Gravitation

- The Phenomenon of attraction btw any two objects in the Universe is called gravitation.
- Gravity — attractive force which is exerted by earth or any body.

Universal law of Gravitation.

$F \propto m_1 m_2$

$F \propto \frac{1}{r^2}$

$F = G \times \frac{m_1 m_2}{r^2}$ → Universal gravitational constant

Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses & inversely proportional to the square of the distance between them.

SI unit of G

$G = \frac{F \times r^2}{m_1 m_2}$

$G = \frac{N \times m^2}{kg^2}$

$G = 6.67 \times 10^{-11}\ Nm^2 kg^{-2}$

Universal gravitational Constant

$F = G \frac{m_1 m_2}{r^2}$

$G = \frac{F r^2}{m_1 m_2}$

$G = \frac{F \times (1m)^2}{1kg \times 1kg}$

$G = F$

Universal gravitational constant is numerically equal to the gravitational force of attraction between two bodies each of mass 1kg kept at unit distance from each other.

Why gravitational constant (G) is known as Universal gravitational constant?

→ The value of G doesn't depend on mass of two bodies, distance btw two bodies, nature, medium, shape, size.

Conditions

i) when two objects each of 1kg and 1m apart

$F = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{1 \times 1}{1} = 6.67 \times 10^{-11} N$

$= 0.000\ldots\ldots 67 N$

ii)

force of Ball to Train $= G \frac{m_1 m_2}{r^2} = \frac{6.2 \times 5000}{1^2}$

$F = 1000.6 N$

Force of Train to Ball $= \frac{G m_2 m_1}{r^2} = \frac{G \times 5000 \times 2}{1^2} = 1000.6 N$

iii)

earth mass $= 6 \times 10^{24}$ kg

Radius of earth $= 6400$ km

$= 6400 \times 1000$ m

$= 6.4 \times 10^6$ m

$F_{BE} = \frac{6.67 \times 10^{-11} \times 1 \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8 N$

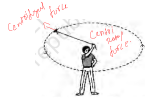
$F_{EB} = 9.8 N$

Newton's 2nd law of motion

$F = ma$

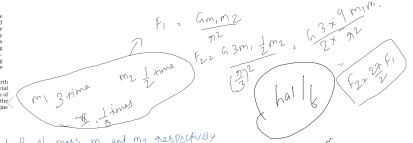
$a_B = \frac{F_{EB}}{m} = \frac{9.8}{1} = 9.8\ ms^{-2}$

$a_{BE} = \frac{F}{m} = \frac{9.8}{6 \times 10^{24}\ kg} = 1.63 \times 10^{-24}\ ms^{-2}$

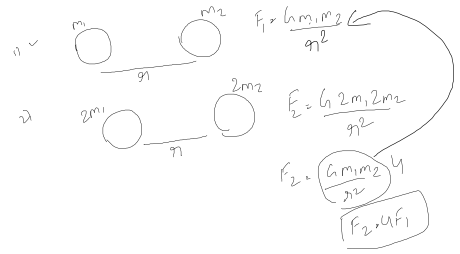


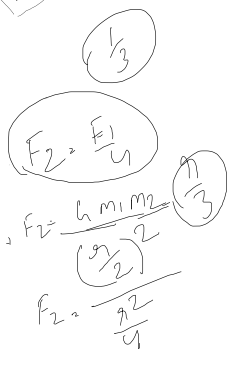
Before the thread is released, the stone moves in a circular path with a certain speed and changes direction at every point. The change in direction involves change in velocity or acceleration. The force that causes this accelerations and keeps the body moving along the circular path is acting towards the centre. This force is called the centripetal (meaning 'centre-seeking') force. In the absence of this

The motion of the moon around the earth is due to the centripetal force. The centripetal force is provided by the force of attraction of the earth. If there were no such force, the moon would pursue a uniform straight line motion.



1) Two particles A and B of mass m_1 and m_2 respectively are placed on some distance. If the mass of each of the two particles is doubled, keeping the distance btw them unchanged, the gravitational force btw them will be.





2) mass of sun is 2×10^{30} kg & mass of earth is 6×10^{24} kg. If the avg distance b/w the sun and the earth is 1.5×10^{8} km. Calculate the gravitational force b/w them.

$F = \frac{Gm_1m_2}{r^2}$

3) The gravitational force b/w two objects is F. If masses of both objects are halved w/o changing distance b/w them, gravitational force would become.

$F_1 = \frac{Gm_1m_2}{d^2}$

$F_2 = \frac{G(m_1/2)(m_2/2)}{d^2} = \frac{Gm_1m_2}{4d^2} = \frac{F_1}{4}$

$F_2 = F_1/4$

4) Gravitational force b/w two objects weighing 20kg and 15kg is 1.5×10^{-10} N. Find distance b/w two objects.

Scientists → Arrangement of planets

Ptolemy → 140 AD → Geocentric theory → Earth is at center

Copernicus → 1543 → Heliocentric theory → Sun at the center & all planets revolve around

Tycho brahe → 1548 → Tychonic system → Moon is at the center.

