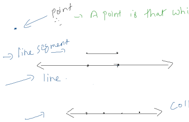
Quadrilaterals

point → A point is that which determines location



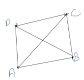
→ line segment

→ line.

→ Collinear points – points lying on same line.

Quadrilaterals

Any enclosed figure made with 4 non-collinear points is known as quadrilateral



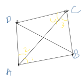
Vertices – A, B, C, D

Sides – AB, BC, CD, DA.

Angles – $\angle A, \angle B, \angle C, \angle D$

Digonals – AC and BD

Thm **8.2 Angle Sum Property of a Quadrilateral**



Given: ABCD is a quadrilateral

To prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

proof:– In ΔABC

$\angle 1 + \angle B + \angle 3 = 180^\circ$ (Angle sum Property of a Δ) –(i)

In ΔADC

$\angle 2 + \angle D + \angle 4 = 180^\circ$ (") –(ii)

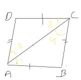
$(\angle 1 + \angle B + \angle 3) + (\angle 2 + \angle D + \angle 4) = 360^\circ$

$(\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^\circ$

$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$

Theorem 8.1 : *A diagonal of a parallelogram divides it into two congruent triangles.*

||gm – Quadrilateral whose opposite sides are ||.



Given – ||gm ABCD.

To prove: $\Delta ABC \cong \Delta ADC$

proof: In ΔABC and ΔADC

AB || DC, AD || BC, AC is a transversal.

$\angle 1 = \angle 2$ (alter interior angles)

$\angle 4 = \angle 3$ (")

$AC = AC$ (Common)

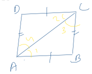
$\therefore \Delta ABC \cong \Delta ADC$ (ASA rule).

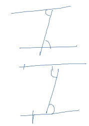
Theorem 8.2 : *In a parallelogram, opposite sides are equal.*

$\Delta ABC \cong \Delta DCA$

$AB = DC$
$AD = BC$ (CPCT)

Theorem 8.3 : *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*





Given: quadrilateral ABCD,

$AB = DC$,

$AD = BC$

To prove: ABCD is a ||gm

Constr: Join A to C

proof:– In ΔABC and ΔCDA

$AB = DC$ (given)

$BC = AD$ (given)

$AC = AC$ (common)

$\therefore \Delta ABC \cong \Delta CDA$ (SSS rule)

$\angle 1 = \angle 2$ (CPCT)

$\angle BAC = \angle DCA$

$\angle 3 = \angle 4$ (CPCT)

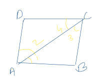
but these are alternate interior angles

$\therefore$ AB || DC

AD || BC

Which implies ABCD is a ||gm.

Theorem 8.4 : *In a parallelogram, opposite angles are equal.*



Given: ABCD is a ||gm, AB || DC

& AD || BC.

To proof: $\angle A = \angle C$ and

$\angle B = \angle D$

proof: AB || CD and AC is transversal.

$\angle 1 = \angle 4$ –(i) (alt interior angles)

AD || BC and AC is transversal

$\angle 2 = \angle 3$ –(ii) (alt interior angles).

Add (i) and (ii)

$\angle 1 + \angle 2 = \angle 4 + \angle 3$

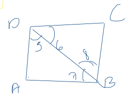
$\boxed{\angle A = \angle C}$

$\angle 5 = \angle 8$ (alt interior)

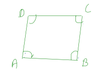
$\angle 6 = \angle 7$ (")

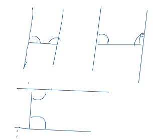
add

$\boxed{\angle D = \angle B}$



Theorem 8.5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*





Given:– $\angle A = \angle C$ and

$\angle B = \angle D$

To proof: ABCD is a ||gm

proof:– $\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Angle sum property of a quadrilateral)

$\angle A + \angle B + \angle A + \angle B = 360^\circ$

$2\angle A + 2\angle B = 360^\circ$

$\angle A + \angle B = 180^\circ$

but these are co-interior angles

$\therefore$ AD || BC –(i)

Similarly

$\angle A + \angle B = 180^\circ$

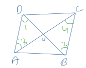
$\angle A + \angle D = 180^\circ$

$\therefore$ AB || CD –(ii)

$\therefore$ From (i) and (ii)

ABCD is a ||gm

Theorem 8.6 : *The diagonals of a parallelogram bisect each other.*



Given:– ||gm ABCD.

To prove: $AO = CO$, $OB = OD$

proof:– In ΔAOD and ΔCOB

$AD = BC$ (opposite sides of ||gm)

$\angle 1 = \angle 2$ (alt interior angles)

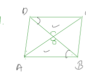
$\angle 3 = \angle 4$ (")

$\therefore \Delta AOD \cong \Delta COB$ (ASA rule)

$AO = CO$ (CPCT)

$OD = OB$ (CPCT)

Theorem 8.7 : *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*



Given: ABCD is a quadrilateral.

$AO = CO$, $OB = OD$.

To prove: ABCD is a ||gm

Proof: In ΔAOB and ΔCOD

$AO = CO$ (given)

$\angle AOB = \angle COD$ (V.O.A).

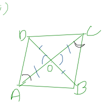
$OB = OD$ (given

$\therefore \Delta AOB \cong \Delta COD$ (SAS)

$\angle ABO = \angle CDO$ (CPCT)

but these are alt interior angles.

