A native compiler for Coq: current status, perspectives and discussion[1]

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February 19, 2013

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Context

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A \equiv_{\beta\delta\iota\zeta} B}{\Gamma \vdash t : B}\ \text{conv}$$

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Most conversion tests involve very little computation.

But some need a bit more:

```
Lemma red000to106 : reducible_in_range 0 106 the_configs.
Proof. CheckReducible. Qed.
```

Current implementations of the conversion rule

Bruno Barras' reduction machine

- Based on Krivine's Abstract Machine, CBN with sharing
- Well-suited for most tests, when smart comparison matters
- Limited performance for (very) heavy computations

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- Call by value
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Weak reduction

```
# let f x = print_endline "Reducing"; (fun y -> y) x;;
val f : 'a -> 'a = <fun>
# f;;
- : 'a -> 'a = <fun>
# f 42;;
Reducing
- : int = 42
```

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val f : 'a -> 'a = <fun>
# f;;
- : 'a -> 'a = <fun>
# f 42;;
Reducing
- : int = 42
```

Strong reduction

```
Coq < Eval compute in (fun (x:nat) => (fun y => y) x).
      = fun x : nat => x
      : nat -> nat
```

Compiling strong reduction

B. Grégoire and X. Leroy remark that strong reduction can be reduced to iterated weak evaluation on open terms.

In this talk, we will:

1. Show that their work is an instance of Normalization by Evaluation
2. Define another instance that can be encoded in regular OCaml
3. Improve over this first attempt to achieve better performance
4. Extend to full CIC
5. Discuss the current state of the implementation

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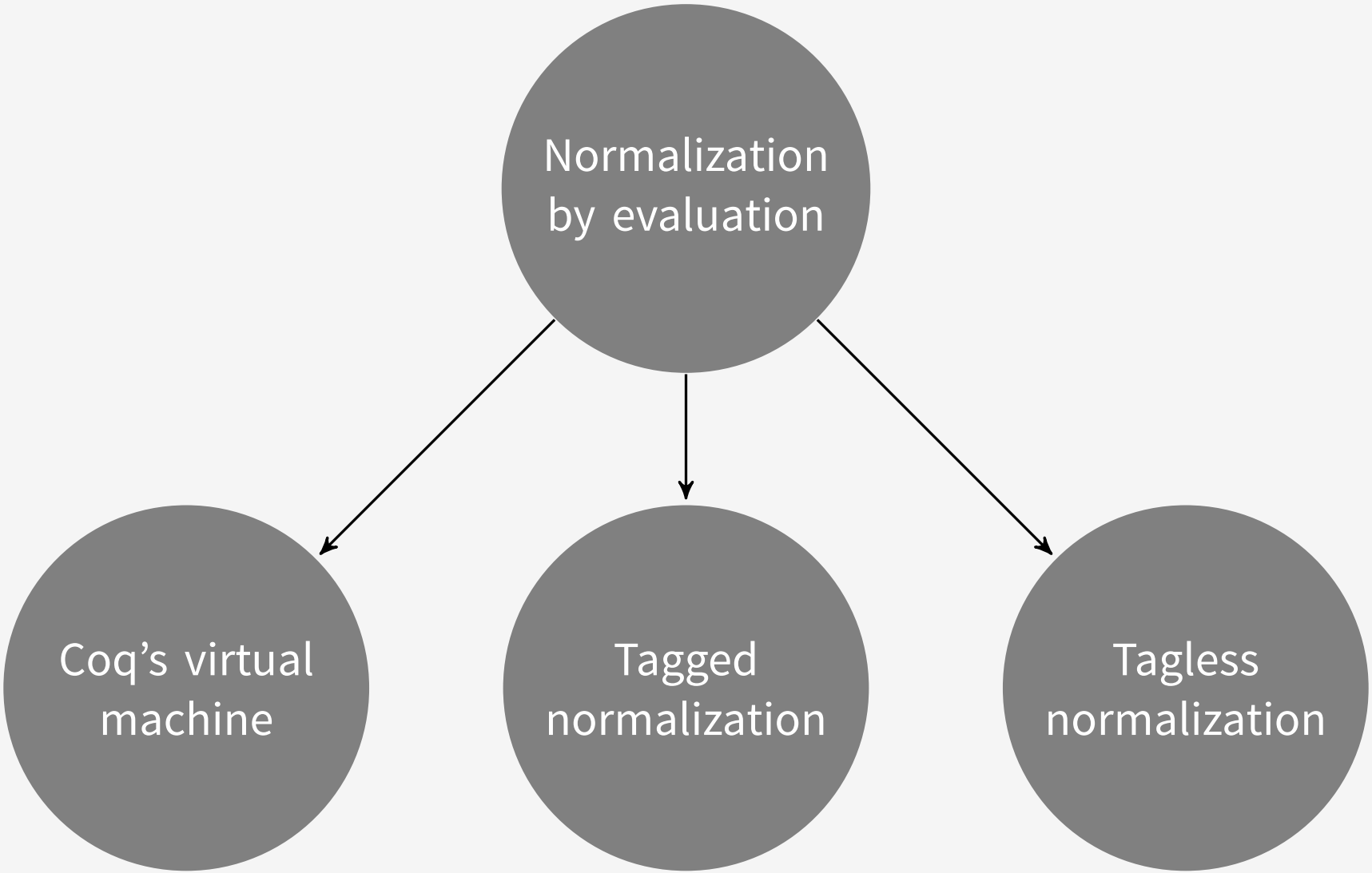
Note: we will focus on normalization, but conversion is easier.

