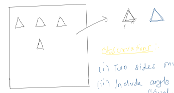
Triangles

i) Congruence – 2 NCERT Science book identical (same in shape, size)

Dairy Milk chocolate

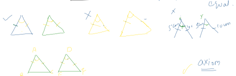
Activity for congruence for Δ



Observations:

(i) Two sides must be equal.

(ii) Include angle will be equal.



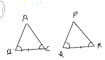
axiom

ΔBAC ≅ ΔEDF (SAS rule)

ΔCAB ≅ ΔFDE

Theorem

ASA



Given: ∠B = ∠Q, BC = QR, ∠C = ∠R

To prove: ΔABC ≅ ΔPQR

Proof: (i) Let AB = PQ

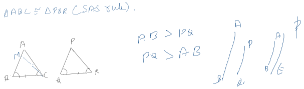
AB = PQ (construction)

∠B = ∠Q (given)

BC = QR (given)

ΔABC ≅ ΔPQR (SAS rule).

(ii)



AB > PQ

PQ > AB

If AB > PQ,

In ΔMBC & ΔPQR,

MB = PQ (by construction)

∠B = ∠Q (Given)

BC = QR (given)

ΔMBC ≅ ΔPQR (SAS rule).

∠MCB = ∠R (CPCT)

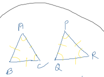
∠C = ∠R (Given)

∠MCB = ∠C

but this is not possible, it's possible only when point M tends to poin A.

BM = BA

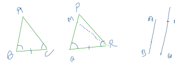
BA = QP



ΔABC ≅ ΔPQR

CPCT (Corresponding parts of congruent triangles)

(iii)



If AB < PQ,

ΔABC and ΔMQR,

AB = MQ (By construction)

∠B = ∠Q (Given).

BC = QR (Given)

ΔABC ≅ ΔMQR (SAS rule).

∠ACB = ∠MRQ (CPCT)

∠C = ∠MRQ (CPCT)

∠C = ∠R (Given)

∠MRQ = ∠R.

This is not possible, only possible when point M tends to point P.

then, MQ = PQ

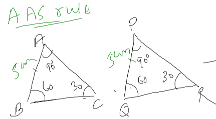
$\boxed{PQ = AB}$

Rules

(i) SAS rule (axiom).

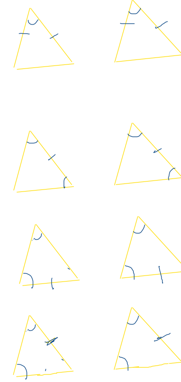
(ii) ASA rule (Thm – 3 cases)

(iii) AAS rule



If two angles are equal, 3rd angle will be equal of Δ's

ΔABC ≅ ΔPQR (AAS rule)



① = SAS rule

= ASA rule

= AAS rule

= AAS rule

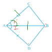
ΔABC ≅ ΔDEF

CPCT

AB = DE, BC = EF, AC = DF

∠ACB = ∠DFE, ∠CAB = ∠FDE

1)



Given :- AC = AD
∠1 = ∠2,

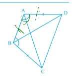
To proof :- ΔABC ≅ ΔABD

Proof :- In ΔABC & ΔABD
AC = AD (given)
AB = AB (common side)
∠CAB = ∠DAB (given)
ΔABC ≅ ΔABD (SAS rule)

BC = BD (CPCT)

---

2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig. 7.17). Prove that
(i) Δ ABD ≅ Δ BAC
(ii) BD = AC
(iii) ∠ABD = ∠BAC.



Given :- AD = BC, ∠DAB = ∠CBA.

To prove :- (i) ΔABD ≅ ΔBAC.
(ii) BD = AC
(iii) ∠ABD = ∠BAC

Proof :- In ΔABD & ΔBAC
AD = BC (given)
∠DAB = ∠CBA (given)
AB = BA (Common)

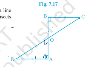
∴ ΔABD ≅ ΔBAC (SAS rule)

BD = AC (CPCT)

∠ABD = ∠BAC (CPCT)

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

Fig. 7.17



Given :- AD = BC, ∠A = ∠B = 90°.

To prove :- CD bisects AB (OA = OB)

Proof :- In ΔOAD and ΔOBC,
AD = BC (given)
∠OAD = ∠OBC (90° each)
∠AOD = ∠BOC (V.O.A)

ΔOAD ≅ ΔOBC (AAS rule)

OA = OB (CPCT) ∴ CD bisects AB.

