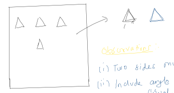
Triangles

1) Congruence – 2 NCERT Science book (identical: same in shape, size)

Dairy Milk chocolate

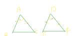
Activity for congruence for Δ



Observations:

(i) Two sides must be equal.

(ii) Include angle will be equal.

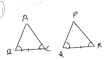


ΔBAC ≅ ΔEDF (SAS rule) – axiom

ΔCAB ≅ ΔFDE

Theorem

(ASA)



Given: ∠B = ∠Q,
BC = QR
∠C = ∠R

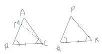
To prove: ΔABC ≅ ΔPQR

Proof: (i) Let AB = PQ

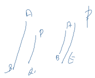
AB = PQ (construction)
∠B = ∠Q (given)
BC = QR (given)

ΔABC ≅ ΔPQR (SAS rule).

(ii)



AB > PQ
PQ > AB



If AB > PQ..

In ΔMBC & ΔPQR,

MB = PQ (by construction)
∠B = ∠Q (Given)
BC = QR (given)

ΔMBC ≅ ΔPQR (SAS rule).

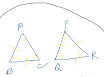
∠MCB = ∠R (CPCT)

∠C = ∠R (Given)

∠MCB = ∠C

but this is not possible,
it's possible only when point
M tends to poin A.

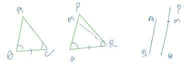
BM = BA
BA = QP



ΔABC ≅ ΔPQR

CPCT (Corresponding parts of Congruent Triangles)

(iii)



If AB < PQ,

ΔABC and ΔMQR,

AB = MQ (By construction)
∠B = ∠Q (Given).
BC = QR (Given)

ΔABC ≅ ΔMQR (SAS rule).

∠ACB = ∠MRQ (CPCT)
∠C = ∠MRQ (CPCT)
∠C = ∠R (Given)

∠MRQ = ∠R.

This is not possible, only possible when
point M tends to point P.

then, MQ = PQ

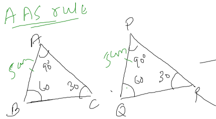
$\boxed{PQ = AB}$

Rules

(i) SAS rule (axiom).

(ii) ASA rule (Thm – 3 cases)

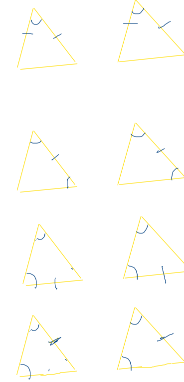
(iii) AAS rule



If two angles are equal, 3rd angle will be equal of Δ's

ΔABC ≅ ΔPQR (AAS rule)

①



= SAS rule

= ASA rule

= AAS rule

= AAS rule

ΔABC ≅ ΔDEF

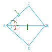
CPCT

AB = DE , ∠ACB = ∠DFE

BC = EF . ∠CAB = ∠FDE

AC = DF ,

1)

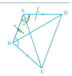


Given :- AC = AD
$\angle 1 = \angle 2$,

To proof :- $\Delta ABC \cong \Delta ABD$

proof :- In ΔABC & ΔABD
AC = AD (given)
AB = AB (Common side)
$\angle CAB = \angle DAB$ (given)
$\Delta ABC \cong \Delta ABD$ (SAS rule)
BC = BD (CPCT)

2. ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$ (see Fig. 7.17). Prove that
(i) $\Delta ABD \cong \Delta BAC$
(ii) BD = AC
(iii) $\angle ABD = \angle BAC$.



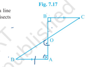
Given :- AD = BC, $\angle DAB = \angle CBA$.

To prove :- (i) $\Delta ABD \cong \Delta BAC$.
(ii) BD = AC
(iii) $\angle ABD = \angle BAC$

proof :- In ΔABD & ΔBAC
AD = BC (given)
$\angle DAB = \angle CBA$ (given)
AB = BA (Common)
$\therefore \Delta ABD \cong \Delta BAC$ (SAS rule)
BD = AC (CPCT)
$\angle ABD = \angle BAC$ (CPCT)

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

Fig. 7.17



Given :- AD = BC, $\angle A = \angle B = 90°$.

