Quadrilaterals

point → A point is that which determines location

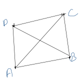
→ line segment

→ line.

Collinear point – Points lying on same line.

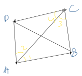
Quadrilaterals

Any enclosed figure made with 4 non-collinear points is known as quadrilateral



Vertices – A, B, C, D
Sides – AB, BC, CD, DA.
Angles – $\angle A, \angle B, \angle C, \angle D$
Diagonals – AC and BD

Thm **8.2 Angle Sum Property of a Quadrilateral**



Given: ABCD is a quadrilateral

To prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

Proof: In ΔABC

$\angle 1 + \angle B + \angle 3 = 180^\circ$ (Angle sum Property of a Δ) – (i)

In ΔADC

$\angle 2 + \angle D + \angle 4 = 180^\circ$ (") – (ii)

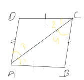
$(\angle 1 + \angle B + \angle 3) + (\angle 2 + \angle D + \angle 4) = 360^\circ$

$(\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^\circ$

$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$

Theorem 8.1 : *A diagonal of a parallelogram divides it into two congruent triangles.*

||gm – Quadrilateral whose opposite sides are ||.



Given – ||gm ABCD.

To prove: $\Delta ABC \cong \Delta ADC$

Proof: In ΔABC and ΔADC

AB || DC, AD || BC, AC is a transversal.

$\angle 1 = \angle 2$ (alter interior angles)

$\angle 4 = \angle 3$ (")

AC = AC (Common)

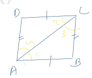
$\therefore \Delta ABC \cong \Delta ADC$ (ASA rule).

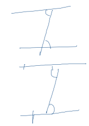
Theorem 8.2 : *In a parallelogram, opposite sides are equal.*

$\Delta ABC \cong \Delta DCA$

AB = DC (CPCT)
AD = BC

Theorem 8.3 : *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*





Given: quadrilateral ABCD,
AB = DC,
AD = BC

To prove: ABCD is a ||gm

Constr: Join A to C

Proof: In ΔABC and ΔCDA

AB = DC (given)
BC = AD (given)
AC = AC (Common)

$\therefore \Delta ABC \cong \Delta CDA$ (SSS rule)

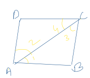
$\angle 1 = \angle 2$ (CPCT)
$\angle BAC = \angle DCA$
$\angle 3 = \angle 4$ (CPCT)

but these are alternate interior angles

$\therefore$ AB || DC
AD || BC

Which implies ABCD is a ||gm.

Theorem 8.4 : *In a parallelogram, opposite angles are equal.*



Given: ABCD is a ||gm, AB || DC
& AD || BC

To proof: $\angle A = \angle C$ and
$\angle B = \angle D$

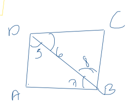
Proof: AB || CD and AC is transversal
$\angle 1 = \angle 4$ – (i) (alt interior angles)

AD || BC and AC is transversal
$\angle 2 = \angle 3$ – (ii) (alt interior angles).

Add (i) and (ii)

$\angle 1 + \angle 2 = \angle 4 + \angle 3$

$\boxed{\angle A = \angle C}$



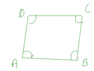
$\angle 5 = \angle 8$ (alt interior)

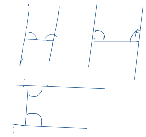
$\angle 6 = \angle 7$ (")

add

$\boxed{\angle D = \angle B}$

Theorem 8.5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*





Given: $\angle A = \angle C$ and
$\angle B = \angle D$

To proof: ABCD is a ||gm

Proof: $\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Angle sum property of a quadrilateral)

$\angle A + \angle B + \angle A + \angle B = 360^\circ$

$2\angle A + 2\angle B = 360^\circ$

$\angle A + \angle B = 180^\circ$

but these are co-interior angles

$\therefore$ AD || BC – (i)

Similarly

$\angle A + \angle B = 180^\circ$

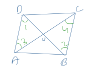
$\angle A + \angle D = 180^\circ$

$\therefore$ AB || CD – (ii)

$\therefore$ From (i) and (ii)

ABCD is a ||gm

Theorem 8.6 : *The diagonals of a parallelogram bisect each other.*



Given: ||gm ABCD.

