DREXEL UNIVERSITY

DEPARTMENT OF PHYSICS

PhD Qualifying Exam **Classical Physics**

Friday, January 12, 2007 9 am - 12 noon

PRINT YOUR NAME________________________

EXAM CODE__________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 6 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

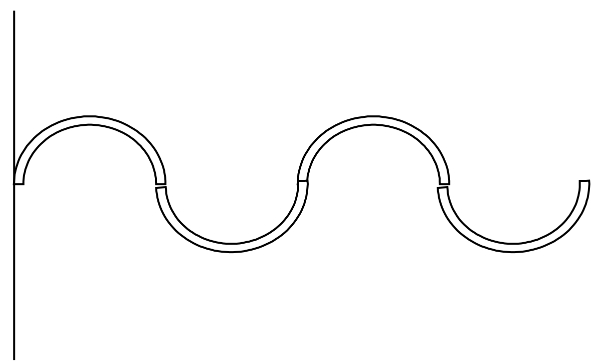
Short questions		*Long Problems*	
circle	grade	*circle*	grade
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____	B3.	_____
7.	_____		

CLASSICAL PHYSICS

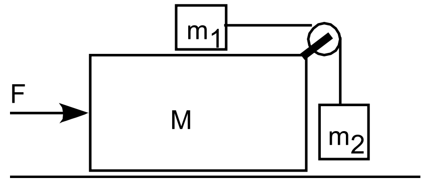
PART I: Short questions (25%)

ANSWER 5 OF 7 QUESTIONS

1. A uniform rod of mass M is bent into four adjacent semicircles all lying in the same plane. What is the moments of inertia of the bent rod about an axis through one end perpendicular to the semicircles and in their plane (see figure)?



2. Masses m_1 and m_2 are connected by a massless string that runs over a frictionless, massless pulley. Mass m_1 rests on a larger mass M. All surfaces are frictionless. What horizontal force F should be applied to M to prevent m_1 from slipping with respect to M?



3. A uniform sphere of mass $m = 0.5$ kg and radius $R = 0.30$ m is projected along a **rough** horizontal surface with an initial velocity $V_o = 6.0$ m/s and no angular velocity. The coefficient of kinetic friction between the sphere and the surface is $\mu_k = 0.80$.
Determine:

(a) the sphere's *linear* acceleration while it initially slides and starts to roll on the surface

(b) the sphere's *angular* acceleration (measured around the center of mass of the sphere) while it initially slides and starts to roll on the surface

4. Starting with Maxwell's equations in charge and current free space, obtain the wave equation satisfied by the electric field $\boldsymbol{E}$ or the magnetic field $\boldsymbol{B}$.

5. A charged particle of mass m and charge q is placed below an infinite conducting sheet near the surface of the earth. At what distance can it be placed such that it is in equilibrium? Is the equilibrium stable or not?

6. What is the cyclotron frequency for a singly-charged ion of atomic mass 5, in a uniform 3T magnetic field?

7. Describe a method (either in the laboratory, or occuring in nature) of producing plane polarized light. You must describe the physical phenomena that is responsible for the polarization and draw a diagram that illustrates the method.

PART II: Long problems (75%)

ANSWER 3 OF 6 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1.

A wheel of fortune found at a carnival consists of a perfectly balanced wheel rotating in a vertical plane about a fixed frictionless horizontal axis. Around the rim of the wheel are 100 equally spaced pegs numbered from one to 100. A flexible "flapper" mechanism can be set to strike the pegs as the wheel rotates. Such a wheel of diameter 1.0 m is set rotating at 50 rpm. At the start, peg No. 1 is directly above the axis, i.e. it is at "12 o'clock".

(See diagram on blackboard.)

What is the velocity and what is the position of peg No. 1 as a function of the time in seconds

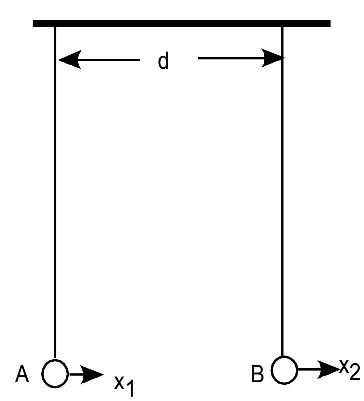
(a) if the wheel is allowed to rotate freely?

(b) if the "flapper" mechanism is engaged and uniformly slows the wheel to a stop in 15 s?

(c) How does the force of the individual impulses from the "flapper" mechanism vary with the speed of the wheel?

A2.

Two small metal spheres of equal mass m and equal charge q are suspended by insulating threads of equal length l from two points at a distance d apart on the same horizontal rigid support. One sphere is pulled aside a distance x_1 and the other sphere a distance x_2 (x_1, $x_2 << d$, l) and released. The two spheres move in the vertical plane formed by the equilibrium positions of the two threads.



(a) How many generalized co-ordinates do you need to solve this problem? Give a brief explanation for your choice.