To prove :- CD bisects AB (OA = OB)

proof :- In ΔOAD and ΔOBC,
AD = BC (given)
$\angle OAD = \angle OBC$ (90° each)
$\angle AOD = \angle BOC$ (V.O.A)
$\Delta OAD \cong \Delta OBC$ (AAS rule)
OA = OB (CPCT) $\therefore$ CD bisects AB.

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that $\Delta ABC \cong \Delta CDA$.

Fig. 7.18

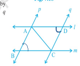


Fig. 7.19

Given :- $l \parallel m$, $p \parallel q$.
Show :- $\Delta ABC \cong \Delta CDA$

proof :- $l \parallel m$ (AD $\parallel$ BC) $p \parallel q$ (AB $\parallel$ CD)
$\therefore$ ABCD is a $\parallel gm$.

In ΔABC & ΔCDA
AB = CD (opp. sides of $\parallel gm$.)
BC = DA (opp sides of $\parallel gm$)
$\angle B = \angle D$ (" angle of $\parallel gm$)

$\Delta ABC \cong \Delta CDA$ (SAS rule)

5. Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see Fig. 7.20). Show that:
(i) $\Delta APB \cong \Delta AQB$
(ii) BP = BQ or B is equidistant from the arms of $\angle A$.

CPCT

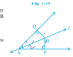
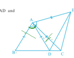


Fig. 7.20

$\Delta APB \cong \Delta AQB$ (AAS rule)
$\angle 1 = \angle 2$ (given)
$\angle P = \angle Q$ (90°)
AB = AB (Common)

6. In Fig. 7.21, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.



Given :- AC = AE, AB = AD, $\angle BAD = \angle EAC$.
To show :- BC = DE
To proof :- In ΔABC and ΔADE.
AC = AE (given)
AB = AD (given)
$\angle BAD = \angle EAC$ (given)
adding $\angle DAC$ both sides.
$\angle BAD + \angle DAC = \angle EAC + \angle DAC$
$\angle BAC = \angle EAD$

$\Delta ABC \cong \Delta ADE$ (SAS rule)
BC = DE (CPCT)

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (see Fig. 7.22). Show that
(i) $\Delta DAP \cong \Delta EBP$
(ii) AD = BE

Fig. 7.21

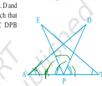


Fig. 7.22

Given :- AP = PB (P is mid point), $\angle BAD = \angle ABE$, & $\angle EPA = \angle DPB$.

To show :- (i) $\Delta DAP \cong \Delta EBP$,
(ii) AD = BE

proof :- In ΔDAP and ΔEBP
$\angle PAD = \angle PBE$ (Given $\angle BAD = \angle ABE$)
AP = BP (P is mid point)

$\angle EPA = \angle DPB$ (given).
Add $\angle EPD$ both sides.

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$
$\angle DPA = \angle EPB$
$\therefore \Delta DAP \cong \Delta EBP$ (ASA rule)
AD = BE (CPCT)

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:
(i) $\Delta AMC \cong \Delta BMD$
(ii) $\angle DBC$ is a right angle.
(iii) $\Delta DBC \cong \Delta ACB$
(iv) $CM = \frac{1}{2} AB$

Fig. 7.22

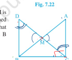


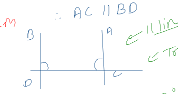
Fig. 7.23

Given :- $\angle C = 90°$, BM = AM, DM = CM

To show :- (i) $\Delta AMC \cong \Delta BMD$
(ii) $\angle DBC$ is 90°
(iii) $\Delta DBC \cong \Delta ACB$.
(iv) $CM = \frac{1}{2} AB$

proof :- (i) In ΔAMC and ΔBMD
AM = BM (M is mid point)
CM = DM (Given)
$\angle CMA = \angle DMB$ (V.O.A)
$\Delta AMC \cong \Delta BMD$ (SAS rule)