To prove: AO = CO, OB = OD

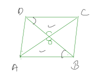
Proof: In ΔAOD and ΔCOB

AD = BC (opposite sides of ||gm)
$\angle 1 = \angle 2$ (alt interior angles)
$\angle 3 = \angle 4$ (")

$\therefore \Delta AOD \cong \Delta COB$ (ASA rule)

AO = CO (CPCT)
OD = OB (CPCT)

Theorem 8.7 : *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*



Given: ABCD is a quadrilateral.
AO = CO, OB = OD.

To prove: ABCD is a ||gm

Proof: In ΔAOB and ΔCOD

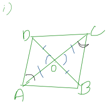
AO = CO (given)
$\angle AOB = \angle COD$ (V.O.A).
OB = OD (given)

$\therefore \Delta AOB \cong \Delta COD$ (SAS)

$\angle ABO = \angle CDO$ (CPCT)

but these are alt interior angles.

AB || DC – (i)



Now, In ΔAOD and ΔBOC

AO = CO (given).
$\angle AOD = \angle BOC$ (V.O.A).
OD = OB (given).

$\therefore \Delta AOD \cong \Delta BOC$ (SAS rule)

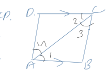
$\angle DAO = \angle BCO$ (CPCT)

but these are alt interior angles

$\therefore$ AD || BC – (ii)

$\therefore$ From (i) and (ii)
ABCD is a ||gm

Theorem 8.8 : *A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.*



Given: In a quadrilateral ABCD.
AB || DC, AB = DC

To proof: ABCD is a ||gm.
AD || BC,

Const: Join A to C

Proof: In ΔABC and ΔCDA.

AB = CD (given)
$\angle 1 = \angle 2$ (alt. interior angles)
AC = AC (Common)

$\therefore \Delta ABC \cong \Delta CDA$ (SAS)

$\angle 3 = \angle 4$ (CPCT)

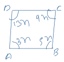
but these are alt. interior angles

$\therefore$ AD || BC

$\therefore$ ABCD is a ||gm

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

let the angles are 3n, 5n, 9n and 13n.



∠A + ∠B + ∠C + ∠D = 360°

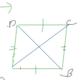
(Angle sum property of quadrilateral)

3n + 5n + 9n + 13n = 360°
30n = 360
n = 12

∠A = 36°
∠B = 60°
∠C = 108°
∠D = 156°

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Given: ABCD is a ||gm,
AC = BD.



To prove: ABCD is a rectangle

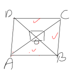
oppo sides equal & 1 angle 90°

proof:

In ΔABD and ΔBAC
AD = BC (oppos sides of ||gm are equal)
AB = BA (common)
BD = AC (Given)
∴ ΔABD ≅ ΔBAC (SSS rule)
∠BAD = ∠ABC (CPCT)
∠BAD + ∠ABC = 180° (co interior angles).
∠BAD + ∠BAD = 180°
2∠BAD = 180°
$\boxed{\angle BAD = 90}$
∴ ABCD is a rectangle

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Given: OA = OC, OB = OD,
Diagonal bisect at right angle

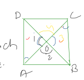


To prove: ABCD is a rhombus,
AB = BC = CD = AD

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Given: ABCD is a Square.

To prove: (i) AC = BD.
(ii) AC and BD bisect each other at right angle.



