A study of empire fragmentation with agent-based modelling

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Abstract: This study tries to investigate the cases in which an empire subject to revolutionary movements. For this purpose an agent based model is developed.In our model the central authority wants to suppress decentralized rebellions to observed the evolution of this model and, possibly, a convergence. In addition to this, we set tunable parameters in order to understand how the outcome changes depending on them. We found that one interesting parameter is the legitimicy of empire (L). We found out that the value of L plays and important role in determining whether a revolution will happen or not, regardless the values of other parameters such as jail time (police brutality). In particular, in the scenarios that we studied, if L decreases through time, a rebellion will happen. This means that make punishments harsher is not a good solution to avoid the empire fragmentation. As this can only postpone the revolution. Hence, to prevent rebelion it is better to increase the popularity of the government instead.

Contents

1 Introduction

In this project we develop an agent-based model to investigate the potential emerging patterns and fragmentation of an empire that is subject to revolutionary movements. The code repository link[1] can be found in [1]. We analyze these patterns as a function of the citizens perception. We initially expect a fragmentation outcome in terms of a segregation into pro-empire and anti-empire clusters.

We assume that a central authority wants to suppress the activities of the rebellious citizens, who are decentralized. To model this, we create two types of agents, which we collectively call *natives*. Firstly, we have the ones who represent the *citizens* with a state of $s_i \in \{0, 1\}$, where $i = 1, 2, \dots C$ enumerates the agent and C is the total number of citizens. State 0 represents their support for the empire while state 1 corresponds to a revolutionary attitude that leads the citizen to take action. We call these states "pro-empire" and "anti-empire", respectively. Additionally, we define a second kind of agent, the *propagandist*. The propagandist has no state attribute since he remains pro-empire across time. These agents represent the government and they will hand in any anti-empire citizen (rebel) to the authority, which will consequently put him to jail. No political or social order is represented in the model, our focus would be the decentralized upheaval instead. The agents are placed in a periodic grid.

This work is motivated by the following scientific

[1] At the timing of writing the repository is private. You can get in touch to be granted access.

questions:

1. Are there any macroscopic dynamics that can emerge from the interaction of the citizens and propagandists? E.g., will there be periodicity with respect to pro-empire and rebel agent densities on the grid?

2. Does the model converge to a certain spatial configuration? Are specific stable fragmentation patterns observable?

3. In which cases does a *revolution* occur? We define revolution as $f_r \geq 0.5$ where

$$f_r = \lim_{t \to \infty} \frac{1}{C_a} \sum_{i}^{C_a} s_i(t), \tag{1}$$

where C_a is the number of active citizens, i.e., not imprisoned, on the grid. In the discrete time steps of the simulation, we approximately this limit as the final time-step where the model seems to have converged numerically. This will be discussed in more detail in Sec. 3.

4. How does the model evolution depend on the initial conditions?

5. Can police brutality, namely, increasing the number of propagandists and the time of imprisonment, help dictators control and ensure the stability of the empire?

2 Agents and Model description

In this section we define the attributes of the agents, that describe their perception of the situation and that determine their behavior of taking action or not. The agents are placed on a two dimensional periodical grid in which they can move to adjacent cells.

2.1 Citizen attributes

Firstly, we describe two agent attributes, one being intrinsic and one being universal: *hardship* and *government legitimacy*, respectively. These attributes influence the decisions of the agent to rebel. These two quantities combined give us the *perceived grievance*. They are defined as follows.

H: perceived hardship. This quantity represents the personal challenges that the agent faces because of the government. They could be for instance related to financial problems and infringements to individual or social freedom. The citizens are inhomogeneous, each having their own value of hardship. We assume that H follows a uniform distribution in the interval (0,1). For the simulation, the values for the individual hardship are randomly initialised for each agent and stay constant for all time steps.

L: perceived legitimacy. This attribute corresponds to the popularity of the central authority according to the agents. As far as for its value, we study two cases. In the first scenario, it is a constant between 0 and 1 and set at the beginning. In the second case, L is decreasing, having in mind that the more time passes from the beginning of the revolutionary movements the less the government is perceived as a legitimate authority. Specifically, the value of L is decreased at each step dividing to a factor of 1.2. In this way, we well have a discreet of the exponential function. In any case the value of L will be equal for all agents.