AB || DC –(i)



Now, In ΔAOD and ΔBOC

$AO = CO$ (given).

$\angle AOD = \angle BOC$ (V.O.A).

$OD = OB$ (given).

$\therefore \Delta AOD \cong \Delta BOC$ (SAS rule)

$\angle DAO = \angle BCO$ (CPCT)

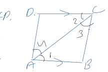
but these are alt interior angles

$\therefore$ AD || BC –(ii)

$\therefore$ From (i) and (ii)

ABCD is a ||gm

Theorem 8.8 : *A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.*



Given:– In a quadrilateral ABCD.

AB || DC, AB = DC

To proof: ABCD is a ||gm.

AD || BC,

Const:– Join A to C

Proof:– In ΔABC and ΔCDA.

$AB = CD$ (given)

$\angle 1 = \angle 2$ (alt. interior angles)

$AC = AC$ (Common)

$\therefore \Delta ABC \cong \Delta CDA$ (SAS)

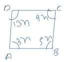
$\angle 3 = \angle 4$ (CPCT)

but these are alt. interior angles

$\therefore$ AD || BC

$\therefore$ ABCD is a ||gm

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.



let the angles are 3n, 5n, 9n and 13n.

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

(Angle sum property of quadrilateral)

$3n + 5n + 9n + 13n = 360^\circ$

$30n = 360$

$n = 12$

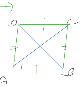
$\angle A = 36^\circ$

$\angle B = 60^\circ$

$\angle C = 108^\circ$

$\angle D = 156^\circ$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Given: ABCD is a ||gm, AC = BD.

To prove: ABCD is a rectangle

oppo sides equal & 1 angle is 90°

Proof: In $\triangle ABD$ and $\triangle BAC$

AD = BC (oppos sides of ||gm are equal)

AB = BA (common)

BD = AC (Given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle BAD = \angle ABC$ (CPCT)

$\angle BAD + \angle ABC = 180^\circ$ (Co interior angles).

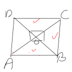
$\angle BAD + \angle BAD = 180^\circ$

$2\angle BAD = 180^\circ$

$\boxed{\angle BAD = 90}$

$\therefore$ ABCD is a rectangle

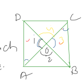
3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.



Given: OA = OC, OB = OD,
Diagonal bisect at right angle

To prove: ABCD is a rhombus,
AB = BC = CD = AD

4. Show that the diagonals of a square are equal and bisect each other at right angles.



Given: ABCD is a Square.

To prove: (i) AC = BD.
(ii) AC and BD bisect each other at right angle.

Proof: In $\triangle ABD$ and $\triangle BAC$

AD = BC (Given)

AB = AB (Common)

$\angle A = \angle B$ (90°)

$\therefore \triangle ABD \cong \triangle BAC$ (SAS rule)

AC = BD (CPCT)

(ii) In $\triangle AOD$ and $\triangle COB$

AD = BC (given)

$\angle OAD = \angle OCB$ (alt interi angles)

$\angle AOD = \angle COB$ (V.O.A)

$\therefore \triangle AOD \cong \triangle COB$ (AAS rule)

OD = OB, OA = OC (CPCT)

In $\triangle AOD$ and $\triangle AOB$

AD = AB (Given)

AO = AO (Common)

OD = OB (proved above)

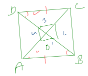
$\triangle AOD \cong \triangle AOB$ (SSS rule), $\angle AOD = \angle AOB$ (CPCT)

$\angle 1 = \angle 2$

$\angle 1 + \angle 2 = 180^\circ$ (linear pair)

$\angle 1 = \angle 2 = 90^\circ = \angle 3 = \angle 4$

5) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Given: AC = BD,
OD = OB, OA = OC.
$\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90^\circ$

To prove: ABCD is a square,
AB = BC = CD = AD, $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Proof: In $\triangle AOB$ and $\triangle COD$

OA = OC, OB = OD (Given)

$\angle AOB = \angle COD$ (V.O.A)

$\triangle AOB \cong \triangle COD$ (SAS rule)

AB = CD (CPCT)

$\angle OBA = \angle ODC$ (CPCT)

but those are alternate interior angles

AB || CD

ABCD is a ||gm (oppo. sides equal and parallel).

BC = AD - (i), AB = CD - (ii)

In $\triangle AOB$ and $\triangle BOC$

OA = OC (Given)

$\angle AOB = \angle BOC$ (each 90°)

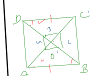
OB = OB (Common)

$\therefore \triangle AOB \cong \triangle BOC$ (SAS rule)

AB = BC (CPCT) - (iii)

From (i), (ii) & (iii)

$\boxed{AB = CD = AD = BC}$



In $\triangle ABD$ and $\triangle BAC$

AD = BC (oppo sides of ||gm)

AB = BA (Common)

BD = AC (given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle A = \angle B$ (CPCT) - (v)

$\angle A + \angle B = 180^\circ$ (Co-interior angles)

$\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = 90^\circ$

$\therefore$ ABCD is a square.

6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig. 8.19). Show that

(i) it bisects $\angle C$ also,

(ii) ABCD is a rhombus.