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (see Fig. 7.19). Show that Δ ABC ≅ Δ CDA.

Fig. 7.18

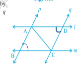


Fig. 7.19

Given :- *l* ∥ *m*, *p* ∥ *q*.

Show :- ΔABC ≅ ΔCDA

Proof :- *l* ∥ *m* (AD ∥ BC) *p* ∥ *q* (AB ∥ CD)
∴ ABCD is a ∥gm.

In ΔABC & ΔCDA

AB = CD (opp. sides of ∥gm.)
BC = DA (opp sides of ∥gm)
∠B = ∠D (" angle of ∥gm)

ΔABC ≅ ΔCDA (SAS rule)

5. Line *l* is the bisector of an angle ∠ A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ∠ A (see Fig. 7.20). Show that:
(i) Δ APB ≅ Δ AQB
(ii) BP = BQ or B is equidistant from the arms of ∠ A.

CPCT

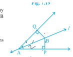
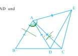


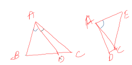
Fig. 7.20

ΔAPB ≅ ΔAQB (AAS rule)

∠1 = ∠2 (given)
∠P = ∠Q (90°)
AB = AB (Common)

6. In Fig. 7.21, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.





Given :- AC = AE, AB = AD, ∠BAD = ∠EAC.

To show :- BC = DE

To proof :- In ΔABC and ΔADE.
AC = AE (given)
AB = AD (given)
∠BAD = ∠EAC (given)

adding ∠DAC both sides.
∠BAD + ∠DAC = ∠EAC + ∠DAC
∠BAC = ∠EAD

ΔABC ≅ ΔADE (SAS rule)
BC = DE (CPCT)

Fig. 7.21

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠ BAD = ∠ ABE and ∠ EPA = ∠ DPB (see Fig. 7.22). Show that
(i) Δ DAP ≅ Δ EBP
(ii) AD = BE

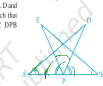


Fig. 7.22

Given :- AP = PB (P is mid point),
∠BAD = ∠ABE, & ∠EPA = ∠DPB.

To show :- (i) ΔDAP ≅ ΔEBP,
(ii) AD = BE

Proof :- In ΔDAP and ΔEBP
∠PAD = ∠PBE (Given ∠BAD = ∠ABE)
AP = BP (P is mid point)

∠EPA = ∠DPB (given).
Add ∠EPD both sides.

∠EPA + ∠EPD = ∠DPB + ∠EPD

∠DPA = ∠EPB

∴ ΔDAP ≅ ΔEBP (ASA rule)

AD = BE (CPCT)

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:
(i) Δ AMC ≅ Δ BMD
(ii) ∠ DBC is a right angle.
(iii) Δ DBC ≅ Δ ACB
(iv) $CM = \frac{1}{2} AB$

Fig. 7.22

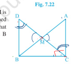


Fig. 7.23

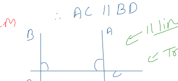
Given :- ∠C = 90°, BM = AM, DM = CM

To show :- (i) ΔAMC ≅ ΔBMD
(ii) ∠DBC is 90°
(iii) ΔDBC ≅ ΔACB.
(iv) $CM = \frac{1}{2} AB$

Proof :- (i) In ΔAMC and ΔBMD
AM = BM (M is mid point)
CM = DM (Given)
∠CMA = ∠DMB (V.O.A)

ΔAMC ≅ ΔBMD (SAS rule)

AC = BD (CPCT)
∠ACM = ∠BDM (CPCT)
but these are alternate interior angles.
∴ AC ∥ BD



∥ lines
Transversal

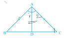
∠DBC + ∠ACB = 180° (sum of interior angles)
∠DBC + 90° = 180°
∠DBC = 90°

(iii) In ΔDBC and ΔACB.
DB = AC (proved above)
∠B = ∠C (each 90°)
BC = CB (Common)
ΔDBC ≅ ΔACB (SAS rule)

(iv) CD = BA (CPCT)
CM + DM = AB
CM + CM = AB (CM = DM given)
2CM = AB
$\boxed{CM = \frac{1}{2} AB}$

Theorem 7.2 : *Angles opposite to equal sides of an isosceles triangle are equal.*

