



DEPARTMENT OF PHYSICS

PhD Qualifying Exam

Friday, September 17, 1999

Classical Physics

9 am - 12 noon

PRINT YOUR NAME _____

EXAM CODE _____

PUT YOUR EXAM CODE, **NOT YOUR NAME**, ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. Put a circle around the numbers below to indicate which questions you have answered—don't write on the lines.

Short questions

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

Long Problems

A1. _____

A2. _____

A3. _____

B1. _____

B2. _____

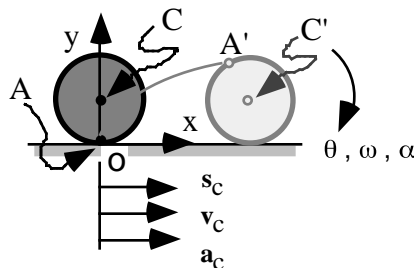
CLASSICAL PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. A fused silica fiber has an attenuation of 0.2 db/km. How far can a signal travel along the fiber before the power level drops by half?

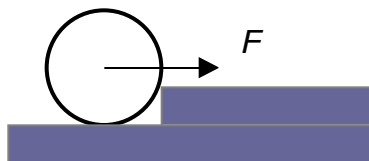
2. A wheel of radius R rolls without slipping on a horizontal surface, starting at point O with point A on the wheel's rim in contact with the surface at O . See the figure right.



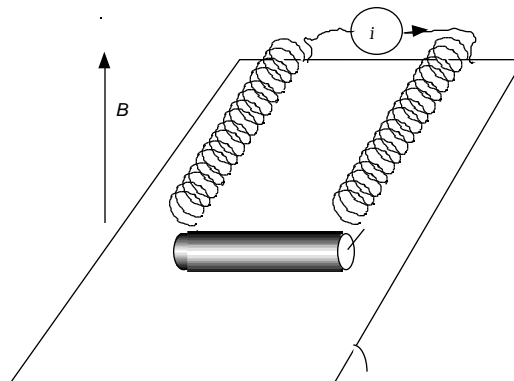
Use the coordinate system shown, with origin fixed at O

- (a) Write the expressions for the x and y coordinates of point C 's position, s_c , velocity, v_c , and acceleration, a_c , in terms of the wheel's angular values of θ and ω .
- (b) Write the expressions for the x and y coordinates of point A 's position, s_A , velocity, v_A , and acceleration, a_A , in terms of the wheel's angular values of θ and ω .

3. A wheel of mass M and radius R rests upright against a step of height $h < R$. What minimum horizontal force F is required, acting on the wheel's center, to have the wheel rise up over the step.



4. A copper bar of mass M and length L is attached to two steel springs, which can be considered massless here. The springs are narrow and identical and have spring constant k . The two springs and bar rest on a smooth wooden plane inclined to the horizontal with angle θ . The springs and bar are attached to a source of constant current i . A uniform magnetic field pointing vertically is turned on as the bar is released from the unstretched position of the springs. Write its equations of motion (don't solve them.)



5. Two large parallel plates, of thickness t_1 and t_2 , and area A , are filled with two media, characterized by dielectric constants ϵ_1 and ϵ_2 and conductivity σ_1 and σ_2 . The top plate is a positive potential V above the lower plate. Show that the resistance across (perpendicular to) the plates is given by

$$R = (\epsilon_2 t_1 + \epsilon_1 t_2) / \sigma_1 \sigma_2$$

6. A plano-convex lens rests on an optical flat, with the plano side up and the convex side in contact with the glass below. If it is illuminated with monochromatic light, incident from the top, is the center (contact) region bright or dark, and why?
7. Suppose the density of a certain astronomical object depends on distance r from its center in the following fashion:

$$\rho = \text{constant}, \quad r < a, \quad \rho = B/r, \quad b > r > a, \quad \rho = 0, \quad r > b.$$

Compute and sketch the gravitational force as a function of distance from the center.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. Great Balls of Fire

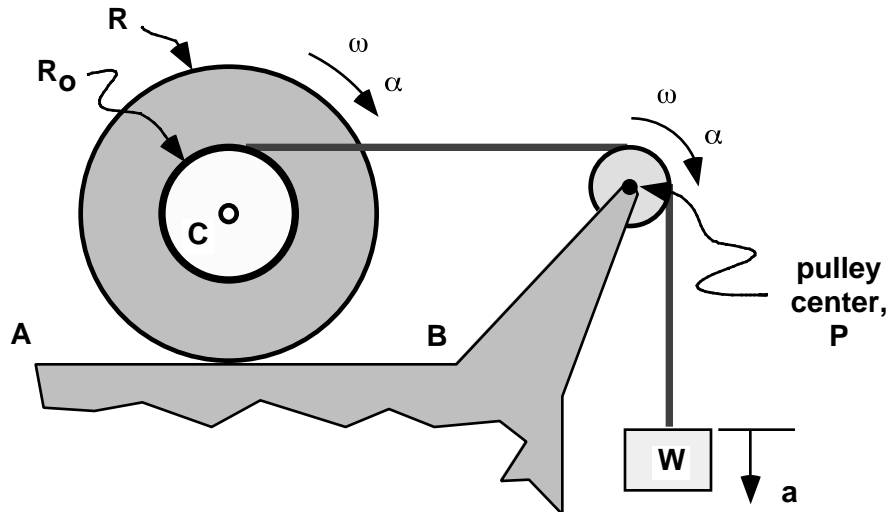
You are at Penn's Landing watching fireworks on the 4th of July. One is launched vertically upward. At the top of its flight it is ignited in a dazzling display of pyrotechnics. As the fiery fragments fall to Earth, a newscaster is moved to describe the spectacle as a **freefalling, uniformly expanding** ball of fire. As a physicist you suspect that there is more literal truth to this description than the newscaster is aware of. Confirm your suspicion.

