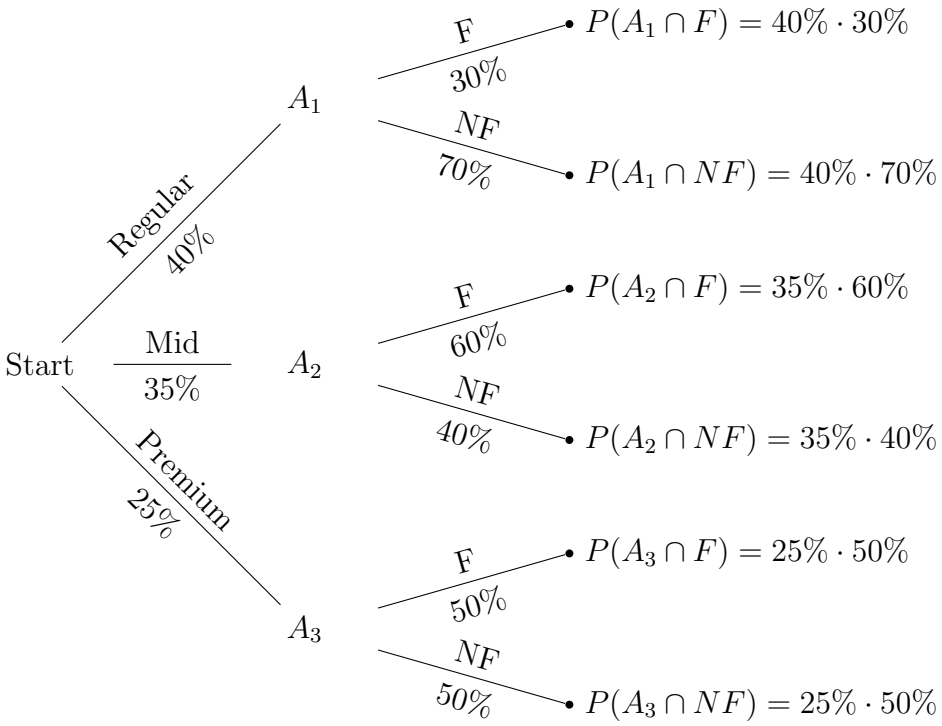
# Vaibhav Beohar - W203 - Section 4 (Fall 2019)
# Statistics for Data Science
# Unit 2 Homework: Probability Theory

September 9, 2019

1. **Gas Station Analytics** At a certain gas station, 40% of customers use regular gas (event R), 35% use mid-grade (event M), and 25% use premium (event P). Of the customers that use regular gas, 30% fill their tanks (Event F). Of the customers that use mid-grade gas, 60% fill their tanks, while of those that use premium, 50% fill their tanks. Assume that each customer is drawn independently from the entire pool of customers.

   (a) What is the probability that the next customer will request regular gas and fill the tank?




   Probability for next customer to request regular gas and fill the tank
   $(A_1 \cap F) = 40\% \cdot 30\% = 0.12$

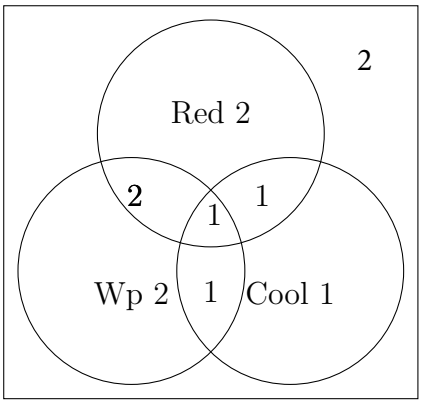
   (b) What is the probability that the next customer will fill the tank?
   $P(A_1 \cap F) + P(A_2 \cap F) + P(A_3 \cap F) = 40\% \cdot 30\% + 35\% \cdot 60\% + 25\% \cdot 30\% = 0.455$

   (c) Given that the next customer fills the tank, what is the conditional probability that they use regular gas?
   $P(A_1|F) = P(A_1 \cap F)/P(F) = 0.12/0.455 = 0.264$

2. **The Toy Bin**

In a collection of toys, 1/2 are red, 1/2 are waterproof, and 1/3 are cool. 1/4 are red and waterproof. 1/6 are red and cool. 1/6 are waterproof and cool. 1/6 are neither red, waterproof, nor cool. Each toy has an equal chance of being selected.

(a) Draw an area diagram to represent these events.




(b) What is the probability of getting a red, waterproof, cool toy?

$P(Wp \cup R \cup C) = P(Wp) + P(R) + P(C) - P(Wp \cap R) - P(R \cap C) - P(Wp \cap C) + P(Wp \cap R \cap C)$

Deriving from this for $P(Wp \cap R \cap C)$ we get
$P(Wp \cap R \cap C) = 1/12 = 0.0833$

(c) You pull out a toy at random and you observe only the color, noting that it is red. Conditional on just this information, what is the probability that the toy is not cool?
$P(C'|R) = P(C' \cap R)/P(R) = [(1/3)/(1/2)] = 2/3 = 0.66$

(d) Given that a randomly selected toy is red or waterproof, what is the probability that it is cool?
$P(C) = P(C \cap R) + P(C \cap Wp) = 1/3 = 0.33$

3. **On the Overlap of Two Events**

Suppose for events A and B, $P(A) = 1/2$, $P(B) = 2/3$, but we have no more information about the events.

(a) What are the maximum and minimum possible values for $P(A \cap B)$?

Maximum value - Suppose, A and B are having some relationship, in which case, the maximum value of $P(A \cap B)$ will be the minimum probability of either A or B, which in this case is $P(A) = 1/2$

Minimum value - In case of disjointed sets $P(A \cap B) = 0$ will be the minimum value $0 <= P(A \cup B) <= 1; therefore, 1.2 >= P(A \cap B) >= 0.2$ This leads to minimum value for $P(A \cap B)$ as 0.2

(b) What are the maximum and minimum possible values for $P(A|B)$?

Maximum value - $P(A|B) = P(A \cap B)/P(B) = 0.5/0.66 = 0.75$

Minimum value - taken from the above minimum value of $P(A \cap B) = 0.2$, we see that $P(A|B) = P(A \cap B)/P(B) = 0.2/0.66 = 0.303$. Therefore minimum value of $P(A|B) = 0.303$

4. **Can't Please Everyone!** Among Berkeley students who have completed w203, 3/4 like statistics. Among Berkeley students who have not completed w203, only 1/4 like statistics. Assume that only 1 out of 100 Berkeley students completes w203. Given that a Berkeley student likes statistics, what is the probability that they have completed w203?

   This is a case of reverse probability using Baye's theorem.
   $P(Complete) = 1/100 = 0.01$, $P(NotComplete) = 0.99$ and therefore, probability of like is aggregate $P(Like) = (0.01 \times 3/4) + (0.99 \times 1/4) = 0.255$

   Therefore, probability of completion given like 0.0294