QUANTUM MECHANICS I

PHYS 516

MidTerm Exam

Distributed: Feb. 18, 2015

Due: February 25, 2015

1a. Write down the relativistic dispersion relation for a spinless particle of mass m.

1b. Convert this to a second order PDE using Schrödinger's prescription $p \rightarrow (\hbar/i)\nabla$.

1c. The particle moves in a scalar potential V. Modify the PDE of part **1b.** to include the potential.

1d. Assume the potential is spherically symmetric: $V(\mathbf{x}) = V(r, \theta, \phi) = V(r)$. Write the Laplacian in the appropriate separation of variables form.

1e. Assume the wavefunction separates: $\psi(r, \theta, \phi) = u(r)Y_m^l(\theta, \phi)$. Write down the equation for the radial wavefunction $u(r)$.

1f. Simplify this equation using the substitution $u(r) = \frac{1}{r}R(r)$. What is the equation for $R(r)$?

1g. Transform this equation to dimensionless form using the substitution $r = \gamma z$.

1h. Compare this dimensionless equation with those that appear in Table **22.6** in Abramowitz and Stegun. Relate your physical variables with their integer variables.

1i. Show that the combination $(l + \frac{1}{2})^2 - \alpha^2$ occurs naturally. $\alpha = e^2/\hbar c$

1j. Solve for the bound state spectrum $E(n, l)$. You should find $E = mc^2/\sqrt{1 + (\alpha/N(\alpha))^2}$. You should also find $N(\alpha) = n + \frac{1}{2} + \sqrt{(l + \frac{1}{2})^2 - \alpha^2}$

1k. Use this information to determine what the scaling constant γ is.

1l. Show that this theory cannot be correct, for it fails for nuclei with charge $Z > Z_{\rm min}$. How does it fail and what is Z_{min} ?

1m. Return to **1c**. Assume the energy E is near the rest energy mc^2, so that you can write $E = mc^2 + W$, where W is a "nonrelativistic" energy. Assuming $W + V \ll mc^2$, what is the nonrelativistic limit of this relativistic wave equation?