DEPARTMENT OF PHYSICS
Ph.D. Qualifying Exam - Classical Physics
Friday, September 21, 2007
9:00 a.m. - 12:00 noon

PRINT YOUR NAME ______________________________

EXAM CODE __________

INSTRUCTIONS

PUT YOUR EXAM CODE, NOT YOUR NAME, ON EACH PIECE OF PAPER YOU HAND IN. (This allows us to grade each student only on the work presented.)

Do each problem or question on a separate sheet of paper. (This allows us to grade them simultaneously.)

You may consult only the references provided or approved in advance by the examiner. You may use a calculator.

In Part I, answer 5 of 7 short answer questions. In Part II, answer 3 of 6 longer problems, including at least one problem from Group A and one from Group B. *Circle the numbers below* to indicate which questions you have answered. Write nothing on the lines below (your grades go there).

Short Problems		Long Problems	
circle	grade	*circle*	grade
1.	________	A1.	________
2.	________	A2.	________
3.	________	A3.	________
4.	________	B1.	________
5.	________	B2.	________
6.	________	B3.	________
7.	________		

PART I: SHORT PROBLEMS (25%)

ANSWER 5 OF 7 QUESTIONS

1. An inverted garbage can of weight W is suspended above the ground by water from a fountain. The water emerges from the ground at constant rate $R = dm/dt$, with initial speed v_0, and rebounds elastically when it strikes the can. Find the height of the water column for which the can is in mechanical equilibrium. Hint: The water does not strike the can at speed v_0.

2. A conveyor belt of mass M is moving at a constant horizontal speed v, while sand is continuously dropping on the belt from a stationary hopper at the rate dm/dt. Transfer of mass to the belt reduces the speed of the belt and hence power has to be supplied to the belt by applying a force F to keep the belt moving at a uniform speed v. Show that the power P is twice the rate at which the kinetic energy is increasing.

3. A ladder of length L with negligible mass leans against a wall. The ladder forms an angle θ with the floor. The static coefficient between the ladder and the floor and between the ladder and the wall is μ. A child of mass m walks up the ladder. What is the maximum distance d_{max} that the child can walk up the ladder (distance along the length of the ladder) before the ladder starts to slip?

4. A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire if the current is distributed in such a way that the current density J is proportional to distance from the axis r.

5. A rectangular plate (with corner points ABCD) has length L, width W, thickness D, and conductivity σ. When electrodes are connected to edges AB and CD, the resistance of the plate is measured as R_1. The electrodes are then connected to edges AD and BC and the resistance measured as R_2. Find an expression for the value of the product $R_1 R_2$.

6. When an unpolarized electromagnetic wave is incident on a medium at an incident angle $\theta_i = 55^\circ$, the reflected rays are completely polarized. What is the refractive index of the medium?

7. Explain in your own words the *physical* reason why, if an electromagnetic wave of frequency ν is incident on a transparent dielectric medium, the reflected and transmitted waves have exactly the same frequency.

PART II: LONG PROBLEMS (75%)

ANSWER 3 OF 6 QUESTIONS, WITH AT LEAST ONE FROM GROUP A AND ONE FROM GROUP B.

A1. Gravity Rules the Galaxy

(a) The disk of the Milky Way galaxy may be approximated as an infinite sheet of matter of negligible thickness and constant surface density σ. Assuming that the sheet lies in the x–y plane, use Gauss's law to derive an expression for the gravitational acceleration (magnitude *and* direction) at location (x, y, z).

(b) A particular population of stars is observed orbiting near the Galactic plane. Their perpendicular velocities (i.e. in the z direction) are measured at $v_z = 3 \times 10^4\,\mathrm{ms}^{-1}$ as they cross the plane, and they are observed to have a maximum departure above (or below) the plane of $1.6 \times 10^{19}\,\mathrm{m}$. Approximating the disk as an infinite gravitating sheet of matter, estimate its surface density σ in kg m^{-2}. Take $G = 6.67 \times 10^{-11}$ SI units.

A2. Rolling Down the Plane

A sphere and a disk with the same radius R are released at the same time at the top of an inclined plane that makes an angle $\theta = 30$ degrees with the horizontal. They immediately roll without slipping down the inclined plane. Note that the disk is set on its edge so that it rolls.

(a) Which rigid body will be farthest down the plane after one second? (Obviously, answer this with equations, not narrative!)

(b) How far apart will their centers be (in meters) at this time? (You DO have enough information to give a numerical result.)

A3. String and Pulley Physics

Two masses m and $M = 2m$ are connected by a light inextensible string which passes without slipping over a uniform circular pulley of mass $2m$ and radius r. The pulley is free to rotate about a frictionless horizontal axis of rotation.

(a) Using the angular position of a point on the rim of the pulley as a generalized co-ordinate, write down the Lagrangian equation.

