

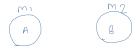
Gravitation

The phenomenon of attraction 

Between any two objects in the Universe is called gravitation.

Gravity — attractive force which is exerted by earth on any body.

Universal Law of Gravitation.



$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

$$F = G \times \frac{m_1 m_2}{r^2}$$

Everybody in the universe attracts every other body with an force which is directly proportional to the product of their mass & inversely proportional to the square of the distance between them.

Universal gravitational constant

$$G = \frac{(F \times r^2)}{m_1 m_2}$$

$$G = \frac{N \cdot m^2}{kg^2}$$

$$G = 6.67 \times 10^{-11} N \cdot m^2 \cdot kg^{-2}$$

SI unit of G

$F = G \frac{m_1 m_2}{r^2}$

$G = \frac{F \cdot r^2}{m_1 m_2}$

$G = F \times (m)^2$

$G = F \times 1 kg$

$G = F$

Universal gravitational constant is numerically equal to the gravitational force of attraction between two bodies each of mass 1 kg kept at unit distance from each other.

Why gravitational constant (G) is known as Universal gravitational constant?

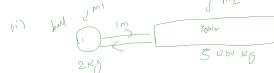
→ The value of G doesn't depend on mass of two bodies, distance b/w two bodies, nature, medium, shape/size.

Conditions

It's within two objects each of 1 kg and 1 m apart.



$$F = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{1 \times 1}{1} = 6.67 \times 10^{-11} N$$



$$\text{Force of Ball to Train} = G \frac{m_1 m_2}{r^2} = 6.67 \times 5000 \frac{1}{1^2} = 33350 N$$

$$\text{Force of Train to Ball} = G \frac{m_1 m_2}{r^2} = 6.67 \times 1 \frac{5000}{1^2} = 33350 N$$

(iii)

Earth mass = $5 \times 10^{24} kg$
Radius of earth = 6300 km
= $6300 \times 10^3 m$
 $\approx 6.3 \times 10^6 m$

$$F_{AE} = G \frac{m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times (6.3 \times 10^6)^2}{(6.3 \times 10^6)^2} = 9.8 N$$

$$F_{EA} = 9.8 N$$

Newton's 2nd law of motion

$$F = ma$$

$$\checkmark \frac{F}{m} = \frac{F}{m} = \frac{9.8}{1} = 9.8 m s^{-2}$$

$$\checkmark a_{AE} = \frac{F}{m} = \frac{9.8}{6 \times 10^{24} kg} = 1.63 \times 10^{-24} m s^{-2}$$



Before the person is released, the earth moves in a circular path with certain speed and changes direction at every certain point due to centripetal force or acceleration. The force that causes this acceleration is called centripetal force. The centripetal force is acting towards the centre. This is the reason why we feel a force pushing us towards the center while moving in a circular path.

In case of the forces around the earth there are no such forces as centripetal force or acceleration. If there were no such force, the person would just move straight like a projectile.

- i) Two particles A and B of mass m_1 and m_2 respectively are placed on some distance r from the center of the earth. Keeping the distance b/w them unchanged, the gravitational force b/w them will be:

$$F_1 = G \frac{m_1 m_2}{r^2}$$

$$F_2 = G \frac{m_1 m_2}{(2r)^2}$$

$$F_2 = G \frac{m_1 m_2}{4r^2}$$

$$F_2 = \frac{1}{4} F_1$$

$$F_2 = \frac{G m_1 m_2}{(3r)^2}$$

$$F_2 = \frac{1}{9} F_1$$

2) mass of sun is $2 \times 10^{30} \text{ kg}$ & mass of earth is $6 \times 10^{24} \text{ kg}$
 At an avg distance b/w the sun and the earth $1.5 \times 10^8 \text{ km}$,
 Calculate the gravitational force b/w them.

$$F = G \frac{m_1 m_2}{r^2}$$

3) The gravitational force b/w two objects is F . If masses of both objects are halved upto same distance b/w them, gravitational force would become:

$$F_1 = G \frac{m_1 m_2}{r^2}$$

$$F_2 = G \frac{(m_1/2)(m_2/2)}{r^2} = \frac{1}{4} F_1 = \frac{F}{4}$$

v) gravitational force b/w two objects weighing 2.0g and 1.8g is $1.8 \times 10^{-10} \text{ N}$. Find distance b/w two objects

Scientists → Arrangement of planets

Ptolemy → Geocentric Theory
 Two AD

Earth
is at
center
& all planets
revolve around

Copernicus → Heliocentric Theory
 1543

Sun at the
center

Tycho Brahe → Tychoic system
 1576

Moon is at the
center

Galileo → Sun is at the center
 1610
 Heliocentric theory
 telescope

Kepler's law of planetary motion

We stated three laws which govern the motion of planets around the sun. These are known as Kepler's law of planetary motion:



Kepler's 1st law of motion

→ Planets move in elliptical orbits around the sun, with the sun at one of two foci of the elliptical orbit.