Outline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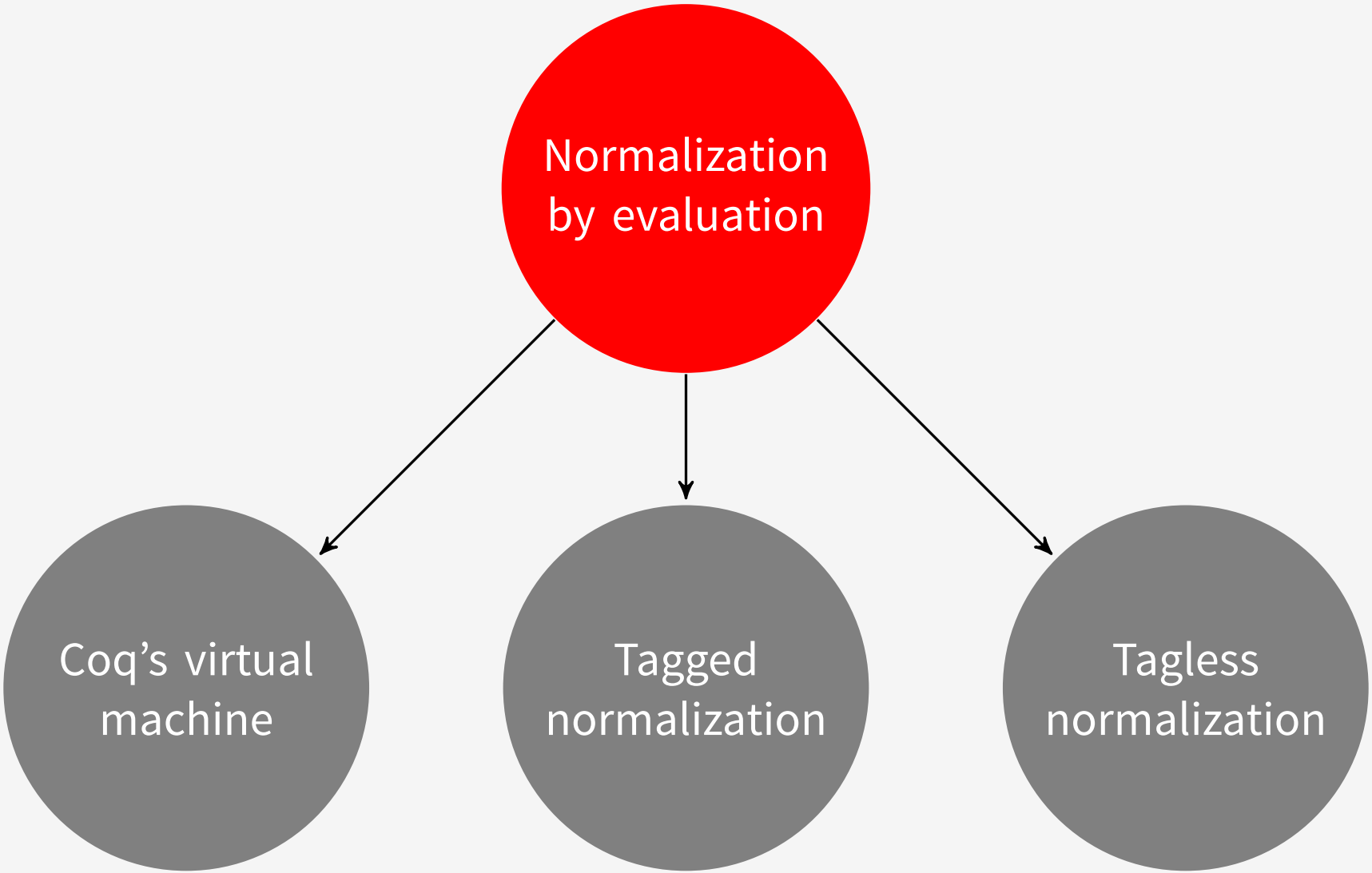
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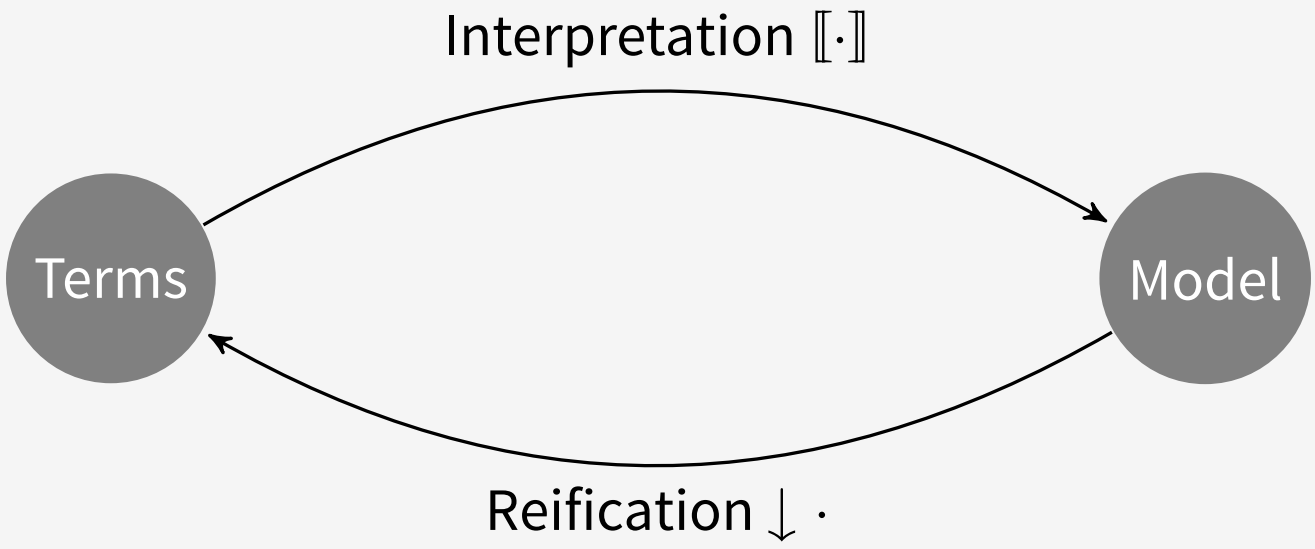
Normalization by evaluation



With:

1. if $t \longrightarrow t'$ then $[\![t]\!] = [\![t']\!]$ (soundness) ;
2. if t is in normal form, then $\downarrow [\![t]\!] = t$ (reproduction).

Model: the calculus of symbolic reduction

$$\mathbf{Term} \ni t ::= x \mid t_1\, t_2 \mid v$$
$$\mathbf{Val} \ni v ::= \lambda x.t \mid [\tilde{x}\, v_1 \ldots v_n]$$

$$(\lambda x.\, t)\, v \to t\{x \leftarrow v\} \qquad (\beta_v)$$
$$[\tilde{x}\, v_1 \ldots v_n]\, v \to [\tilde{x}\, v_1 \ldots v_n\, v] \qquad (\beta_s)$$
$$\Gamma(t) \to \Gamma(t') \quad \text{if } t \to t' \quad (\text{with } \Gamma ::= t\ \square \mid \square\ v) \qquad \text{context}$$

Interpretation and reification

$$[\![\cdot]\!] = \mathcal{V}(\ulcorner\cdot\urcorner)$$

$\ulcorner\cdot\urcorner$: λ-terms $\longrightarrow$ closed symbolic terms

$\mathcal{V}(\cdot)$: weak symbolic reduction

