

# Coupling microwave and photonic systems through nanomechanical resonators

Quantum Mechanics III final  
presentation

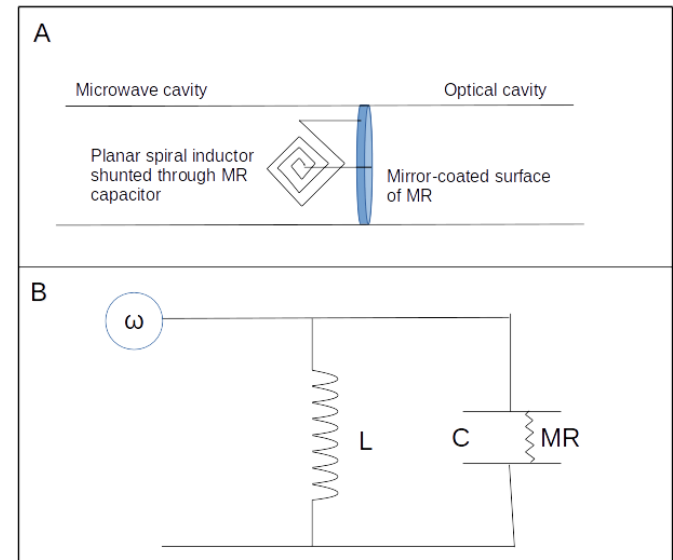
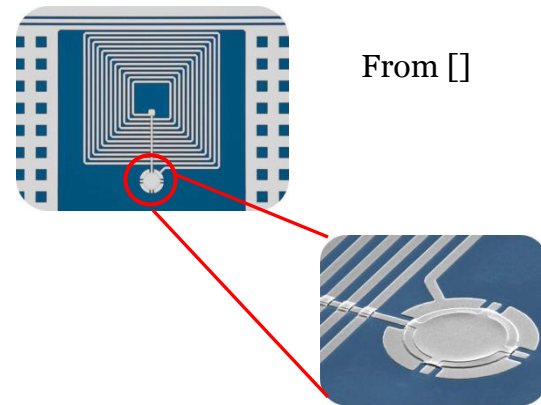
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# Motivation

- Quantum entanglement normally studied at optical (THz) frequencies
- Entanglement at Microwave (GHz) frequencies opens new areas of study and applications
  - Quantum computing
  - Quantum illumination
- Recent advances in the theory and construction of nanomechanical resonators allow coherent coupling of optical and microwave systems

