Quadrilaterals

point → A point is that which determines location

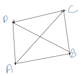
→ line segment

→ line

→ Collinear point – Points lying on same line.

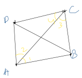
Quadrilaterals

Any enclosed figure made with 4 non-collinear points is known as quadrilateral



Vertices – A, B, C, D
Sides – AB, BC, CD, DA.
Angles – $\angle A, \angle B, \angle C, \angle D$
Diagonals – AC and BD

Thm **8.2 Angle Sum Property of a Quadrilateral**



Given: ABCD is a quadrilateral

To prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

Proof: In $\triangle ABC$
$\angle 1 + \angle B + \angle 3 = 180^\circ$ (Angle sum Property of a $\triangle$) – (i)

In $\triangle ADC$
$\angle 2 + \angle D + \angle 4 = 180^\circ$ (") – (ii)

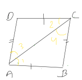
$(\angle 1 + \angle B + \angle 3) + (\angle 2 + \angle D + \angle 4) = 360^\circ$

$(\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^\circ$

$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$

Theorem 8.1 : *A diagonal of a parallelogram divides it into two congruent triangles.*

||gm – Quadrilateral whose opposite sides are ||.

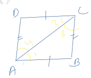


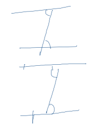
Given – ||gm ABCD.
To prove: $\triangle ABC \cong \triangle ADC$
Proof: In $\triangle ABC$ and $\triangle ADC$
AB || DC, AD || BC, AC is a transversal.
$\angle 1 = \angle 2$ (alter interior angles)
$\angle 4 = \angle 3$ (")
AC = AC (Common)
$\therefore \triangle ABC \cong \triangle ADC$ (ASA rule).

Theorem 8.2 : *In a parallelogram, opposite sides are equal.*

$\triangle ABC \cong \triangle DCA$
AB = DC, AD = BC (CPCT)

Theorem 8.3 : *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*





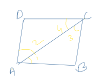
Given: quadrilateral ABCD,
AB = DC,
AD = BC

To prove: ABCD is a ||gm

Constr: Join A to C

Proof: In $\triangle ABC$ and $\triangle CDA$
AB = DC (given)
BC = AD (given)
AC = AC (Common)
$\therefore \triangle ABC \cong \triangle CDA$ (SSS rule)
$\angle 1 = \angle 2$ (CPCT)
$\angle BAC = \angle DCA$
$\angle 3 = \angle 4$ (CPCT)
but these are alternate interior angles
$\therefore$ AB || DC
AD || BC
Which implies ABCD is a ||gm.

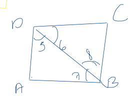
Theorem 8.4 : *In a parallelogram, opposite angles are equal.*



Given: ABCD is a ||gm, AB || DC & AD || BC

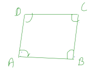
To proof: $\angle A = \angle C$ and $\angle B = \angle D$

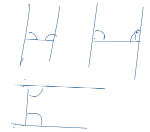
Proof: AB || CD and AC is transversal
$\angle 1 = \angle 4$ – (i) (alt interior angles)
AD || BC and AC is transversal
$\angle 2 = \angle 3$ – (ii) (alt interior angles).
Add (i) and (ii)
$\angle 1 + \angle 2 = \angle 4 + \angle 3$
$\boxed{\angle A = \angle C}$



$\angle 5 = \angle 8$ (alt Interior)
$\angle 6 = \angle 7$ (")
add
$\boxed{\angle D = \angle B}$

Theorem 8.5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*



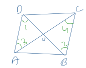


Given: $\angle A = \angle C$ and $\angle B = \angle D$
To proof: ABCD is a ||gm
Proof: $\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Angle sum property of a quadrilateral)
$\angle A + \angle B + \angle A + \angle B = 360^\circ$
$2\angle A + 2\angle B = 360^\circ$
$\angle A + \angle B = 180^\circ$
but these are co-interior angles
$\therefore$ AD || BC – (i)