AC = BD (CPCT)
$\angle ACM = \angle BDM$ (CPCT)
but these are alternate interior angles.
$\therefore$ AC $\parallel$ BD



$\angle DBC + \angle ACB = 180°$ (sum of interior angles)
$\angle DBC + 90° = 180°$
$\angle DBC = 90°$

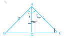
(iii) In ΔDBC and ΔACB.
DB = AC (proved above)
$\angle B = \angle C$ (each 90°)
BC = CB (Common)
$\Delta DBC \cong \Delta ACB$ (SAS rule)

(iv) CD = BA (CPCT)
CM + DM = AB
CM + CM = AB (CM = DM given)
2CM = AB
$\boxed{CM = \frac{1}{2} AB}$

Theorem 7.2 : *Angles opposite to equal sides of an isosceles triangle are equal.*

Given :- AB = AC

To prove :- $\angle B = \angle C$



Construction :- AD is angle bisector of $\angle A$.

Sol :- ΔBAD and ΔCAD.

AB = AC (given)

$\angle 1 = \angle 2$ (By construction)

AD = AD (common)

$\Delta BAD \cong \Delta CAD$ (SAS rule)

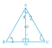
$\angle B = \angle C$ (CPCT)

Theorem 7.3 : *The sides opposite to equal angles of a triangle are equal.*

Given : $\angle B = \angle C$

To prove : AB = AC

Construct : AD is angle bisector of $\angle A$ $(\angle 1 = \angle 2)$



Proof :- ΔBAD and ΔCAD

$\angle B = \angle C$ (given)

$\angle 1 = \angle 2$ (By construction)

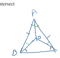
AD = AD (Common)

$\Delta BAD \cong \Delta CAD$ (AAS rule)

AB = AC (CPCT)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that :

(i) OB = OC (ii) AO bisects $\angle A$



Given Follow

Sol : ΔABC, AB = AC

$\angle B = \angle C$ (Angle opposite to equal sides are equal)

$\frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\angle OBC = \angle OCB$

In ΔOBC if $\angle OBC = \angle OCB$

OB = OC (sides opposite to equal angles are equal)

In ΔAOB and ΔAOC

AB = AC (given)

$\angle ABO = \angle ACO$ $(\frac{1}{2}\angle B = \frac{1}{2}\angle C)$

OB = OC (proved above)

$\therefore \Delta AOB \cong \Delta AOC$ (SAS rule)

$\angle BAO = \angle CAO$ (CPCT)

$\therefore$ AO bisects $\angle A$

2. In Δ ABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that Δ ABC is an isosceles triangle in which AB = AC.

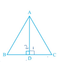


Fig. 7.30

Given : $\angle 1 = \angle 2 = 90°$ &

BD = DC

To prove : AB = AC

Proof : ΔABD and ΔACD

BD = CD (given)

$\angle 2 = \angle 1$ (90)

AD = AD (common)

$\Delta ABD \cong \Delta ACD$ (SAS rule)

AB = AC (CPCT)

$\therefore \Delta ABC$ is an isosceles Δ

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

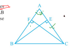
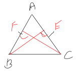


Fig. 7.31



Given : AB = AC

To prove : CF = BE

Proof : In ΔACF and ΔBAE

$\angle CAF = \angle BAE$ (Common)

$\angle F = \angle E$ (90°)

CA = BA (given)

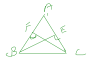
$\therefore \Delta ACF \cong \Delta BAE$ (AAS rule)

BE = CF (CPCT)

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) Δ ABE $\cong$ Δ ACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.



In ΔABE and ΔACF

$\rightarrow$ $\angle A = \angle A$ (common)

$\angle E = \angle F$ (90°)

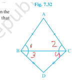
BE = CF (given)

$\therefore \Delta ABE \cong \Delta ACF$ (AAS rule)

AB = AC (CPCT)

Fig. 7.32

5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that $\angle$ ABD = $\angle$ ACD.