G: Grievance. It encodes the influence of the perceived illegitimacy $(1-L)$ and of the perceived hardship in order to describe the unsatisfactory conditions of the citizens. For simplicity, and in absence an argument for a more complicated functional expression, these two quantities are multiplied to define the negative emotion, i.e., grievance of each agent at every time step

$$G \coloneqq H(1 - L). \tag{2}$$

R: level of risk aversion. This intrinsic attribute of the citizens quantifies how much an agent is inclined to take risks. It is assumed, that $R \sim U(0, 1)$, as in the case of H. This value is fixed during the entire evolution of the model. The agents evaluate the risk of taking action based on the states of the surrounding agents. The more propagandists there are in the neighbourhood of the agent, the higher the risk to be arrested. On the other hand, the more rebels surround the agent, the less likely it would be for it to be caught. To model this, we further define the following features of our simulation.

(X,Y): grid position of the agent. It indicates where the agent is placed on the two dimensional periodical grid (50x50 cells). At every time step the agents move simultaneously to an empty cell in their neighborhood.

$\boldsymbol{\nu}$: set of the agent's neighbours. Every agent is able to check the state of his neighbourhood, namely, of the agents in $\boldsymbol{\nu}$. For this model we consider a Moore neighbourhood, i.e., each citizen has access to the information of the eight agents that surround it. Hence, we are dealing with a model that the information flow is localised.

With the above quantities, we define the personal likelihood estimate of the agent to be arrested

$$\alpha(t) := 1 - k \exp\left(-\left\lfloor \frac{A(t)}{P(t)} \right\rfloor\right), \tag{3}$$

where $A(t)$ is the number of anti-empire in the agent's neighborhood $\boldsymbol{\nu}$, $P(t)$ is number of propagandists in the neighborhood $\boldsymbol{\nu}$, k is a constant that is set to 2.3 to ensure that $\alpha(t) \in [0, 1]$, and $\lfloor \cdot \rfloor$ is the floor function.

Combining the risk aversion of the agents, R with Eq. 2.1 $\mathbf{R}$ and $\boldsymbol{\alpha}$ we define the *net risk*.

$$N(t) = R \cdot \alpha(t) \tag{4}$$

The value of N determines whether the agent will be rebel or not in the following time-step.

2.2 Citizen behavioral rules

In the previous section, the quantities that describe the state of the model and internal degrees of freedom of the agents were defined. Subsequently, we now present the decision rule of the agents.

The state of an citizen at the next time step, $s_i(t+1)$, is determined by the following equation

$$s_i(t+1) = \Theta[z_i(t)] \quad | \quad z_i(t) := G_i(t) - N_i(t) - T_i, \tag{5}$$

where $\Theta(\cdot)$ is the Heaviside step function. Furthermore, time dependence and dependence on the individual agent attributes through the index i, was introduced. The above expression can be summarised as follows. The decision rule of the agent is:

If $G_i(t) - N_i(t) > T_i$ be pro-empire; otherwise, be a rebel.

2.3 Propagandist attributes and behavioral rules

Propagandists are less complex agents than citizens in that they don't have an internal state that changes through time. Nevertheless, the same attributes apply with respect to the neighborhood and their movement. We notice that the information received by the propagandists is local in the same manner it is for the citizens. They inspect the state of the citizens in their $\boldsymbol{\nu}$ and then pick a random anti-empire agent, if at least one exists, in order to arrest it and send it to jail for some time steps. The citizens and propagandists move simultaneously.

2.4 Model

Additional features of our model are the following. When an anti-empire agent is caught by a propagandist, that citizen is removed from the grid temporarily. The amount of steps that the agents will spend off the grid is a tunable parameter of the model J, which stands for Jail. We imagine this value as the years spent in prison, once an agent is caught. Moreover, we say that there is a *revolution* if at the final step we obtain a fraction of rebels that is higher than 0.5, as defined in Eq. 3.

