Quantum II HW4

Vincent Baker

May 8, 2015

1 Problem 1

a. Since the system is in a single state the density matrix has a single 1 on the diagonal, all other elements are 0.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.1}$$

b. To follow the time evolution of the system we transform to the total momentum basis. We transofrm from the orbit/spin basis $|0\ \frac{1}{2}\rangle$ to the total momentum basis $|\frac{3}{2}\ \frac{1}{2}\rangle$, $|\frac{1}{2}\ \frac{1}{2}\rangle$ using the appropriate Clebsch-Gordon coee-ficients. The pure states then evolve in time under the $\mathbf{L}\cdot\mathbf{S}$ Hamiltonian which is just the total angular momentum operator $\mathbf{J}$.

$$|J\ M\rangle = \sqrt{\frac{2}{3}}\,|\frac{3}{2}\ \frac{1}{2}\rangle - \sqrt{\frac{1}{3}}\,|\frac{1}{2}\ \frac{1}{2}\rangle \tag{1.2}$$

$$E = \hbar^2 J(J+1) \tag{1.3}$$

$$E_1 = \hbar^2\frac{15}{4},\ E_2 = \hbar^2\frac{3}{4} \tag{1.4}$$

$$|J\ M\rangle = \sqrt{\frac{2}{3}}e^{-i\hbar\frac{15}{4}t}\,|\frac{3}{2}\ \frac{1}{2}\rangle - \sqrt{\frac{1}{3}}e^{-i\hbar\frac{3}{4}t}\,|\frac{1}{2}\ \frac{1}{2}\rangle \tag{1.5}$$

We now transform back to the orbit/spin basis $|0\ \frac{1}{2}\rangle, |1\ -\frac{1}{2}\rangle$. We ignore the other states that do not have total z momentum $\frac{1}{2}$.

$$\Psi = (\frac{2}{3}e^{-i\hbar\frac{15}{4}t} + \frac{1}{3}e^{-i\hbar\frac{3}{4}t})\,|0\ \frac{1}{2}\rangle + (\frac{\sqrt{2}}{3}e^{-i\hbar\frac{15}{4}t} + \frac{\sqrt{2}}{3}e^{-i\hbar\frac{3}{4}t})\,|1\ -\frac{1}{2}\rangle \tag{1.6}$$

c. We can write the density matrix as $|\Psi\rangle\langle\Psi|$.

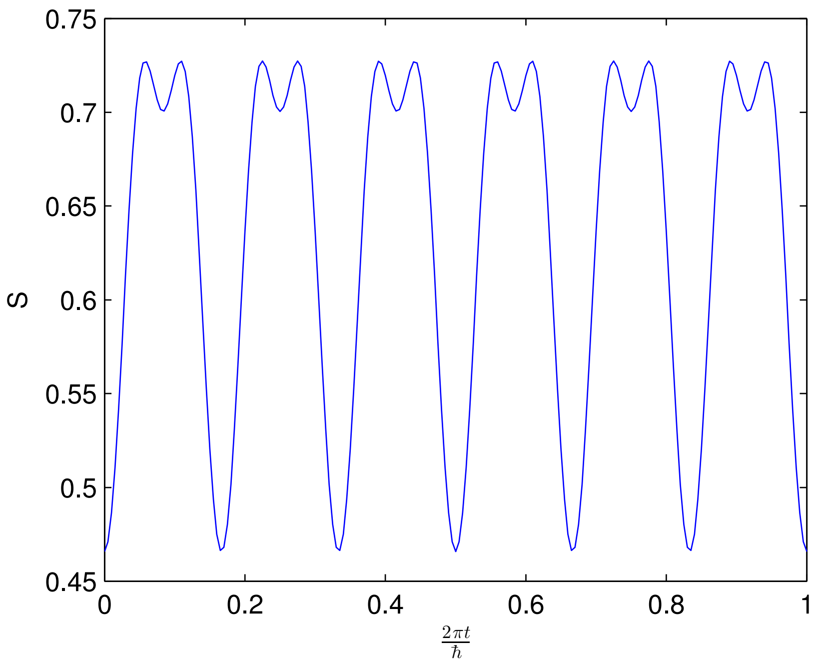
$$\begin{bmatrix} \frac{5}{9}+\frac{2}{9}(e^{i\hbar 3t}+e^{-i\hbar 3t}) & \frac{\sqrt{2}}{3}+\frac{2\sqrt{2}}{9}e^{i\hbar 3t}+\frac{\sqrt{2}}{9}e^{-i\hbar 3t} \\ \frac{\sqrt{2}}{3}+\frac{2\sqrt{2}}{9}e^{-i\hbar 3t}+\frac{\sqrt{2}}{9}e^{i\hbar 3t} & \frac{4}{9}+\frac{2}{9}(e^{i\hbar 3t}+e^{-i\hbar 3t}) \end{bmatrix} \tag{1.7}$$

d. We calculate the entropy from the state probabilities through the eigenvalues of ρ.
e. We calculate the reduced spin density matrix by taking the partial trace over m_ℓ. Since we have a 2x2 matrix and the two rows/column indices have different values of m_ℓ this is just the sum of the diagonal elements.

$$\rho_{spin} = \begin{bmatrix} \frac{5}{9}+\frac{2}{9}(e^{i\hbar 3t}+e^{-i\hbar 3t})+\frac{4}{9}+\frac{2}{9}(e^{i\hbar 3t}+e^{-i\hbar 3t}) & 0 \\ 0 & 0 \end{bmatrix} \tag{1.8}$$

f. The entropy is $\sum -\rho_i \ln \rho_i$. Using an exponential/trig identity we can write the entropy as:

$$S = -\frac{5}{9}\cos(6\hbar t)\ln(\frac{5}{9}\cos 6\hbar t) - \frac{4}{9}\cos(6\hbar t)\ln(\frac{4}{9}\cos 6\hbar t) \tag{1.9}$$



g. h. The reduced orbital density matrix contains the same sum of diagonal elements of the total density matrix. The results are the same as parts e and f.
i. The system has entropy S. The observed entropy should be the same regardless of which state in the mixture we choose to measure.