Quantum II midterm question

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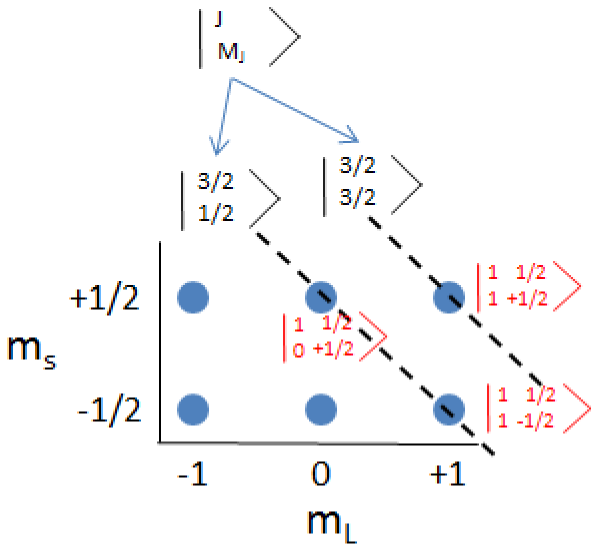
1 Problem 1: "Graph thing with the lines"

Recall the lowering operator $L_- |j,\ m\rangle = \sqrt{(j+m)(j-m+1)}\,|j,\ m-1\rangle$. The lowering operator is the same for any angular momentum state (orbit, spin or total).
The 6 states of an electron with orbital angular momentum 1, spin $\frac{1}{2}$ are shown below.



The top-right state $\left|\begin{smallmatrix}\ell & s\\ m_\ell & m_s\end{smallmatrix}\right\rangle = \left|\begin{smallmatrix}1 & 1/2\\ 1 & 1/2\end{smallmatrix}\right\rangle$ can be written in the TOTAL angular momentum basis as $\left|\begin{smallmatrix}J\\ M_J\end{smallmatrix}\right\rangle = \left|\begin{smallmatrix}3/2\\ 3/2\end{smallmatrix}\right\rangle$.
a) In the total angular momentum basis apply the lowering operator J_- to the top-right state basis to find the next lower state.

$$J_- \left|\begin{smallmatrix}3/2\\ 3/2\end{smallmatrix}\right\rangle = \alpha \left|\begin{smallmatrix}3/2\\ 1/2\end{smallmatrix}\right\rangle \tag{1.1}$$

b) Also apply the lowering operator $(L_- + S_-)$ to the top-right state in the spin-orbit basis:

$$(L_- + S_-) \left|{}^{1}_{1}\ {}^{1/2}_{1/2}\right\rangle = \beta \left|{}^{1}_{0}\ {}^{1/2}_{1/2}\right\rangle + \gamma \left|{}^{1}_{1}\ {}^{1/2}_{-1/2}\right\rangle \tag{1.2}$$

c) Use 1.1 and 1.2 to write the total angular momentum state $\left|{}^{3/2}_{1/2}\right\rangle$ as a linear combination of the spin-orbit states $\left|{}^{1}_{0}\ {}^{1/2}_{1/2}\right\rangle$ and $\left|{}^{1}_{1}\ {}^{1/2}_{-1/2}\right\rangle$.

2 Solution

a) Applying J_- we find:

$$J_- \left|{}^{3/2}_{3/2}\right\rangle = \sqrt{3} \left|{}^{3/2}_{1/2}\right\rangle \tag{2.1}$$

b) Applying $(L_- + S_-)$ we find:

$$(L_- + S_-) \left|{}^{1}_{1}\ {}^{1/2}_{1/2}\right\rangle = \sqrt{2} \left|{}^{1}_{0}\ {}^{1/2}_{1/2}\right\rangle + \sqrt{1} \left|{}^{1}_{1}\ {}^{1/2}_{-1/2}\right\rangle \tag{2.2}$$

c) We can therefore write the total angular momentum state as:

$$\left|{}^{3/2}_{1/2}\right\rangle = \sqrt{\frac{2}{3}} \left|{}^{1}_{0}\ {}^{1/2}_{1/2}\right\rangle + \sqrt{\frac{1}{3}} \left|{}^{1}_{1}\ {}^{1/2}_{-1/2}\right\rangle \tag{2.3}$$