Kepler's 2nd law of motion



✓ Each planet revolves around the sun in such a way that the line joining the planet to the sun sweeps over equal areas in equal intervals of time.

✓ A planet moves faster when it is closer to the sun & moves slowly when it is farther away from the sun.

Kepler's 3rd law of motion

✓ The cube of the mean distance of a planet from the sun is directly proportional to the square of time, it takes to move around the sun.

$$\frac{r^3}{T^2} = \text{constant}$$

$$r^3 = \text{constant } T^2$$

$$\frac{r^3}{T^2} = \text{constant}$$

mean distance of the planet from the sun.
 Time period of the planet around the sun.

Free fall and acceleration due to gravity (g)

The falling of a body (or object) from a height towards the earth under gravitational force of earth (with no other force acting on it), such a body is called **freely falling body**.

$$\text{out: change in velocity}$$

$$F = ma$$

Acc due to gravity (g)

→ The uniform acc produced in a freely falling body due to gravitational force of the earth is known as **acc due to gravity**.

$$\rightarrow \text{SI unit} = \text{m s}^{-2}$$

→ g doesn't depend on the mass of the falling object.



Calculation of acc due to gravity

Force exerted by earth

$$F = G \frac{Mm}{R^2}$$

$$F = m a$$

$$G \frac{Mm}{R^2} = m a$$

$$a = \frac{GM}{R^2}$$

$$a = \frac{GM}{R^2}$$

$$g = \frac{GM}{R^2}$$

$$g = 9.8 \text{ m s}^{-2}$$

This force by earth
produces acc in the
stone due to which
stone moves downwards

$$g = \frac{GM}{R^2}$$

$$g = \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times (6 \times 10^{24} \text{ kg})}{(6.4 \times 10^6)^2}$$

$$g = 9.8 \text{ m s}^{-2}$$

Variation of acc due to gravity (g)



- Earth is not a perfect sphere.
- Gravitational force is not constant.
- Gravity is highest at the equator.
- Gravity is lowest at the poles.

$$g = G \frac{M}{R^2}$$

Radius ratio = Radius of equator / Radius of pole

→ The value of g is not constant at all the places on the surface of earth.

→ As earth is not a perfect sphere so the value of radius R is not the same at all the places on its surface.

→ Radius of earth at the poles is minimum. Value of g is maximum.

→ Radius of earth at the equator is maximum. Value of g is minimum.

→ But this formula is not applicable at any point inside the surface of earth.

→ The value of g decreases as we go down inside the earth.

→ Value of g becomes zero at the center of earth.

Equation of Motion for freely falling bodies

Equation of motion \rightarrow for horizontal motion

$$\begin{aligned} V &= U + at \\ S &= Ut + \frac{1}{2}at^2 \\ V^2 - U^2 &= 2as \end{aligned}$$

$$\begin{aligned} \text{Motion due to gravity} \\ V &= U + gt \\ h &= Ut + \frac{1}{2}gt^2 \\ V^2 - U^2 &= 2gh \end{aligned}$$

$$\begin{aligned} \text{Upward motion} \\ \text{P.V.O.C.} \\ U = 5 \text{ m/s}, V = 10 \text{ m/s} \\ a = \frac{V-U}{t} = \frac{10-5}{2} = 2.5 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \text{Parabola} \\ U = 10 \text{ m/s}, V = 5 \text{ m/s} \\ a = \frac{V-U}{t} = \frac{5-10}{2} = -2.5 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \text{when } a = \frac{V-U}{t} \text{ (V>U)} \\ \text{when } a = \frac{U-V}{t} \text{ (U>V)} \end{aligned}$$

- when a body is falling vertically downwards, its velocity is increasing so the acceleration due to gravity is taken as positive
- $a = 9.8 \text{ m/s}^2$
- therefore a is upwards.
- negative acceleration.
- $a = -9.8 \text{ m/s}^2$