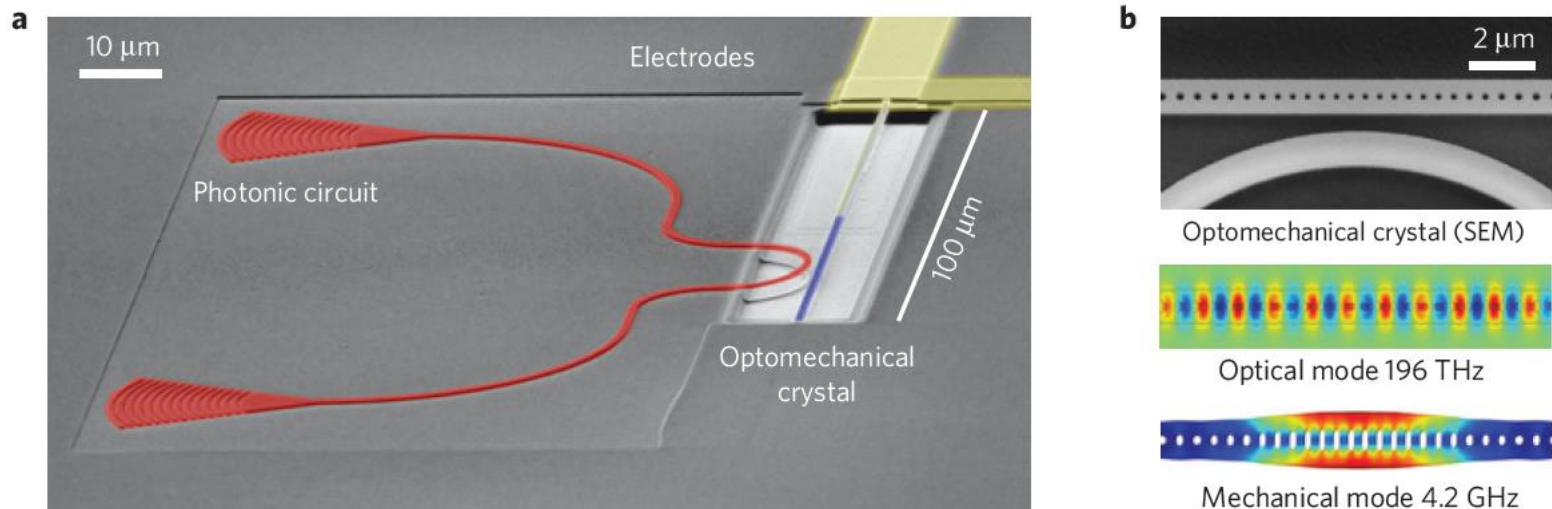
# Example 1: Drum-head capacitor

- Drum-head capacitor coupled to microwave stripline by a planar inductor
- Proposal: mirror coat other surface, put at one end of an optical cavity
- Would result in coupled microwave/optical signals



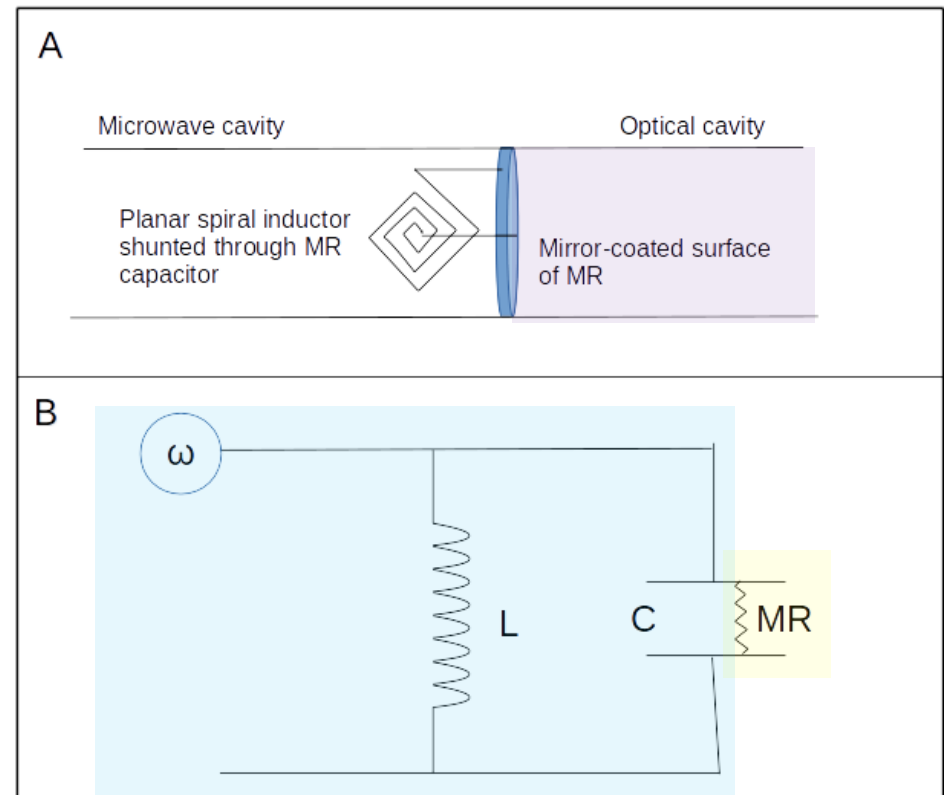
## Example 2: Opto-mechanical crystal<sup>[2]</sup>