5:33

Given : AB = AC, BD = CD

To show : $\angle ABD = \angle ACD$

In ΔABC

If AB = AC (given)

$\angle 1 = \angle 2$ (Angle opposite to equal sides are equal) (1)

If ΔDBC

BD = CD (given)

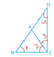
$\angle 3 = \angle 4$ (" ") (2)

Add (1) and (2)

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle ABD = \angle ACD$

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that $\angle$ BCD is a right angle.



Given : AB = AC

AB = AD

To prove : $\angle BCD$ is 90°

Proof : In ΔABC,

AB = AC (given) – (i)

$\angle 1 = \angle 2$ (angle opposite to equal sides are equal)

AD = AB – (ii)

From (i) and (ii)

AD = AC

$\angle 3 = \angle 4$ (" ")

In ΔBCD,

$\angle B + \angle C + \angle D = 180°$ (Angle sum property)

$\angle 1 + \angle 4 + \angle 2 + \angle 3 = 180°$

$(\angle 1 + \angle 4) + \angle 2 + \angle 3 = 180°$

$\angle 2 + \angle 3 + \angle 2 + \angle 3 = 180°$

$2(\angle 2 + \angle 3) = 180°$

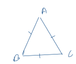
$\angle 2 + \angle 3 = 90°$

$\angle DCB = 90°$

$\therefore \angle BCD$ is a right angle.

7. ABC is a right angled triangle in which $\angle$ A = 90° and AB = AC. Find $\angle$ B and $\angle$ C. $\rightarrow$ Angle Sum property

8. Show that the angles of an equilateral triangle are 60° each.



Given : AB = AC = BC

To prove : $\angle B = \angle C = \angle A = 60°$

Proof : AB = BC

$\angle B = \angle C$ (angle opposite to equal sides) – (i)

If BC = AC

$\angle B = \angle A$ (") – (ii)

From (i) and (ii)

$\angle A = \angle B = \angle C$

In ΔABC

$\angle A + \angle B + \angle C = 180°$ (Angle sum property)

$\angle A + \angle A + \angle A = 180°$

$3\angle A = 180°$

$\angle A = 60°$, $\angle B = \angle C = 60°$

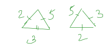
(i) SAS rule

(ii) ASA rule

(iii) AAS rule

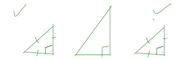
Thm

iv)



(SSS Rule)

(v)

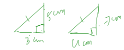


(RHS rule)

Right angle

Hypotenuse

Side



EXERCISE 7.3

1. Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i) Δ ABD $\cong$ Δ ACD

(ii) Δ ABP $\cong$ Δ ACP

(iii) AP bisects $\angle$ A as well as $\angle$ D.

(iv) AP is the perpendicular bisector of BC.

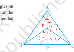


Fig. 7.39

Given : AB = AC, DB = DC

To prove :

Proof : (i) In ΔABD and ΔACD

AB = AC (given)

BD = CD (given)

AD = AD (common)

$\therefore \Delta ABD \cong \Delta ACD$ (SSS rule)

$\angle DAB = \angle CAD$ (CPCT)

$\angle 1 = \angle 2$

$\therefore$ AP bisects $\angle A$.

In ΔBPD and ΔCPD

BP = CP (proved above)

DP = DP (common)

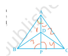
BD = CD (given)

$\therefore \Delta BPD \cong \Delta CPD$ (SSS rule)

$\angle BDP = \angle CDP$ (CPCT)

$\angle 5 = \angle 6$ (CPCT)

$\therefore$ AP bisects $\angle D$



(ii) In ΔABP and ΔACP

AB = AC (given)

$\angle 1 = \angle 2$ (proved above)

AP = AP (common side).

$\therefore \Delta ABP \cong \Delta ACP$ (SAS rule).

BP = CP (CPCT)

AP bisects BC

$\angle 3 = \angle 4$ (CPCT)

$\angle 3 + \angle 4 = 180°$ (linear pair)

$\angle 3 + \angle 3 = 180°$

$2\angle 3 = 180°$

$\angle 3 = 90°$

$\angle 4 = 90°$

$\therefore$ AP is $\perp$ bisector to BC

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that
(i) AD bisects BC (ii) AD bisects ∠ A.