$$\mathcal{N}(t) = \mathcal{R}(\mathcal{V}(\ulcorner t \urcorner))$$

$$\mathcal{R}(\lambda x.t) = \lambda y.\, \mathcal{R}(\mathcal{V}((\lambda x.t)\ [\tilde{y}])) \text{ where } y \text{ is fresh}$$

$$\mathcal{R}[\tilde{x}\, v_1 \dots v_n] = x\, \mathcal{R}(v_1) \dots \mathcal{R}(v_n)$$

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- Implement the β_v and β_s rules
- Distinguish kinds of values
- Apply abstractions
- Retrieve names of variables from accumulators

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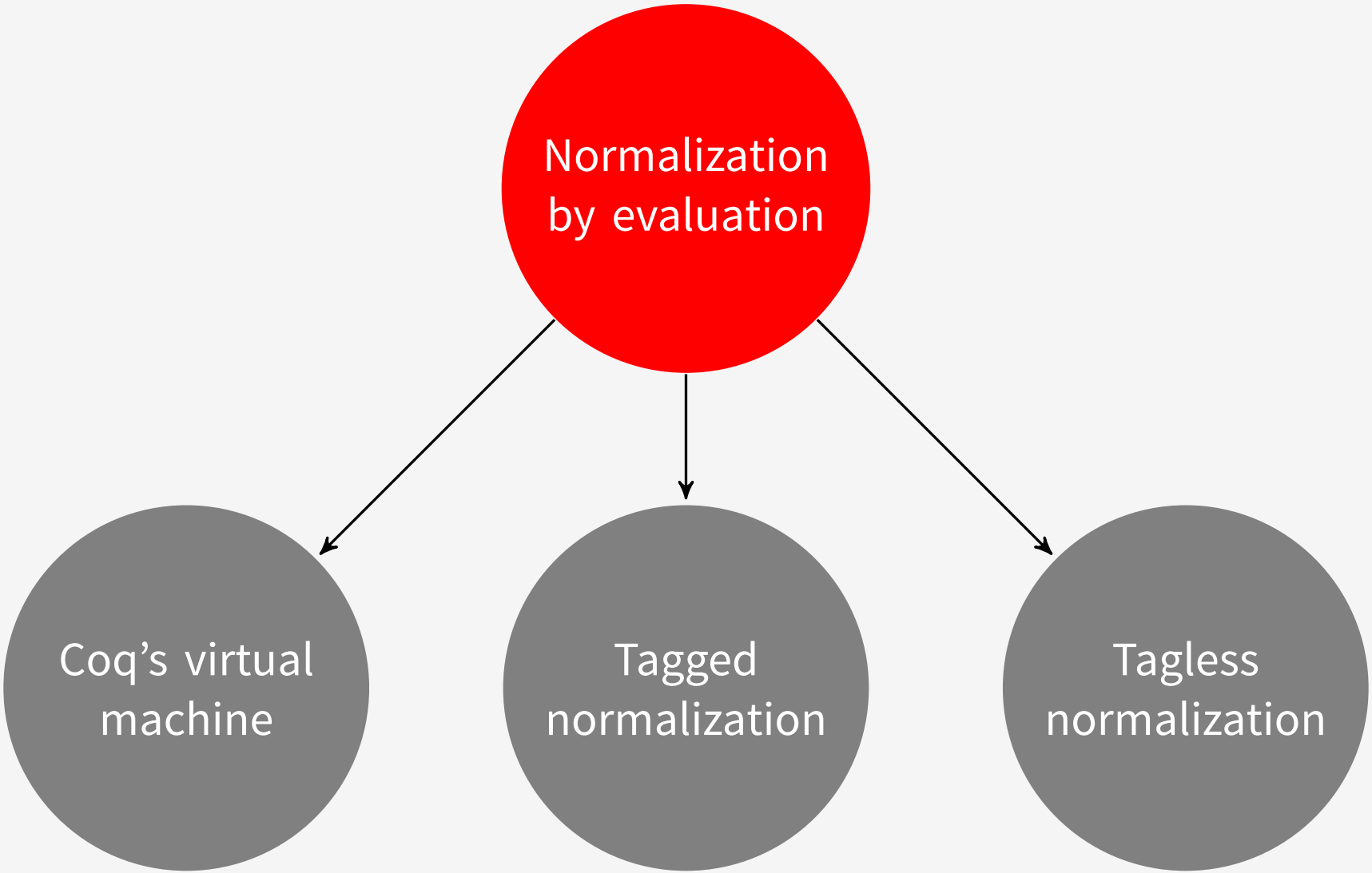
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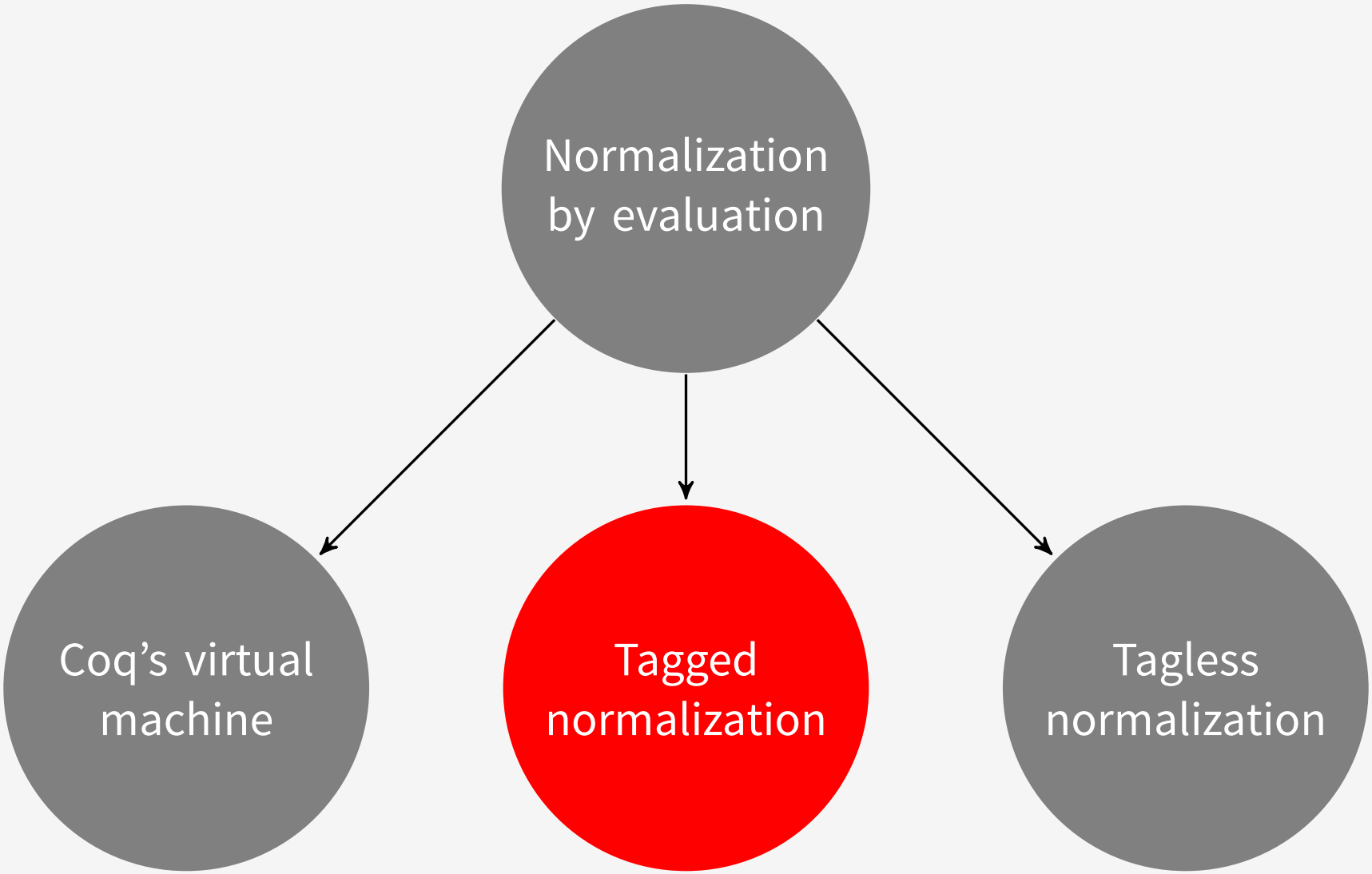
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  | Var of var
type term =
  | Lam of term -> term
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Tagged normalization

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let app t v = match t with
  | Lam f -> f v
  | Accu(a, args) -> Accu(a,v::args)
```

