DREXEL
UNIVERSITY

DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam **Classical Physics**

Friday, September 18, 1998 9 am - 12 noon

PRINT YOUR NAME________________________

EXAM CODE__________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. Put a circle around the numbers below to indicate which questions you have answered.

Short questions	*Long Problems*
1. _____	A1. _____
2. _____	A2. _____
3. _____	A3. _____
4. _____	B1. _____
5. _____	B2. _____
6. _____	
7. _____	

CLASSICAL PHYSICS

PART I: Short answers (25%)

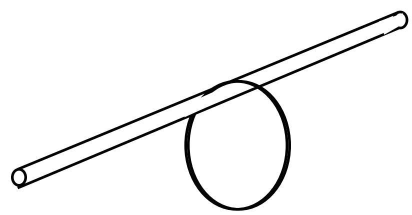
ANSWER 5 OF 7 QUESTIONS

1. A baseball is thrown vertically with a velocity v_o at time $t = 0$. It reaches the high point of its trajectory at time t_1 and returns to its original position at time $t_1 + t_2$. Explain why $t_2 > t_1$. If a heavy object (like a shotput) is thrown vertically under the same conditions, explain why $t_2 \approx t_1$.

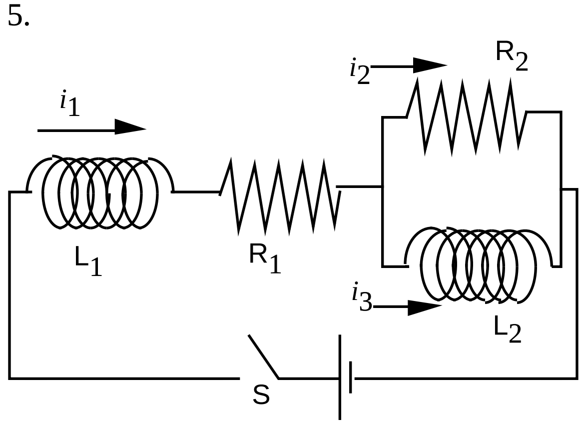
2. A cylinder of mass m and radius R has a string wrapped around its circumference many times. If the end of the string is held fixed while the cylinder is allowed to fall, show that its downward acceleration is $2g/3$.

3. A classical particle of mass m moves along the x axis under the influence of a force given by $F = -kx^3$. If the particle passes through the origin with speed v_o along x>0 what is the maximum displacement of the particle?

4. A thin ring of mass m and radius R is hung on a horizontal rod of negligible radius. What is the ratio of the frequency of oscillations in a plane perpendicular to the rod to the frequency of oscillations along the direction of the rod.



5.



Write three simultaneous equations and their initial conditions that describe the time dependent currents in the circuit to the left after switch S is closed (t=0).
(You do not need to solve the equations.)

Make a rough drawing of the currents i_1 and i_3 as a function of time. Be sure to indicate the values at t=0 and as $t \to \infty$.

6. How would Maxwell's equations be modified if magnetic monopoles were discovered?

7. A thin aluminum ring is placed horizontally flat in the x-y plane. At t=0, a magnetic field pointing in the $-z$ direction is turned on that has the form $B_z = -B_o e^{-\alpha z}\left(1 - e^{-\beta t}\right)$. In words, describe what happens to the ring and why.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1 *Left with no leg to stand on.*

A circular tabletop of radius 1 m and mass 3 kg is supported by three equally spaced legs on its circumference.   A valuable vase is placed on the table.   The legs now support 1, 2 and 3 kg, respectively.

(a) How heavy is the vase?
(b) Where is it located on the table?
(c) What is the lightest vase which might upset the table, and where must it be placed?

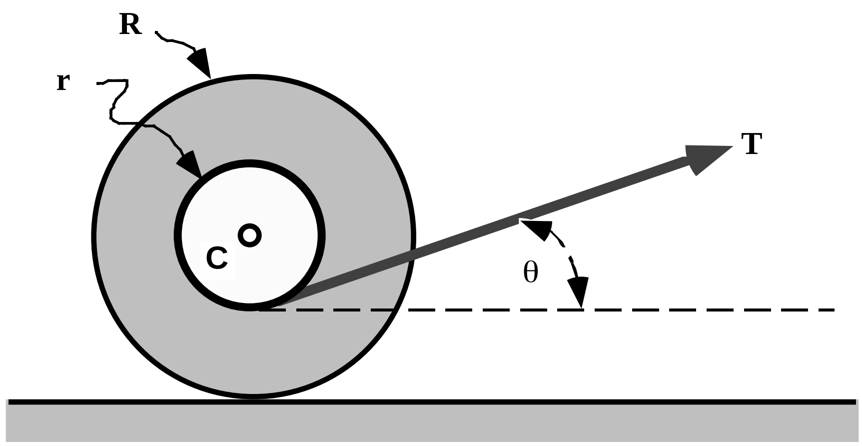
A2. *Splitting Atoms*

The two atoms in a diatomic molecule are separated by distance r , have mass m_1 and m_2 and interact through potential energy

