

The equations: (Good-Parts version: what you really need to remember, cont'd.)

A light wave ray diffracted from a molecule in a crystal:

A wave is defined by an Amplitude $|F_{hkl}|$ and a Phase: $e^{i\phi_{hkl}}$

Each diffracted “wave” is a sum of contributions from ALL atoms:

$$\sum_n^N (\text{Amplitude-factors}) \cdot (\text{Phase-factor}) \quad \dots \text{the sum is over all atoms.}$$

Amplitude-factors are a property of the atom, including uncertainty about its position.

(Uncertainties are put into the B-factor. Many people think they understand the B-factor because they remember hearing about the Temperature Factor from Physics, for macromolecules the temperature part of the B-factor is only a minor part, most of the effect comes from other kinds of disorder.)

Phase-factor is just dependent on the position of the atom.

And now the equation we will derive (and that crystallographers should be familiar with):

Remember this equation for your prelim!

$$\vec{F} = |F_{hkl}| \cdot e^{i\phi_{hkl}} = \sum_n^N O_n \cdot f_{n,\theta_{hkl}} \cdot e^{-B_n (\sin\theta_{hkl}/\lambda)^2} \cdot e^{i2\pi(hx_n + ky_n + lz_n)}$$

h, k, ℓ : integer index numbers of a particular Bragg Plane, a diffracted ray, a “reflection”.

θ_{hkl} : angle of incident beam to the h, k, ℓ Bragg plane

$|F_{hkl}|$: amplitude of the hkl^{th} diffracted ray

ϕ_{hkl} : the phase of the hkl^{th} diffracted ray

B : B-factor (historically the Temperature Factor, but dominated by other uncertainties)

$f_{n,\theta_{hkl}}$: individual atomic scattering factor of the n^{th} atom as a function of θ_{hkl}

x_n, y_n, z_n : coordinates of the n^{th} atom

N : total number of atoms in the repeating unit of the crystal

O_n : occupancy of the n^{th} atom. (e.g. for a half-occupied ligand, sidechain alternate rotamer, etc.)