- To estimate the height of Rishabh bridge over river, a stone is dropped freely in the river from the bridge. The stone takes 2 seconds to touch the water surface in the river. Calculate the height of the bridge from water level.

$$\begin{aligned} h &= Ut + \frac{1}{2}gt^2 \\ h &= 0 \times 2 + \frac{1}{2} \times 9.8 \times 2^2 \\ h &= 19.6 \text{ m} \end{aligned}$$

QUESTION 10 A ball is thrown vertically upwards with a speed of 10 m/s . How high it go before it begins to fall?

ANSWER

$$\begin{aligned} \text{Initial velocity } U &= 10 \text{ m/s} \\ \text{Acceleration } a &= -9.8 \text{ m/s}^2 \\ \text{Final velocity } V &= ? \\ \text{Time } t &= 0.5 \text{ s} \\ \text{Distance } h &= ? \end{aligned}$$

(i) $V = U + gt$
 $0 = 10 - 9.8 \times 0.5$
 $0 = 10 - 4.9$
 $4.9 = 10$

(ii) $h = Ut + \frac{1}{2}gt^2$
 $h = 10 \times 0.5 + \frac{1}{2} \times 9.8 \times 0.5^2$
 $h = 5 + 1.225$
 $h = 6.225 \approx 6.2 \text{ m}$

Example 10.2 An object is thrown vertically upwards and reaches a height of 10 m above the point from which the object was thrown. Find the initial velocity with which the object was thrown.

$$\begin{aligned} S &= 10 \text{ m} \\ V &= 0 \text{ m/s} \\ g &= 9.8 \text{ m/s}^2 \quad (\text{Upward motion}) \\ V^2 - U^2 &= 2gh \\ (0)^2 - U^2 &= 2 \times 9.8 \times 10 \\ U^2 &= 196 \\ U &= \sqrt{196} \\ &= 14 \text{ m/s} \\ (i) \quad V &= U + gt \\ 0 &= 14 - 9.8t \\ t &= \frac{14}{9.8} = 1.43 \text{ s} \end{aligned}$$

- B. A stone is dropped from a height of 20 m
- i) How long will take to reach the ground?
- ii) What will be its speed when it hits the ground?
- $\Rightarrow h = Ut + \frac{1}{2}gt^2$
 $20 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$
 $20 = 4.9t^2$
 $t^2 = \frac{20}{4.9}$
 $t = \sqrt{\frac{20}{4.9}} = 2.02 \text{ s}$
- iii) A stone is thrown vertically upwards with a speed of 20 m/s . How high it go before it begins to fall?

Mass and Weight

Mass of the body is the quantity of matter (or mass) contained in it.

- scalar quantity (only magnitude and no direction).
- SI unit is kg.
- Mass of body is constant, doesn't change from place to place.
- Mass of body can't be zero.

Weight

Weight of a body is the force with which it is attracted towards the center of the earth.

$$F = \frac{GMm}{R^2}$$

\rightarrow By Newton's 2nd law of motion
 $F = ma$
 $W = mg$

\rightarrow SI unit of weight is Newton.

- g is a vector quantity.
- Weight of a body acts in vertically downward direction.

\rightarrow Weight depends on g therefore, weight of a body is not constant.

\rightarrow Wt in poles \neq Wt in equator

In Weighing m/c weight or mass?

Liquid sensor measures the force which your hand applying because of the gravitational pull of earth.

$$\begin{aligned} F &= W = mg \\ W &\rightarrow m \text{ in kg} \end{aligned}$$

Weight of an object on the moon

- Weight of an object on the moon is the force with which the moon attracts the object.
- Value of g differs on earth and moon.

$g_{\text{moon}} = \frac{1}{6} g_{\text{earth}}$

Celestial Body	Mass (kg)	Radius (m)
Earth	5.98×10^{24}	6.37×10^6
Moon	7.36×10^{22}	1.73×10^6

$$W_m = \frac{GMm}{R^2}$$

$$W_m = \frac{(6.67 \times 10^{-11})(7.36 \times 10^{22})(m)(6.37 \times 10^6)^2}{(6.37 \times 10^6)^2}$$

$$\frac{2.431 \times 10^2}{1.474 \times 10^2} = 0.165 \approx \frac{1}{6}$$

Weight of an object on the moon

Weight of an object on the earth

Weight of an object on the moon will be about one-sixth of what is on earth.

