DEPARTMENT OF PHYSICS

PhD Qualifying Exam — **Modern Physics**

Friday, September 22, 2000 — 1 pm - 4 pm

PRINT YOUR NAME________________________

EXAM CODE__________________

1. PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

2. Do each problem or question on a separate sheet of paper.
   (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

Short questions		*Long Problems*	
1.	_____	A1.	_____
2.	_____	A2.	_____
3.	_____	A3.	_____
4.	_____	B1.	_____
5.	_____	B2.	_____
6.	_____		
7.	_____		

MODERN PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

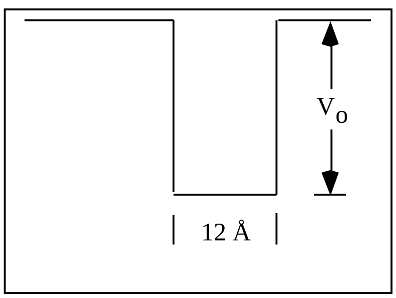
1. Consider the Hamiltonian for one particle in two dimensions

$$H = \frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{1}{2} m\omega^2 \left(X^2 + Y^2\right).$$

Is this system invariant under translations? Why or why not? Is it invariant under rotations around the z-axis? Why or why not? What are the corresponding conserved quantities, if any, in these two cases?

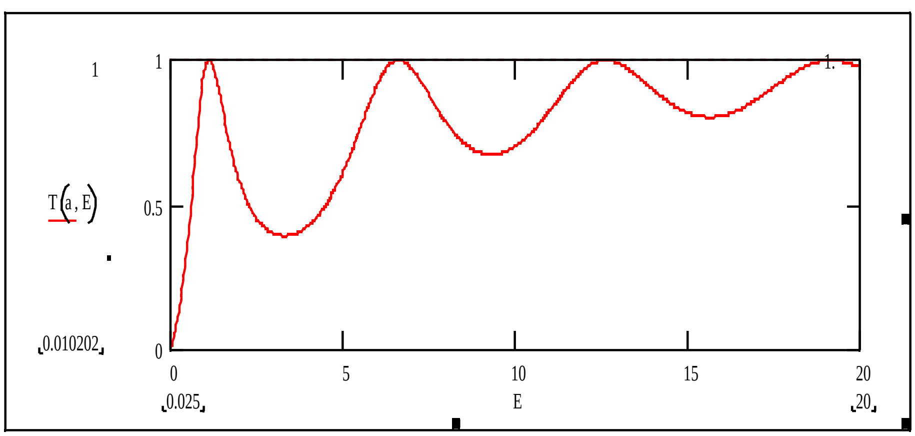
2. In your own words, explain why one needs to symmetrize the state vectors of identical particles in quantum mechanics. What is the form of the symmetric and antisymmetric states for a system of two identical particles (please, define your symbols)?

3. Why is the Pauli exclusion principle automatically satisfied if the state vector of a system is properly symmetrized?



4. An electron is scattered by a one-dimensional square well potential. The width of the square well is 12 Å and the depth of the square well, V_o, is to be determined. A measurement of the transmission coefficient as a function of electron kinetic energy (in eV) is shown. Estimate V_o. Show your work, state any assumptions, and explain your strategy in solving the problem.

Helpful information: $\sqrt{\frac{2m_e}{\hbar^2}} = 0.512\text{Å}^{-1}$



5. In the double helix DNA molecule, each base from one strand bonds to a base in the other strand. The correct matches A-T and G-C are more tightly bound than the incorrect matches (mismatches). At 300 K, the probability for a mismatch to occur is about 10^{-9} per base pair. The DNA molecule can be assumed to be a thermodynamic system in equilibrium with a thermal reservoir. If the probability for a mismatch is determined entirely by the binding energy difference between the correct and incorrect base pairs, what is this energy difference? When the temperature is raised by 20 C, how will the mismatch probability affected?