$$V(r) = \frac{a^2}{4r^4} - \frac{b^2}{3r^3}$$

(a) What is the equilibrium separation of the atoms?
(b) What is the frequency of small oscillations about equilibrium if the molecule is not rotating?
(c) How much energy must be supplied to the molecule in equilibrium in order to break it apart?
(d) Determine the maximum angular momentum which the molecule can possess without breaking up, assuming that the motion is in circular orbits.
(e) Find the separation of the atoms at the break-up angular momentum.

A3 *Yo!*



A yo-yo (above) consists of two solid disks each of mass M and radius $\boldsymbol{R}$ connected by a central spindle of negligible mass and radius r.   A light string is wound on the spindle; its thickness is sufficiently small such that winding or unwinding has no effect on the effective radius of the spindle. The yo-yo is upright on a level horizontal surface and is pulled by a tension T at θ to the horizontal. Assume that the surface is rough enough for the yo-yo to roll without slipping. Derive an equation to show what happens to the direction of the motion when angle θ is varied?

B1 *Ions take a dip*

Consider an ion solvated in water (such as a sodium or chloride ion that you get when you add salt to your soup).

(a) In terms of molecular interactions, explain why the ion finds it energetically favorable to be deep in the water rather than near the surface (above which is air).

(b) Let the dielectric constants of air and water be ε_1 and ε_2, respectively. As far as the electric field in the air is concerned, it is as though the ion is in a dielectric medium with dielectric constant $(\varepsilon_1 + \varepsilon_2)/2$ (rather than in the water with dielectric constant ε_1). Based this fact, what is the magnitude of the image charge of the ion for the purpose of calculating the electric field in the water? (Hint: the image charge is located at equal distance to the air-water surface as the ion.)

(c) Suppose the ion is at a distance of d from the air-water surface. What is the force on the ion?

B2. *Shaping up spectra*

In the classical approach to spectral transitions, an electron of mass m and charge e is considered in a harmonic potential with a frictional damping force.

Thus the equation of motion for the electron has the form

$$m\ddot{x} + \gamma\dot{x} + m\omega_o^2 x = 0$$

(a) If $x = x_o$ and $x = 0$ at $t = 0$, what is the motion of the electron?
(b) Classically this would involve the emission of radiation. What is the shape of the frequency spectrum of the intensity of the radiation $I(\omega)$? (You may assume $\gamma/m \ll \omega_o^2$. You do not need to determine the scale of I.)
(c) Now suppose $\gamma \to 0$, and the only damping is due to the emission of radiation, which was neglected above. Assume the energy U decays exponentially, so that

$$U = U_o \exp(-\Gamma\, t)$$

and determine Γ. You may assume that the decay takes very many cycles.

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DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam **Modern Physics**

Friday, September 18, 1998 1 pm - 4 pm

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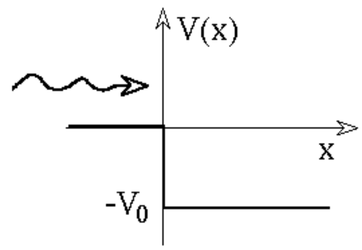
MODERN PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. A particle of mass m and kinetic energy $E > 0$ approaches (from the left) the potential drop shown schematically in the figure to the right.

   Calculate the probability that the particle will be reflected back if $E = V_0/3$.



2. If two or more distinct solutions of the time-independent Schroedinger equation have the same energy E, these states are said to be degenerate. (For example, the free particle states are doubly degenerate, one solution representing motion to the right, and the other motion to the left.) However, in one dimension, there are no degenerate <u>bound</u> states. Provide a convincing argument or proof of this statement.

3. The ground state wavefunction for a certain potential $V(x)$ is $\psi(x) = \dfrac{A}{\cosh \lambda x}$. A and λ are constants. Determine $V(x)$, assuming $V(x) \to 0$ as $x \to \infty$.