Given :- AB = AC
To prove :- ∠B = ∠C

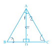


Construction :- AD is angle bisector of ∠A

Sol :- ΔBAD and ΔCAD.
AB = AC (given)
∠1 = ∠2 (By construction)
AD = AD (Common)
ΔBAD ≅ ΔCAD (SAS rule)
∠B = ∠C (CPCT)

Theorem 7.3 : *The sides opposite to equal angles of a triangle are equal.*

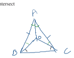
Given : ∠B = ∠C
To prove :- AB = AC
Construct :- AD is angle bisector of ∠A (∠1 = ∠2)



Proof :- ΔBAD and ΔCAD
∠B = ∠C (given)
∠1 = ∠2 (By Construction)
AD = AD (Common).
ΔBAD ≅ ΔCAD (AAS rule)
AB = AC (CPCT)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠ B and ∠ C intersect each other at O. Join A to O. Show that :

(i) OB = OC (ii) AO bisects ∠ A



Given
To show

Sol :- ΔABC, AB = AC
∠B = ∠C (Angle opposite to equal sides are equal.).
$\frac{1}{2}$∠B = $\frac{1}{2}$∠C
∠OBC = ∠OCB

In ΔOBC, ∠OBC = ∠OCB
OB = OC (sides opposite to equal angles are equal.

In ΔAOB and ΔAOC
AB = AC (given)
∠ABO = ∠ACO ($\frac{1}{2}$∠B = $\frac{1}{2}$∠C)
OB = OC (proved above)
∴ ΔAOB ≅ ΔAOC (SAS rule)
∠BAO = ∠CAO (CPCT)
∴ AO bisects ∠A

2. In Δ ABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that Δ ABC is an isosceles triangle in which AB = AC.

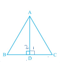


Fig. 7.30

Given :- ∠1 = ∠2 = 90° &
BD = DC.
To prove : AB = AC
Proof :- ΔABD and ΔACD
BD = CD (given)
∠2 = ∠1 (90°)
AD = AD (Common)
ΔABD ≅ ΔACD (SAS rule)
AB = AC (CPCT)
∴ ΔABC is an isosceles Δ

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

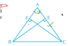
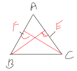


Fig. 7.31

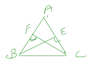


Given :- AB = AC
To prove :- CF = BE
Proof :- In ΔACF and ΔBAE
∠CAF = ∠BAE (Common)
∠F = ∠E (90°)
CA = BA (given)
∴ ΔACF ≅ ΔBAE (AAS rule)
BE = CF (CPCT)

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) Δ ABE ≅ Δ ACF

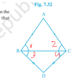
(ii) AB = AC, i.e., ABC is an isosceles triangle.



In ΔABE and ΔACF
→ ∠A = ∠A (Common)
∠E = ∠F (90°)
BE = CF (given)
∴ ΔABE ≅ ΔACF (AAS rule)
AB = AC (CPCT)

Fig. 7.32

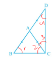
5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ABD = ∠ACD.



5:33

Given :- AB = AC, BD = CD
To show :- ∠ABD = ∠ACD
In ΔABC
In AB = AC (given)
∠1 = ∠2 (Angle Opposite to equal sides are equal) (1)
In ΔDBC
BD = CD (given)
∠3 = ∠4 (" ") (2)
Add (1) and (2)
∠1 + ∠3 = ∠2 + ∠4
∠ABD = ∠ACD

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠ BCD is a right angle.

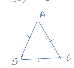


Given :- AB = AC
AB = AD
To prove :- ∠BCD is 90°
Proof :- In ΔABC,
AB = AC (given) – (i)
∠1 = ∠2 (angle opposite to equal sides are equal)
AD = AB – (ii)
From (i) and (ii)
AD = AC
∠3 = ∠4 (" ")

In ΔBCD,
∠B + ∠C + ∠D = 180° (Angle sum property)
∠1 + ∠4 + ∠2 + ∠3 = 180°
(∠1 + ∠4) + ∠2 + ∠3 = 180°
∠2 + ∠3 + ∠2 + ∠3 = 180°
2 (∠2 + ∠3) = 180°
∠2 + ∠3 = 90°
∠DCB = 90°
∴ ∠BCD is a right angle.

7. ABC is a right angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C.
→ Angle Sum property

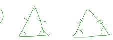
8. Show that the angles of an equilateral triangle are 60° each.



