TA.THFL+SIM.H15

Numerical methods: **Problems 6 (unsteady state heat transfer)**

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1. **1d unsteady heat transfer**:

   Given is a wall with constant thermal conductivity a, thickness $L = 0.15\,m$ and boundary temperatures $\vartheta(0,t) = 5°C$ and $\vartheta(L,t) = 25°C$. The initial temperatur is assumed to be constant with $\vartheta(x,0) = 40°C$. Thermal conductivities $a = \lambda/(\rho \cdot c_p)$ for different materials:

   Concrete: $0.54 \cdot 10^{-6}\mathrm{m}^2/\mathrm{s}$
   Steel: $22.8 \cdot 10^{-6}\mathrm{m}^2/\mathrm{s}$
   Gold: $124 \cdot 10^{-6}\mathrm{m}^2/\mathrm{s}$

   (a) Explicit FDM: Choose $\Delta x = 0.1\mathrm{m}$ for the spatial discretization. Calculate the critical time steps Δt_{max} for the tree different materials. Implement the explicit scheme in MATLAB.

   (b) Implicit FDM: Implement the implicit scheme in MATLAB.

   (c) Optional: Implement the *Crank-Nicolson* scheme.

2. **1d unsteady heat transfer (non-constant material properties)**:

   If the the wall has non-constant thermal conductivities given by a function $a = a(x, y, z)$ or by a wall consisting of different materials a_1,a_2,.., the unsteady state heat transfer equation is given by

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(a \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(a \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(a \cdot \frac{\partial T}{\partial z}\right) \tag{1}$$

   or in one spatial dimension x

$$\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(a(x) \cdot \frac{\partial T(x,t)}{\partial x}\right) \tag{2}$$

   (a) Find a simple, explizit FDM scheme for equation (2). Hint: Choose forward- or backward differences.

   (b) Solve the problem in Matlab for a wall constisting of two materials

$$a(x) = \begin{cases} a_1 & \text{for } x < L/2 \\ a_2 & \text{for } x > L/2 \end{cases}$$