

Bragg's Law

$$n\lambda = 2d_{\text{unit cell}} \sin(\theta)$$

$$\lambda = 2d_{h,k,l} \sin(\theta_{h,k,l})$$

smallest $d_{h,k,l}$ is resolution

$$\frac{1}{d_{h,k,l}} = \frac{2 \sin(\theta_{h,k,l})}{\lambda} \quad \text{so} \quad \frac{\sin(\theta_{h,k,l})}{\lambda} \text{ is a measure of resolution}$$

Scalar value of amplitude equals the square root of the intensity (energy) of the wave.

$$|F_{hkl}| = (I_{hkl})^{1/2} \quad I_{hkl} = |F_{hkl}|^2$$

“Intensity” has been corrected for polarization and other consequences of data collection method.

A diffracted ray is the sum of contributions from all atoms.

$$F \Rightarrow |F_{hkl}| \cdot e^{i\phi_{hkl}} = \sum_n^N O_n \cdot f_{n,\theta} \cdot e^{-B_n (\sin \theta / \lambda)^2} \cdot e^{i 2\pi (hx_n + ky_n + lz_n)}$$

A diffracted ray is the sum of contributions from all electron density.

$$|F_{hkl}| \cdot e^{i\phi_{hkl}} = \text{Volume}_{\text{(of repeating unit)}} \sum_x \sum_y \sum_z \rho_{xyz} \cdot e^{i 2\pi (hx + ky + lz)}$$

Electron density is the Fourier transform of all diffracted rays.

$$\rho_{xyz} = (\text{Vol})^{-1} \sum_h \sum_k \sum_\ell m_{hkl} \cdot |F_{hkl}| \cdot e^{i\phi_{hkl}} \cdot e^{-i 2\pi (hx + ky + lz)}$$

Patterson map is the Fourier transform of the intensities.

$$P_{xyz} = (\text{Vol})^{-2} \sum_h \sum_k \sum_\ell |F_{hkl}|^2 \cdot e^{-i 2\pi (hx + ky + lz)}$$

product of all electron densities separated by the x,y,z vector distance
same shape and dimension unit cell

Residuals, R-values, assess agreement between datasets. (here model vs experimental)

$$R_{\text{cryst}} = \frac{\sum ||F_{\text{obs}}| - |F_{\text{calc}}||}{\sum |F_{\text{obs}}|}$$

R_{free} calculated from
otherwise unused 5%

λ	: wavelength of radiation
d	: interplanar spacing, the effective distance associated with a particular diffracted ray
θ_{hkl}	: angle of incident beam to the h, k, ℓ Bragg plane
h, k, ℓ	: integer index numbers of a particular Bragg Plane, a diffracted ray, a “reflection”, index of a point of the reciprocal lattice “reciprocal space”
$ F_{hkl} $: amplitude of the hkl^{th} diffracted ray
ϕ_{hkl}	: the phase of the hkl^{th} diffracted ray
B	: B-factor (historically the Temperature Factor, but dominated by other uncertainties)
$f_{n,\theta_{hkl}}$: individual atomic scattering factor of the n^{th} atom as a function of θ_{hkl}
x_n, y_n, z_n	: coordinates of the n^{th} atom
m_{hkl}	: figure of merit for phase of the hkl^{th} diffracted ray
N	: total number of atoms in the repeating unit of the crystal
O_n	: occupancy of the n^{th} atom.
Vol	: volume of the repeating unit, the unit cell
ρ_{xyz}	: electron density at coordinates x, y, z in the crystal “real space”
P_{xyz}	: Patterson function value at coordinates x,y,z : vector distances-between atom positions