Stat mech II HW8

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1 Problem 7.21

we start with the equation:

$$C_P - C_V = TV\kappa_T \left(\frac{\partial P}{\partial T}\right)^2 \tag{1.1}$$

$$C_P - C_V = T\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V^2 \tag{1.2}$$

$$\tag{1.3}$$

To find an expression for V we calculate the form of the Helmholtz free energy H. Since the Debye specific heat $C_V \propto T^3$ at low temperature, U has the form:

$$U \simeq T^4 + f(V) \tag{1.4}$$

Using the thermodynamic relations:

$$dS = \frac{1}{T}dU + \frac{P}{T}dV \tag{1.5}$$

$$S \simeq VT^3 + g(V) \text{ (constant volume)} \tag{1.6}$$

$$-\left(\frac{\partial A}{\partial T}\right)_V = S \tag{1.7}$$

$$A \simeq VT^4 + g(V) \tag{1.8}$$

$$P = -\left(\frac{\partial A}{\partial V}\right)_T \simeq T^4 + g(V) \tag{1.9}$$

Where $f(V), g(V)$ are different arbitrary functions of V only. We can now calculate the derivatives of the pressure:

$$\left(\frac{\partial P}{\partial T}\right)_V \simeq T^3 \tag{1.10}$$

$$\left(\frac{\partial P}{\partial V}\right)_T \simeq g(V) \tag{1.11}$$

Putting these into equation (2) we find:

$$C_P - C_V = Tg(V)(T^3)^2 \propto T^7 \tag{1.12}$$

2 Problem 7.34

We consider an n-dimensional Debye system and assume the propagation speed is the same in all directions. Taking the lattice vibrations as phonons with momentum $p = \hbar\mathbf{k}$ and $\omega = Ak$ we can find $g(\omega)\ d\omega$:

$$g(k)\ dk = \frac{L^n}{(2\pi)^n} k^{n-1} dk \tag{2.1}$$

$$g(\omega) d\omega = \frac{L^n}{(2\pi)^n} \frac{\omega^{n-1}}{A^n} d\omega \tag{2.2}$$

The factor of ω^{n-1} will carry into the calculation of ω_D:

$$\int_0^{\omega_D} g(\omega) d\omega = nN \tag{2.3}$$

$$g(\omega) \propto \frac{1}{\omega_D^n} \tag{2.4}$$

The temperature dependence of C_V comes from the