Magnetic Mirrors and their role in Fermi Acceleration

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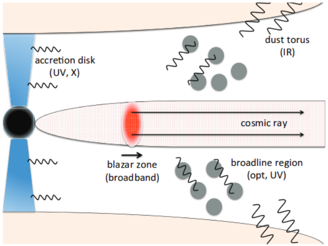
Fermi Acceleration

Two types:

- First order – astrophysical shock waves
- Second order – moving magnetized gas clouds

Can produce UHECR $\rightarrow$ pp, $p\gamma$



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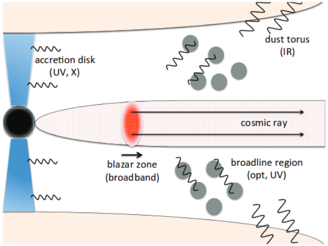
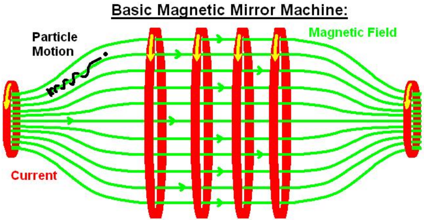


Consider: Magnetic fields alone can only change direction, not increase energy

- Electron dense plasmas effectively short out static electric fields
- Increased energy must come from changing magnetic fields $\rightarrow \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

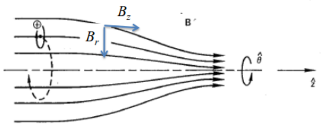
Magnetic Mirror



- Constitutes an electric current loop with a dipole moment
- Magnetic moment invariant

$$\mu = \frac{\frac{1}{2} m {v_{\perp}}^2}{B}$$

Magnetic Mirror

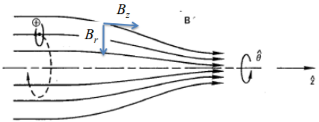


- Cylindrical symmetry: $\mathbf{B} = B_r\hat{\mathbf{r}} + B_z\hat{\mathbf{z}}$
- Using Maxwell's $\nabla \cdot \mathbf{B} = 0$:

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0$$

$$\rightarrow \frac{\partial}{\partial r}(rB_r) = -r\frac{\partial B_z}{\partial z}$$

Magnetic Mirror



If $\frac{\partial B_z}{\partial z}$ is more or less constant with r, then we can integrate:

$$B_r = -\frac{1}{2} r \left[\frac{\partial B_z}{\partial z} \right]_{r=0}$$

Lorentz force?

Magnetic Mirror

In absence of an electric field, the Lorentz force, $F = q(v \times B)$, provides the following terms:

$$\mathbf{F}_r = q(v_\theta B_z - v_z B_\theta), \quad \mathbf{F}_\theta = q(-v_z B_z + v_z B_r), \quad \mathbf{F}_z = q(v_r B_\theta - v_\theta B_r)$$

(1) (2) (3) (4) (5) (6)

- Orientation such that $B_\theta = 0$ [(2) and (5)]
- Produces regular gyroscopic motion [(1) and (3)] and radial shift (4)

B_r substituted into (6) to find:

$$\mathbf{F}_z = -q v_\theta B_r = \frac{q v_\theta r}{2} \frac{\partial B_z}{\partial z}$$

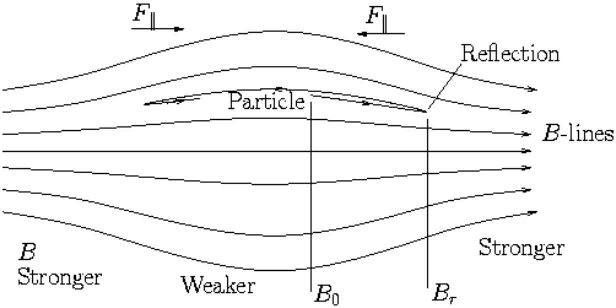
Magnetic Mirror

Averaging $\mathbf{F}_z$ over one gyro-orbit and defining $v_\theta = \mp v_\perp$ yields:

$$\mathbf{F}_z = \mp \frac{q v_\perp r}{2} \frac{\partial B_z}{\partial z} = \mp \frac{q {v_\perp}^2}{2\omega_c} \frac{\partial B_z}{\partial z} = -\frac{1}{2} \frac{m v_\perp r}{B} \frac{\partial B_z}{\partial z}$$

Or, more succinctly, $\mathbf{F}_z = -\mu \frac{\partial B_z}{\partial z}$, where $\mu = \frac{1}{2} \frac{m {v_\perp}^2}{B}$ is the magnetic moment, which is invarient in time for a particle's orbit.

Consequence: if $B \uparrow$, $v_\perp \uparrow$, requiring $v_\parallel \downarrow$ to conserve energy. Eventually $v_\parallel \to 0$ and the particle is reflected back into the weaker magnetic field!



Now that we accept a denser magnetic field may reflect a charged particle, let's consider a massive, moving dense magnetic field.

Acceleration

A particle at velocity v reflected by a magnetic mirror with velocity V at angle θ has a center of momentum frame which is the rest frame of the magnetic mirror.