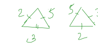
Given :- AB = AC = BC
To prove :- ∠B = ∠C = ∠A = 60°
Proof :- AB = BC
∠B = ∠C (angle opposite to equal sides) – (i)
If BC = AC
∠B = ∠A (") – (ii)
From (i) and (ii)
∠A = ∠B = ∠C
In ΔABC
∠A + ∠B + ∠C = 180° (Angle Sum property)
∠A + ∠A + ∠A = 180°
3∠A = 180°
∠A = 60°, ∠B = ∠C = 60°

(i) SAS rule
(ii) ASA rule
(iii) AAS rule

Thm

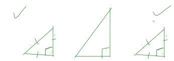


iv)

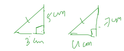


(SSS Rule)

(v)



(RHS rule)
Right angle
Hypotenuse
Side



EXERCISE 7.3

1. Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i) Δ ABD ≅ Δ ACD

(ii) Δ ABP ≅ Δ ACP

(iii) AP bisects ∠ A as well as ∠ D.

(iv) AP is the perpendicular bisector of BC.

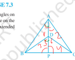
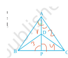


Fig. 7.39

Given :- AB = AC, DB = DC
To prove :-
Proof :- (i) In ΔABD and ΔACD
AB = AC (given)
BD = CD (given)
AD = AD (common)
∴ ΔABD ≅ ΔACD (SSS rule).
∠BAD = ∠CAD (CPCT)
∠1 = ∠2
∴ AP bisects ∠A.

In ΔBPD and ΔCPD
BP = CP (proved above)
DP = DP (Common)
BD = CD (given)
∴ ΔBPD ≅ ΔCPD (SSS rule)
∠BDP = ∠CDP (CPCT)
∠5 = ∠6 (CPCT)
∴ AP bisects ∠D



(ii) In ΔABP and ΔACP
AB = AC (given)
∠1 = ∠2 (proved above)
AP = AP (common side).
∴ ΔABP ≅ ΔACP (SAS rule).
BP = CP (CPCT)
AP bisects BC
∠3 = ∠4 (CPCT)
∠3 + ∠4 = 180° (linear pair)
∠3 + ∠3 = 180°
2∠3 = 180°
∠3 = 90°
∠4 = 90°
∴ AP is ⊥ bisector to BC

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that
(i) AD bisects BC (ii) AD bisects ∠ A.

Given: ΔABC, $AB = AC$

To show: (i) AD bisects BC ($BD = DC$)
(ii) AD bisects $\angle A$ ($\angle 1 = \angle 2$)

Sol: ΔABD and ΔACD

$\angle 3 = \angle 4$ (each $90°$)

$AB = AC$ (Given)

$AD = AD$ (Common)

$\Delta ABD \cong \Delta ACD$ (RHS rule).

$BD = CD$ (CPCT)

$\therefore$ AD bisects BC

$\angle DAB = \angle DAC$ (CPCT)

($\angle 1 = \angle 2$)

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR (see Fig. 7.40). Show that:
(i) Δ ABM ≅ Δ PQN SSS rule
(ii) Δ ABC ≅ Δ PQR

SAS rule.

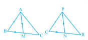
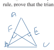


Fig. 7.40

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Given: $BE = CF$

To prove: ΔABC is isosceles.

Proof: ΔBCF and ΔCBE

$CF = BE$ (given)

$\angle BFC = \angle CEB$ ($90°$)

$BC = BC$ (Common)

$\Delta BCF \cong \Delta CBE$ (RHS rule)

$\angle B = \angle C$ (CPCT)

$AB = AC$ (sides opposite to equal angles are equal)

$\therefore \Delta ABC$ is an isosceles Δ.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠ B = ∠ C.

ΔABP and ΔACP,

$AB = AC$ (Common)

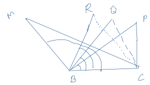
$\angle APB = \angle APC$ ($90°$)

$AP = AP$ (Common)

$\therefore \Delta ABP \cong \Delta ACP$ (RHS rule)

$\angle B = \angle C$ (CPCT)

In Scalene Δ



$\hookrightarrow CM > CR > CQ > CP$

$\angle MBC > \angle RBC > \angle QBC > \angle PBC$

Theorem 7.6 : *If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater).*

Thm
(i) Angle opposite to the longer side is greater.
(ii) Side opposite to the greater angle is longer.

Theorem 7.7 : *In any triangle, the side opposite to the larger (greater) angle is longer.*

1. Show that in a right angled triangle, the hypotenuse is the longest side.

Given: $\angle B = 90°$

To show: AC (hypotenuse) is the longest side.