During our work for this project, a similar study by Epstein [2] came to our attention. It has a similar direction in formalizing what we are interested in, hence, we took inspiration from it for naming and defining variables such as the Grievance and Net Risk, in lack of more suitable definitions leading to complex model outcomes. Our work deviates from the article mainly because we considered the case of decreasing L, which is not treated in the aforementioned work. Furthermore, our grid is periodic and the initial population of agents is treated as an initial condition of the simulation. Specifically, we define the fraction of cells that are occupied by the propagandists and citizens, respectively, at the beginning of the simulation. We further study the stability of the empire, that is, the emergence of a revolution or not, as a function of these initial conditions.

It is important to highlight that the agents possess both stochastic attributes and deterministic ones. The rule of changing agent's internal state is deterministic, however, the movement of both citizens and propagandists on the grid is random in their neighbourhood, following a discrete uniform distribution. Therefore, the computed grievance and risk at every time step exhibit stochasticity due to their dependence on the (randomly) chosen move of the agent.

3 Results and discussion

In this section results will be presented. Some images will be presented from the visualization of the simulations and each of them will be discussed in details.There are two types of pictures:first, those that show the percentage of rebels at the end of the simulation; Second: those who depict the dynamic of the model throughout the time steps. The darker the colour is, the smaller the percentage of rebel is and vice versa. The red colour corresponds to the limit situation of what we defined as rebellion, namely half of the population is made up of rebel citizens.

3.1 The role of the parameter L

Fig. 1 illustrates different initial conditions regarding fraction of propagandist (x axis) and fraction of citizens (y axis). As is shown we obtained a specific fraction of rebels at the end of simulation. Each subplot corresponds to different values of Jail time J and legitimacy L. The three columns from left to right correspond to the jail time of $J = 10$, $J = 15$ and $J = 20$. Rows, on the other hand, show the fixed values of L in the following order from top to bottom: $L = 0.8$, $L = 0.5$ and $L = 0.2$. The results indicate that, a phase transition happen with the government legitimacy of 0.5. At this value, the jail time (police brutality) of 20 years is the value that can maintain the solidarity of the empire. Less jail time, as a result, will end up in empire fragmentation. It is worth mentioning that 20 years jail time should be considered a very harsh punishment. With purpose of making the model more realistic a government legitimacy set to be decreased throughout the time. This assumption is supported by the idea that always police brutality will results in the decrease of the legitimacy. Therefore, the harsher the punishment, the faster decrease in the legitimacy. This setup is illustrated in Fig. 2.

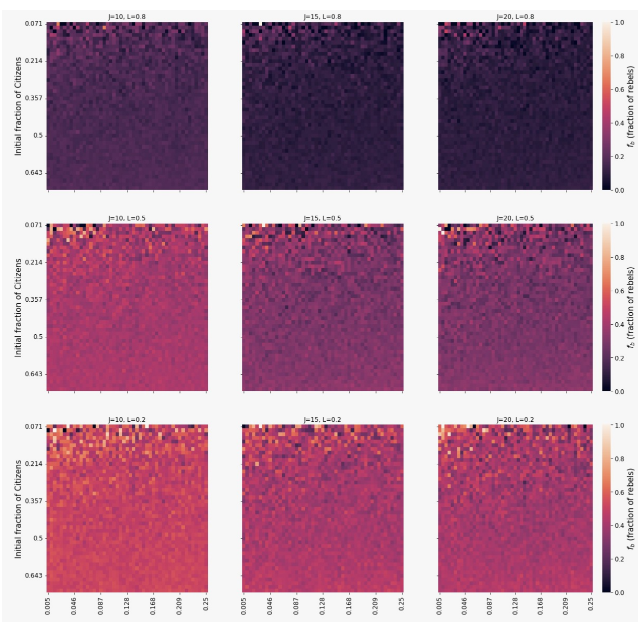

Figure 1: L value of the rows from above to below: $L = 0.8$, $L = 0.5$, $L = 0.2$; J value of columns from left to right: $J = 5$, $J = 15$, $J = 20$; x axis: initial fraction of propagandist; y axis: initial fraction of citizens. The colour represents the percentage of rebels

In Fig. 2 the column corresponds to the same jail time as in Fig. 1.