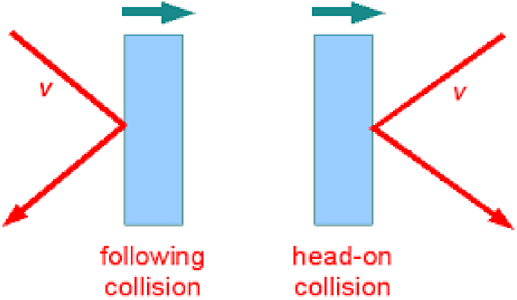
In COM frame, before collision:

$$E' = \gamma v(E + Vp\cos\theta)$$

$$p'_x = p'\cos\theta' = \gamma v(p\cos\theta + \frac{VE}{c^2})$$

After collision, p'_x reversed, energy conserved. Transforming back:

$$E'' = \gamma v(E' + Vp'_x)$$



Acceleration

The resulting change in particle energy is:

$$E'' - E = \Delta E = \frac{2Vv\cos\theta}{c^2} + 2\left(\frac{V}{c}\right)^2$$

which has a first order acceleration term with the velocity of the mirror.

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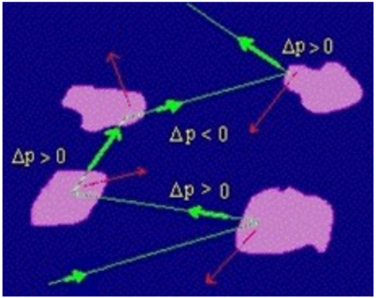
...however, this assumes the angle is fixed.

Second-Order Fermi Acceleration

Fermi's original idea in 1949:

- Clumps of plasma distort magnetic field
- Magnetic inhomogeneities provide stochastic acceleration of particles
- On average, more head on collisions than tail on collisions: net gain for particles



Second-Order Fermi Acceleration

For relativistic particles of $v \approx c$, the probability of collision is proportional to $\gamma v[1 + (\frac{V}{c})\cos\theta]$. Integrating:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{8}{3}\left(\frac{V}{c}\right)^2$$

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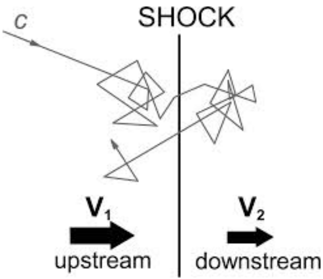
It turns out Fermi's result is of second order in $\frac{V}{c}$. But what if all collisions were head on?

First order Fermi Acceleration

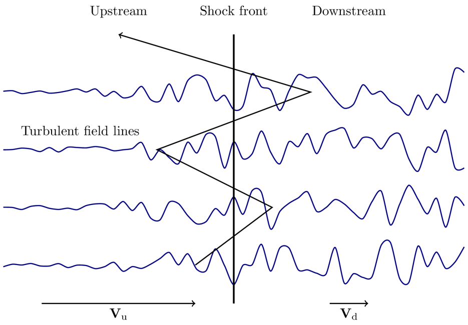
Consider the media around a shock wave emitted from an astrophysical object, which segregates the media on either side of it.

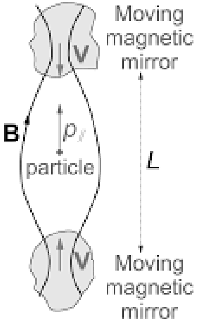


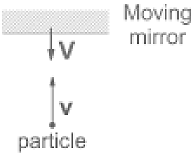
- The medium on each side of the shock sees the other side moving *toward* it
- Upstream (with isotropic velocity distribution) sees downstream (with magnetic inhomogeneities) as speeding toward it with velocity $V2 - V1$
- Downstream sees upgoing doing the same thing
- A particle traversing the shock will encounter a head on collision

A particle caught in this circumstance will continually bounce over the shock, gaining energy from each pass to first-order in velocity.

First order - Fermi Acceleration







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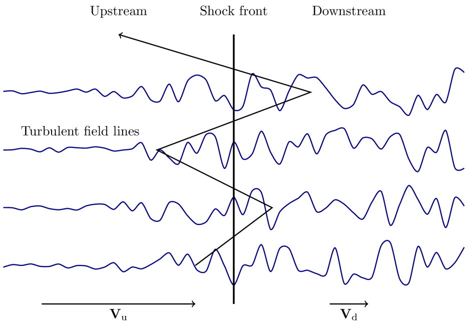

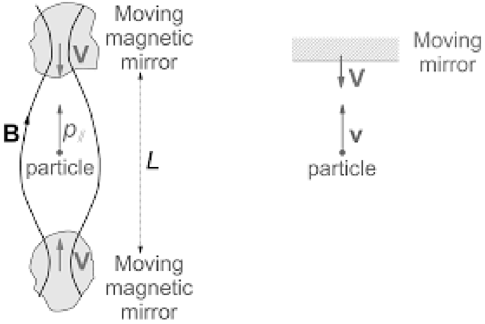

Only nonthermal cosmic rays can cross the shock to undergo this acceleration – this mechanism is currently not well understood.

First order - Fermi Acceleration

The mechanism for energy transfer is the same as before. Assuming $v \approx c$ and ignoring the second order term:

$$E'' - E = \Delta E = \frac{2V\cos\theta}{c}$$

This time, the angle of the magnetic mirrors is fixed, so we needn't integrate.

First Order - Fermi Acceleration

To preserve the number of particles on either side of the shock, $\rho_1 V_1 = \rho_2 V_2$. This gives us an expression for the ratio between velocities for strong shocks:

$$\frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}$$

which is solely dependent on the adiabatic index, γ. For a fully ionized plasma $\gamma = \frac{5}{3}$ so $\frac{V_1}{V_2} = 4$.

Questions?