Given: ΔABC, AB = AC
To show: (i) AD bisects BC (BD = DC)
(ii) AD bisects ∠A (∠1 = ∠2)

Sol: ΔABD and ΔACD
∠3 = ∠4 (each 90°)
AB = AC (Given)
AD = AD (Common)
ΔABD ≅ ΔACD (RHS rule).
BD = CD (CPCT)
∴ AD bisects BC
∠DAB = ∠DAC (CPCT)
(∠1 = ∠2)

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR (see Fig. 7.40). Show that:
(i) Δ ABM ≅ Δ PQN SSS rule
(ii) Δ ABC ≅ Δ PQR
SAS rule.

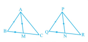
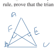


Fig. 7.40

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

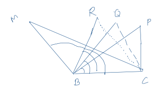


Given: BE = CF
To prove: ΔABC is isosceles.
Proof: ΔBCF and ΔCBE
CF = BE (given)
∠BFC = ∠CEB (90°)
BC = BC (Common)
ΔBCF ≅ ΔCBE (RHS rule)
∠B = ∠C (CPCT)
AB = AC (sides opposite to equal angles are equal)
∴ ΔABC is an isosceles Δ.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠ B = ∠ C.

ΔABP and ΔACP,
AB = AC (Common)
∠APB = ∠APC (90°)
AP = AP (Common)
∴ ΔABP ≅ ΔACP (RHS rule)
∠B = ∠C (CPCT)

In Scalene Δ



↳ CM > CR > CQ > CP
∠MBC > ∠RBC > ∠QBC > ∠PBC

Theorem 7.6 : *If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater).*

Thm
(i) Angle opposite to the longer side is greater.
(ii) Side opposite to the greater angle is longer.

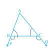
Theorem 7.7 : *In any triangle, the side opposite to the larger (greater) angle is longer.*

1. Show that in a right angled triangle, the hypotenuse is the longest side.

Given: ∠B = 90°
To show: AC (hypotenuse) is the longest side.

Proof: ∠B > ∠A (∠B = 90°)
AC > BC —(i) (side opposite to the greater angle is longer)
∠B > ∠C
AC > AB —(ii)
From (i) & (ii)
AC is the longest side.

2. In Fig. 7.48, sides AB and AC of Δ ABC are extended to points P and Q respectively. Also, ∠PBC < ∠QCB. Show that AC > AB.



Given: ∠PBC < ∠QCB
To show: AC > AB.

∠ABC > ∠ACB
⇓
AC > AB

Sol: ∠ABC + ∠PBC = 180° (Linear Pair)
∠ABC = 180° − ∠PBC —(i)

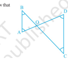
∠ABC = 180° − ∠PBC

∠ACB + ∠QCB = 180° (Linear Pair)
∠ACB = 180° − ∠QCB —(ii)

∠ACB = 180° − ∠QCB

∠PBC < ∠QCB (given)
Multiplying by −1 both sides.
−∠PBC > −∠QCB
∠ABC − 180° > ∠ACB − 180°
∠ABC > ∠ACB
AC > AB

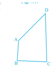
3. In Fig. 7.49, ∠ B < ∠ A and ∠ C < ∠ D. Show that AD < BC.

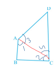


Given: ∠B < ∠A and ∠C < ∠D
To show: AD < BC
Proof: ∠B < ∠A
AO < BO —(i) (sides opposite to greater angle are longer)
∠C < ∠D
OD < OC —(ii) (")
Add (i) & (ii)
AO + OD < BO + OC
AD < BC

V.I.

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50). Show that ∠ A > ∠ C and ∠ B > ∠ D.