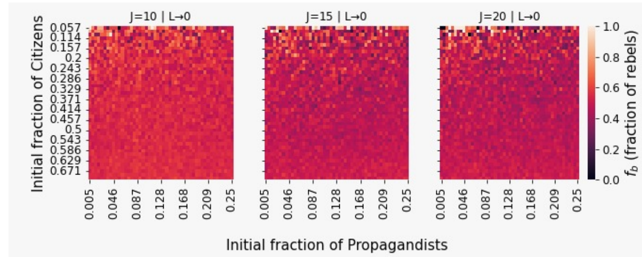

Figure 2: The value of L decreases to 0.; J value from left to right: $J = 5$, $J = 15$, $J = 20$; x axis: initial fraction of propagandist; y axis: initial fraction of citizens. The colour represents the percentage of rebels.

As is illustrated in Fig. 2, interestingly for any jail time (even the harshest one) rebellions will take over the country. In extreme cases that the jail time is at its highest value of more than 20 years the number of rebel is very close to the number of pro-empire citizens but in such cases the the fraction of citizens that are in the jail is higher than those are free. This scenario can be also considered as a fragmentation. In conclusion, in an empire with a decreasing legitimacy, we typically end up with a fragmentation no matter what is the jail time and initial fraction of cops and citizens. To elaborate if the popularity of a government decreases, it does not matter how brutal policy is or how much a ruler forces people to endure higher sufferings, the rebellion will take over the empire. In addition to this, we can say that the agents have an inherent need to rebel if the condition around them are in favour, independently from the jail time value.

On the other hand, when the value of L is fixed a different scenario comes out. For the values $L = 0.8$ and $L = 0.5$ again first we have that the agents has the tendency to rebel but the legitimacy is high enough and, even if it is easy for them to rebel, they prefer to stay quiet. Even if there is a low net risk in taking action, namely the number of propagandist is low and there are a lot of rebels, the rebellion does not happen. For 0.2 we observe a colour that is more similar to red, so the situation is close to a rebellion, but most of the times this rebellion is not as aggressive as the case in which L is decreasing. Something that the Fig. 1 and 2 have in common is that in both we can observe the fact that the jail time (police brutality) is not a huge differentiating factor for keeping citizens quiet.

3.2 Time dependence analysis

In this subsection each scenario will be discussed in a deeper manner. In particular, the effect of each parameter on the dynamics of the model will be discussed.

The following figures will be of two kinds. The figures of the first kind depict the two-dimensional 50 by 50 grid (with periodic boundary condition) that each lattice can be either empty (white), occupied by an anti-empire citizen (red), occupied by a pro-empire citizen (blue) or controlled by a propagandist (black). Hence, the colour of the lattices changes through time at each time step. The change in color represents the movement of the agents on the grid. The second type of figures is consisting of a graph that keeps the count of the number of agents of each type in each time step. In this way the exact number of each type of the agents will be tracked.

In Fig.3 a fixed value of L is chosen, namely $L = 0.8$ and we have Jail time of $J = 15$. Hence, we are referring to the second grid of the first row in Fig. 1. In particular, our initial conditions are:

- initial fraction of propagandist = 0.1
- initial fraction of citizens = 0.7

This is what we observe once we have run all the steps of the simulation.

