DREXEL UNIVERSITY

DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam
Classical Physics

Friday, September 25, 1992 — 9 am - 12 noon

PRINT YOUR NAME______________________

EXAM CODE________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN.
(This allows us to grade each student only on the work presented.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. Indicate in the spaces below which questions you have answered.

Short questions	*Long Problems*
1. _____	1. _____
2. _____	2. _____
3. _____	3. _____
4. _____	4. _____
5. _____	5. _____
6. _____	
7. _____	

CLASSICAL PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. Two identical neutron stars of mass M and radius R fall toward each other for a head on collision. Neglecting tidal distortion, and assuming no initial velocity, what is the velocity of the stars in the center of mass frame at the moment of contact.

2. What are the action - angle variables and why are they useful?

3. An explosive shell is fired into the air at a non-zero angle. It explodes at some height above the ground. Why does the center of mass of the fragments continue in the same trajectory as the shell would have taken if it did not explode?

4. In August 1992, the crew of the shuttle Atlantis in orbit above the Earth tried to deploy a satellite which would remain tethered to the shuttle through a wire of length 12 miles. This configuration, travelling at 5 miles per second, was to generate electricity in conjunction with an electron gun firing from the shuttle. Explain how this is possible, and comment on the magnitudes of the quantities involved.

5. Classically, why are circular electron orbits around the nucleus unstable?

6. A radio capacitor consists of a stack of 5 equally spaced plates. The top (#1) and bottom plates (#5) are connected by a wire. Plates #2 and #4 are connected by a different wire. The center plate (#3) is either connected to #2 and 4 (configuration X) or to #1 and 5 (configuration Y). What is the ratio of capacitance between X and Y?

7. A collection of charges pulsates radially. Can this be remotely detected? If so, how? If not, why not?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *See you around.*

A particle of known mass m moves under the action of an attractive central force. The potential energy is $U(r) = kr^2/2$, where k is a known positive constant, and r is the distance from the center of force of the particle.
(a) Show that the vector angular momentum ($\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$) is a constant of the motion.
(b) What does the fact that $\boldsymbol{L}$ is a constant vector allow you to conclude about the motion?
(c) Show that for a given total energy E and angular momentum L the motion of the particle takes place between two circles, and determine the radii of these circles.

A2. *Rope tricks.*

A smooth rope of length L and mass m is placed above a hole in a table. One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope. The height of the table is greater than the length of the rope.
(a) Find the velocity of the rope as a function of the distance to the end of the rope, x. (Ignore friction as the rope unwinds).
(b) Find the acceleration of the falling rope.
(c) Find the energy lost from the system as the end of the rope leaves the table.

A3. *Skyhook*

A satellite can be placed in an orbit above the plane of the earth's equator such that it remains above the same point on earth continuously. This is called a synchronous orbit.
(a) Show that the satellite must have a distance from the center of the earth given by

$$r_s = \left(\frac{G M_E}{\omega^2}\right)^{\frac{1}{3}} = 42\,100\ km$$

where $GM_E = 3.99 \times 10^{14}\ m^3 s^{-1}$ and $\omega = 2\pi/86160\ s^{-1}$.

(b) Suppose a satellite is placed in orbit with a rope suspended beneath it, of length L which extends from the satellite, at orbital distance r_s', to the surface of the earth, r_E. (Science-fiction writer R. A. Heinlein named such a satellite a "skyhook".) The satellite/skyhook can orbit synchronously at a distance $r_s' > r_s$. If the rope has a uniform linear density of μ, find an expression for the differential tension in the rope, $dT(r)$.

(c) Using the boundary condition $T(r_E) = 0$, find an expression for $T(r)$.

(d) Locating the satellite at the point where $T(r) \approx 0$, show that the length of rope is given by

$$L = \frac{r_E}{2}\left[\left(1 + \frac{8\,G\,M_E}{\omega^2 r_E^3}\right)^{1/2} - 3\right] = 144\,000\ km$$

for $r_E = 6380$ km.

