Gravitation

- The Phenomenon of attraction b/w any two objects in the Universe is called gravitation.
- Gravity — attractive force which is exerted by earth on any body.

Universal law of Gravitation.

m_1 A, m_2 B, r

Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses & inversely proportional to the square of the distance between them.

$F \propto m_1 m_2$

$F \propto \frac{1}{r^2}$

$$\boxed{F = G \times \frac{m_1 m_2}{r^2}}$$

G → Universal gravitational constant.

SI unit of G

$$G = \frac{(F \times r^2)}{m_1 m_2}$$

$$G = \frac{N \times m^2}{Kg^2}$$

$$G = 6.67 \times 10^{-11}\ Nm^2\ Kg^{-2}$$

Universal gravitational Constant

$$F = G \frac{m_1 m_2}{r^2}$$

$$G = \frac{F r^2}{m_1 m_2}$$

$$G = \frac{F \times (1m)^2}{1kg \times 1kg}$$

$$\boxed{G = F}$$

Universal gravitational constant is numerically equal to the gravitational force of attraction between two bodies each of mass 1kg kept at unit distance from each other.

Why gravitational Constant (G) is known as Universal gravitational Constant?

→ The value of G doesn't depend on mass of two bodies, distance b/w two bodies, nature, medium, shape / size.

Conditions

(i) when two objects each of 1kg and 1m apart.

1kg, 1kg, 1m

$$F = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{1 \times 1}{1} = 6.67 \times 10^{-11} N$$

$$= 0.000\ldots\ldots 67 N$$

(ii) ball m_1, m_2 Train, 1m, 2kg, 5000 kg

$$\text{Force of Ball to Train} = G \frac{m_1 m_2}{r^2} = \frac{G\, 2 \times 5000}{(1)^2}$$

$$F = 1000\, G N$$

$$\text{Force of Train to Ball} = \frac{G m_2 m_1}{r^2} = \frac{G\, 5000 \times 2}{(1)^2} = 1000\, G N$$

(iii) 1kg

earth mass = 6×10^{24} kg.

Radius of earth = 6400 km.

$= 6400 \times 1000$ m.

$= 6.4 \times 10^6$ m

$$F_{AE} = \frac{6.67 \times 10^{-11} \times 1 \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = \boxed{9.8 N}$$

F_{EA} = 9.8N

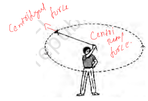
Newton's 2nd law of motion

$F = ma$.

✓ $a_{EA} = \frac{F}{m} = \frac{9.8}{1} = 9.8 ms^{-2}$

✓ $a_{AE} = \frac{F}{m} = \frac{9.8}{6 \times 10^{24} kg} = 1.63 \times 10^{-24} ms^{-2}$

Centrifugal force, Centripetal force.



Earth, moon

Before the thread is released, the stone moves in a circular path with a certain speed and changes direction at every point. The change in direction involves change in velocity or acceleration. The force that causes this acceleration and keeps the body moving along the circular path is acting towards the centre. This force is called the centripetal (meaning 'centre-seeking') force. In the absence of this

The motion of the moon around the earth is due to the centripetal force. The centripetal force is provided by the force of attraction of the earth. If there were no such force, the moon would pursue a uniform straight line motion.

m_1 3 time, m_2 $\frac{1}{2}$ time, ~~r~~ $\frac{1}{3}$ times

$$F_1 = \frac{G m_1 m_2}{r^2}$$

$$F_2 = \frac{G\, 3m_1 \frac{1}{2} m_2}{\left(\frac{r}{3}\right)^2} = \frac{G\, 3 \times 9\, m_1 m_2}{2 \times r^2}$$

$$F_2 = \frac{27}{2} F_1$$

half, 6, 1 cm, $\frac{1}{3}$

1) Two particles A and B of mass m_1 and m_2 respectively are placed on some distance. If the mass of each of the two particles is doubled, keeping the distance b/w them unchanged, the gravitational force b/w them will be.

