DREXEL
UNIVERSITY

DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam **Classical Physics**

Friday, September 23, 1994 9 am - 12 noon

PRINT YOUR NAME______________________

EXAM CODE________________

PUT YOUR EXAM CODE, **NOT YOUR NAME,** ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. By circling the numbers below, indicate which questions you have answered.

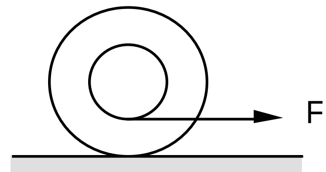
Short questions	*Long Problems*
1. _____	A1. _____
2. _____	A2. _____
3. _____	A3. _____
4. _____	B1. _____
5. _____	B2. _____
6. _____	
7. _____	

CLASSICAL PHYSICS

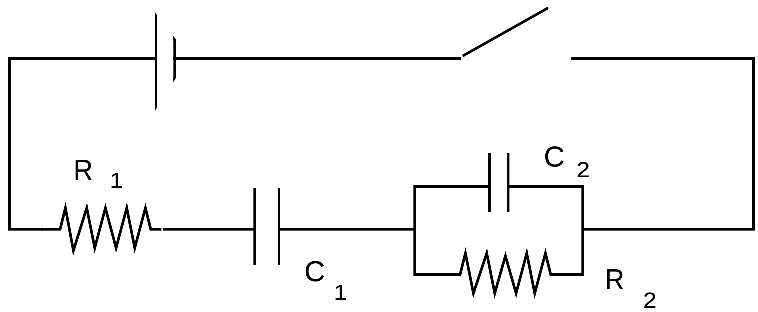
PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. A yo-yo is shown at right. If the string is pulled with a force F as shown, which way does the yo-yo roll, and why?



2. Consider the circuit shown to the right.



The switch, which was open, is closed at t=0. Make a qualitative sketch of the currents through R_1 and R_2, and the voltages across C_1 and C_2 as a function of time. Briefly explain your reasoning.

3. Accelerating electric charges produce electromagnetic radiation. We define the electric dipole moment as

$$d_j = \sum_{A=1}^{N} e_A x_j^A$$

where j= 1,2,3 and the sum is over N point charges. For a nonrelativistic charge distribution, the leading order radiation is dipole radiation, with luminosity proportional to $\sum_{j=1}^{3} \ddot{d}_j \ddot{d}_j$ where a dot indicates a time derivative. General relativity predicts that accelerating masses will produce *gravitational* radiation. Accordingly the mass dipole moment is defined by

$$D_j = \sum_{A=1}^{N} m_A x_j^A$$

with the sum over point masses. Show that the luminosity of gravitational dipole radiation is zero, i.e.,

$$\sum_{j=1}^{3} \ddot{D}_j \ddot{D}_j = 0$$

(The leading term for gravitation radiation is quadrupolar.)

4. In a certain textile manufacturing process, small metal needles may accidently get caught in the textile as it moves along the conveyer line. Could you develop an electromagnetic method for detecting these needles? Describe the principles of your method. (Hint: consider metal detectors as used to detect buried coins at the beach.)

5. Suppose you know the Schwarzschild radius (the distance of no escape) for a black hole of 1 solar mass. Using dimensional arguments, how large should the radius be for a black hole of 1000 solar masses?

6. A long cylinder made of a dielectric material (ε_1) is immersed in a dielectric fluid (ε_2). A uniform magnetic field is applied to the system. What will be the orientation of the cylinder? What happens if we use a disk of the same material instead of the cylinder?

