DEPARTMENT OF PHYSICS

PhD Qualifying Exam **Classical Physics**

Friday, September 21, 2001 9 am - 12 noon

PRINT YOUR NAME________________________

EXAM CODE__________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

Short questions		*Long Problems*	
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

CLASSICAL PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. "Gemini 4, piloted by Jim McDivitt and Ed White, reached orbit on June 3, 1965. In accordance with his flight plan, McDivitt immediately tried to maneuver close to the spent second stage of the launch rocket, which was drifting about 650 feet away.... He found that simply pointing the craft toward the target and firing his rear attitude thrusters did not help him overtake the spent stage.... Indeed, the distance between them actually increased, as if he were pressing a car's gas pedal in forward gear and the car was moving in reverse." (from "Docking in space", by R. Zimmerman, Invention and Technology, Fall 2001, p. 12). Explain. (You may approximate the orbits as circular).

2. In a famous, recent experiment (reported even in the *New York Times*) Lene Hau, of Harvard University, sent a pulse of light through a dilute gas (density $\approx 10^{13}$ atoms/cm^3) and measured its group velocity to be a mere 17 m/s (about the speed of a bicycle). Because the index of refraction of such dilute gas is virtually equal to unity, how can we can understand such a remarkably small group velocity?

3. Two cannons A and B on a level surface are going to shoot cannon balls that will collide. Cannon A points at an angle θ_A above the horizontal, aiming in the direction of B. Cannon B points back in the direction of A, with an angle θ_B above the horizontal. Ignoring all friction and air resistance, derive an equation for the conditions on the velocities v_A and v_B and the angles θ_A and θ_B such that the cannon balls collide in mid-air.

4. A thin flat, plate of dielectric with parallel faces has been manufactured by modern techniques so that its index of refraction is a function of distance horizontally along the surface as well as wavelength, i.e., $n = n(x, \lambda)$. Describe the behavior of a beam of white light that strikes the plate at normal incidence (i.e. travelling in the z direction).

5. Which exerts more force on an umbrella: rain drops or hailstones of the same mass and size as the raindrops falling from the same height? Explain your answer.

6. A solid copper box can shield a piece of equipment inside it from radio waves. If the box has holes in it, how large can the holes be and still allows the shielding to be generally effective.

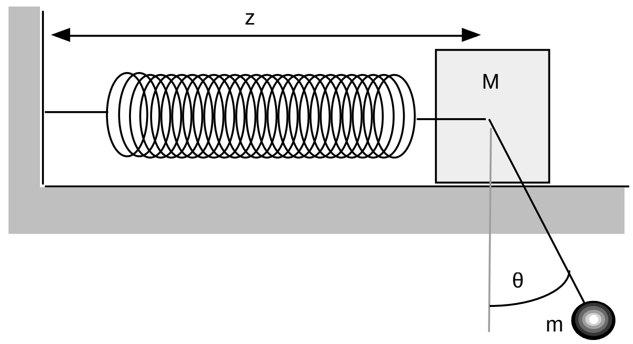
7. When a bowling ball is sent down a bowling lane, it first slides without rolling, and then begins to roll. What determines when rolling begins?

PART II: Long problems (75%)

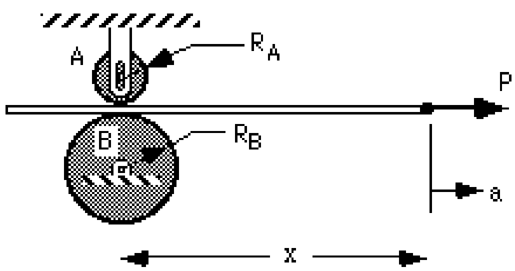
ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. Spring, as normal.

A block of mass M is attached to a spring (spring constant k) and slides along a frictionless table. Attached to the side of the block, and extending below the table is a mass m attached to a rigid rod of length l. Find the Lagrangian and the equations of motion for the system (as shown). Using the variable z and θ. Find the normal modes of small vibrations for the system.



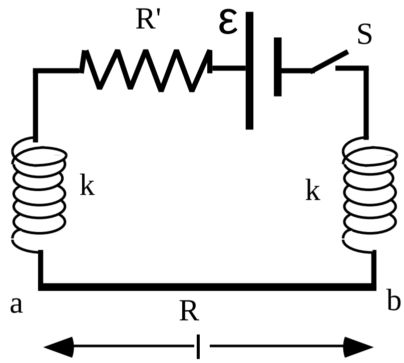
A2. Pardon me, but your belt is slipping



A belt of negligible mass passes between two solid cylinders and is pulled to the right with a force of P. It has acceleration "a" to the right, as shown below. The top cylinder, cylinder "A" of radius R_A = 50 mm, is free to move vertically, as well as rotate, in the bearing blocks that support its shaft; its mass is 6 kg. Cylinder "B" of radius R_B = 120 mm is held in its bearings so it only rotates; its mass is 24 kg. Coefficients of friction between the belt and the cylinders are unknown, but it is known that only rolling occurs, without any slippage of the cylinders on the belt. Neglect the friction in the bearings on the shafts of the cylinders.