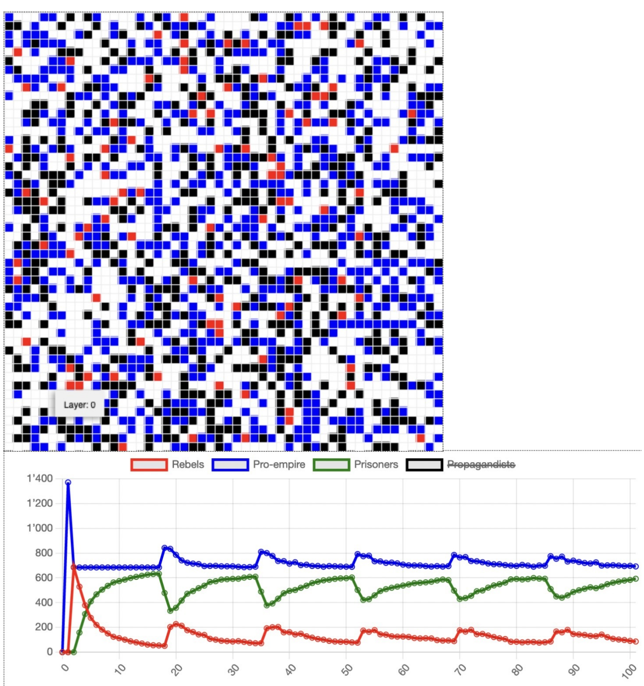

Figure 3: Fixed value of L=0.8, Jail time $J = 15$; x axis: number of steps, y axis: population

From Fig.3, four observations can be concluded. First, we notice that at the end there is no revolution as we expected from the results of section 3.1. Secondly, at the minimum of the curve that represents the jailed agents a maximum number of anti-empire people happens. In addition to this, we can see a periodic pattern repeating throughout the time. Lastly, at the very beginning of the simulation we observe a spike in the curve that describes the rebel. Fig. 4 represents the grid at the third time step of the simulation which correspond to the explosion of the rebellion. The following picture is the frame of the grid that corresponds to this peak of rebels. We can in fact observe a relevant quantity of red boxes.

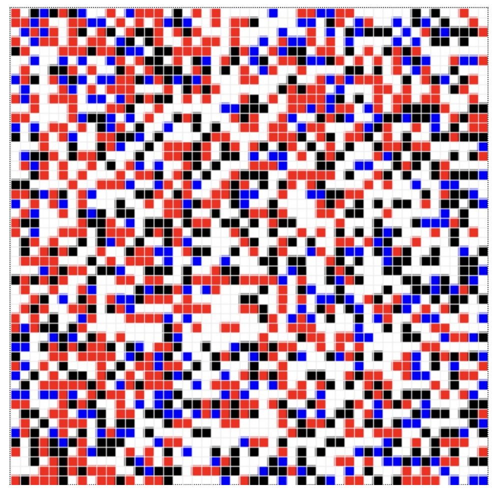

Figure 4: Third step of the simulation with fixed value of L=0.8, Jail time $J = 15$;x axis: number of steps, y axis: population

So as is observed the higher jail time can decrease this initial burst but it can not help the empire to avoid fragmentation. From this can be inferred that, harsher police brutality will just postpone the revolution and it cannot save the country at the end.

In Fig. 5 the value of the legitimacy is fixed as in the previous case. This time we choose $L = 0.2$, while we kept the other parameters unchanged. From what we have seen in subsection 3.1, we expect a moderate rebellion. This figure corresponds to the central grid in the last row of Fig. 1.

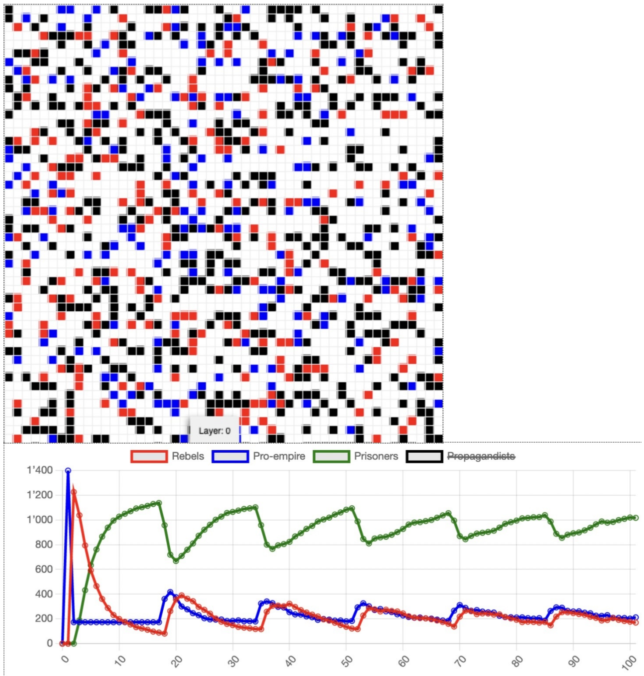

Figure 5: Fixed value of L=0.2, Jail time J = 15; x axis: number of steps, y axis: population