Similarly
$\angle A + \angle B = 180^\circ$
$\angle A + \angle D = 180^\circ$
$\therefore$ AB || CD – (ii)

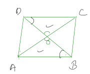
$\therefore$ From (i) and (ii)
ABCD is a ||gm

Theorem 8.6 : *The diagonals of a parallelogram bisect each other.*

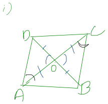


Given: ||gm ABCD.
To prove: AO = CO, OB = OD
Proof: In $\triangle AOD$ and $\triangle COB$
AD = BC (opposite sides of ||gm)
$\angle 1 = \angle 2$ (alt interior angles)
$\angle 3 = \angle 4$ (")
$\therefore \triangle AOD \cong \triangle COB$ (ASA rule)
AO = CO (CPCT)
OD = OB (CPCT)

Theorem 8.7 : *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*



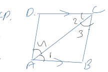
Given: ABCD is a quadrilateral.
AO = CO, OB = OD.
To prove: ABCD is a ||gm
Proof: In $\triangle AOB$ and $\triangle COD$
AO = CO (given)
$\angle AOB = \angle COD$ (V.O.A).
OB = OD (given)
$\therefore \triangle AOB \cong \triangle COD$ (SAS)
$\angle ABO = \angle CDO$ (CPCT)
but these are alt interior angles
AB || DC – (i)



Now, In $\triangle AOD$ and $\triangle BOC$
AO = CO (given).
$\angle AOD = \angle BOC$ (V.O.A).
OD = OB (given).
$\therefore \triangle AOD \cong \triangle BOC$ (SAS rule)
$\angle DAO = \angle BCO$ (CPCT)
but these are alt interior angles
$\therefore$ AD || BC – (ii)

$\therefore$ From (i) and (ii)
ABCD is a ||gm

Theorem 8.8 : *A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.*



Given: In a quadrilateral ABCD,
AB || DC, AB = DC

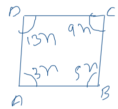
To proof: ABCD is a ||gm.
AD || BC,

Const: Join A to C

Proof: In $\triangle ABC$ and $\triangle CDA$.
AB = CD (given)
$\angle 1 = \angle 2$ (alt. interior angles)
AC = AC (Common)
$\therefore \triangle ABC \cong \triangle CDA$ (SAS)
$\angle 3 = \angle 4$ (CPCT)
but these are alt. interior angles
$\therefore$ AD || BC
$\therefore$ ABCD is a ||gm

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

let the angles are $3x, 5x, 9x$ and $13x$.



$\angle A + \angle B + \angle C + \angle D = 360^\circ$

(Angle sum property of quadrilateral)

$3x + 5x + 9x + 13x = 360^\circ$

$30x = 360$

$x = 12$

$\angle A = 36^\circ$

$\angle B = 60^\circ$

$\angle C = 108^\circ$

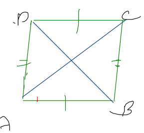
$\angle D = 156^\circ$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Given: ABCD is a ||gm, $AC = BD$.

oppo sides equal & 1 angle is 90°

To prove: ABCD is a rectangle



proof:

In $\triangle ABD$ and $\triangle BAC$

$AD = BC$ (oppos sides of ||gm are equal)

$AB = BA$ (common)

$BD = AC$ (Given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle BAD = \angle ABC$ (CPCT)

$\angle BAD + \angle ABC = 180^\circ$ (co interior angles).

$\angle BAD + \angle BAD = 180^\circ$

$2\angle BAD = 180^\circ$

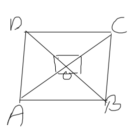
$\boxed{\angle BAD = 90}$

$\therefore$ ABCD is a rectangle

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Given:- $OA = OC$, $OB = OD$,

Diagonal bisect at right angle



To prove:- ABCD is a rhombus,

$AB = BC = CD = AD$