A2. A little bounce in your swing.

A spring pendulum consists of a mass m attached to one end of a massless spring with spring constant k . The other end of the spring is tied to a fixed support. When no weight is on the spring, its length is l . Assume that the motion of the system is confined to a vertical plane. You may assume that the spring and mass always lie along a straight line.

- (a) Derive the equations of motion by Lagrange's approach.
- (b) Derive the equations by a direct application of Newton's laws.
- (c) Solve the equations of motion in the approximation of small angular and radial displacements from equilibrium.

A3. Roll Playing



In the diagram above, a 42-lb spool is connected by a cord to a load W . The cord passes over a pulley of radius R_p . The moment of inertia of the pulley (about a horizontal axis through its center) is I_{pulley} . The spool starts from rest and rolls without slipping along the horizontal surface AB . The linear acceleration of the

load is a . The angular accelerations of the spool and the pulley are α_s and α_p respectively. The moment of inertia of the spool about a horizontal axis through its center C is I_{spool} ; its smaller radius is R_o and the larger, outer radius R .

(a) Write down expressions relating the linear acceleration a of the load to the angular accelerations α_p and α_s of the pulley and spool respectively.

For the rest of the problem, assume: $R_p = 4$ in, $R_o = 8$ in, $R = 12$ in, $W = 60$ lb., $I_{\text{pulley}} = 0.4$ slug ft² and $g = 32.2$ ft s⁻².

(b) Find the angular accelerations α_s and α_p of the spool and the pulley, if the linear acceleration of the load is $g/6$ downward.

(c) Determine the tensions in the cord.

(d) What is the frictional force at the point of contact of the spool with the floor? Is it static or kinetic?

(Remember to identify the forces on the spool, pulley and the load.)

B1. Get the drift?

A particle of mass m and charge q moves in the (x,y) plane under the influence of a magnetic field $\vec{B} = B \hat{z}$ in the z direction. Assume the motion is non-relativistic.

(a) If \vec{B} is uniform, with $B = B_o = \text{constant}$, show that the particle's motion is circular and that its velocity $\vec{v} = (v_x, v_y, 0)$ is related to its position $\vec{r} = (x,y,0)$ in the (x,y) plane by

$$\vec{v} = -\frac{qB_o}{m} \hat{z} \times \vec{r}$$

where $\omega = qB_o/m$. Calculate the radius R of the circular orbit.

(b) Now suppose that \vec{B} is slightly non-uniform with $B = B_o + B_1(x)$ where $B_1(x) = \alpha B_o x$ and $\alpha R \ll 1$.

Denote by \vec{r}_o and \vec{v}_o the position and velocity solutions to the unperturbed problem in part (a) and write

$$\vec{r} = \vec{r}_o + \vec{r}_1 \quad \text{and} \quad \vec{v} = \vec{v}_o + \vec{v}_1$$

where \vec{r}_1 and \vec{v}_1 are small quantities of order αR . Substitute these expressions into the equations of motion of the particle to obtain a first-order differential equation for \vec{v}_1 . By considering the time averaged solution to this equation (or otherwise) show that, superimposed on the motion in part (a), the particle acquires a net drift velocity

$$\langle \vec{v}_1 \rangle = \frac{mv_o^2}{2qB_o} \hat{y}$$

B2. Two raisins in a piece of bread

Consider two point charges, $+q$ and $-q$, embedded in a dielectric sphere with a dielectric constant ϵ_i . The dielectric sphere in turn is embedded in an infinite dielectric medium with a dielectric constant ϵ . The two charges are a distance d apart and a distance r away from the center of the dielectric sphere. The radius of the dielectric sphere is R . Suppose $\epsilon \gg \epsilon_i$.

- (a) If you were to move the two charges from a radial distance r to $r' > r$ ($r' < R$ and the charges remain a distance d apart), would the charges have a higher energy or a lower energy? Explain your answer.
- (b) For simplicity, let us assume the infinite medium surrounding the dielectric sphere is a conductor. Calculate the electrostatic energies due to the interactions of the image charge of $+q$ with: (i) $+q$ itself; (ii) charge $-q$.

You may or may not like to know that a conductor can be modeled as a dielectric medium and an infinite dielectric constant.