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$$\ulcorner x \urcorner^{B} = \begin{cases} x & \text{if } x \in B \\ \mathsf{Accu}(\mathsf{Var}\, x) & \text{otherwise} \end{cases}$$

$$\ulcorner \lambda x.t \urcorner^{B} = \mathsf{Lam}\,(\mathsf{fun}\, x \to \ulcorner t \urcorner^{B \cup \{x\}})$$

$$\ulcorner t_1\ t_2 \urcorner^{B} = \mathsf{app}\ \ulcorner t_1 \urcorner^{B}\ \ulcorner t_2 \urcorner^{B}$$

Unsatisfactory solution

Pros:

- No need to maintain an ad-hoc compiler and interpreter
- Easily portable

Cons:

- Breaking compiler optimizations has a negative impact on performance
- Efficient interpretation of pattern matching is unclear

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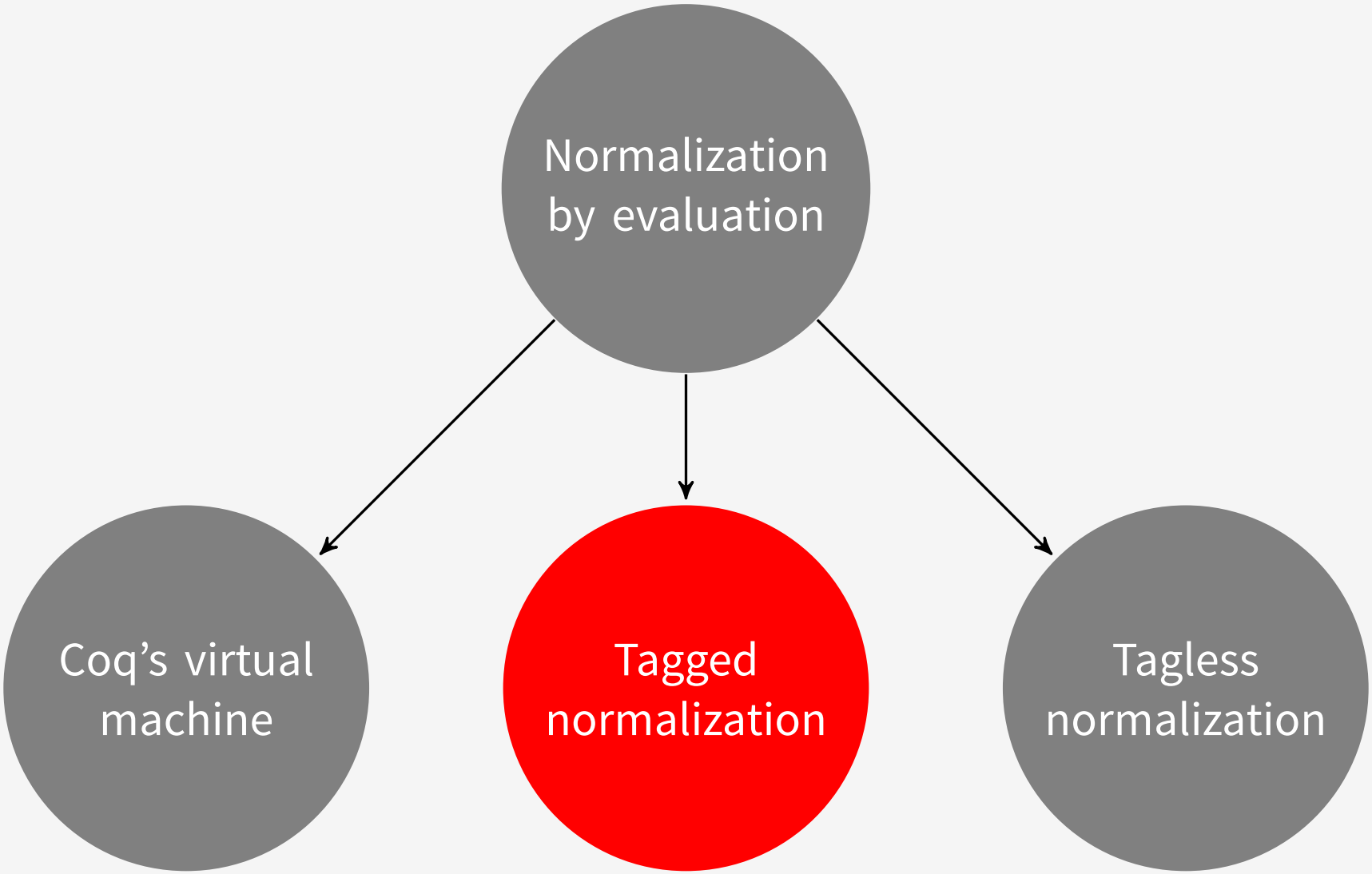
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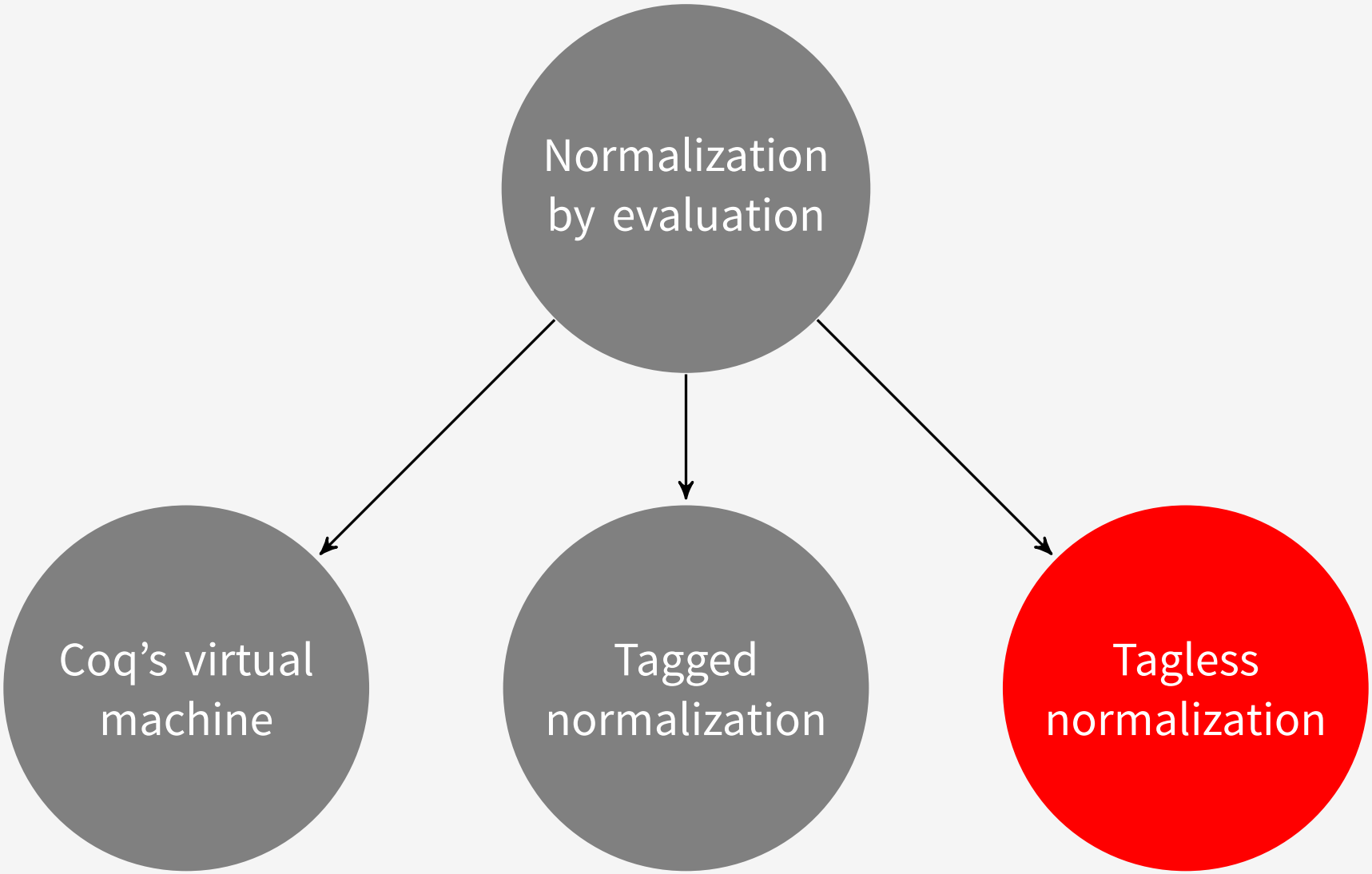
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Getting rid of tags