7. Two roughly spherical atoms associate into a dumbbell shaped molecule. Describe the degrees of freedom before and after the association.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Close encounters.*

(a) A test particle of mass m moves in a central conservative field described by a potential $\Phi(r)$, where $\Phi \to 0$ as $r \to \infty$. The particle's velocity at infinity is v_∞ and its trajectory is such that its closest approach to the origin ($r = 0$) in the absence of any interaction would be b. Show that the actual distance of closest approach r_{min} satisfies

$$\left(\frac{b}{r_{min}}\right)^2 + \frac{2\Phi(r_{min})}{m v_\infty^2} = 1.$$

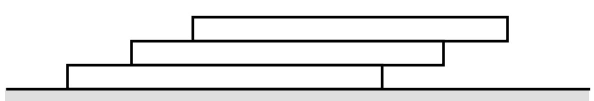
In the particular case of a particle moving in an attractive Coulomb potential, $\Phi(r) = -k\, m\, r^{-1}$, show that the maximum value of b for which approach within radius R of the origin is possible is

$$b_{max} = R\sqrt{1 + \frac{2k}{v_\infty^2 R}} .$$

(b) A star of mass 5×10^{30} kg and radius 10^6 km is moving at velocity 50 km/s through a field of stationary test particles (with negligible masses and radii) having spatial density 10^{15} particles per cubic meter. Estimate the rate at which particles strike the surface of the star. (Take the gravitational constant G to be 6.67×10^{-11} kg^{-1} m^3 s^{-2}).

A2. *Plate Tectonics?*

Wooden plates, all having the same rectangular shape, are stacked one above the other as shown in the figure below:



(a) If the length of each plate is $2L$, prove that the equilibrium conditions will prevail if the $(N+1)$th plate extends no more than a distance of L/N beyond the Nth plate, where $N = 1, 2, 3$ ...

(b) What is the maximum horizontal distance which can be reached if more and more plates are added?

(c) What would your answers be if the plates were stacked on a *sphere* of radius R instead of on a flat surface?

A3. *Jupiter's Greatest Hits*

Earlier this summer, the remnants of comet Schoemaker Levy collided with Jupiter in a spectacular extraplanetary fireworks display. During its previous close encounter with Jupiter, it was torn apart into the string of "cometesimals" which actually collided with Jupiter. Estimate how close the comet had to come to Jupiter in order to be torn apart.

Jupiter's mass is 1.989 x 10^{27} kg; it's radius is 7.14 x 10^{4} km.
You may take the density of the comet as 1 gm / cm^3.

B1. *Bright ideas.*

An ordinary 3 V flashlight bulb draws roughly 0.25 A, converting about 1% of the dissipated power into light. If the beam has a cross-sectional area of 10 cm^2,

(a) The beam is actually emitted as a stream of quanta which have energy E= hf, where h is Planck's constant and f is the frequecy. Estimate how many photons are emitted per second?

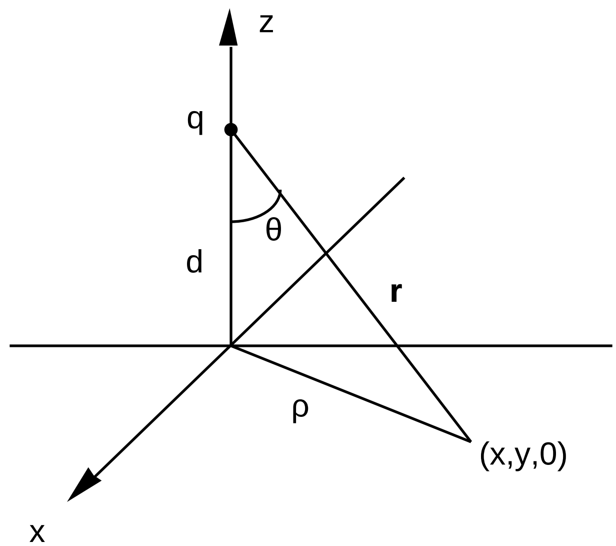
(b) What is the intensity of the beam?

(c) What are the electric and magnetic field amplitudes?

(d) What pressure (in atmospheres) would the beam exert on an absorbing surface?

B2. *Served on a silver platter?*

A positive point charge q is placed at the point (0,0,d).



