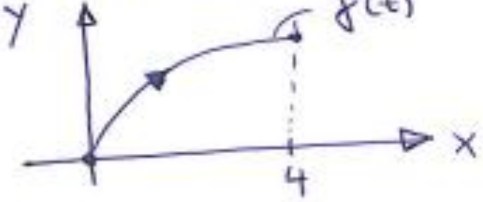
Aufgabe 1:

a) $\vec{F} = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$, $\vec{\gamma}(t) = \begin{pmatrix} t \\ \sqrt{t} \end{pmatrix}$ mit $0 \le t \le 4$

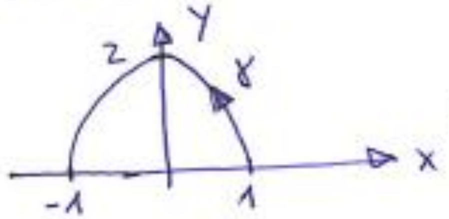


Lsg:

$$\dot{\vec{\gamma}}(t) = \begin{pmatrix} 1 \\ \frac{1}{2\sqrt{t}} \end{pmatrix}$$

$$\Rightarrow \int_0^4 \begin{pmatrix} t^2 \\ t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \frac{1}{2\sqrt{t}} \end{pmatrix} dt = \int_0^4 \left(t^2 + \frac{1}{2}\sqrt{t}\right) dt = \underline{\underline{24}}$$

b) $\vec{F} = \begin{pmatrix} 2x \\ -y \end{pmatrix}$, $\vec{\gamma}(t) = \begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix}$, $t \in [0, \pi]$



Lsg:

$$\dot{\vec{\gamma}}(t) = \begin{pmatrix} -\sin t \\ 2\cos t \end{pmatrix}$$

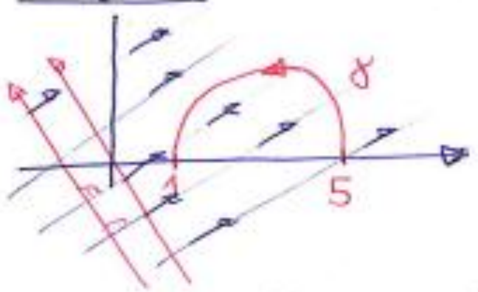
$$\int_\gamma \vec{F} \cdot \dot{\vec{\gamma}}\, dt = \int_0^\pi \begin{pmatrix} 2\cos t \\ -2\sin t \end{pmatrix} \cdot \begin{pmatrix} -\sin t \\ 2\cos t \end{pmatrix} dt$$

$$= \int_0^\pi -2\sin t \cdot \cos t - 2\sin t \cos t \, dt = \int_0^\pi -4\sin t \cdot \cos t \, dt$$

$$= \underline{\underline{0}}$$

Aufgabe 2:

$\vec{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \vec{\gamma}(t) = \begin{pmatrix} 2\cos t + 3 \\ 2\sin t \end{pmatrix}$ $\quad t \in [0, \pi]$



Lsg: $\dot{\vec{\gamma}}(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix}$

a) $$\int_0^\pi \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix} dt = \int_0^\pi 2\cos t - 4\sin t \, dt$$

$$= 2\sin t + 4\cos t \Big|_0^\pi = 0 - 4 - (0 - 4)$$

$$= \underline{\underline{-8}}$$

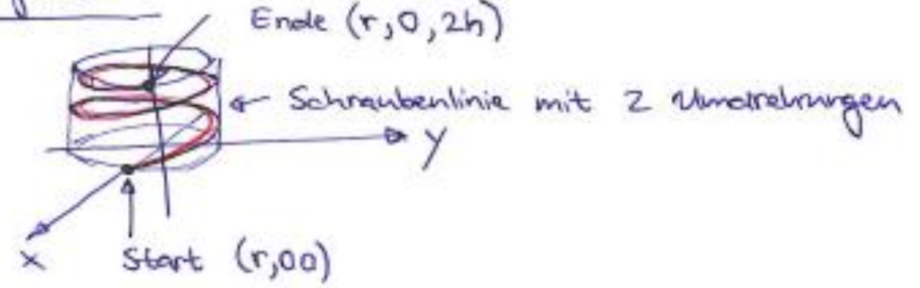
b) i) $\int_\gamma \vec{F} d\vec{r} = 0$ falls Kurve geschlossen $\Rightarrow$ $\vec{F}$ konservativ

ii) $\int_\gamma \vec{F} \cdot d\vec{r} = 0$ falls $\vec{\gamma}(t) \perp \vec{F}$ siehe Skizze