- Optical crystal made from piezoelectric material
- Microwave and optical resonances
- Experimental measurement of phase/amplitude fidelity and optical transparency controlled by microwave signal



# System dynamics <sup>[3]</sup>

$$\begin{aligned}
 H = & \frac{p_x^2}{2m} + \frac{m\omega_m^2 x^2}{2} \\
 & + \frac{\Phi^2}{2L} + \frac{Q^2}{2(C + C_0(x))} - e(t)Q \\
 & + \hbar\omega_c a^\dagger a - \hbar G_{0c} a^\dagger a x \\
 & + i\hbar E_c \left( a^\dagger e^{-i\omega_0 c t} - a e^{i\omega_0 c t} \right)
 \end{aligned}$$



# Quantum Langevin equations <sup>[3]</sup>

- Thermal noise is important in microwave regime
- QLEs: differential equations for system evolution (“slow” system plus “fast” noise terms)

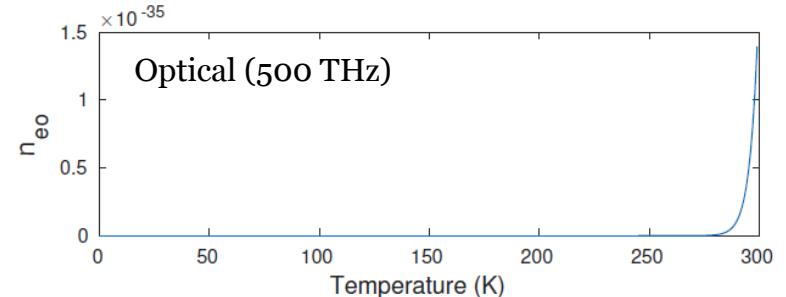
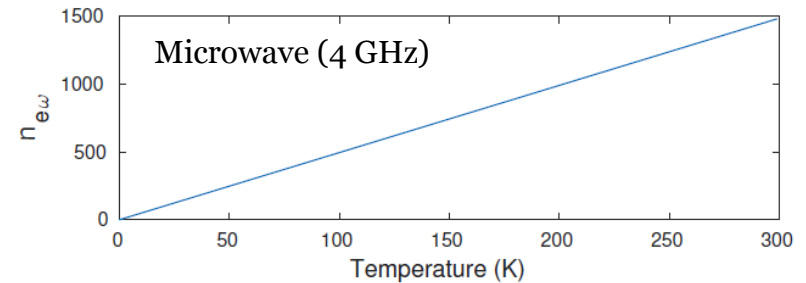
$$\dot{q} = \omega_m p \quad (3.5)$$

$$\dot{p} = -\omega_m q - \gamma_m p + G_{0c} a^\dagger a + G_{0w} b^\dagger b + \xi \quad (3.6)$$

$$\dot{a} = -(\kappa_c + i\Delta_{0c})a + iG_{0c}qa + E_c \quad (3.7)$$

$$\begin{aligned} \dot{b} = & -(\kappa_w + i\Delta_{0w})b + iG_{0w}qb + E_w \\ & + \sqrt{2\kappa_w}b_{in} \end{aligned} \quad (3.8)$$

$$\langle n \rangle_e = \frac{1}{z^{-1}e^{\beta\hbar\omega} - 1}$$



# Entanglement metrics

- Systems are separable (not entangled) if we can write state as a tensor product