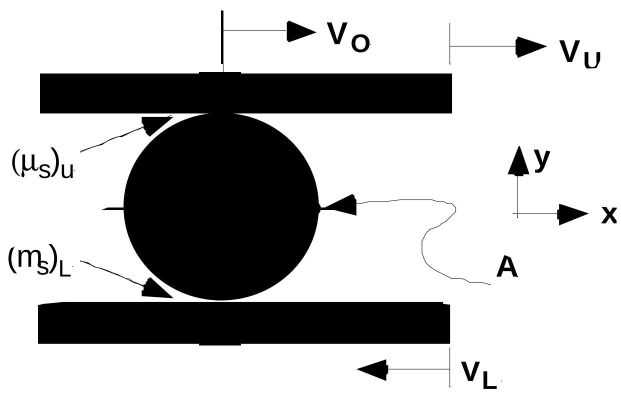
(A2, continued)

(b) Write down expressions for the kinetic and the potential energy of the system.

(c) Write down the Lagrange's equations of motion for the system and determine the resulting motion of the two spheres.

A3.

A roller bearing rolls without sliding between two moving metal plates (the bearing "races") as shown. The plates, each weighing 15 kg., move at constant horizontal velocities (relative to the ground) of $V_U = 6$ m/s and $V_L = 4$ m/s, in the directions shown, and the roller's radius is $r = 0.15$ m and it weighs 60 kg. Point "O" is the center of the bearing. Points "T" and "B" are at the top and bottom of the roller bearing at a given instant. Assume $g = 9.80$ m/s^2



[m_S in diagram should be \mu_S. Also note location of points T, B, O]

Determine the following

(a) velocity V_{TB} of point "T" relative to point "B"

(b) velocity V_{OB} of point "O" relative to point "B"

(c) velocity V_{OG} of point "O" relative to the ground coordinate system shown in the diagram

(d) roller's angular velocity of revolution about its center, O

(e) roller's total kinetic energy of motion; Assume: Moment of inertia of the roller $I = (2/5)Mr^2$

(f) total velocity of point A at the periphery of the roller, on its horizontal centerline (you must give the vector components of the velocity)

(g) If the coefficients of static friction between the plates and the roller are $\mu_s = 0.8$, find the maximum horizontal force that can be applied to each one of the plates before any slipping occurs.

B1.

An insulating sphere of radius R has a spherical cavity of radius r carved out at $x=-R/2$. The object has a uniform charge density ρ .

For this problem, we will consider only point P which is a height *d* above the center of the sphere.

(a) What is the electric field at P?

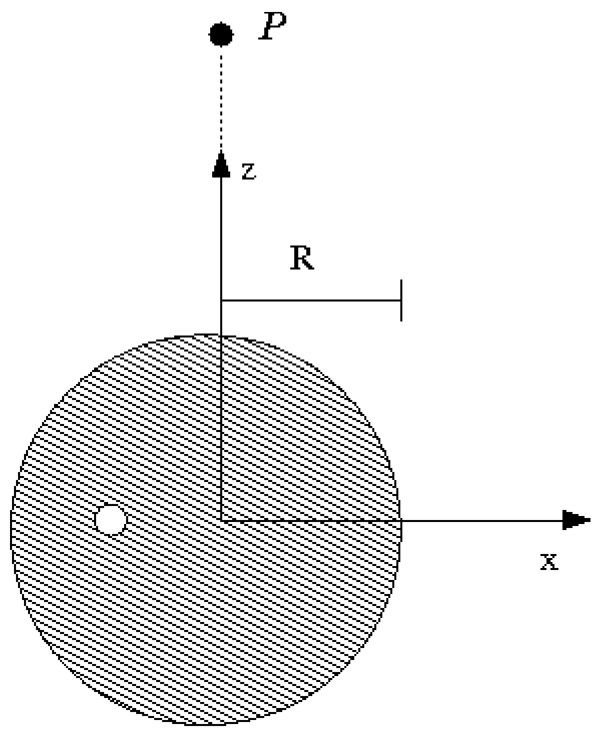
(b) At time *t*=0, the sphere begins to rotate round its z axis with angular frequency ω . At a retarded time $t'=t-d/c$, what is the vector potential at P? You may assume $d\gg R$ and $R\gg r$.

(c) What is the *time variable* part of the electric field at P from the rotating sphere?

(d) What is the magnetic field at P? Hint: This is much easier if you note that

$$\mathrm{B} = \frac{1}{c}\hat{r} \times \mathbf{E}$$

(e) What is the Poynting vector evaluated at P?



B2.

A spherical charge distribution that (roughly) models light nuclei is described by a charge density
$\rho(r)=\rho(0)\left(1-r^2/a^2\right), r<a$

and

$\rho(r)=0, r>a$

(a) Calculate the total charge Q.

(b) Find the electric field for $r>a$.

(c) Find the electric field for $r<a$.

(d) At what radius is the magnitude of the electric field a maximum?

B3.

Consider a long, hollow metal pipe of rectangular cross section with perfectly conducting walls of cross-sectional dimensions a and b ($a > b$) on the xy-plane.

(a) Show that it is possible for EM waves to be propagated along the z-axis. Begin by considering a (transverse electric) solution of Maxwell's equations inside the cavity of the form:

$E_z = 0$
$E_x = E_1(x,y) \exp[i(kz-\omega t)]$
$E_y = E_2(x,y) \exp[i(kz-\omega t)]$

Using the wave equation and appropriate boundary conditions, determine the allowed frequencies.

(b) Determine an expression for the cut-off frequency, if it exists, below which no waves are propagated. Calculate the frequency, in units of Hz if a = 1 cm and b = 1 cm.

(c) In general, what is the physical meaning of phase and group velocities (draw a picture). Determine the phase and group velocities of the propagating wave. How are these two velocities mathematically related to each other ?