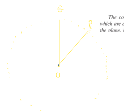
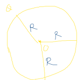
Ch-10 Circles



The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

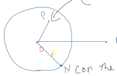
O - Fixed point

OP, OQ ... - Constant distance



O - Fixed point (Centre)

OP - Radius



(Interior point)

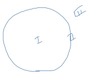
M (exterior point)

N (on the circle).

OM > R

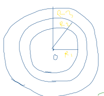
ON = R

OP < R.



→ A circle divides a plane into 3 parts:

- Interior, on the circle & exterior.

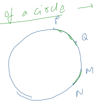


Cocentric circles

→ Radius different

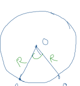
→ Centre same.

Arc of a circle → Continuous curves



PQ arc

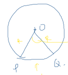
MN arc.



→ Arc containing central angle Minor arc

→ Arc excluding central angle are called Major arc.

Central angle



→ Central angle

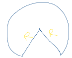
minor arc

Sector

→ Two radii

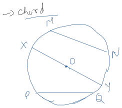
→ One arc.

↑ minor sector.

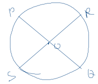


← major sector.

→ chord

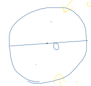


→ line joining any two parts in a circle is called chord.



Diameter - chord passing through centre

→ Semi circle



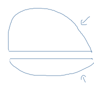
Semi circle

→ Diameter divides circle into two equal parts.

→ Each part is called semi circle.

→ Semi circle's end is diameter end.

→ Segment → chord + arc



← Major Segment

↑ Minor Segment

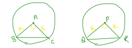
EXERCISE 10.1

1. Fill in the blanks:
   (i) The centre of a circle lies in <u>interior</u> of the circle. (exterior/ interior)
   (ii) A point, whose distance from the centre of a circle is greater than its radius lies in <u>exterior</u> of the circle. (exterior/ interior)
   (iii) The longest chord of a circle is a <u>diameter</u> of the circle.
   (iv) An arc is a <u>semi circle</u> when its ends are the ends of a diameter.
   (v) Segment of a circle is the region between an arc and <u>chord</u> of the circle.
   (vi) A circle divides the plane, on which it lies, in <u>3</u> parts.

2. Write True or False: Give reasons for your answers.
   (i) Line segment joining the centre to any point on the circle is a radius of the circle. ✓
   (ii) A circle has only finite number of equal chords. ✗
   (iii) If a circle is divided into three equal arcs, each is a major arc. ✗
   (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle. ✓
   (v) Sector is the region between the chord and its corresponding arc. ✗
   (vi) A circle is a plane figure. ✓

EXERCISE 10.2

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.



Given - BC = QR.
AB = AC = PQ = PR (Radii of congruent circles).

To prove:- ∠A = ∠P

Proof:- In ΔABC and ΔPQR.
AB = PQ
AC = PR (Given) {Radii of congruent circles}
BC = QR (given)
ΔABC ≅ ΔPQR (SSS rule).
∠BAC = ∠QPR (CPCT)
∠A = ∠P

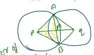
2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

To prove:- BC = QR.

Given - ∠A = ∠P

Proof:- In ΔABC and ΔPQR
AB = PQ
AC = PR {Radii of congruent circles}
∠BAC = ∠QPR (given).
ΔABC ≅ ΔPQR (SAS rule).
BC = QR (CPCT)

3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.



To prove:- AM = BM
∠3 = ∠4 = 90°

let P be center of 1st circle & Q be ... 2nd

In ΔAPQ and ΔBPQ.
AP = BP (Radii of same circle)
AQ = BQ (")
PQ = PQ (common)
∴ ΔAPQ ≅ ΔBPQ (SSS rule)
∠1 = ∠2 (CPCT)

Now in ΔAPM and ΔBPM
AP = BP (Radii of same circle)
∠1 = ∠2 (proved above)
PM = PM
∴ ΔAPM ≅ ΔBPM (SAS rule)
AM = BM (CPCT)
→ PQ bisects AB
∠AMP = ∠BMP
∠3 = ∠4 (CPCT) —(i)