(a) If the space $z \le 0$ is filled with *conducting* material, use the method of images to show that the charge density induced on the surface is given by

$$\sigma = -\frac{q\,d}{2\pi\,(\rho^2 + d^2)^{3/2}} \qquad (1)$$

by recalling that

$$\sigma = -\,\varepsilon\,\frac{\partial V}{\partial z}$$

where V is the potential due to q and its image.

(b) The electric field above the z = 0 plane *could* be evaluated by integrating over the conducting surface as

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\int \sigma\,\frac{\mathbf{r}}{r^3}\,d\tau$$

where $d\tau$ is an element of area located at (x,y,0). The force on the charge density easily follows. However, this integration can be avoided by using the potential obtained by the method of images to find the force on the charge q. Find the force on q.

(c) The material in the space $z \le 0$ is now replaced by a *dielectric* material of susceptibility χ, so that the charge density on the plane z = 0 is

$$\sigma_d = \varepsilon\,\chi\,E_z$$

where E_z is the z component of the total electric field just below the z=0 surface. This filed includes the contribution from the charge located at (0,0,d) and the plane (i.e., $-\sigma_d/2\varepsilon$). What is the charge density σ_d induced on the surface of the dielectric? Express this in the form of equation 1, with q replaced by q'.

(d) Continuing from part c, and noting that the electric field is still related to the surface charge as in equation (1), what is the force on the charge located at (0,0,d)?

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DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE

PhD Qualifying Exam **Modern Physics**

Friday, September 23, 1994 1 pm - 4 pm

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MODERN PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

1. According to the third law of thermodynamics, absolute zero is not attainable. Is there a similar theoretical reason that excludes states of negative temperature in [a] a classical system, and [b] a quantum system ?

2. Describe how the process of adiabatic demagnetization can be used to obtain very low (<1K) temperatures. Use diagrams involving appropriate thermodynamic parameters but avoid writing too many equations.

3. An optical diffraction grating has 20,000 rulings per centimeter. Suppose that we use such a grating for electron diffraction. What would be the angular separation between the principal interference maxima if the electron energy is 100 eV

4. The specific heat (constant volume) of a 3 dimensional electron gas is given by the formula

$$C_v = \left(\frac{\partial E}{\partial T}\right)_V = \frac{\pi^2}{3}\,\rho(E_F)\,k_B^2\,T$$

where E is the energy, $\rho(E_F)$ is the density of electronic states at the Fermi energy, k_B is the Boltzmann constant, and T is the temperature (in K). This result dominates the specific heat of metals at low temperature. At higher temperatures the phonon specific heat dominates, but we will not consider that here. Since the energy of an electron is proportional to T, it might have been expected that the electron gas specific heat would be *independent* of temperature. What other considerations must be made to arrive at the correct result, given by the above equation?

5. What is the Pauli exclusion principle?

6. In the presence of degeneracy eigenstates must be labelled by some extra indices in order to keep them distinct from each other. Thus, for example, consider the following table of eigenvalues and corresponding eigenvectors

Eigenvalues	Eigenvectors
$\omega_1 = 3$	$\|3,a$
$\omega_2 = 3$	$\|3,b$
$\omega_3 = 5$	$\|5$

The indices a, b in the first two eigenvectors are necessary, otherwise the symbol $|3$ by itself would be ambiguous; these extra indices are typically associated to other observables of the system.

Now, suppose that a quantum state is given by

$$|\psi\rangle = \frac{1}{2}|3,a\rangle + \frac{1}{\sqrt{2}}|3,b\rangle + \frac{1}{2}|5\rangle$$

What is the probability that a measurement of this system, initially in the state $|\psi\rangle$, will produce the eigenvalue 3?

7. What is meant by a Bose-Einstein Condensation?

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. *Born Free*

The wave function of a free particle is given by

$$\psi(x) = \begin{cases} N e^{i p_0 x/\hbar} \sin\left(\dfrac{\pi x}{a}\right) & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

where a is a real positive constant and N is a real normalization factor.