(b) Determine the acceleration of mass M.

B1. Circuit Fun

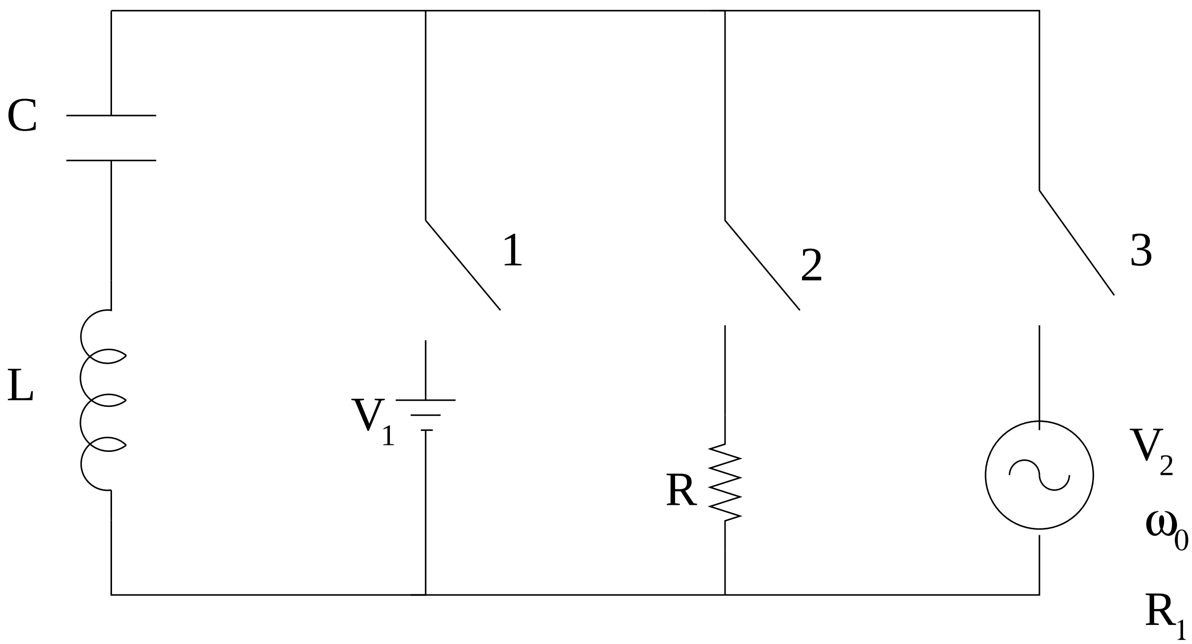

Figure 1: Circuit diagram for problem B1

For this problem, all answers may be expressed algebraically in terms of C, the capacitance of the capacitor, L, the inductance of the inductor, and so forth, as indicated in the figure above. You may also find it useful to define new derived quantities, rather than writing them out each time.

Prior to $t = 0$, all three switches are open, and the capacitor is uncharged. At $t = 0$, switch 1 is thrown closed. Assume that the battery has zero resistance.

(a) What is the charge on the capacitor at time t after switch 1 is closed?

(b) How much energy has been put into the circuit (from the battery) as a function of time? What fraction of that energy is magnetic and what fraction electrical?

(c) Eventually, the system reaches equilibrium. At that time, $t' = 0$, we open switch 1 and close switch 2. What the the charge on the capacitor as a function of time, t'?

(d) Assuming R is much smaller than the critical value for an RLC circuit, what is the power output from the resistor as a function of time (for simplicity, you may average over an oscillation period)?

(e) Surprise, surprise, at some still yet later time (after the circuit has once again died down), we flip switch 2 open and close switch 3. The AC supply is calibrated such that $\omega_0 = 1/\sqrt{LC}$. Assuming R is relatively small, what is the maximum current through the circuit?

B2. Transforming Maxwell

(a) Show that Maxwell's equations in vacuum are invariant under the transformation:

$$E' = E\cos\theta + kB\sin\theta \quad B' = B\cos\theta - E\sin\theta/k$$

Where k is a constant that depends on which unit system one is using for expressing Maxwell's equations (Gaussian, MKS, etc.). Also find the value of k for the expression of Maxwell's equations that you use.

(b) Now apply the transformation above to Maxwell's equations with charges and currents (you can assume that $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$), and find how Maxwell's equations should be modified to remain invariant under the transformation, *and* how the charges and currents transform.

B3. Charged Sphere

A sphere of radius R carries a charge density $\rho = kr$, where k is a constant.

(a) Calculate the electric field inside the sphere, $r < R$.

(b) Calculate the electric field outside the sphere, $r > R$.

(c) Find the electrostatic energy of this configuration.

(d) What happens to the electrostatic energy if $R \to 0$? Justify your answer (I was under the impression that the energy of a charged sphere approached infinity when the sphere became a point particle).