$\Updownarrow$ $\dot{\vec{\gamma}} \cdot \vec{F} = 0$

Aufgabe 3:

a)



$$\Rightarrow \vec{\gamma}(t) = \begin{pmatrix} r\cos t \\ r\sin t \\ \frac{h}{2\pi} t \end{pmatrix} \quad \text{mit } t \in [0, 4\pi]$$

$$\text{und } \dot{\vec{\gamma}}(t) = \begin{pmatrix} -r\sin t \\ r\cos t \\ \frac{h}{2\pi} \end{pmatrix}$$

b) $\int_\gamma \vec{F} \circ d\vec{r}$ für $\vec{F} = \vec{F}_G = C \cdot \begin{pmatrix} 0 \\ 0 \\ \frac{1}{(z-R_E)^2} \end{pmatrix}$ mit $C := -GMm$

$$\int_\gamma \vec{F} \cdot d\vec{r} = C \int_0^{4\pi} \begin{pmatrix} 0 \\ 0 \\ \frac{1}{(\frac{h}{2\pi}t - R_E)^2} \end{pmatrix} \cdot \begin{pmatrix} -r\sin t \\ r\cos t \\ \frac{h}{2\pi} \end{pmatrix} dt$$

$$= C \int_0^{4\pi} \frac{h}{2\pi} \cdot \frac{1}{(\frac{h}{2\pi}t - R_E)^2}\, dt = \frac{C \cdot h}{2\pi} \int_0^{4\pi} \frac{1}{(\frac{h}{2\pi}t - R_E)^2}\, dt$$

$$= \frac{C \cdot \cancel{h}}{\cancel{2\pi}} \cdot \frac{\cancel{2\pi}}{\cancel{h}} \, \frac{-1}{\frac{h}{2\pi}t - R_E} \Bigg|_0^{4\pi} = -C \cdot \frac{1}{\frac{h}{2\pi}t - R_E} \Bigg|_{t=0}^{4\pi}$$

$$= C \left(\frac{1}{-R_E} - \frac{1}{2h - R_E} \right) = C \left(\frac{2h - \cancel{R_E} + \cancel{R_E}}{-R_E(2h - R_E)} \right)$$

$$\underline{\underline{W_G = \frac{2GMmh}{R_E(2h - R_E)}}} = \underline{\underline{-3.918\ \text{MJ}}}$$

c) da $\vec{F}_G$ nahe der Erdoberfläche fest konstant ist mit $\vec{F}_G = -m \cdot g$ $(g = 9{,}81\ m/s^2)$

$$\Rightarrow \text{Arbeit} = -m \cdot g \cdot \underbrace{\Delta h}_{2h} = \underline{\underline{-3{,}918\ MJ}} \quad \text{(Potentielle Energie)}$$

d) $$\vec{F}_W = 50 \begin{pmatrix} -y + 20 \\ x \\ -2z \end{pmatrix}$$

$$W_{Wind} = \int_\gamma \vec{F}_W \cdot \dot{\vec{\gamma}}\, dt = 50 \int_0^{4\pi} \begin{pmatrix} -r\sin t + 20 \\ r\cos t \\ -\frac{h}{\pi} \cdot t \end{pmatrix} \cdot \begin{pmatrix} -r\sin t \\ r\cos t \\ \frac{h}{2\pi} \end{pmatrix} dt$$

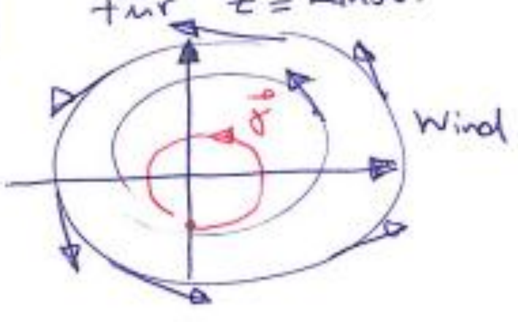
$$= 50 \int_0^{4\pi} \Big(\underbrace{r^2 \sin^2 t + r^2 \cos^2 t}_{= r^2} \cancel{+} - 20 r \sin t - \frac{h^2}{2\pi^2} t \Big) dt$$

$$= 50 \int_0^{4\pi} \Big(r^2 - 20 r \sin t - \frac{h^2}{2\pi^2} t \Big) dt = \underline{\underline{656\ MJ}}$$

$$W_{Total} = W_G + W_{Wind} = -3{,}92 + 656\ MJ$$
$$= \underline{\underline{622{,}4\ MJ}}$$

↳ Energiegewinn

e) für $z = \text{konst}$:



Antwort: <u>Rückenwind</u>