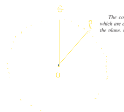
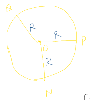
Ch-10 circles



The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

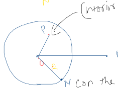
O - Fixed point

OP, OQ ... - Constant distance



O - Fixed point (Centre)

OP - Radius.



(Interior point)

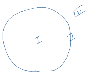
M (exterior point)

N (on the circle).

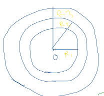
OM > R

ON = R

OP < R.



→ A circle divides a plane into 3 parts :
- Interior, on the circle & exterior.

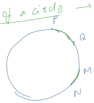


Cocentric circles

→ Radius different

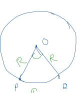
→ Centre same.

Arc of a circle → Continuous curves



PQ arc

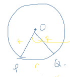
MN arc.



→ Arc containing central angle Minor arc

→ Arc excluding central angle are called Major arc.

central angle



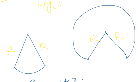
central angle

minor arc

Sector

→ Two radii

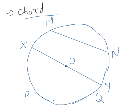
→ one arc.



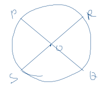
minor sector.

major sector.

→ chord

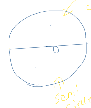


→ line joining any two points in a circle is called chord.



Diameter - chord passing through centre.

→ Semi circle



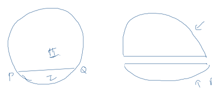
Semi circle

→ Diameter divides circle into two equal parts.

→ Each part is called semi circle.

→ Semi circle's end is diameter end.

→ Segment → chord + arc



Major segment

Minor segment

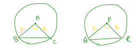
EXERCISE 10.1

1. Fill in the blanks:
   (i) The centre of a circle lies in ___interior___ of the circle. (exterior/ interior)
   (ii) A point, whose distance from the centre of a circle is greater than its radius lies in ___exterior___ of the circle. (exterior/ interior)
   (iii) The longest chord of a circle is a ___diameter___ of the circle.
   (iv) An arc is a ___semi circle___ when its ends are the ends of a diameter.
   (v) Segment of a circle is the region between an arc and ___chord___ of the circle.
   (vi) A circle divides the plane, on which it lies, in ___3___ parts.

2. Write True or False: Give reasons for your answers.
   (i) Line segment joining the centre to any point on the circle is a radius of the circle. ✓
   (ii) A circle has only finite number of equal chords. ✗
   (iii) If a circle is divided into three equal arcs, each is a major arc. ✗
   (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle. ✓
   (v) Sector is the region between the chord and its corresponding arc. ✗
   (vi) A circle is a plane figure. ✓

EXERCISE 10.2

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.



Given - BC = QR.
AB = AC = PQ = PR (Radii of congruent circles)

To prove :- ∠A = ∠P

Proof :- In △ABC and △PQR.
AB = PQ
AC = PR (Given) {Radii of congruent circles}
BC = QR (given)
△ABC ≅ △PQR (SSS rule).
∠BAC = ∠QPR (CPCT)
∠A = ∠P

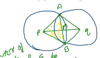
2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

To prove :- BC = QR.

Given - ∠A = ∠P

Proof :- In △ABC and △PQR
AB = PQ
AC = PR {Radii of congruent circles}
∠BAC = ∠QPR (given).
△ABC ≅ △PQR (SAS rule)
BC = QR (CPCT)

3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.



To prove :- AM = BM
∠3 = ∠4 = 90°

let P be center of 1st circle & Q be 2nd

In △APQ and △BPQ.
AP = BP (Radii of same circle)
AQ = BQ (")
PQ = PQ (common)
∴ △APQ ≅ △BPQ (SSS rule)
∠1 = ∠2 (CPCT)

Now in △APM and △BPM
AP = BP (Radii of same circle)
∠1 = ∠2 (proved above)
PM = PM
∴ △APM ≅ △BPM (SAS rule)
AM = BM (CPCT)
→ PQ bisects AB
∠AMP = ∠BMP
∠3 = ∠4 (CPCT) —(i)

Now, ∠3 + ∠4 = 180° (linear pair)
∠3 + ∠3 = 180° (From (i))
2∠3 = 180°
∠3 = 90°
∠3 = ∠4 = 90°

EXERCISE 10.3

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

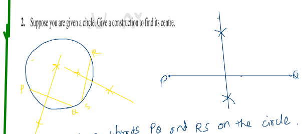
— No point in common

— 1 point in common

— 2 point in common.