1) ✓ m_1, m_2, r

$$F_1 = \frac{G m_1 m_2}{r^2}$$

2) $2m_1$, $2m_2$, r

$$F_2 = \frac{G\, 2m_1 2m_2}{r^2}$$

$$F_2 = \frac{G m_1 m_2}{r^2} 4$$

$$\boxed{F_2 = 4F_1}$$

$$F_2 = \frac{F_1}{4}$$

$$F_2 = \frac{G m_1 m_2}{\left(\frac{r}{2}\right)^2}$$

$$F_2 = \frac{r^2}{4}$$

$\frac{1}{3}$, $\frac{r}{3}$

2) mass of sun is 2×10^{30} kg & mass of earth is 6×10^{24} kg. If the avg distance b/w the sun and the earth is 1.5×10^{8} km, Calculate the gravitational force b/w them.

$F = \frac{G m_1 m_2}{r^2}$

3) The Gravitational force b/w two objects is F. If masses of both objects are halved w/o changing distance b/w them, gravitational force would become.

$F_1 = \frac{G m_1 m_2}{d^2}$

$F_2 = \frac{G\left(\frac{m_1}{2}\right)\left(\frac{m_2}{2}\right)}{d^2} = \frac{G m_1 m_2}{d^2 \, 4} = \frac{F_1}{4}$

$F_2 = \frac{F_1}{4}$

4) Gravitational force b/w two objects weighing 20 kg and 15 kg is is 150×10^{-10} N2. Find distance b/w two objects.

Scientists → Arrangement of planets

Ptolemy $\xrightarrow[\text{140 AD}]{}$ Geocentric theory — Earth is at center

Copernicus $\xrightarrow[\text{1543}]{}$ Heliocentric theory — Sun at the center — & all planets revolve around

Tycho brahe $\xrightarrow[\text{1548}]{}$ Tychonic system — Moon is at the center.

Galileo $\xrightarrow{1609}$ Sun is at the center

Heliocentric theory

↳ telescope

Kepler's law of planetary motion

He stated three laws which govern the motion of planets around the sun. These are known as Kepler's law of planetary motion.

Circle – 1 center

1 Focus (radius are equal)

Ellipse → 2 center

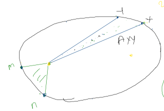
Focus, Foci

Kepler's 1st law of motion

– Planets move in elliptical orbits around the sun, with the sun at one of two foci of the elliptical orbit.

Kepler's 2nd law of motion

2 m/s



$F = \frac{G m_1 m_2}{R^2}$

✓ Each planet revolves around the sun in such a way that the line joining the planet to the sun sweeps over equal areas in equal intervals of time.

✓ A planet moves faster when it is closer to the sun & moves slowly when it is further away from the sun.

Kepler's 3rd law of motion

✓ The cube of the mean distance of a planet from the sun is directly proportional to the square of time, it takes to move around the sun.

$r^3 \propto T^2$

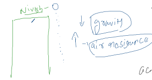
$r^3 = \text{constant}\ T^2$

$r^3/T^2 = \text{constant}$

r = mean distance of planet from the sun.

T = time period of the planet around the sun.

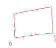
Free Fall and acceleration due to gravity (g)



The falling of a body (or object) from a height towards the earth under gravitational force of earth (with no other force acting on it), such a body is called Freely Falling body.

acc^n = change in velocity

Force



$F = ma$

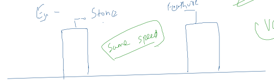
Acc^n due to gravity (g)

→ The uniform acc^n produced in a freely falling body due to gravitational force of the earth is known as acc^n due to gravity.

→ SI unit = $m s^{-2}$

→ g doesn't depend on the mass of the falling object.

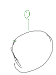
Eg –



Calculation of acc^n due to gravity

→ Force exerted by earth on the body

$F = GMm/R^2$ – (i)



→ $F = ma$ – (ii)

This force by earth produces acc^n in the stone due to which stone moves downwards

$GMm/R^2 = m \times a$

$a = \frac{GM\cancel{m}}{\cancel{m} \cdot R^2}$

Here, $a = \frac{GM}{R^2}$.

$\boxed{g = \frac{GM}{R^2}}$

$g = \frac{(6.67 \times 10^{-11}\ N m^2 kg^{-2} \times 6 \times 10^{24}\ kg)}{(6.4 \times 10^6)^2}$

$\boxed{g = 9.8\ m s^{-2}}$

Variation of acc^n due to gravity (g)

poles

- Earth is not a perfect sphere.
- Geoid shape.

✓ poles are flat.