Proof:

In ΔABD and ΔBAC
AD = BC (Given)
AB = AB (Common)
∠A = ∠B (90°)
∴ ΔABD ≅ ΔBAC (SAS rule)
AC = BD (CPCT)

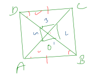
(ii) In ΔAOD and ΔCOB
AD = BC (given)
∠OAD = ∠OCB (alt interi angles)
∠AOD = ∠COB (V.O.A)
∴ ΔAOD ≅ ΔCOB (AAS rule)
OD = OB } (CPCT)
OA = OC }

In ΔAOD and ΔAOB
AD = AB (Given)
AO = AO (Common)
OD = OB (proved above)
ΔAOD ≅ ΔAOB (SSS rule), ∠AOD = ∠AOB (CPCT)

∠1 = ∠2
∠1 + ∠2 = 180° (linear pair)
∠1 = ∠2 = 90° = ∠3 = ∠4

5) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Given: AC = BD,
OD = OB, OA = OC.
∠1 = ∠2 = ∠3 = ∠4 = 90°

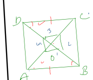


To prove: ABCD is a square,
AB = BC = CD = AD, ∠A = ∠B = ∠C = ∠D = 90°

Proof: In ΔAOB and ΔCOD
OA = OC (Given)
OB = OD
∠AOB = ∠COD (V.O.A)
ΔAOB ≅ ΔCOD (SAS rule)
AB = CD (CPCT)
∠OBA = ∠ODC (CPCT)
but these are alternate interior angles
AB || CD
ABCD is a ||gm (oppo. sides equal and parallel).
BC = AD -(i) AB = CD -(ii)

In ΔAOB and ΔBOC
OA = OC (Given)
∠AOB = ∠BOC (each 90°)
OB = OB (Common)
∴ ΔAOB ≅ ΔBOC (SAS rule)
AB = BC (CPCT) - (iii)

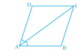
From (i), (ii) & (iii)
$\boxed{AB = CD = AD = BC}$



In ΔABD and ΔBAC
AD = BC (oppo sides of ||gm)
AB = BA (Common)
BD = AC (given)
∴ ΔABD ≅ ΔBAC (SSS rule)
∠A = ∠B (CPCT) -(V)

∠A + ∠B = 180° (co-interior angles)
∠A + ∠A = 180°
2∠A = 180°
∠A = 90°
∴ ABCD is a square.

6. Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig. 8.19). Show that
(i) it bisects ∠C also,
(ii) ABCD is a rhombus.



6–10

Fig. 8.21

11. In Δ ABC and Δ DEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22). Show that
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD ∥ CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) Δ ABC ≅ Δ DEF.

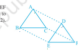


Fig. 8.22

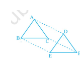
Given: AB = DE, AB || DE,
BC = EF, BC || EF.

(i) In quad ADED, AB = DE, also AB || DE (given)
∴ ABED is a ||gm (oppo side are equal and ||)

(ii) In quad BEFC, BC = EF, BE || EF (given)
∴ BEFC is a ||gm (oppo sides are equal & ||)

(iii) If ABED is a ||gm,
AD || BE -(ii) AD = BE -(iv)
BEFC is a ||gm,
CF || BE -(V) CF = BE (vi)

From (iii) and (v)
AD || CF
From (iv) and (vi)
AD = CF



(iv) In quad ACFD,
AD || CF and AD = CF (proved above)
∴ ACFD is a ||gm

(V) AC = DF (oppo sides of ||gm ACFD.

(vi) In ΔABC and ΔDEF.
AB = DE
BC = EF (Given)
AC = DF (proved above)
∴ ΔABC ≅ ΔDEF (SSS rule)

12. ABCD is a trapezium in which AB ∥ CD and AD = BC (see Fig. 8.23). Show that

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) Δ ABC ≅ Δ BAD

(iv) diagonal AC = diagonal BD

[*Hint*: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

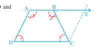
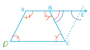


Fig. 8.23

Given :- ABCD is a trapezium, AB ∥ CD, AD = BC

Const :- extend AB to E, Join C to E.
CE ∥ DA and CE = DA



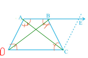
Sol :- CE ∥ DA, & AE is transversal
∠A + ∠E = 180° – (i) (co interior angles)
∠CBA + ∠CBE = 180° – (ii) (linear pair)