$$\rho^{ab} = \sum_i c_i \rho_i^a \otimes \rho_i^b$$

- If the entangled systems are in a pure state, can use reduced density matrix

$$\rho^a \equiv \text{Tr}_b(\rho^{ab})$$

$$S = -\text{Tr}(\rho^a \ln \rho^a)$$

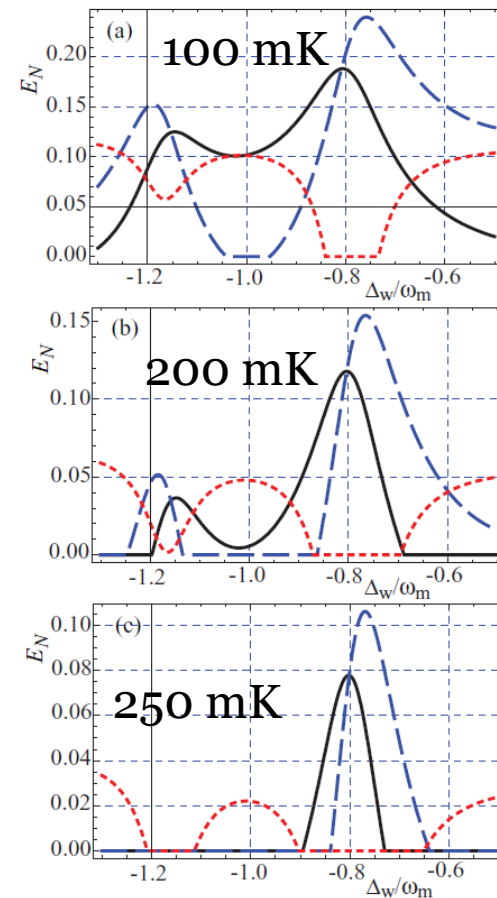
- For a general mixed state it's more complicated
- One computable metric is the negativity [6]

$$N(\rho) \equiv \frac{\|\rho^{T_a}\|_1}{2}$$

# Entanglement results [3]

- Correlation analysis from QLEs (fluctuations about steady state)
- Log-negativity shows substantial entanglement between optical and microwave modes
- Enhancing OC-MC entanglement
  - Minimize resonator mass
  - Low temperature

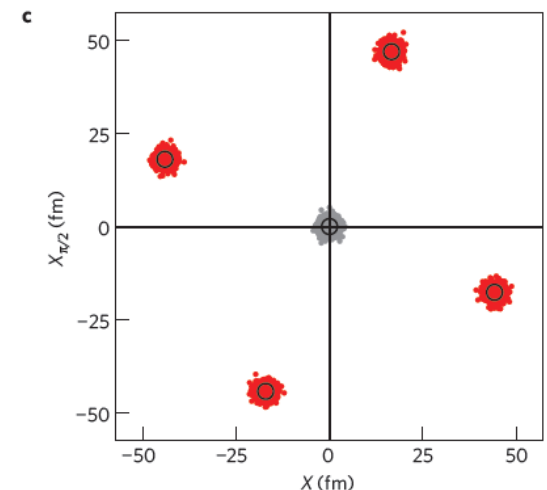
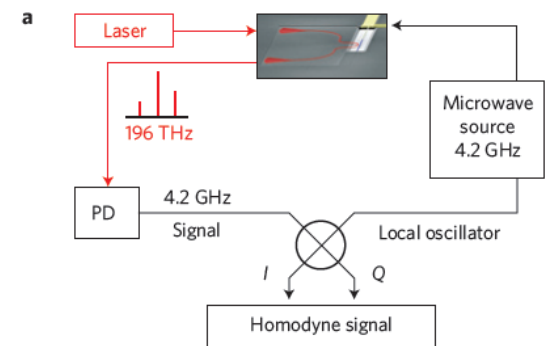
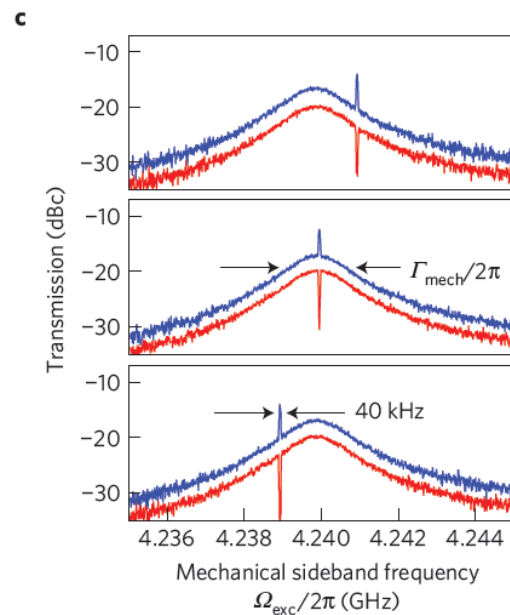
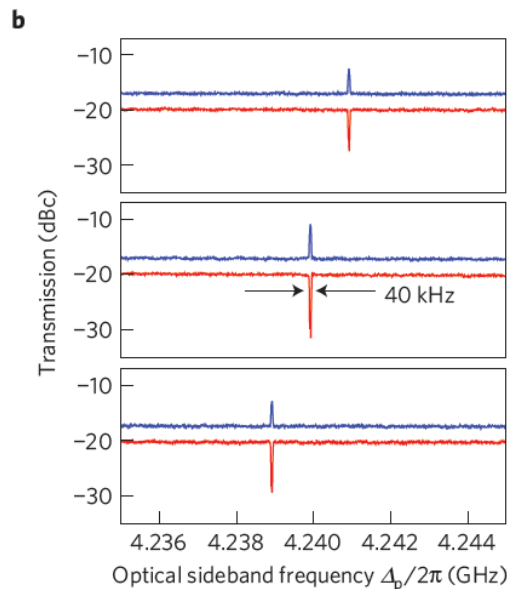
— OC-MC  
- - - MC-MR  
- - - OC-MR