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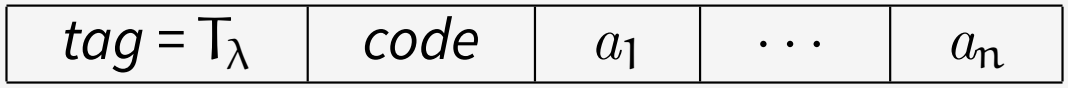

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- Apply abstractions ?
  Using the native application of the language
- Retrieve information from accumulators ?
  Using `Obj.get_field` (we know where the atom and arguments are)

Extension to CIC

Pattern matching

```
Definition is_zero n :=
  match n with
  | O => true
  | S _ => false
  end.
```

Pattern matching

```
let     const_is_zero  x_n_0                    =
  begin match            x_n_0  with


  | Construct_nat_0 ->
                Construct_bool_0
  | Construct_nat_1 _ ->
                Construct_bool_1
  end
```

Pattern matching

```
let rec const_is_zero  x_n_0                          =
  begin match             x_n_0  with
  | Accu_nat _ ->
    mk_sw_accu [...] (cast_accu x_n_0) (pred_is_zero_0)
    const_is_zero
  | Construct_nat_0 ->
                Construct_bool_0
  | Construct_nat_1 _ ->
                Construct_bool_1
  end
```