(a) Calculate N under the constraint that the wave function be properly normalized.

(b) What is the probability of finding the particle inside the infinitesimal interval dx around the position x=a/2?

(c) What the average velocity of the particle?

(d) For which value of the momentum is the momentum probability distribution maximum?

The following integral may be useful: $\int dx \sin^2 x = -\frac{1}{4}\sin 2x + \frac{1}{2}x$

A2. *Operator assistance required*

A measuring device is represented by the Hermitian operator

$$A = \begin{vmatrix} 1 & i \\ -i & 1 \end{vmatrix}$$

(a) Calculate the eigenvalues and normalized eigenvectors of the operator A.

(b) Verify that the two eigenvectors are orthogonal to each other

(c) A system is described by the state

$$|\psi\rangle = \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

We carry out a measurement of this state with the device A above. What is the probability of recording each of the two eigenvalues of the measuring system (of course, one eigenvalue at a time, because this is all the device can do)?

(d) If we carry out this measurement many times what is the expectation value of the result? Does this result make sense?

A3. *Helium, from the bottom up*

Consider the simplest multiple electron system, the helium atom. The Schroedinger equation of the helium atom is given by

$$-\frac{\hbar^2}{2m}\left(\nabla_1^2 + \nabla_2^2\right) - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} + \frac{e^2}{r_{12}}\psi = E\,\psi$$

where r_1 and r_2 are the distance to the nucleus of electrons 1 and 2, respectively, and r_{12} is the inter-electronic distance. $Z = 2$. Use the hydrogen-like 1s wavefunction,

$$\psi_{1s} = \sqrt{\frac{Z^3}{\pi a_o^3}}\; e^{-Zr/a_o}$$

(with a_o = Bohr radius) and the following steps to find an approximate value of the helium ground state energy, based on first-order perturbation theory.

Neglecting spin, we write the wave function ψ in the form of the product

$$\psi(1,2) = \psi_{1s}(1)\,\psi_{1s}(2)$$

and treat e^2/r_{12} as a perturbation.

(a) Calculate the zero-order energy of the helium atom.

(b) Calculate the first order electron-electron interaction term, using

$$\int \psi_{1s}^2(1)\,\psi_{1s}^2(2)\,\frac{e^2}{r_{12}}\,d\tau_1\,d\tau_2 = e^2\,\frac{32}{a_o^3}\int_0^\infty dr\,r^2\,e^{-4r/a_o}\left[\frac{1}{r} - e^{-4r/a_o}\left(\frac{2}{a_o} + \frac{1}{r}\right)\right]$$

(c) Find the ground state energy of helium based on your results and compare it with the experimental value of -79.0 eV.

(d) Calculate the first ionization potential, that is, the energy require to remove one electron to infinity from the ground state.

B1. *Gas in the magnet*

Consider a monatomic ideal gas maintained at the absolute temperature T.

(a) calculate the average kinetic energy of N atoms of such a gas.

(b) What is the specific heat, C_v?

Now suppose that each particle of this gas has spin 1/2 and magnetic moment μ, and the gas is placed in a magnetic field B, such that each particle has two possible magnetic energies, $\pm \mu B$, corresponding to the two possible orientations of its spin.

(c) What is the average energy of N such atoms at temperature T? Include contributions of both kinetic and magnetic energies.

(d) What is the new specific heat, C_V? Sketch C_V as a function of T.

B2. *A Bit Sticky*

Suppose a dilute classical gas interacts with a surface. A surface site has a binding energy for the gas molecules of $-\varepsilon$.

(a) Write an expression for the chemical potential of the gas in terms of its pressure.

(b) Write an expression for the chemical potential of the gas on the surface in terms of the fraction of the surface sites which have gas molecules attached. (You may assume that less than the entire surface is covered.)

(c) Write an expression for the pressure of the gas in terms of the fraction of the surface covered.