Maximum

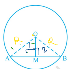
2. Suppose you are given a circle. Give a construction to find its centre.



- Take two chords PQ and RS on the circle.
- Draw ⊥r bisector of PQ and RS.
- Intersection point of ⊥ bisector of PQ and RS gives required centre.

Theorem 10.3 : *The perpendicular from the centre of a circle to a chord bisects the chord.*

Given - O is centre of circle,
$\angle 1 = \angle 2$ (90°)
AB is a chord.



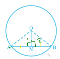
To proof: AM = BM

Proof :- In ΔOAM and ΔOBM
OA = OB [Radii of the same circle]
$\angle 1 = \angle 2$ (each 90°)
OM = OM (common)
$\Delta OAM \cong \Delta OBM$ (RHS rule)
AM = BM (CPCT)

Converse

Theorem 10.4 : *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*

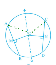
Given : M is midpoint.



Proof :- In ΔOAM and ΔOBM
OA = OB (Radii of same circle)
OM = OM (common)
AM = BM (M is midpoint)
$\therefore \Delta OAM \cong \Delta OBM$ (SSS rule)
$\angle 1 = \angle 2$ (CPCT)
$\angle 1 + \angle 2 = 180°$ (linear pair)
$\angle 1 + \angle 1 = 180°$
$2\angle 1 = 180°$
$\boxed{\angle 1 = 90°}$ $\boxed{\angle 2 = 90°}$

Theorem 10.6 : *Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).*

Given :- AB = CD
To prove :- OM = ON
Const :- $OM \perp AB$ and $ON \perp CD$

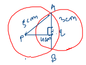


Proof :- AB = CD (given)
$\frac{1}{2}AB = \frac{1}{2}CD$ (⊥r drawn from the centre bisects the chord)
AM = CN

In ΔOAM and ΔOCN
OM = ON (Radii of same circle)
$\angle M = \angle N$ (90°)
AM = CN (proved above)
$\therefore \Delta OAM \cong \Delta OCN$ (SAS rule)
OM = ON (CPCT)

EXERCISE 10.4

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.



let P be center of 1st circle &
Q be " 2nd circle.

In ΔAPQ,
$(5)^2 = (3)^2 + (4)^2$
$25 = 9 + 16$
$25 = 25$

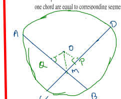
As it satisfies Pythagoras thm,
$\therefore$ we can conclude that ΔAQP is a right angle Δ.

$\angle Q = 90°$

Since P is the centre of 1st circle & AB is the chord, &
therefore PQ is ⊥r drawn from center of circle bisects the chord.
$\therefore$ AQ = BQ

AB = 2 × AQ
= 2 × 3
= 6 cm.

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.



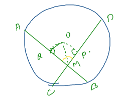
Given : AB = CD.
To prove : AM = DM
CM = BM.

Const : Draw OQ and OP ⊥r from centre O.

In ΔOQM and ΔOPM
OQ = OP (equal chords are equidistant from the centre).
$\angle Q = \angle P$ (90°)
OM = OM (Common)
$\therefore \Delta OQM \cong \Delta OPM$ (RHS rule)
QM = PM (CPCT) — (1)
AB = CD (given)
$\frac{1}{2}AB = \frac{1}{2}CD$
AQ = PD — (2)

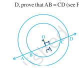
Add (1) and (2)
AQ + QM = PM + PD.
$\boxed{AM = DM}$ — (3)
AB = CD — (4)
Subtract (3) from (4)
AB − AM = CD − DM.
$\boxed{BM = CM}$

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



$\angle QMO = \angle PMO$

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 10.25).

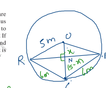


Const :- ⊥r line OM.

$OM \perp BC$, $OM \perp AD$
BM = CM (⊥r from the center to the chord bisect chord.) — (i)
AM = MD — (ii) (")

Subtract (1) from (2)
AM − BM = MD − CM
$\boxed{AB = CD}$

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?