As we expect, this situation is the edge-case (phase transition). In fact, we can talk about a fragmentation of the empire and about a rebellion because firstly there are a lot of empty boxes that create blank areas and secondly because the fraction of rebels oscillates around 0.5, which we can consider a revolution. The same pattern related to local minima of the jailed citizens and the maxima of the rebels can be observed as in the previous case.

We now analyze some of the cases in which we have considered the value of L decreasing through time. In the following case (Fig. 6) we have kept all the parameters as in the Fig. 5 except for the value of the L.

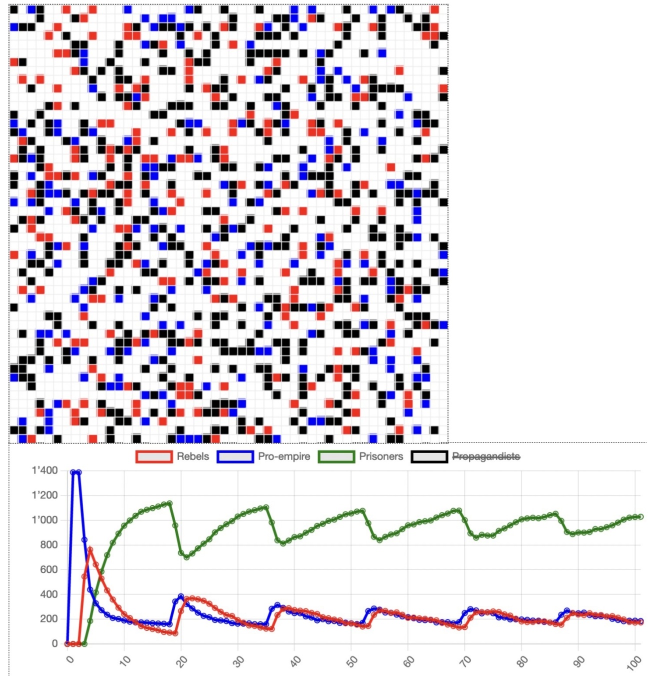

Figure 6: Value of decreasing to 0, Jail time J = 15; x axis: number of steps, y axis: population

As we expected from our conclusions in section 3.1, we have a rebellion, bu also in this case we can observe an edge case, as in the latter picture studied. We again can see a high percentage of population that is jailed and hence a lot of blank spaces in the grid which cause the empire to be fragmented because of the rebels. We want to see now what happens if we decrease jail time, setting the parameters J (jail time-police brutality) equal to 5.

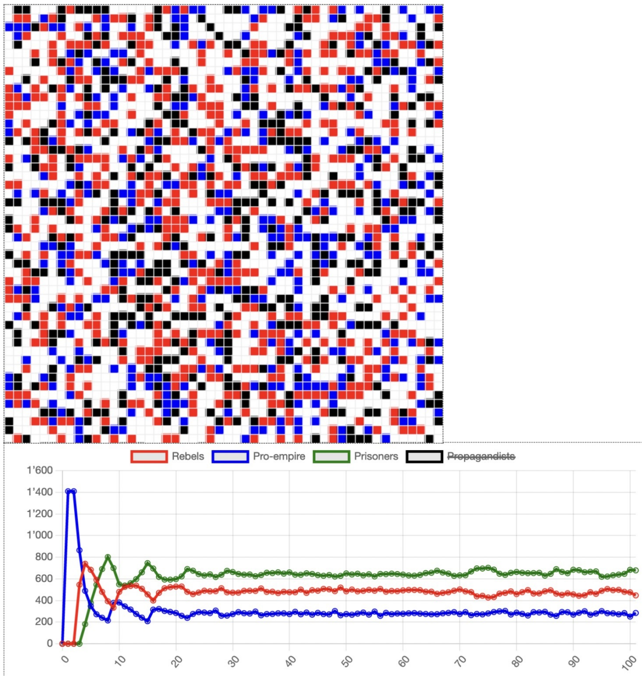

Figure 7: Value of L decreasing to 0, Jail time $J = 5$; x axis: number of steps, y axis: population