Proof: $\angle B > \angle A$ ($\angle B = 90°$)

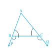
$AC > BC$ —(i) (side opposite to the greater angle is longer)

$\angle B > \angle C$

$AC > AB$ —(ii)

From (i) & (ii) AC is the longest side.

2. In Fig. 7.48, sides AB and AC of Δ ABC are extended to points P and Q respectively. Also, ∠ PBC < ∠ QCB. Show that AC > AB.



Given: $\angle PBC < \angle QCB$.

To show: $AC > AB$.

($\angle ABC > \angle ACB$
$AC > AB$)

Sol: $\angle ABC + \angle PBC = 180°$ (linear Pair)

$\angle ABC = 180° - \angle PBC$ —(i) → $-\angle PBC = \angle ABC - 180°$

$\angle ACB + \angle QCB = 180°$ (linear Pair)

$\angle ACB = 180° - \angle QCB$ —(ii)

$-\angle QCB = \angle ACB - 180°$

$\angle PBC < \angle QCB$ (given)

Multiplying by -1 both sides.

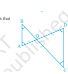
$-\angle PBC > -\angle QCB$

$\angle ABC - 180° > \angle ACB - 180°$

$\angle ABC > ACB$

$AC > AB$

3. In Fig. 7.49, ∠ B < ∠ A and ∠ C < ∠ D. Show that AD < BC.



Given: $\angle B < \angle A$ and $\angle C < \angle D$

To show: $AD < BC$

Proof: $\angle B < \angle A$

$AO < BO$ —(i) (sides opposite to greater angle are longer)

$\angle C < \angle D$

$OD < OC$ —(ii) (")

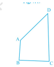
Add (i) & (ii)

$AO + OD < BO + OC$

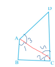
$AD < BC$

V.I.

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50). Show that ∠ A > ∠ C and ∠ B > ∠ D.



Given: AB is smallest, CD is longest.



To show: (i) $\angle A > \angle C$
(ii) $\angle B > \angle D$

Proof-(i) Construction :- Join A to C.

In ΔABC,

AB is smallest,

$BC > AB$

$\angle 1 > \angle 2$ —(i) (Angle opposite to longer side is greater)

In ΔACD,

CD is largest,

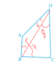
$CD > AD$

$\angle 3 > \angle 4$ —(ii) (")

Add (i) and (ii)

$\angle 1 + \angle 3 > \angle 2 + \angle 4$

$\angle A > > C$



In ΔABD

AB is smallest side,

$AD > AB$

$\angle 5 > \angle 6$ —(i) (Angle opp. to larger side is greater).

In ΔBCD,

CD is largest,

$CD > BC$

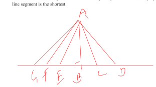
$\angle 7 > \angle 8$ —(ii) (" ")

Add (i) and (ii)

$\angle 5 + \angle 7 > \angle 6 + \angle 8$

$\angle B > \angle 8$

6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.



In ΔAGB.

$\angle A + \angle G + \angle B = 180°$ (Angle sum property)

$\angle A + \angle G = 90°$

$\angle G < \angle B$

$AB < AG$ ($\because$ AB is smallest)

V.I.

5. In Fig 7.51, PR > PQ and PS bisects ∠ QPR. Prove that ∠ PSR > ∠ PSQ.

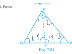


Fig. 7.51

Given: $PR > PQ$, $\angle 1 = \angle 2$

To prove: $\angle PSR > \angle PSQ$

$\angle 4 > \angle 5$

Proof: $\angle 1 = \angle 2$ (given)

In ΔPQR

$PR > PQ$

$\angle 6 > \angle 3$ —(i) (Angle opposite to longer side is greater).

In ΔPSR

$\angle 5 = \angle 2 + \angle 3$ (exterior angle property) —(ii)

In ΔPSQ

$\angle 4 = \angle 1 + \angle 6$ (")

$\angle 4 = \angle 2 + \angle 6$ ($\angle 1 = \angle 2$ given) —(iii)

$\angle 6 > \angle 3$ —(i)

$\angle 5 = \angle 2 + \angle 3$ —(ii)

$\angle 4 = \angle 2 + \angle 6$ —(iii)

$5 = 2 + 3$

$4 = 2 + 6$

From (i), (ii) & (iii)

$\angle 4 > \angle 5$