Gravitational force of the moon is about one-sixth that of the earth.

1.2 Mass of an object is 10kg. What is weight on the earth?

$$\rightarrow W = mg \\ 10 \times 9.8 = 98N$$

Example 10.5 An object weighs 10 N when measured on the surface of the earth. What would be its weight when measured on the surface of the moon?

$$\text{wt of obj on the moon} = \left(\frac{1}{6}\right) \times \text{wt of the earth} \\ = \frac{1}{6} \times 10 = \frac{10}{6} = 1.67N$$

3) what is the mass of an object whose weight is 10N?

$$\rightarrow W = mg$$

$$m = \frac{W}{g} = \frac{10}{9.8} = 5 \text{ kg} \quad (g = 9.8 \text{ ms}^{-2})$$

4) A man weighs 600N on the earth. what is his mass?

If he were taken to the moon, his weight would be 100N. What is his mass on the moon? what is the acc due to gravity on the moon?

$$W = mg \\ 600 = m \times 9.8 \\ m = \frac{600}{9.8} = 61.2 \text{ kg}$$

moon's weight

moon's acc due to gravity

$W = mg \\ 100 = m \times 1.63 \\ m = \frac{100}{1.63} = 61.2 \text{ kg}$

5) How much would a 70kg man weigh on the moon? what would be his mass on the earth and the moon? (acc due to gravity on moon = 1.63 ms^{-2})

$$\text{Weight} \propto \text{mass} \propto \text{constant} \\ W = mg \\ 70 \times 1.63 = 114.1N$$

10.5 Thrust and Pressure



Pressure is the force acting on a unit area of the object.



Tip: You stand on your head. Your feet glide into the world. Now, by force of the law of conservation of momentum, you go that way as you want. In both cases, the body continues to move in the height of your body.

→ The effect of a force depends on the area of the object on which it acts

→ same force can produce different pressures depending on the area over which it acts

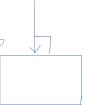
→ When a force acts over large contact area of an object, it produces a small pressure.



$$\rightarrow \text{SI unit of pressure} : p = \frac{F}{A} = \frac{N}{m^2} = \text{N m}^{-2} \quad (\text{pascal}) \\ \rightarrow 1 \text{ Pa} = 1 \text{ N m}^2$$

Thrust

The force acting on a body perpendicular to its surface.



Ques: In the given figure, ABCD is a trapezium. M is the midpoint of AB and PQ is a line such that AD is parallel to PQ and parallel to BC. Prove that $\triangle PMB \cong \triangle QMC$ and $PM = QM$.



→ In $\triangle PMB$ and $\triangle QMC$,
 $PM = QM$ ($AM \cong BM$)
 $\angle PMB = \angle QMC$ (90°)
 $\angle AMB = \angle CMQ$ (v.o.t.)
 $AM = QM$ (M is mid point of AB)

$\therefore \triangle PMB \cong \triangle QMC$ (ASA)

$PM = QM$ (CPCT)

$\therefore \triangle PMB \cong \triangle QMC$ (ASA)

$\therefore CB + BA > PB + PA$ (triangle inequality)
 $\therefore CP > PB + PA$ (transitivity)

(ii)

In $\triangle PBM$ and $\triangle QDM$,

$\angle PBM = \angle QDM$ (90°)
 $\angle BMP = \angle DMQ$ (v.o.t.)

$BM = QM$ (M is mid point of PB)

$\therefore \triangle PBM \cong \triangle QDM$ (ASA)

$PM = QM$ (CPCT)

In the following figure, ABC is a triangle and D is any point in its interior. Show that $BD + DC < AB + AC$.



On $\triangle BAE$

Sum of any Two sides of a triangle is greater than the third side

$AB + AE > BE$ (i)

from Pythagoras' theorem
 $AB^2 + AE^2 = BE^2$ (ii)

$AB + AE > BE$ (i)

$AB + AE > BD + DE$ (iii)

$AB + AE + CE > BD + DE + EC$ (iv)

On $\triangle ADE$
 $CE + ED > CD$ (v)

Add (iii) and (v)

$AB + AE + CE + ED > BD + DE + EC$
 $AB + AE + CE + ED > BD + DE + EC$

$AB + AC > BD + DC$