Galileo → 1609 → Sun is at the center → Heliocentric theory → Telescope

Kepler's law of planetary motion

He stated three laws which govern the motion of planets around the sun. They are known as Kepler's law of planetary motion.

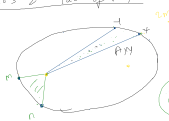
Circle – 1 center
1 Focus (radius are equal)

Ellipse – 2 center
Foci, Foci

Kepler's 1st law of motion

– Planets move in elliptical orbits around the sun, with the sun at one of two foci of the elliptical orbit.

Kepler's 2nd law of motion



✓ Each planet revolves around the sun in such a way that the line joining the planet to the sun sweeps over equal areas in equal intervals of time.

✓ A planet moves faster when it is closer to the sun & moves slowly when it is further away from the sun.

Kepler's 3rd law of motion

✓ The cube of the mean distance of a planet from the sun is directly proportional to the square of time, it takes to move around the sun.

$r^3 \propto T^2$

r^3/T^2 = constant f^2

r^3/T^2 = constant

r → mean distance of planet from the sun.

T → time period of the planet around the sun.

Free fall and acceleration due to gravity (g)

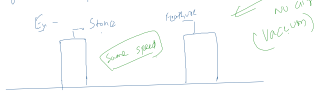
The falling of a body (or object) from a height towards the earth under gravitational force of earth (with no other force acting on it), such a body is called freely falling body.

Acc^n due to gravity (g)

→ The uniform acc^n produced in a freely falling body due to gravitational force of the earth is known as acc^n due to gravity.

→ SI unit = ms^{-2}

→ g doesn't depend on the mass of the falling object.



Calculation of acc^n due to gravity

Force exerted by earth on the body

$F = GMm/R^2$ – (i)

This force by earth produces acc^n in the stone due to which stone moves downwards

$F = ma$ – (ii)

$GMm/R^2 = m \times a$

$a = \frac{GMm}{mR^2}$

Now, $a = \frac{GM}{R^2}$

$g = \frac{GM}{R^2}$

$g = \frac{(6.67\times10^{-11}\ Nm^2kg^{-2} \times 6\times10^{24}\ kg)}{(6.4\times10^{6})^2}$

$g = 9.8\ ms^{-2}$

Variation of acc^n due to gravity (g)

- Earth is not a perfect sphere.
- Geoid shape
- ✓ poles are flat
- ✓ Equator are bulged.

$g = \frac{GM}{R^2}$ ↑↓

Radius pole < Radius of equator

✓ The value of g is not constant at all the places on the surface of earth.

✓ As earth is not a perfect sphere so the value of Radius R is not the same at all the places on its surface.

✓ Radius of earth at the poles is minimum the value of g is maximum.

✓ " " " equator is maximum ∴ minimum.

✓ But this formula is not applicable at any point inside the surface of earth.

✓ The value of g decreases as we go down inside the earth.

✓ Value of g becomes zero at the center of earth.

Equation of Motion for freely falling bodies

Equation of motion → for horizontal motion

$v = u + at$

$S = ut + \frac{1}{2}at^2$

$v^2 - u^2 = 2as$

Motion due to gravity

$v = u + gt$

$h = ut + \frac{1}{2}gt^2$

$v^2 - u^2 = 2gh$

horizontal motion

+ve accn

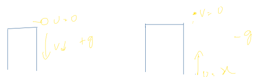
$u = 5 m/s$, $v = 10 m/s$

$a = \frac{v-u}{t} = \frac{10-5}{2} = 2.5 ms^{-2}$

Retardation

$u = 10 m/s$, $v = 5 m/s$

$a = \frac{v-u}{t} = \frac{5-10}{2} = -2.5 ms^{-2}$



✓ $a = \frac{v-u}{t} =$ (+ve)

✓ $a = \frac{v-u}{t} = \frac{0-n}{t} =$ (−ve)

✓ when a body is falling vertically downwards, it's velocity is increasing. So the accn due to gravity is taken as positive $g = +9.8 ms^{-2}$.

✓ " " thrown " upwards, " " " negative decreasing, " $g = -9.8 ms^{-2}$

18. To estimate the height of the bridge over a river, a stone is dropped freely in the river from the bridge. The stone takes 2 seconds to touch the water surface in the river. Calculate the height of the bridge from water level.

$u = 0$

$t = 2 sec.$

$g = 9.8$

$h = ut + \frac{1}{2}gt^2$

$h = 0 \times t + \frac{1}{2} \times 9.8 \times 2 \times 2$

$h = 19.6 m$

Example 10.2 A car falls off a ledge and drops to the ground in 0.5 s. Let $g = 10 m s^{-2}$ (for simplifying the calculations).
(i) What is its speed on striking the ground?
(ii) What is its average speed during the 0.5 s?
(iii) How high is the ledge from the ground?