Now, ∠3 + ∠4 = 180° (linear pair)
∠3 + ∠3 = 180° (from (i))
2∠3 = 180°
∠3 = 90°
∠3 = ∠4 = 90°

EXERCISE 10.3

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

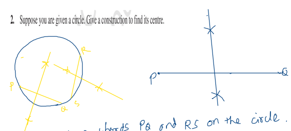
— No point in common

— 1 point in common

— 2 point in common. ↑ Maximum

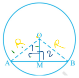
—

2. Suppose you are given a circle. Give a construction to find its centre.



- Take two chords PQ and RS on the circle.
- Draw ⊥r bisector of PQ and RS.
- Intersection point of ⊥r bisector of PQ and RS gives required centre.

Theorem 10.3 : *The perpendicular from the centre of a circle to a chord bisects the chord.*



Given - O is center of circle, $\angle 1 = \angle 2$ (90°) AB is a chord.

To proof: AM = BM

Proof :- In $\triangle OAM$ and $\triangle OBM$

OA = OB (Radii of the same circle)

$\angle 1 = \angle 2$ (each 90°)

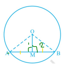
OM = OM (common)

$\triangle OAM \cong \triangle OBM$ (RHS rule)

AM = BM (CPCT)

Converse

Theorem 10.4 : *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*



Given : M is midpoint.

Proof :- In $\triangle OAM$ and $\triangle OBM$

OA = OB (Radii of same circle)

OM = OM (common)

AM = BM (M is midpoint)

$\therefore \triangle OAM \cong \triangle OBM$ (SSS rule)

$\angle 1 = \angle 2$ (CPCT)

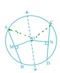
$\angle 1 + \angle 2 = 180°$ (linear pair)

$\angle 1 + \angle 1 = 180°$

$2\angle 1 = 180°$

$\boxed{\angle 1 = 90°}$ $\boxed{\angle 2 = 90°}$

Theorem 10.6 : *Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).*



Given :- AB = CD

To prove :- OM = ON

Const :- $OM \perp AB$ and $ON \perp CD$

Proof :- AB = CD (given)

$\frac{1}{2} AB = \frac{1}{2} CD$ ($\perp$ drawn from the centre bisects the chord)

AM = CN

In $\triangle OAM$ and $\triangle OCN$

OM = ON (Radii of same circle)

$\angle M = \angle N$ (90°)

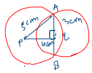
AM = CN (proved above)

$\therefore \triangle OAM \cong \triangle OCN$ (SAS rule)

OM = ON (CPCT)

EXERCISE 10.4

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.



let P be center of 1st circle & Q be " 2nd circle.

In $\triangle APQ$,

$(5)^2 = (3)^2 + (4)^2$

$25 = 9 + 16$

$25 = 25$

As it satisfies Pythagoras thm, $\therefore$ we can conclude that $\triangle APQ$ is a right angle $\triangle$.

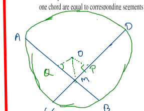
$\angle Q = 90°$

Since P is the centre of 1st circle & AB is the chord, & therefore PQ is $\perp$ drawn from center of circle bisects the chord.

$\therefore$ AQ = BQ

$\boxed{AB = 2 \times AQ = 2 \times 3 = 6cm.}$

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.



Given :- AB = CD.

To prove : AM = DM

CM = BM.

Const :- Draw OQ and OP as $\perp$ from centre O.

In $\triangle OQM$ and $\triangle OPM$

OQ = OP (equal chords are equidistant from the centre).

$\angle Q = \angle P$ (90°)

OM = OM (Common)

$\therefore \triangle OQM \cong \triangle OPM$ (RHS rule)

QM = PM (CPCT) — (1)

AB = CD (given)

$\frac{1}{2} AB = \frac{1}{2} CD$

$\Rightarrow$ AQ = PD — (2)

Add (1) and (2)

AQ + QM = PM + PD.

$\boxed{AM = DM}$ — (3)

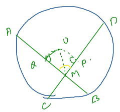
AB = CD — (4)

Subtract (3) from (4)

AB − AM = CD − DM.

$\boxed{BM = CM}$

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



$\angle QMO = \angle PMO$