4. An infinite one dimensional square well potential is divided into two wells (call them A and B) by a delta function barrier which is not necessarily at the center. What is required to permit a particle, placed in A, to tunnel to B?

5. A Bose-Einstein condensation is said to be controlled by the product $\lambda^3\rho$ where λ is the thermal deBroglie wavelength and ρ is the particle density. What is the physical reason that this product is so important?

6. The grand canonical partition function for Fermi particles and for Bose particles are given by (respectively)

   $$\mathcal{Q}(z,V,T) = \prod_p (1 + ze^{-\beta\varepsilon_p}) \quad \text{and} \quad \mathcal{Q}(z,V,T) = \prod_p \frac{1}{1 - ze^{-\beta\varepsilon_p}}$$

   where $z = \exp(\beta\mu)$ and μ is the chemical potential, $\beta = 1/kT$, with T the absolute temperature, and k being Boltzmann's constant. ε_p are the energies of the various states. If I make a small parameter expansion of the Bose statistics partition function, it appears I arrive at the Fermi result! What is the physical meaning of this?

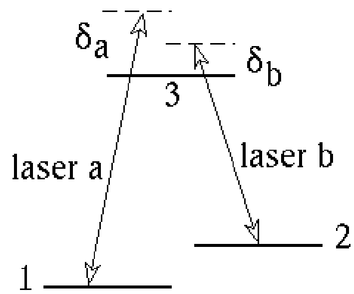
7. What is meant by an *ideal* gas?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Between the Lines*

The hyperfine-split ground state levels of an alkali atom such as sodium are coupled to one of the excited states by two tunable lasers, as shown schematically in the diagram below (not to scale, to be sure).



In the case of sodium, for example, 1 and 2 are the hyperfine levels of the ground state $3S_{1/2}$, and are separated by a bit more than 1.5 GHz; level 3 could be one of the two hyperfine states of level $3P_{1/2}$ whose spontaneous emission back to the ground state generates orange light with a wavelength of about 600 nm.

Take the model Hamiltonian of this system to have the standard form

$$H = H_0 - pE(t), \qquad H_0|\varphi_n\rangle = E_n|\varphi_n\rangle, \qquad \text{(n=1,2,3)}$$

where H_0 is the unperturbed atomic Hamiltonian, and E_n is the unperturbed energy of the stationary state $|\varphi_n\rangle$. The operator p is the atomic dipole moment operator whose only non-zero matrix elements are

$$\langle\varphi_3|p|\varphi_1\rangle = \langle\varphi_1|p|\varphi_3\rangle = \mu_{31}$$

$$\langle\varphi_3|p|\varphi_2\rangle = \langle\varphi_2|p|\varphi_3\rangle = \mu_{32}$$

and $E(t)$ is the electric field amplitude of an arbitrary external electromagnetic field.

If we ignore every state of the atom except those which are coupled by the nearly resonant laser fields, we can represent the atomic state vector as follows

$$|\psi(t)\rangle = \sum_{n=1}^{3} A_n(t)\exp\left(-i\frac{E_n}{\hbar}t\right)|\varphi_n\rangle.$$

(a) Starting from the Schroedinger equation

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

construct the equations of motion for the three amplitudes $A_n(t)$ (n=1,2,3), if the atom is driven by an arbitrary electromagnetic wave whose electric field amplitude is given by $E(t)$.

(b) Assume the field to be created by two lasers with carrier frequencies ω_a and ω_b, i.e. let

$$E(t) = E_a\left(e^{-i\omega_a t} + e^{i\omega_a t}\right) + E_b\left(e^{-i\omega_b t} + e^{i\omega_b t}\right)$$

where E_a and E_b are real, for simplicity. Substitute the above expression for $E(t)$ in the amplitude equations derived in part (a), neglect all the terms which oscillate at optical frequencies, and derive the slowly varying equations for the atomic amplitudes $A_n(t)$ (n=1,2,3). As you do so, define the small detuning parameters

$$\delta_a \equiv \omega_a - \omega_{31}, \qquad \delta_b \equiv \omega_b - \omega_{32}$$

(here, "small detuning" means small as compared to the optical transition frequencies).