As CE = DA and CE ∥ DA
∴ AECD is a llgm / opp sides equal and ∥).
AD = CE (oppo sides of llgm) – (iii)
AD = BC (given) – (iv)
From (iii) & (iv)
∴ BC = CE
∴ ∠CBE = ∠CEB (angle oppo to equal sides)
∠CBA + ∠CBE = 180° (using (ii))
∠CBA + ∠CEB = 180° (vi)
∠A + ∠E = 180° – (using (i))
∠A + ∠E = ∠CEB + ∠CBA.
∠A + ~~∠E~~ = ~~∠E~~ + ∠B

$\boxed{\angle A = \angle B}$

(ii) ABCD is a trapezium, AB ∥ CD, AD is transversal
∠A + ∠D = 180° (co-interior angles)
AB ∥ CD, BC is transversal,
∠B + ∠C = 180° (co-interior angles)

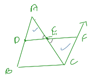
~~∠A~~ + ∠D = ~~∠B~~ + ∠C. (∠A = ∠B proved above)
∠D = ∠C



(iii)
In ΔABC and ΔBAD
BC = AD (given) ∠A = ∠B (proved above) ΔABC ≅ ΔBAD
AB = AB (common) (SAS rule)
(iv) AC = BD (CPCT)

Theorem 8.9 : *The line segment joining the mid-points of two sides of a triangle is parallel to the third side.*

Given :- ΔABC, D is midpoint of AB, E is midpoint of AC



To prove :- DE ∥ BC, DE = $\frac{1}{2}$ BC

Const :- CF ∥ AB

Proof :- In ΔAED and ΔCEF.
∠AED = ∠CEF (VOA)
∠DAE = ∠ECF (alt interior angles)
AE = EC (E is mid point)
∴ ΔAED ≅ ΔCEF (ASA rule).
DE = EF (CPCT)
AD = CF
but AD = DB (D is midpoint).
∴ DB = CF also DB ∥ CF
∴ DBCF is a llgm (∵ oppo sides equal and ∥)

$\boxed{\therefore BC \parallel DE}$

also BC = DF
BC = DE + EF
BC = DE + DE
BC = 2DE
$\boxed{DE = \frac{1}{2} BC}$

EXERCISE 8.2

1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that :

(i) SR ∥ AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

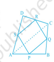


Fig. 8.29

Given :- ABCD is a quad. P, Q, R and S are mid points.
AC is diagonal.

To prove (i), (ii), (iii)

Proof :- In ΔDAC,
S is midpoint of AD,
R is midpoint of DC
SR ∥ AC. SR = $\frac{1}{2}$AC (By mid point theorem) – (i)

(ii) In ΔBAC,
P is midpoint of AB
Q is midpoint of BC
PQ ∥ AC, PQ = $\frac{1}{2}$AC (By MPT) – (ii)

From (i) and (ii)
SR ∥ PQ, SR = PQ
∴ PQRS is a llgm (A pair of opposite side is equal and ∥)

If D & E are mid points,
DE ∥ BC, DE = $\frac{1}{2}$BC

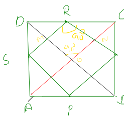
DE ∥ BC
DE = $\frac{1}{2}$ BC

Converse

If D is mid point & DE ∥ BC
→ E is mid point

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

PQRS → similar to 1st qn
is a llgm



Const :- Join D to B
as (SR ∥ AC)

In ΔDBC,
Q is the midpoint of BC
R is the " of DC
QR ∥ BD (Using MPT)
∴ RN ∥ MO

SR ∥ AC (proved above)
∴ MR ∥ ON

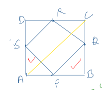
∴ MORN is a llgm (opposite sides of a quadrilateral are ∥).