In ΔRNS,
$(RS)^2 = (RN)^2 + (SN)^2$
$(6)^2 = RN^2 + (5-x)^2$
$36 = RN^2 + 25 + x^2 - 10x$
$RN^2 = 36 - 25 - x^2 + 10x$
$(RN)^2 = 11 - x^2 + 10x$ — (1)

In ΔRON,
$(OR)^2 = (ON)^2 + (RN)^2$ from (1)
$(5)^2 = (x)^2 + (RN)^2$
$25 = x^2 + 11 - x^2 + 10x$
$25 - 11 = 10x$ $\boxed{x = 1.4m}$

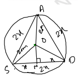
Now putting x in (1) eqn
$(RN)^2 = 11 - (1.4)^2 + 10 \times 1.4$
$= 11 - 1.96 + 14$
$(RN)^2 = 25 - 1.96$
$RN = \sqrt{23.04}$
RN = 4.8m

RM = 2 × RN (⊥r drawn from center of circle bisect chord)
RM = 2 × 4.8
= 9.6 m.

4.8
4 | 23.04
16
88 | 704
704
×

6
88
× 8
704

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.



Const - Draw OM ⊥ to SD

In Δ SOM, using Pyth thm

$(OS)^2 = (OM)^2 + (SM)^2$

$(20)^2 = (x)^2 + (OM)^2$ — (1)

In Δ ASM, using Pyth thm

$(AS)^2 = (AM)^2 + (SM)^2$

$(2x)^2 = (AM)^2 + (x)^2$

$4x^2 - x^2 = AM^2$

$3x^2 = AM^2$

$AM = x\sqrt{3}$

$OM = AM - AO$

$OM = x\sqrt{3} - 20$

Now putting OM in (1) eqn

$(20)^2 = (x)^2 + (x\sqrt{3} - 20)^2$

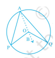
$\cancel{400} = x^2 + 3x^2 + \cancel{400} - 40\sqrt{3}x$

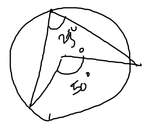
$40\sqrt{3}\cancel{x} = 4x^{\cancel{2}}$

$x = \frac{\cancel{40}\sqrt{3}}{\cancel{4}} = 10\sqrt{3}$ m

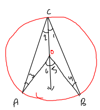
length of string $= 2 \times 10\sqrt{3}$

$= 20\sqrt{3}$ m





Theorem 10.8 : *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*



To prove : $\angle AOB = 2\angle ACB$

proof : In Δ OCB

$\angle 5 = \angle 1 + \angle 4$ [Sum of two interior angle equal to exterior angle] — (i)

In Δ OCA.

$\angle 6 = \angle 2 + \angle 3$ (") — (ii)

In Δ OCB,

OC = OB (Radii of same circle)

$\angle 1 = \angle 4$ (Angle opposite to equal sides are equal) — (iii)

In Δ OAC,

OC = OA (Radii of same circle)

$\angle 2 = \angle 3$ (Angle oppo to equal sides are equal) — (iv)

Add (i) and (ii)

$\angle 5 + \angle 6 = \angle 1 + \angle 4 + \angle 2 + \angle 3$

$\angle AOB = \angle 1 + \angle 1 + \angle 2 + \angle 2$

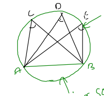
($\angle 1 = \angle 4$ — (iii)

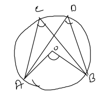
$\angle 2 = \angle 3$ — (iv))

$\angle AOB = 2(\angle 1 + \angle 2)$

$\boxed{\angle AOB = 2\angle ACB}$

Theorem 10.9 : *Angles in the same segment of a circle are equal.*





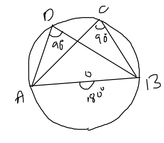
To prove : $\angle ACB = \angle ADB$.

Proof : $\angle AOB = 2\angle ACB$ (Thm 10.8) — (i)

$\angle AOB = 2\angle ADB$ (" — (ii))

$\cancel{2}\angle ACB = \cancel{2}\angle ADB$

Thm - Angle in a semi circle is right angle.

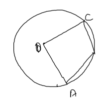


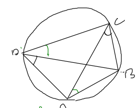
$\angle AOB = 2\angle ACB$ (Thm 10.8).