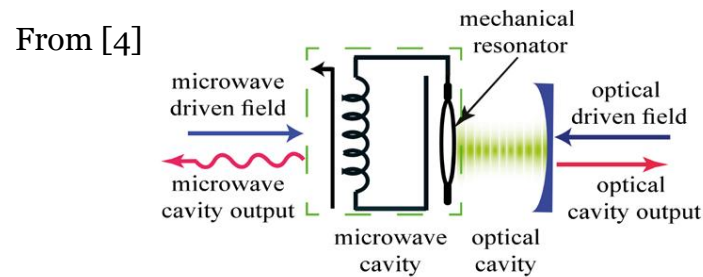
# Photonic crystal result <sup>[2]</sup>

- Experimentally showed phase/amplitude transfer from microwave to optical carriers
- Also showed microwave-controlled optical sideband transparency

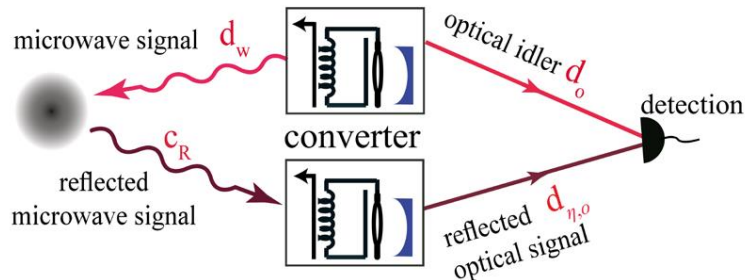


# Quantum illumination

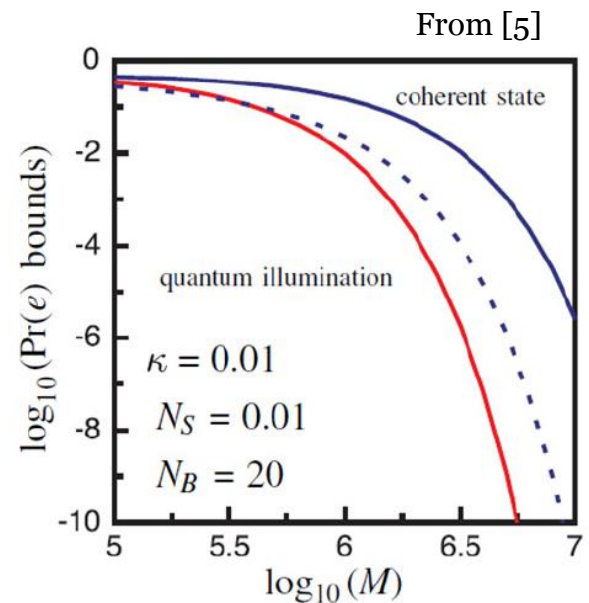
- Uses entangled photons for enhanced sensing
- Create entangled pair, retain one “idler”, correlate with received signal