6. A gas obeys the equation of state

$$P = \frac{NkT}{V} + \frac{B(T)}{V^2}$$

where B is a function of temperature T *only*. Find the work done if the gas, initially at T and volume V is expanded isothermally and reversibly to volume 2V.

7. By considering the number of microstates of a (small) system A in thermal contact with a larger reservoir A', derive the ratio of probabilities $P(U_1)/P(U_2)$ of finding A with energy U_1 or U_2, in terms of temperature T. State your assumptions.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. You call this exciting?

Consider a one-dimensional simple harmonic oscillator of mass m and angular frequency ω in its ground state at $t = -\infty$. It is acted upon by a spatially uniform by time dependent **force** *(not potential)* given by

$$F(t) = \frac{F_o \tau}{\omega(\tau^2 + t^2)} \qquad \text{for} \quad -\infty < t < \infty .$$

(A) Using time-dependent perturbation theory to first order calculate the probability that the oscillator is found in the first excited state at $t = \infty$.
(B) Again, with time-dependent perturbation theory to first order calculate the probability that the oscillator is found in any other excited state at $t = \infty$.
(C) Show that for $\tau >> 1/\omega$, the probability for excitation is essentially negligible.

Hint: if x is the position operator, $\langle n' | \; x | n \rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2} \left(\sqrt{n}\;\; \delta_{n',n-1} + \sqrt{n+1}\;\; \delta_{n',n+1}\right)$

A2. Well, since you put it that way…

Consider a particle in the state described by the wave function

$$\psi = N(x + 2z)\exp\left[-\alpha\left(x^2 + y^2 + z^2\right)\right],$$

where N is a known normalization factor.

(A) With the help of the explicit expressions given below, write the wave function as a linear superposition of spherical harmonics.

(B) For the given wave function, what are the possible measurable pairs of values (ℓ, m) and what are the respective probabilities?

Hint: $Y_1^{\pm 1}(\theta,\varphi) = \mp\left(\frac{3}{8\pi}\right)^{1/2} \sin\theta\left(\cos\varphi \pm i\sin\varphi\right) = \mp\left(\frac{3}{4\pi}\right)^{1/2} \frac{x \pm iy}{\sqrt{2}r}$

$$Y_1^0(\theta,\varphi) = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta = \left(\frac{3}{4\pi}\right)^{1/2} \frac{z}{r}$$

A3. A new angle

Compute the angle between the bonds described by the wavefunctions ψ_1 and ψ_2 :

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ x\phi_2 \\ y\phi_2 \\ z\phi_2 \end{bmatrix}$$

Here $\phi_1(r)$ and $\phi_2(r)$ are real, spherically symmetric functions normalized by

$$\int \phi_1^2 d\,V = 1 \qquad \int \left(x\phi_2\right)^2 d\,V = 1$$

B1. Hot or Cold?

Consider a system of N (>>1) non interacting particles. U, the energy of the total system is fixed. Each particle has energy either 0 or ε (>0). n_o and n_1 are the occupation numbers of the two states.

(A) What is the entropy of the system?
(B) Derive a relationship for the temperature as a function of U.
(C) Under what range of values of n_o is the temperature T < 0?
(D) Suppose such a system (call it I) with T<0 is brought into contact with another system (call it II) with T >0. Explain the direction in which heat flows.

B2. Do you hear what I hear?

At T=0, the velocity of sound u in a spin -1/2 Fermi gas (i.e. an electron gas) is given by

$$u^2 = (\partial P / \partial \rho)_{T=0}$$

where $\rho = mn$, in which m is the mass of the gas particles and n is the number density.

(A) If μ is the chemical potential, show that

$$(\partial P / \partial \rho)_T = (n/m)\ (\partial \mu / \partial n)$$

(B) Find an expression for the total number of electrons in terms of fundamental constants and the momentum of the Fermi level, p_F.
(C) Find an expression for the total energy E of the gas.
(D) Using the equation of state for a Fermi gas, PV = (2/3) E, calculate the sound velocity in the limit of zero temperature using the results of parts A-C, and expressing your results in terms of n and m.