Pattern matching

```
let rec const_is_zero (x_n_0 : term) =
  begin match Obj.magic (x_n_0) with
  | Accu_nat _ ->
    mk_sw_accu [...] (cast_accu x_n_0) (pred_is_zero_0)
    const_is_zero
  | Construct_nat_0 ->
    (Obj.magic Construct_bool_0 : term)
  | Construct_nat_1 _ ->
    (Obj.magic Construct_bool_1 : term)
  end
```

Fixpoints

```
Fixpoint id (n : nat) :=
  match n with
  | O => O
  | S p => S (id p)
  end.

let norm_id (x_id_1 : term) (x_n_0 : term) =
  case_id x_id_1 x_n_0

let const_id =
  let rec x_id_1 (x_n_0 : term) =
    if is_accu x_n_0 then
      mk_fix_accu [...] norm_id x_n_0
    else
      norm_id x_id_1 x_n_0
  in
  x_id_1
```

Fixpoints (with optimizations)

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    | Construct_nat_0 ->
      (Obj.magic (Construct_nat_0) : term)
    | Construct_nat_1 x_p_2  ->
      (Obj.magic (Construct_nat_1 (x_id_1 x_p_2)) : term)
    end
  in
  x_id_7
```

Other extensions

We have also implemented:

- Coinductive types and cofixpoints
- Machine integers (native-coq branch only)
- Persistent arrays (native-coq branch only)
- Various optimizations (e.g. completely constructed values)

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We have also implemented:

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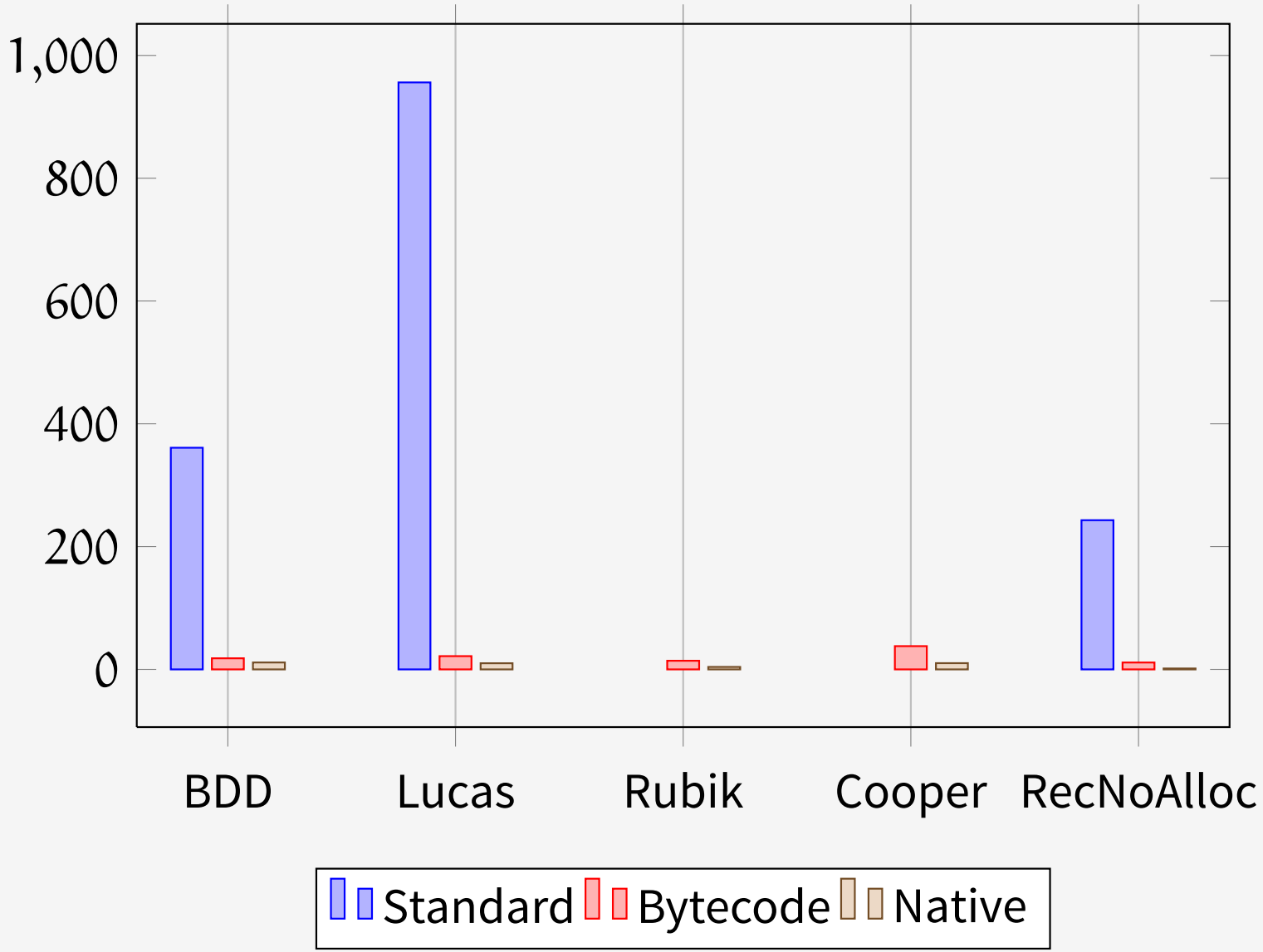
No modification of the runtime system required!

Benchmarks (32 bits)



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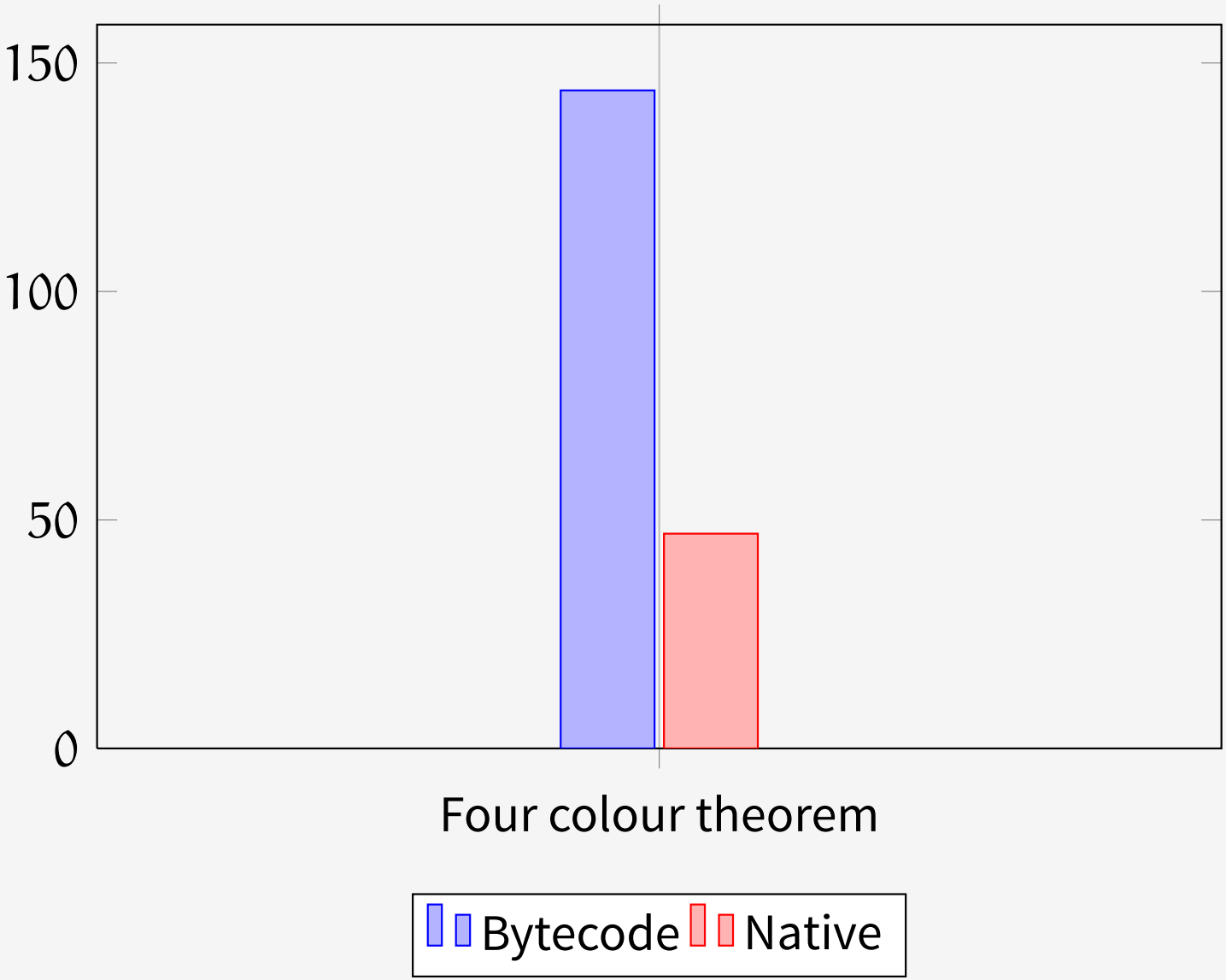

Current state of the implementation

Available as a branch (native-coq) on github since 2011.

