls.

(b) A particle of mass m is dropped into the well and eventually settles into the ground state. Write down its wavefunction.

(c) A thin barrier is then inserted into the middle of the well. What is the probability that the particle is in the left half and the right half of the divided potential? Sketch the wavefunctions.

(d) A measurement reveals that the particle is in the left half of the well, in its ground state. Sketch the wavefunction after the measurement. What is the energy of the particle? From where did the extra energy come?

(e) The barrier is now suddenly removed from the middle of the original well. Sketch the wavefunction immediately after the barrier is removed.

(f) Following on part (e), compute the probability that the electron is in each of the four lowest states of the *original* potential.

(g) Write down a formal expression for the probability that the particle is in the right half of the original well ($0<x<a$) for time $t > 0$. <u>Do not attempt to evaluate this probability.</u> Is the probability 0 or >0? Why?

B1. *Big things in small packages.*

Consider a system of N independent particles maintained at the absolute temperature T.

(a) Derive the following expression for the specific heat

$$C_v = \frac{N}{kT^2}\sigma_\varepsilon$$

where σ_ε is the fluctuation in ε, the energy of a single particle.
σ_ε is defined by

$$\sigma_\varepsilon^{\ 2} = [\langle\varepsilon^2\rangle - \langle\varepsilon^2\rangle]$$

in which $\langle\varepsilon\rangle$ is the average energy of a single particle, and similarly $\langle\varepsilon^2\rangle$ is the average of ε^2.

(b) Show that the fluctuation is ε is not small, and in fact is given by

$$\frac{\sigma_\varepsilon}{\langle\varepsilon\rangle} = \sqrt{N}\frac{\sigma_E}{\langle E\rangle}$$

where E is the total energy of the system, σ_E is its fluctuation, and $\langle E\rangle = N\langle\varepsilon\rangle$. Thus even though the fluctuation in E is very small for a macroscopic system, the fluctuation in ε is of the same order as ε itself.

B2. *Solid or Gas?*

(a) Starting with the partition function, derive an expression for the specific heat of an ideal gas.

(b) Again, starting with the partition function, derive an expression for the specific heat of a solid in the Einstein model.

(c) What is the physical reason that the high temperature results are the same?