a. If the acceleration of the belt is a = 1.5 m/s^2 to the right, determine the angular accelerations α_A and α_B of the cylinders A and B. Indicate the directions of these.
b. What is the static frictional force between the belt and the cylinder **B** so there is no slippage? (Static friction provides the necessary torque.)
c. What is the static frictional force between the belt and the cylinder A so there is no slippage? (Static friction provides the necessary torque.)
d. What is the magnitude of the required force P to perform this task?
e. If the maximum acceleration of the belt is a_{max} = 2.0 m/s^2 before any slipping occurs, find the coefficient of static friction, μ_s, between cylinder B and the belt.
f. What is the maximum angular acceleration of cylinder "A" for a_{max} = 2.0 m/s^2 ?

A3. **Unexpected support**



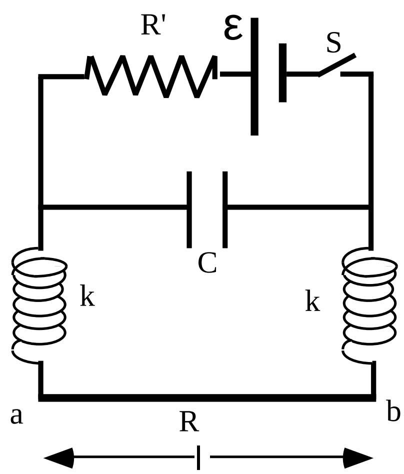
A battery of emf ε = 24 V and internal resistance r = 0.45 Ω is connected in series with a variable resistance R', two conducting vertical springs each of spring constant k = 19.62 N / m, and a linear conductor of length l = 120 cm, mass m = 80 gram and resistance R = 2.55 Ω as shown. Neglect the mass and the resistance of the springs. (The internal resistance is not drawn.)

a. What is the extension (Δy) in each spring due to the weight of the conductor?

b. Switch S is closed, and a uniform magnetic field of 0.436 T is applied to reduce the elastic force on the spring to zero. Find the direction of the applied magnetic field and the magnitude of current needed to reduce the elastic force on the spring to zero. Also determine the value of the variable resistor R'.

c. The magnitude and direction of the applied magnetic field remain unchanged as above. The value of R' is adjusted to (7 / 3) Ω. What is the length of the spring if the unstretched length for each spring is 60 cm?

d. Magnetic field stays the same as in part (b) with B = 0.436 Tesla and the variable resistance is set at 7 Ω . Switch S is kept open. A capacitor C = 50 μF is connected across the circuit as in figure below. Clock is reset to t = 0 and at this instant the switch S is closed. Find the initial current through the resistor R, the initial extension or compression in the spring and the charge on the capacitor C.



B1. **Doing the wave with a group**

Maxwell's wave equation for the electric field in a non-magnetic medium has the well known form

$$_2\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{c^2 \varepsilon_0}\frac{\partial^2 \vec{P}}{\partial t^2}, \qquad (1)$$

with the conventional meaning of the symbols. If the medium is a dilute dielectric with a single resonance, it may be modeled as a distribution of N positive and N negative charges per unit volume. The positive charges are immobile and each negative charge is tied to a positive charge by an elastic force with resonance frequency ω_0. Thus, the equation of motion for each negative charge is

$$\left(\frac{d^2}{dt^2} + \gamma\frac{d}{dt} + \omega_0^2\right)\vec{R} = -\frac{q}{m}\vec{E}\,, \tag{2}$$

where m is the mass, γ a phenomenological damping constant, $\vec{R}$ the displacement of the negative charge from the corresponding positive charge, and q is the absolute value of the charge; $\vec{E}$ is the electric field at the location of the charge. The macroscopic induced polarization in the medium is given by

$$\vec{P} = -qN\vec{R}$$

at the location of the charge under consideration.

a) We want to solve the coupled equations (1) and (2) at steady state for the case of plane polarized electric and polarization waves, propagating in the positive direction of the z-axis, i.e.

$$\vec{E}(z,t) = \hat{i}E_{st}e^{i(kz-\omega t)} + c.c.\,, \tag{3a}$$

$$\vec{P}(z,t) = \hat{i}P_{st}e^{i(kz-\omega t)} + c.c. \tag{3b}$$

Find the dispersion relation $k = k(\omega)$ so that the above waves (3a,b) satisfy Eqs. (1) and (2).

b) Under the generally valid assumption $\gamma << \omega_0, \omega$, and in the frequency range $|\omega - \omega_0| \approx \gamma$, calculate the phase and group velocity of the electromagnetic wave in the medium.

Hint for part (b): for a dilute medium (e.g. a gas) the index of refraction is very close to unity.

B2. **Charging an egg**

(a) How does charge distribute itself on and within an isolated conductor?
(b) How and why does such a distribution occur?
(c) A certain conductor has the shape of an ellipsoid described by the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

where a < b < c. What is the ratio of the maximum to minimum charge densities on the conductor's surface?