B1. *Who left the dipole out again?*

An electric dipole is located at a perpendicular distance d from a grounded conducting infinite plate. It is perpendicular to the plane, and pointed away. The electric dipole moment is represented by $\boldsymbol{p}$.
(a) Find the electrostatic potential on the side of the plate in the space in which the dipole resides for all points except of course the point right at the dipole.
(b) How would you find the electric field? (Don't carry out the actual calculation.)
(c) How would you determine the induced charge distribution on the plate? (Again, don't carry out the details.)

B2. *The waves come in, the waves come out...*

A plane polarized electromagnetic wave, polarized so that $\boldsymbol{E}_{\text{incident}}$ lies along the $+x$ direction, is initially moving in the the $+z$ direction in a medium characterized by ε_o and μ_o. The wave is incident on a slab of material of thickness d and ε_1 and μ_o. The medium on the far side of the slab again has characteristic constants ε_o and μ_o.
Set up the equations from which you can solve for the amplitudes of the reflected and transmitted waves. (Don't actually solves the equations, it would take too much time.)

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MODERN PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. Under the assumption that all stellar bodies have an average mass of 10^{30} kg, and move with random velocities of about 10 km/s, what is the average rms velocity of a 1000 kg asteroid after a very long time if it is NOT captured?

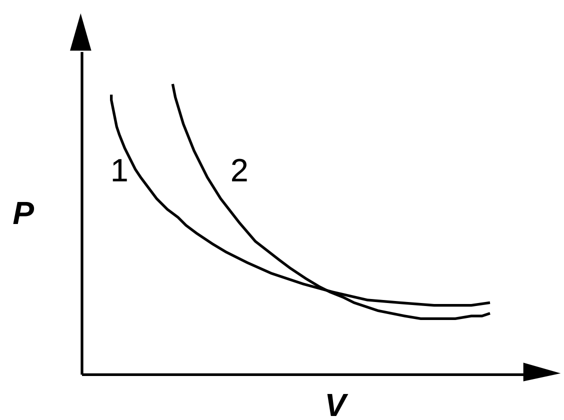
2. Three spin 1/2 particles occupy the corners of an equilateral triangle, and interact via a spin Hamiltonian proportional to $(\sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_3 + \sigma_3 \cdot \sigma_1)$. List the energy levels, giving total spin values and degeneracies. What is the partition function?

3. What is the physical basis for the selection rule prohibiting transitions between states of zero total angular momentum (J) with the emission of a photon? Is there any way to obtain light from a zero-zero transition?

4. Why does the recently discovered variation in the cosmic background radiation represent inhomogeneity in the mass distribution of the early universe?

5. How do the electromagnetic spectra emitted by earth and the sun differ? How are these differences related to the "greenhouse" warming of the earth?

6. Why can curves 1 and 2 (in the PV diagram at right) not both be adiabatic curves? Explain how it would violate the laws of thermodynamics.



7. Current models for the nuclear physics of the sun predict far more neutrinos arriving at earth than are observed in standard neutrino detectors. It has been proposed that neutrino oscillations account for this discrepancy. What is meant by "neutrino oscillations" in quantum mechanics?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Bouncing off the wall*

(a) In what respect are the quantum mechanical solutions for the following two potentials the same:

$$V(x) = \begin{cases} 0 & -\frac{L}{2} \le x \le \frac{L}{2} \\ \infty & otherwise \end{cases}$$

$$W(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & otherwise \end{cases}$$

(b) Why is it that parity plays no role in your answer to (a)?
(c) Suppose a particle is in the first excited state of $V(x)$. Suddenly the width of the potential is increase from L to $2L$ (i.e., $-L \le x \le L$). At that instant, what are the probabilities of the particle in the ground, 1st and 2nd excited states of the new potential?

A2. *Double Trouble*

A well collimated stream of mono-energetic electrons strikes two thin parallel slits separated by a distance d from each other. The electrons are stopped by a sensitive plate a distance L away from the slits. Their arrival can be monitored both by watching tiny flashes of light as the electrons hit the plate, or by processing the plate at the end of the experiment in a way that yields essentially a photographic pattern of the hits.