As is shown in Fig.7 the rebels outnumber the pro-empire citizens after some initial steps. We can see that the grid has a higher elements. The fraction of rebel is strictly higher than 0.5, hence the revolution happens.

Now we have a broad picture of what is the dependence of the rebellion on the jail time value. We can state that in the case of decreasing legitimacy, the revolution will happen independently from jail time. However, the value of this parameter influence the value of the amount of revolutionary citizen that would exist, hence the brutality of the rebellion. In conclusion, there is dependence in how many rebels you have active at the end but if legitimacy is decreasing no matter how many jail years there are, there will always been at least 50Another outcome that we can highlight from these graphs is that we have a periodicity in time but not in space. One of our goals was to understand if there could have been a convergence to a spatial pattern, however we have understood that this is not the case. On the other hand, we obtained patterns in the time dependence graphs, which we find interesting.

3.3 Limitations of the model

After analyzing the evolution of the model throughout the time, we can state some limitations of the model that we observed.

- We can observe that if the initial conditions are more brutal, more people will be in jail so the fraction of active rebels doesn't increase as in less brutal condition. However one can observe that in a real life scenario, if the rebels overtake, they will also free the jailed ones, which will lead to bigger fluctuations than that we observed. In addition to this, it is not likely that Jails can contain an amount of people that is about half of the entire population.
- Propagandists are always loyal to the empire, and hence their number is constant. If we would like to make it more realistic, we could set a threshold for the value of fraction of rebels and when this threshold is exceeded even propagandists can become rebels.
- In our model the information spreads only locally. This means that the single agent does not have a global picture about how many people are taking action while in real life situation this information can influence the decision to rebel or not.

4 Conclusion

After this study of different scenarios, now we can go back to the scientific questions that motivated our project. This study is motivated by the following scientific questions:

1. There are some macroscopic dynamics that emerge from the interaction between the citizens and propagandists. We did observe periodicity in time with respect to pro-empire and rebel agent densities on the grid.
2. We did not observe convergence to a certain spatial configuration on the grid.
3. In the case in which L is chosen as decreasing, independently from the value of the jail time J, there will always be a revolution. If we keep the value of L fixed, the higher is this value, the more stable will be the empire and hence the smallest will be the percentage of rebels. With this choice for the parameter L, if we set a lower value for the parameter jail time, there will be an higher percentage of revolutionary people. This dynamics can be observed in the case of decreasing value of L but in that case we will end up with at least a moderate revolution while with a fixed value of L the rebellion can be sometimes avoided.
4. The model evolution depends more heavily on the initial conditions related to the values of L and J than the initial fractions of propagandists and citizens.

5. Police brutality, i.e., increasing the number of propagandists and the time of imprisonment won't help dictators control and nor will it ensure the stability of their empires if the legitimacy of their kingdom decreases over time. On the other hand, if the legitimacy is fixed and it is very high through all the steps, an increase in time jail corresponds to the suppression of the riots.

In addition to this, in the time-dependence graphs we observed that when there is a local minimum of the amount of prisoners, there is a maximum number of rebels, and when prisoners are back to the grid there is much rebellion. This type of oscillation might have been approximately reproduced with predator-pray dynamics [3] but we didn't have time to investigate the differential equations and identify their stable solutions. For the moment we can notice that our model is more stable, but this analysis should be developed in depth study.

References

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[3] Érika Diz-Pita and M. Victoria Otero-Espinar. Predator–prey models: A review of some recent advances. *Mathematics*, 9(15):1783, July 2021.