(a)



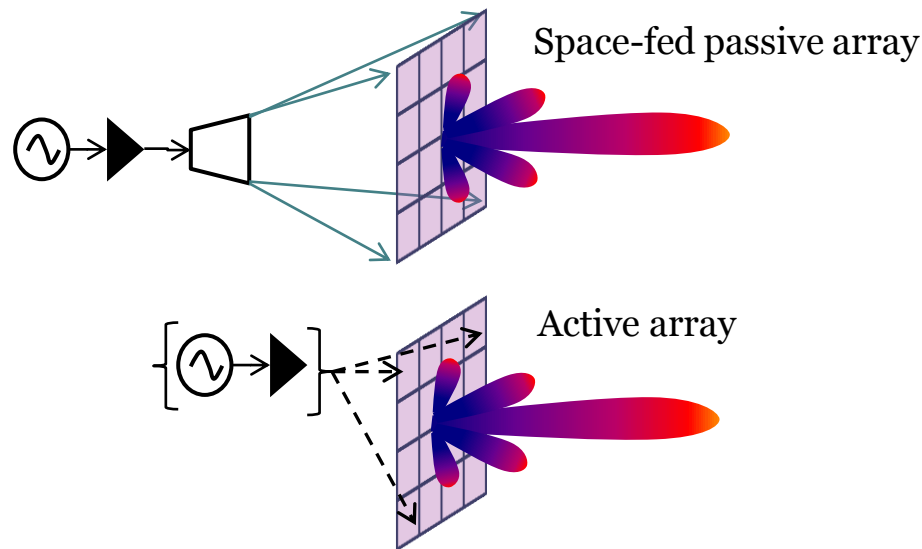
(b)



- Quantum upper bound
- - - Classical lower bound
- Classical upper bound

# Application to radar systems

- Current analyses focus on thermal noise, but radar systems usually limited by unwanted reflections (“clutter”)
  - Space sensing might be an exception
- Not clear how to use single idler to search multiple range gates
- Not clear how to apply to electronically steered arrays



# References

- [1] J. Bochmann A. Vainsencher D. Awschalom A. Cleland. "Nanomechanical coupling between microwave and optical photons". In: Nature Physics 9 (2013), pp. 712{716. doi: 10.1038/NPHYS2748.
- [2] Integrated Photonics Institute for Manufacturing Innovation. url: <http://manufacturing.gov/ipimi.html>.
- [3] Sh. Barzanjeh D. Vitali P. Tombesi G.J. Milburn. "Entangling optical and microwave cavity modes by means of a nanomechanical resonator". In:
- [4] Sh. Barzanjeh S. Guha C. Weedbrook D. Vitali J. Shapiro S. Pirandola. «Microwave quantum illumination". In: Physical Review Letters 114 (2015). doi:
- [5] S. Tan B. Erkmen V. Giovannetti S. Guha S. Lloyd L. Maccone S. Pirandola J. Shapiro. "Quantum Illumination with Gaussian State". In: Physical Review Letters 101.253601 (2008). doi: <http://dx.doi.org/10.1103/PhysRevLett.101.253601>.
- [6] G. Vidal R.F. Wener. "Computable measure of entanglement". In: Physical Review A 65 (2002). doi: <http://dx.doi.org/10.1103/PhysRevA.65.032314>.