(a) give a qualitative description of the spatial distribution of the individual flashes of light on the plate as each electron ends its journey, and describe your expectation of the final distribution of the electrons at the plate after a large number of events have been observed. The experiment is done twice: first with a fairly high flux of incident electrons, i.e. many electrons can be expected to be located simultaneously in the region of space between the slits and the plate, and second, with such a low flux that, at any given time, only one electron can be found in this region. What differences do you expect to observe in both experiments?

(b) With the help of the Schroedinger equation, give a quantitative account of the above expectations. In particular, address the issue of a possible mathematical description of the individual flashes of light and of the long term exposure of the plate.

(c) Now consider the following gedanken extension of the previous situation. The output of each slit is fitted with a hollow Faraday cage in the form of a metallic tube with open ends to allow the passage of electrons. A potential difference is maintained between the two cages, which can be varied from zero to positive or negative values. The Faraday cages are ideal (no fringe effects at the ends) so that the electric fields are exactly zero inside each of them. Describe the long term exposure pattern of the electrons at the plate when the potential difference between the cages is zero and when it is different from zero.

A3. *Take it for a spin*

Consider a system with spin $\boldsymbol{S}$. The spin projection in an arbitrary direction $\hat{u}$ is $S_u = \boldsymbol{S} \cdot \hat{u}$ where $\hat{u} = \sin\theta \cos\phi \; \hat{\imath} + \sin\theta \sin\phi \; j + \cos\theta \; k$. The simultaneous eigenvectors of S^2 and S_u are given by

$$|{}^1/_2, +{}^1/_2\rangle_u = \cos\theta/2 \; e^{-i\varphi/2} \; |{}^1/_2, {}^1/_2\rangle + \sin\theta/2 \; e^{i\varphi} \; |{}^1/_2, -{}^1/_2\rangle \quad \text{and}$$

$$|{}^1/_2, -{}^1/_2\rangle_u = -\sin\theta/2 \; e^{-i\varphi/2} \; |{}^1/_2, {}^1/_2\rangle + \cos\theta/2 \; e^{i\varphi} \; |{}^1/_2, -{}^1/_2\rangle$$

where $|{}^1/_2, \pm{}^1/_2\rangle$ are the Pauli basis (i.e., eignevectors of $\mathbf{S}^2$ and S_z).

(a) The system is prepared by measuring S_y and obtaining the value $\hbar/2$. What is this state in the Pauli basis.

(b) At times $t \geq 0$, the Hamiltonian of the system is $H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$ where $\boldsymbol{B} = B_0 \,\hat{\imath}$ is a constant vector, and $\boldsymbol{\mu}_S = (q/m) \; \boldsymbol{S}$. q/m is constant. What are the eigenvalues of H?

(c) The system starts as prepared in part (a), then evolves under the Hamiltonian H of part (b). Give an expression for the state of the system at times $t > 0$.

(d) At the time $t > 0$, what is the probability that a measurement of S_z will yield the value of $\hbar/2$.

B1. *Piling up electrons*

(a) Show that the density of states (or of orbitals) $D(\varepsilon)$ for non-interacting electrons of energy ε and mass m in a volume V is given by

$$D(\varepsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2}\varepsilon^{1/2}$$

(b) If the electrons in a metal behaved classically, the heat capacity of the metal would be much larger than it really is. Use the above to explain this discrepancy by an order of magnitude argument. (Hint: consider the width of the Fermi-Dirac distribution function with energy.)

B2. *Hot and Cold Hydrogen molecules*

Diatomic hydrogen molecules in states with an even rotational quantum number are referred to as parahydrogen. Molecules in a state with odd J are called orthohydrogen.

(a) Which has greater degeneracy and why?

(b) Which will have greater specific heat? Why?

(c) In the figure below, the rotational heat capacity (specific heat) as a function of temperature is shown for orthohydrogen and parahydrogen (solid lines). Dashes show the equilibrium theoretical curve. The crosses show an experiment in which a room temperature mixture was cooled to the desired temperature and measured. Can you suggest why the theory (later confirmed) and this experiment might not match?