



DEPARTMENT OF PHYSICS

PhD Qualifying Exam

Friday, September 20, 2002

Classical Physics

9 am - 12 noon

PRINT YOUR NAME _____

EXAM CODE _____

PUT YOUR EXAM CODE, **NOT YOUR NAME**, ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

Answer 5 of 7 short answer questions in part I and 3 of 5 longer problems in part II, with at least one problem from Group A and one from Group B. ***Circle the numbers*** below to indicate which questions you have answered—write nothing on the lines (your grades go there).

Short questions

circle *grade*

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

Long Problems

circle *grade*

A1. _____

A2. _____

A3. _____

B1. _____

B2. _____

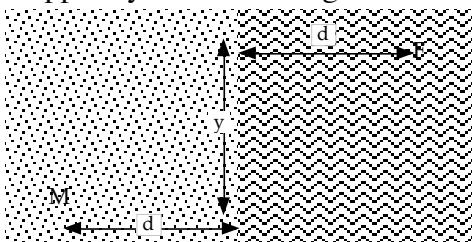
CLASSICAL PHYSICS

PART I: Short answers (25%)

ANSWER 5 OF 7 QUESTIONS

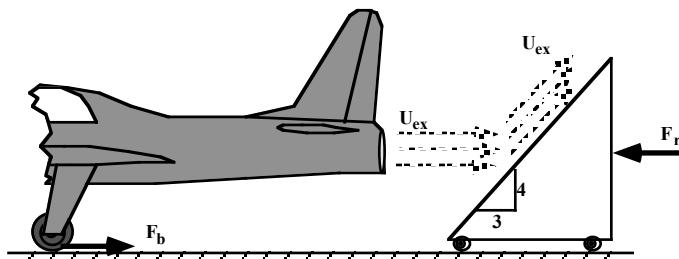
1. Suppose you were to launch a space-craft from the Earth's equator by abruptly (and magically) transferring all of the rotational angular momentum (not orbital!) from Earth to the space-craft. What is the launch velocity of the spacecraft, in terms of the earth's mass M , its radius R , its rotational period T , and the spacecraft mass m ?
2. Two identical race-cars approach a curve in the track that bends to the right. The right car follows the shortest possible path, hugging the inside curb, which has a radius r_{in} . The left car follows the longest possible path, hugging the outside curb which has radius r_{out} . Each car drives the turn at the maximum speed it can without sliding. By what factor does the time around the curve differ for the two cars?

3. Suppose you are standing on the beach at position M at a distance d from the water. You see a friend struggling in the water at position F, d from shore, and a distance y along the shore (from the point where you would enter the water if you ran straight towards it.) At what angle should you run in order to reach your friend in the minimum time? Your speed on the sand is 10 mph, and in the water is only 5 mph.



4. The jet exhaust from a military fighter plane, sitting at rest on the runway, is exhausting 4 kg/s at a velocity of $u_{ex} = 200$ m/s, assumed to be horizontal, as shown in the drawing.

The jet is backed-up by a portable noise and heat deflector that deflects all of the exhaust gases upward, parallel to the ramp on the deflector, also as shown. The slope of the deflector is $4/3$. Ignoring friction in the deflector's wheels, find the force, \mathbf{F}_r , required to maintain the deflector in the position shown?



5. A perfectly conducting plane stretches horizontally in the x - y direction. Below it is a dipole, composed of charges $+Q$ and $-Q$, separated by distance d . The dipole is perpendicular to the plane (and thus in the z direction), and the $+$ charge is nearest the plane, at distance a . If the mass of the dipole is m , for what values of a will the dipole move downward, away from the plane.

6. A conducting sphere of radius a is placed in a uniform electric field, $\vec{E} = E_0 \hat{k}$ and the sphere is grounded. The resulting potential of the system outside the sphere in polar

$$\phi(r, \theta) = -E_0 r (1 - (a/r)^3) \cos \theta$$

coordinates is

Find the surface charge density induced on the sphere.

7. Special relativity leaves the form of Maxwell's equations unchanged in all inertial frames of reference. Thus, the field equations can be expressed in terms of a tensor field that unites \vec{E}

$$E_r = \gamma(E_r - v B_\theta)$$

and \vec{B} . In cylindrical coordinates, this implies that if \vec{E} and \vec{B} are the electric and magnetic fields in a frame S , then the radial electric field in S' in cylindrical coordinates is where

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

and v is the speed of S' relative to S , along the z axis.

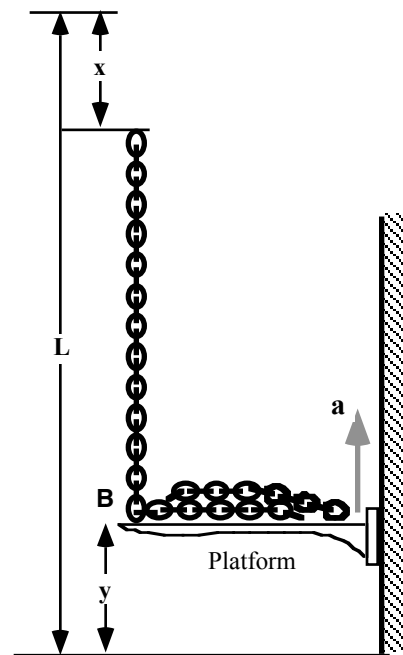
Now consider an infinite cylinder of charge density λ and current I , at rest in frame S . Find the speed v of S' where the electric field becomes zero.

PART II: Long problems (75%)

ANSWER 3 OF 5 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. Chained to the platform.