ABCD is a rhombus (given)
rhombus diagonal bisect each other at 90°

∠O = 90°
∠O = ∠R (opp angles of llgm)
∴ PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Given :- ABCD is a ▭
To prove :- PQRS is a rhombus.
all sides equal & diagonal bisect each other at 90°.



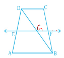
In ΔSAP and ΔQBP
AP = BP (P is mid point)
∠SAP = ∠QBP (90°)
AD = BC (oppo sides of rectangle)
$\frac{1}{2}$AD = $\frac{1}{2}$BC
AS = BQ
∴ ΔSAP ≅ ΔQBP (SAS rule)
SP = QP (CPCT)
∴ PQRS is a rhombus

Proof :- PQRS is a llgm –

In ΔADC,
S is mid pt AD
R is " DC
∴ SR ∥ AC, SR = $\frac{1}{2}$AC

In ΔABC
P is mid point of AB
Q is " of BC
∴ PQ ∥ AC, PQ = $\frac{1}{2}$AC (MPT)

4. ABCD is a trapezium in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.



Given :- AB ∥ DC, E is the mid point of AD, EF ∥ AB

To show :- F is midpoint of BC

Sol :- In ΔDAB, E is the mid point of AD,
and EG ∥ AB (as EF ∥ AB)
∴ G is midpoint by converse of MPT

In ΔBCD, G is midpoint of BD (proved above)
If, AB ∥ DC
and AB ∥ EF,
∴ DC ∥ EF,
DC ∥ GF
∴ F is midpoint by converse of MPT.

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.31). Show that the line segments AF and EC trisect the diagonal BD.

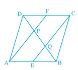
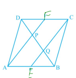


Fig. 8.31

Given :- ABCD is a ∥gm, E is midpoint of AB &
F is midpoint of CD.

To prove :- DP = PQ = QB

Sol :- ABCD is a ∥gm,
If AB ∥ DC, AB = DC
then, AE ∥ FC $\frac{1}{2}$AB = $\frac{1}{2}$DC
AE = FC



∴ AECF is a ∥gm (a pair of oppo. sides are equal and ∥).

⇒ EC ∥ AF

In ΔDQC, F is mid point of DC,
FP ∥ CQ
∴ P is midpoint of DQ (by converse of MPT)

$\boxed{DP = PQ}$ — (i)

In ΔABP, E is midpoint of side AB, EQ ∥ AP
∴ Q is mid point of AD (by converse MPT)

$\boxed{PQ = QB}$ — (ii)

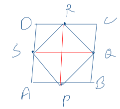
From (i) and (ii)

$\boxed{DP = PQ = BQ}$

~~Fig. 8.31~~

6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Given :- ABCD is a quad,
P, Q, R and S are
midpoints



To prove :- PR and QS
bisect each other

Proof :- From 1st question, PQRS is a ∥gm
∴ Diagonals of ∥gm bisect each other
∴ PR and QS bisect each other

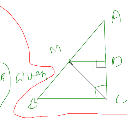
7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC (ii) MD ⊥ AC

(iii) CM = MA = $\frac{1}{2}$AB

Given

To show

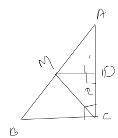


Proof :- (i) In ΔABC,
M is midpoint of AB
& MD ∥ BC } Given

∴ D is mid point of AC (by converse of MPT)

(ii) MD ∥ BC (given)
∴ ∠D = ∠C (corresponding angles)
$\boxed{\angle D = 90^\circ}$ ∴ MD ⊥ AC
∴ ∠1 = ∠2 = 90°

(iii)



In ΔMDA and ΔMDC
AD = DC (D is midpoint)
∠1 = ∠2 (90°)
MD = MD (common)
∴ ΔMDA ≅ ΔMDC (SAS rule)
AM = CM (CPCT)
AM = $\frac{1}{2}$AB (M is mid point)
∴ CM = $\frac{1}{2}$AB

$\boxed{AM = CM = \frac{1}{2}AB}$