5:29

$u = 0$

$t = 0.5 s$

$g = +10 ms^{-2}$

(i) $v = u + gt$
$= 0 + 0.5 \times 10$
$= 5 ms^{-1}$

(ii) Avg = $\frac{\text{Initial velocity} + \text{Final velocity}}{2} = \frac{0+5}{2} = 2.5 ms^{-1}$

(iii) $h = ut + \frac{1}{2}gt^2$

$h = 0 \times t + \frac{1}{2} \times 10 \times (0.5)^2$

$= \frac{1}{2} \times 10 \times \frac{5}{10} \times \frac{5}{10} = 1.25 m$

Example 10.3 An object is thrown vertically upwards and rises to a height of 10 m. Calculate (i) the velocity with which the object was thrown upwards and (ii) the time taken by the object to reach the highest point.

$s = 10 m$

$v = 0 ms^{-1}$

$g = -9.8 ms^{-2}$ (upward motion)

$v^2 - u^2 = 2gh$

$(0)^2 - (u)^2 = 2 \times -9.8 \times 10$

$u^2 = 196$

$u = 14$

(ii) $v = u + gt$

$0 = 14 - 9.8 \times t$

$t = \frac{14}{9.8} = \frac{140}{98} = 1.4 s$

Q. A stone is dropped from a height of 20m
a) How long will it take to reach the ground.
b) what will be its speed when it hits the ground. g(10)

→ $h = ut + \frac{1}{2}gt^2$

$20 = 0 + \frac{1}{2} \times 10 \times t^2$

$t^2 = \frac{20 \times 2}{10} = 4$, $t = 2 sec$

b) $v = u + gt$
$= 0 + 10 \times 2 = 20 ms^{-1}$

Q. A stone is thrown vertically upwards with a speed of 20 ms^{-1}. How high it go before it begins the fall?

Mass and Weight

✓ Mass of the body is the quantity of matter (or material) contained in it.

✓ Scalar quantity (only magnitude and no direction).

✓ SI unit is Kg.

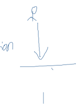
✓ Mass of a body is constant, doesn't change from place to place.

✓ Mass of body can't be zero.

Weight

→ Weight of a body is the force with which it is attracted towards the center of the earth.

$F = \frac{GMm}{R^2}$



→ By Newton's 2nd law of motion

$F = ma$

$W = m \times g$

→ SI unit of weight is Newton.

→ It is a vector quantity.

→ Weight of a body acts in vertically downward direction.

→ Weight depends on g therefore, weight of a body is not constant.

→ Wt in poles ↑ & wt in equator ↓

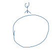
Q. In weighing m/c weight or mass?

↳ load sensor — measures the force which your body is applying because of the gravitational pull of earth.

$F = W = m \times g$

chip $\frac{W}{g} = m$ in Kg

Weight of an object on the moon.



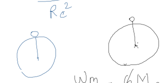
✓ weight of an object on the moon is the force with which the moon attracts that object.

✓ Value of g differs on earth and on moon.

$g = GM/R^2$

Celestial body	Mass (kg)	Radius (m)
Earth	5.98×10^{24}	6.37×10^6
Moon	7.36×10^{22}	1.74×10^6

$W_e = \frac{GM_e m}{R_e^2}$ $W_m = \frac{GM_m m}{R_m^2}$



$\frac{W_m}{W_e} = \frac{GM_m m}{R_m^2} \times \frac{R_e^2}{GM_e m} = \frac{7.36 \times 10^{22}}{(1.74 \times 10^6)^2} \times \frac{(6.37 \times 10^6)^2}{5.98 \times 10^{24}}$

$\frac{2.431 \times 10^{10}}{1.474 \times 10^{11}}$

$= 0.165 \approx \frac{1}{6}$

$\frac{\text{weight of an object on the moon}}{\text{weight of an object on the earth}} = \frac{1}{6}$

weight of an object on the moon will be about one-sixth of what is on earth.

✓ Gravitation force of the moon is about one-sixth that of the earth.

1) Mass of an object is 10kg. What is weight on the earth?

→ $W = m \times g$

$10 \times 9.8 = 98\ N$

2)

Example 10.5 An object weighs 10 N when measured on the surface of the earth. What would be its weight when measured on the surface of the moon?

wt of obj on the moon $= \left(\frac{1}{6}\right) \times$ wt of obj on the earth

$= \frac{1}{6} \times 10 = \frac{10}{6} = 1.67\ N$

3) What is the mass of an object whose weight is 49 N?

→ $W = m \times g$

$m = \frac{490}{98} = 5\ kg$

g (10 m/s²)

4) A man weighs 600 N on the earth. What is his mass? If he were taken to the moon, his weight would be 100 N. What is his mass on the moon? What is the accn due to gravity on the moon?

$W = mg$

$\frac{600}{10} = m$

60 kg

moon

earth

$W = mg$

$100 = 60 \times g$

$g = \frac{10}{6}$ or $1.67\ ms^{-2}$

5) How much would a 70 kg man weigh on the moon? What would be his mass on the earth and the moon? (accn due to gravity on moon = 1.63 ms⁻²)