Porting it to Coq's trunk in two steps:

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…But due to an apparently unrelated air conditioning problem!

Impact on user contributions

A few contribs have been broken because of the -rectypes flag (fixed).

Some others hit OCaml limitations in code size:

- Nijmegen/QArithSternBrocot (fixed)
- Orsay/FingerTree (fixed)
- Nijmegen/CoRN (fixed, but unification issue)
- Amsterdam/Coalgebras (to be fixed)

Apart from that, things went relatively smoothly.

Rough edges

A few issues still have to be addressed:

- `coq_makefile` should take into account generated files
- The checker should be patched to re-plug native casts to regular conversion.
- Filesystems are a bit polluted by generated files at the moment (move them to a subdirectory?)

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- Performance not far from extracted code
- Full target language available, easily extensible
- Another reason why OCaml is a nice language: low-level access to memory objects

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  - Interfacing with OCaml bytecode compiler
- Formally verify the target compiler

Thank you!

Comments, suggestions, discussion welcome!

Boespflug, Mathieu, Maxime Dénès, and Benjamin Grégoire (2011). “Full Reduction at Full Throttle”. In: *Certified Programs and Proofs*. Vol. 7086. LNCS, pp. 362–377.

Grégoire, Benjamin and Xavier Leroy (2002). “A compiled implementation of strong reduction”. In: *International Conference on Functional Programming 2002*. ACM Press, pp. 235–246.