(c) If we select $\delta_a = \delta_b \equiv \delta$, where δ is generally not equal to zero, construct the equations of motion for the three new amplitudes

$$Q(t) \equiv A_3(t)$$

$$R(t) \equiv A_1(t)\cos\theta - A_2(t)\sin\theta$$

$$S(t) \equiv A_1(t)\sin\theta + A_2(t)\cos\theta$$

where

$$\sin\theta = \frac{\mu_{31}E_a}{\Omega}, \quad \cos\theta = \frac{\mu_{32}E_b}{\Omega} \quad \text{and } \Omega^2 = (\mu_{31}E_a)^2 + (\mu_{32}E_b)^2$$

Also show that these equations can be put into the form

$$\frac{dR(t)}{dt} = 0$$

$$\frac{dS(t)}{dt} = \frac{i}{\hbar}\,\Omega\, Q(t) e^{i\delta t}$$

$$\frac{dQ(t)}{dt} = \frac{i}{\hbar}\,\Omega S(t) e^{-i\delta t}$$

(d) So far, for simplicity, we have ignored spontaneous emission. With the inclusion of this important effect, the theory shows that $S(t) \to 0$ and $Q(t) \to 0$ when the time becomes sufficiently long as compared to the atomic spontaneous decay time. What are the physical consequences of this result? In particular, how does the fluorescence produced by the decaying atom behave for sufficiently long times?

Epilogue: This counter-intuitive effect, called coherent population trapping, has been demonstrated experimentally in numerous beautiful experiments.

A2. *As expected*

The wave function

$$\psi(x,t) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left\{ -\frac{m\omega}{2\hbar}\left[x^2 + \frac{a^2}{2}\left(1 + e^{-2i\omega t}\right) + \frac{i\hbar t}{m} - 2axe^{-i\omega t} \right] \right\},$$

satisfies the time-dependent Schroedinger equation for the one-dimensional harmonic oscillator where a is a real constant,

(a) Calculate $|\psi(x,t)|^2$ and describe the motion of the wave packet.

(b) Calculate the expectation values of the position and the momentum operators, $\langle x(t)\rangle$ and $\langle p(t)\rangle$, respectively, and verify the validity of the Ehrenfest equation

$$\frac{d}{dt}\langle p(t)\rangle = -\left\langle \frac{\partial V(x)}{\partial x} \right\rangle$$

A3. *Neutrons scattered about*

Consider the elastic scattering of low energy spin one-half particles (e.g. neutrons) from spin one particles (e.g. deuterons). Assume that the target deuterons are completely polarized with their spins in the positive z-direction. Assume that the force responsible for the scattering depends on the spins of the particles, but in such a way that both the total spin and its z- component are good quantum numbers. Assume finally that this force acts only in the total spin equal to three-half state, and is independent of the value of the z-component of the total spin.

(a) If the incident neutrons are completely polarized with their spins in the negative z-direction (i.e. spin down), show that the *probability* of observing scattered neutrons with spin up (i.e. a spin flip) is twice as great as observing scattered neutrons with their spins remaining down (non spin-flip).

(b) Show that, for incident neutrons whose spin is up, the *scattering cross section* is three times as great as the cross section for incident neutrons whose spin is down.

(c) If the spatial part of the potential responsible for the scattering is a potential of the Yukawa type, i.e.,

$$V(r) = \frac{V_o}{r} exp(-ar)$$

then what is the dependence of these elastic scattering cross sections on the scattering angle θ between the final and the incident neutron directions in the center of mass system. (Hint: use the Born approximation.)

B1. *It takes two...*

The rotational quantum energy levels of diatomic molecules are given by

$$E_j = \frac{\hbar^2}{2I} J(J+1) \qquad \text{j = 0,1,2,...}$$

where I is the moment of inertia of the diatomic molecule, and is assumed constant. J is the rotational quantum number.

(a) Write down an expression for the partition function of the rotational motion.
(b) Find the molar rotational heat capacity, C_{rot},in the high temperature limit.
(c) Find the molar rotational heat capacity, C_{rot},in the low temperature limit.
(d) The moment of inertia of carbon monoxide (CO) has the value of I = 1.3 x 10^{-46} kg m^2. What is the molar rotational heat capacity of CO at room temperature.

B2. *String in the bath*

A thin wire of mass per unit length μ is stretched with tension F between two points a distance L apart.

(a) What are the normal modes of the wire?
(b) What is the average kinetic and potential energy in each mode?

The wire is immersed in a thermal bath of temperature T.

(c) What is the rms fluctuation at the center of the wire and how does it depend on L, F and T?

You may need the fact that $\sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} = \frac{\pi^2}{8}$