✓ Equator are bulged.

$g = \frac{GM}{R^2}$

Radius pole < Radius of equator

✓ The value of g is not constant at all the places on the surface of earth.

✓ As earth is not a perfect sphere so the value of Radius R is not the same at all the places on its surfaces.

✓ Radius of earth at the poles is minimum the value of g is maximum.

✓ " " equator is maximum " " " minimum.

✓ But, this formula is not applicable at any point inside the surface of earth.

✓ The value of g decreases as we go down inside the earth.

✓ Value of g becomes zero at the center of earth.

Equation of Motion for freely falling bodies

Equation of motion → for horizontal motion

$V = u + at$

$S = ut + \frac{1}{2}at^2$

$v^2 - u^2 = 2as$

Motion due to gravity

$V = u + gt$

$h = ut + \frac{1}{2}gt^2$

$v^2 - u^2 = 2gh$

horizontal motion

+ve accn

$u = 5 m/s$, $V = 10 m/s$.

$a = \frac{V-u}{t} = \frac{10-5}{2} = 2.5 ms^{-2}$

Retardation

$u = 10 m/s$, $V = 5 m/s$

$a = \frac{V-u}{t} = \frac{5-10}{2} = -2.5 ms^{-2}$

✓ $a = \frac{V-u}{t} = +ve$

✓ $a = \frac{V-u}{t} = \frac{0-n}{t} = -ve$

✓ when a body is falling vertically downwards, it's velocity is increasing. So the accn due to gravity is taken as positive

$g = +9.8 ms^{-2}$.

✓ " " thrown " upwards, " " " negative decreasing, $g = -9.8 ms^{-2}$

1Q. To estimate the height of the bridge over a river, a stone is dropped freely in the river from the bridge. The stone takes 2 seconds to touch the water surface in the river. Calculate the height of the bridge from water level.

$u = 0$

$t = 2 sec.$

$g = 9.8$

$h = ut + \frac{1}{2}gt^2$

$h = 0 \times t + \frac{1}{2} \times 9.8 \times 2 \times 2$

$\boxed{h = 19.6 m}$

Example 10.2 A car falls off a ledge and drops to the ground in 0.5 s. Let $g = 10 m s^{-2}$ (for simplifying the calculations).
(i) What is its speed on striking the ground?
(ii) What is its average speed during the 0.5 s?
(iii) How high is the ledge from the ground?

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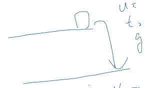

$u = 0$

$t = 0.5 s$

$g = +10 ms^{-2}$

(i) $V = U + gt$

$= 0 + 0.5 \times 10$

$= 5 ms^{-1}$

(ii) Avg = $\frac{\text{Initial veloc + Final velocity}}{2} = \frac{0+5}{2} = 2.5 ms^{-1}$

(iii) $h = ut + \frac{1}{2}gt^2$

$h = 0 \times t + \frac{1}{2} \times 10 \times (0.5)^2$

$= \frac{1}{2} \times 10 \times \frac{5}{10} \times \frac{5}{10} = 1.25 m$

Example 10.3 An object is thrown vertically upwards and rises to a height of 10 m. Calculate (i) the velocity with which the object was thrown upwards and (ii) the time taken by the object to reach the highest point.

$s = 10 m$

$V = 0 ms^{-1}$

$g = -9.8 ms^{-2}$ (Upward motion)

$V^2 - U^2 = 2gh$.

$(0)^2 - (U)^2 = 2 \times -9.8 \times 10$.

$U^2 = 196$

$U = 14$

(ii) $V = U + gt$

$0 = 14 - 9.8 \times t$

$t = \frac{14}{9.8} = \frac{140}{98} = 1.4 s$

Q A stone is dropped from a height of 20m

a) How long will it take to reach the ground.

b) What will be its speed when it hits the ground. (g = 10)

→ $h = ut + \frac{1}{2}gt^2$

$20 = 0 + \frac{1}{2} \times 10 \times t^2$

$t^2 = \frac{20 \times 2}{5}$, $t = 2 sec$

b) $V = U + gt$

$= 0 + 10 \times 2 = 20 ms^{-1}$

Q A stone is thrown vertically upwards with a speed of 20 ms^{-1}. How high it go before it begins to fall?