An open-link chain of total length L and of mass ρ per unit length hangs so that it just reaches a platform below without resting on it. At time $t = 0$ two things happen at once. The chain is released from rest (where $x=0$). The platform starts from rest (where $y = 0$) and moves up with a constant acceleration a . Determine an expression for the total force F exerted on the platform by the chain t seconds after the motion starts. (see figure to right)

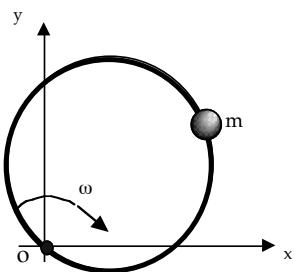


A2. Loop-de-loop, loop-de-lie

With one hand, a small child is launching a ball tied to the end of a string and hanging from her other hand. (i) At first, her push gets the ball to oscillate. (ii) She discovers then that she is able to get the ball to complete a circle if she gives it a certain angular velocity. (iii) For another range she can get an intermediate result in which the ball comes above the level of the hand clutching the string, before the ball leaves its circular path (with the string slackening) and then becoming taut again when it comes back to another part of the circle.

If the string length is L , what velocities correspond to situations i, ii and iii above? With what velocity should the ball be launched so that it will land squarely in the hand holding the string?

A3. Hoop-la



A bead of mass m can slide without friction on a circular wire of radius R . The z -axis is vertical (out of the page). The circular wire rotates clockwise in a horizontal (xy) plane about O with a constant angular velocity ω as shown to the left in a view from above.

[a] Choose appropriate co-ordinates, and write down the Lagrangian for the bead.

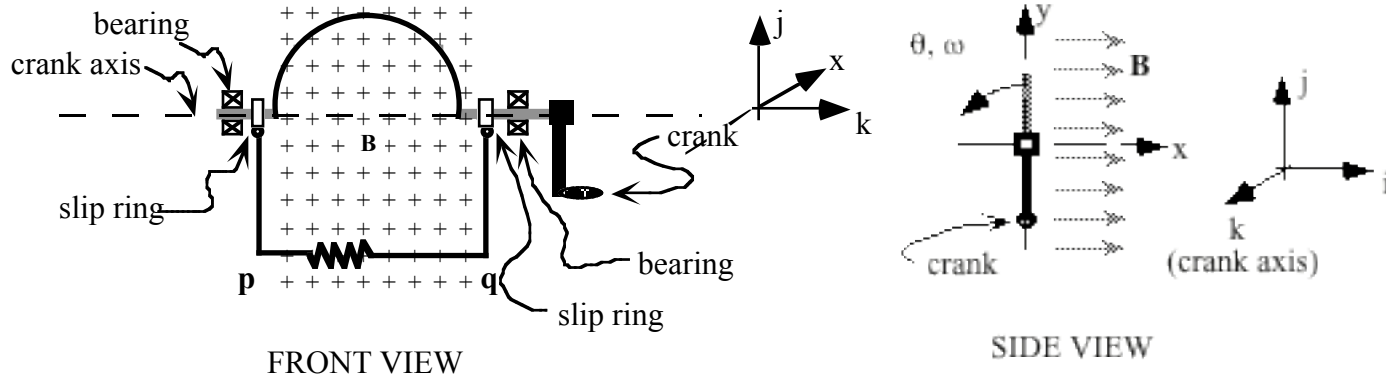
[b] By solving the Lagrange's equation(s), describe the motion of the bead.

B1. A ringer!

(a) A loop of wire of radius a is centered in the x - y plane, and carries a current I . Find the magnetic field on the axis of the loop, a distance of z from the plane. *continues on next pg.*

(b) Suppose we put N such rings concentrically, each carrying a current I , so that a kind of disk is formed in the x - y plane. Suppose the outer radius of the "disk" is b , and the inner radius is a . Now what is the magnetic field at distance z from the plane?

B2. See how it turns out



An electrical circuit consists of a conducting wire bent into a semi-circular shape (radius r_o) and a resistor R as shown in the figure. A constant magnetic field B (perpendicular to and inward the paper along positive x direction) is applied in the region. The conducting wire, bent like a semi-circular shape, is rotated by a hand crank with a constant angular velocity ω in the counterclockwise direction, as viewed from the crank end, and the angle θ' is measured from the vertical position as shown. The area A_o of the segment of the circuit below the crank axis of rotation (i.e. the z -axis) is a constant. See figures below.

Assume that at $t = 0$, the angle θ is zero and the circuit appears as shown in the figure on the left.

- Assuming that the angle $\theta = 0$ at $t = 0$, what is magnetic flux (Φ) linked to the circuit at $t = 0$? Express your answer in terms of the parameters r_o , B and A_o .
- Assuming that the angle $\theta = 0$ at $t = 0$, what is magnetic flux (Φ) linked to the circuit at $t = \pi/\omega$? Express your answer in terms of the parameters r_o , B and A_o .
- What is the emf ($[\mathcal{E}(t)]$ as a function of time) induced across the terminals (at the slip ring) of the conducting wire?
- Between $t=0$ and $t= \pi/\omega$, the area bounded by the circuit decreases. What is the direction of the induced current in the resistor during this time interval? Give reasons to substantiate your answer. What is the magnitude of this induced current?
- What is the smallest time does the clock read when the induced emf is at its maximum value?
- How do your answers to the above questions change if the semi-circular segment of the circuit is kept fixed and the rectangular segment of the circuit shown below the crank axis is rotated at a constant rate?