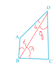


Given: AB is smallest,
CD is longest.
To show: (i) ∠A > ∠C
(ii) ∠B > ∠D

Proof: (i) Construction: Join A to C.
In ΔABC,
AB is smallest,
BC > AB
∠1 > ∠2 —(i) (Angle opposite to longer side is greater)

In ΔACD,
CD is longest,
CD > AD
∠3 > ∠4 —(ii) (")

Add (i) and (ii)
∠1 + ∠3 > ∠2 + ∠4
∠A > ∠C



In ΔABD
AB is smallest side,
AD > AB
∠5 > ∠6 —(i) (Angle opp. to larger side is greater).

In ΔBCD,
CD is largest,
CD > BC
∠7 > ∠8 —(ii) (")

Add (i) and (ii)
∠5 + ∠7 > ∠6 + ∠8
∠B > ∠8

V.I.

5. In Fig 7.51, PR > PQ and PS bisects ∠ QPR. Prove that ∠ PSR > ∠ PSQ.

Fig. 7.51

Given: PR > PQ, ∠1 = ∠2
To prove: ∠PSR > ∠PSQ
∠4 > ∠5

Proof: ∠1 = ∠2 (given)
In ΔPQR
PR > PQ
∠6 > ∠3 —(i) (Angle opposite to longer side is greater).

In ΔPSR
∠5 = ∠2 + ∠3 (exterior angle property) —(ii)

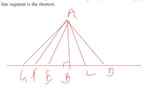
In ΔPSQ
∠4 = ∠1 + ∠6 (")
∠4 = ∠2 + ∠6 (∠1 = ∠2 given) —(iii)

∠6 > ∠3 —(i)
∠5 = ∠2 + ∠3 —(ii)
∠4 = ∠2 + ∠6 —(iii)

5 = 2 + 3
4 = 2 + 6

From (i), (ii) & (iii)
∠4 > ∠5

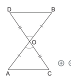
6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.



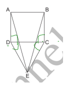
In ΔAGB.
∠A + ∠G + ∠B = 180° (Angle sum property)
∠A + ∠G = 90°
∠G < ∠B
AB < AG
(∵ AB is smallest)

R.S Aggarwal

1) In the given figure, $OA = OB$ and $OC = OD$. Show that:
(i) $\triangle AOC \cong \triangle BOD$ (ii) $AC \parallel BD$.

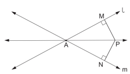


2) In the given figure, $\triangle CDE$ is an equilateral triangle on a side CD of a square $ABCD$. Show that $\triangle ADE \cong \triangle BCE$.

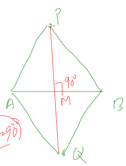


AD = DC
DC = ...
AD = DE
SAS

3) P is a point equidistant from two lines l and m intersecting at a point A, as shown in the given figure. Show that the line AP bisects the angle between them.

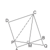


4) In the given figure, AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B. Show that the line PQ is the perpendicular bisector of AB.

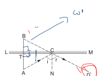


AM = BM
& ∠BMP = ∠AMP = 90°

5) In the given figure, $ABCD$ is a square, M is the midpoint of AB and $PQ \perp CM$ meets AD at P and CB produced at Q. Prove that (i) $PA = QB$ and (ii) $CP = AB + PA$.



6) **23.** The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D, as shown in the figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



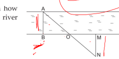
Alternate interior

BT = AT

In △TAC & TBC
∠BTC = ∠ATC (90°)
∠CBT = ∠n (Corresponding angle) – (i)
∠CAT = ∠i (alternate interior angles) – (ii)
From i.. ∠CBT = ∠CAT (From (i) & (ii))
CT = CT (Common)
AAS (rule)

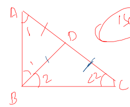
BT = AT (CPCT)

24. In the adjoining figure, explain how one can find the breadth of the river without crossing it.



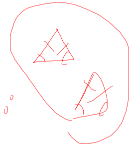
MN = AB

25. In a $\triangle ABC$, D is the midpoint of side AC such that $BD = \frac{1}{2}AC$. Show that $\angle ABC$ is a right angle.



Show

∠1 + ∠2 = 90°



∠1 + ∠1 + ∠2 + ∠2 = 180°
2(∠1 + ∠2) = 180°
∠1 + ∠2 = 90°