Statmech II HW4 problem 8.5

Vince Baker

November 14, 2015

1 Huang 8.5

For a system of N non-interacting harmonic oscillators, the energy levels are given by:

$$E_M = \frac{N}{2}\hbar\omega + M\hbar\omega \tag{1.1}$$

Where M are the number of energy quanta distributed among the harmonic oscillators. For Maxwell-Boltzmann statistics we can calculate the degeneracy by considering the M indistinguishable quanta as a row of marks, with $N-1$ "separators" to put them into bins. Each separator may be in any of M positions, so the degeneracy is given by the binomial coefficient:

$$W = \binom{M+N-1}{M} \tag{1.2}$$

Fixing the number of oscillators at N, the grand parition function is therefore:

$$Z_N = z^N e^{-\beta\frac{N}{2}\hbar\omega} \sum_{M=0}^{\infty} \binom{M+N-1}{M} e^{-\beta M\hbar\omega} \tag{1.3}$$

For Bose-Einstein statistics the oscillators themselves are also indistinguishable. So we can imagine ordering the N harmonic oscillators in a row and filling them sequenctially with energy quanta. There is then one configuration with all M quanta in the first oscillator, there is one configuration with M-1 quanta in the first oscillator and 1 quanta in the second oscillator, and so on. The degeneracy is therefore the "partition" of M in the number theory sense. As an example, $P(3) = (3, 2+1, 1+1+1) = 3$. Therefore, the grand partition function for Bose-Einstein statistics is:

$$Z_N = z^N e^{-\beta\frac{N}{2}\hbar\omega} \sum_{M=0}^{\infty} P(M) e^{-\beta M\hbar\omega} \tag{1.4}$$