$180° = 2\angle ACB$

$\boxed{\angle ACB = 90°}$

Theorem 10.11 : *The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.*





Quad Sum property

$\angle D + \angle C + \angle B + \angle A = 360°$

$180° + \angle C + \angle A = 360°$

$\boxed{\angle A + \angle C = 180°}$

To Prove : $\angle ADC + \angle ABC = 180°$

$\angle DAB + \angle DCB = 180°$ (i)

$\angle ACB = \angle ADB$ (angles in the same segment)

$\angle BAC = \angle BDC$ (") (ii)

Add (i) and (ii)

$\angle ACB + \angle BAC = \angle ADB + \angle BDC$

$\angle ACB + \angle BAC = \angle ADC$

Adding $\angle ABC$ both sides

$\angle ACB + \angle BAC + \angle ABC = \angle ADC + \angle ABC$

angle sum property.

$\boxed{180° = \angle ADC + \angle ABC}$

$180° = \angle D + \angle B$

EXERCISE 10.5

1. In Fig. 10.36, A,B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.

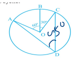
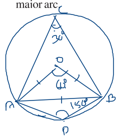


Fig. 10.36

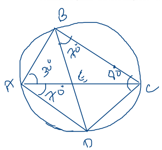
$\angle AOC = \angle AOB + \angle BOC$
$= 60° + 30°$
$= 90°$

$2\angle ADC = \angle AOC$ (.....)
$\angle ADC = 45°$

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.



$\angle DAC = \angle DBC$ (angles in same segm.)

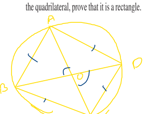
$\angle BAD = \angle BAC + \angle DAC$
$= 30° + 70°$
$= 100°$

$\therefore$ ABCD is cyclic quadr
$\angle BAD + \angle BCD = 180°$
$100° + \angle BCD = 180°$
$\angle BCD = 80°$

Given, AB = BC
$\angle BAC = \angle BCA$.
$30° = \angle BCA$

$\angle ECD = \angle BCD - \angle BCA$
$= 80° - 30°$
$= 50°$

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.



In ΔAOB and ΔCOD
OB = OD (Radii of same circle)
AO = CO (").
$\angle BOA = \angle COD$ (v.o.A).
$\therefore \Delta AOB \cong \Delta COD$ (SAS.)
AB = CD (cpct)

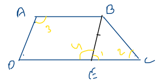
Similarly, $\Delta BOC \cong \Delta AOD$ (SAS).
BC = AD (cpct).

As BD is a diameter, $\angle BOD = 180°$
$\angle BAD = 90°$ (Angle in the semi circle)

$\therefore$ ABCD is a rectangle

Imp

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.



Const - Draw line BE ∥ AD.
BE = AD.

AD = BE (by const). - (i)
AD = BC (Given) - (ii)
From (i) and (ii)
BE = BC
In ΔBEC, $\angle 1 = \angle 2$ (Angle oppo to equal side sides) (iii)

As ABDE is a llgm,
$\angle 3 = \angle 4$ (opp angles of llgm)
$\angle 1 + \angle 4 = 180°$ (linear pair) (iv)
$\angle 2 + \angle 3 = 180°$

From (iii) & (iv) ($\angle 1 = \angle 2$ & $\angle 4 = \angle 3$)
$\therefore$ It is cyclic.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that $\angle ACP = \angle QCD$.

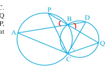
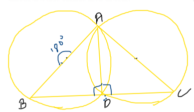


Fig. 10.40

$\angle ACP = \angle ABP$ (Angles in the same segment) - (i)
$\angle QCD = \angle QBD$ (") - (ii)
$\angle ABP = \angle QBD$ (v.o.A) - (iii)

$\boxed{\angle ACP = \angle QCD}$

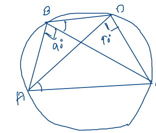
10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.



Given : AB and AC are diameter.

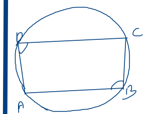
AD is a common chord.
$\angle ADB = 90°$ (angle in semi circle)
$\angle ADC = 90°$ (")
$\angle ADB + \angle ADC = 90° + 90°$
$= 180°$
but its a linear pair.
BDC is a line.
$\therefore$ BC passes through point D.
$\therefore$ point of Intersection D lies on third side BC.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.



$\angle CAD = \angle CBD$ (Angle in same segment)

12. Prove that a cyclic parallelogram is a rectangle.



ABCD is a llgm
$\angle B = \angle D$ - (i)
but ABCD is cyclic llgm
$\angle B + \angle D = 180°$
$\angle B + \angle B = 180°$
$2\angle B = 180°$
$\angle B = 90°$
$\therefore$ ABCD is a rectangle.