Quadrilaterals

point → A point is that which determines location

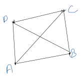
→ line segment

→ line.

Collinear point - Points lying on same line.

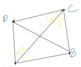
Quadrilaterals

Any enclosed figure made with 4 non-collinear points is known as quadrilateral



Vertices - A, B, C, D
Sides - AB, BC, CD, DA.
Angles - $\angle A, \angle B, \angle C, \angle D$
Digonals - AC and BD

Thm 8.2 Angle Sum Property of a Quadrilateral



Given: ABCD is a quadrilateral

To prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

proof: In $\triangle ABC$

$\angle 1 + \angle B + \angle 3 = 180^\circ$ (Angle sum Property of a $\triangle$) -(i)

In $\triangle ADC$

$\angle 2 + \angle D + \angle 4 = 180^\circ$ (") -(ii)

$(\angle 1 + \angle B + \angle 3) + (\angle 2 + \angle D + \angle 4) = 360^\circ$

$(\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^\circ$

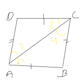
$$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$$

Theorem 8.1 : *A diagonal of a parallelogram divides it into two congruent triangles.*

||gm - Quadrilateral whose opposite sides are ||.

Given - ||gm ABCD.

To prove: $\triangle ABC \cong \triangle ADC$



proof: In $\triangle ABC$ and $\triangle ADC$

AB || DC, AD || BC, AC is a transversal.

$\angle 1 = \angle 2$ (alter interior angles)

$\angle 4 = \angle 3$ (")

AC = AC (Common)

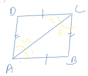
$\therefore \triangle ABC \cong \triangle ADC$ (ASA rule).

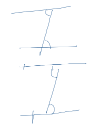
Theorem 8.2 : *In a parallelogram, opposite sides are equal.*

$\triangle ABC \cong \triangle DCA$

AB = DC
AD = BC (CPCT)

Theorem 8.3 : *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*





Given: quadrilateral ABCD,
AB = DC,
AD = BC

To prove: ABCD is a ||gm

Constr: Join A to C

proof: In $\triangle ABC$ and $\triangle CDA$

AB = DC (given)
BC = AD (given)
AC = AC (common)

$\therefore \triangle ABC \cong \triangle CDA$ (SSS rule)

$\angle 1 = \angle 2$ (CPCT)
$\angle BAC = \angle DCA$
$\angle 3 = \angle 4$ (CPCT)

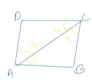
but these are alternate interior angles

$\therefore$ AB || DC
AD || BC

Which implies ABCD is a ||gm.

Theorem 8.4 : *In a parallelogram, opposite angles are equal.*

Given: ABCD is a ||gm, AB || DC & AD || BC.



To proof: $\angle A = \angle C$ and $\angle B = \angle D$

proof: AB || CD and AC is transversal.

$\angle 1 = \angle 4$ -(i) (alt interior angles)

AD || BC and AC is transversal

$\angle 2 = \angle 3$ -(ii) (alt interior angles).

Add (i) and (ii)

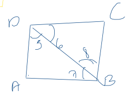
$\angle 1 + \angle 2 = \angle 4 + \angle 3$

$$\boxed{\angle A = \angle C}$$

$\angle 5 = \angle 8$ (alt interior)

$\angle 6 = \angle 7$ (")

add

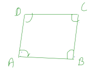


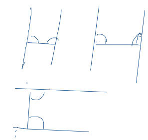
$$\boxed{\angle D = \angle B}$$

Theorem 8.5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*

Given: $\angle A = \angle C$ and $\angle B = \angle D$

To proof: ABCD is a ||gm





Proof: $\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Angle sum property of a quadrilateral)

$\angle A + \angle B + \angle A + \angle B = 360^\circ$

$2\angle A + 2\angle B = 360^\circ$

$\angle A + \angle B = 180^\circ$

but these are co-interior angles

$\therefore$ AD || BC -(i)

Similarly

$\angle A + \angle B = 180^\circ$
$\angle A + \angle D = 180^\circ$

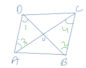
$\therefore$ AB || CD -(ii)

$\therefore$ From (i) and (ii)
ABCD is a ||gm

Theorem 8.6 : *The diagonals of a parallelogram bisect each other.*

Given: ||gm ABCD.

To prove: AO = CO, OB = OD



proof: In $\triangle AOD$ and $\triangle COB$

AD = BC (opposite sides of ||gm)
$\angle 1 = \angle 2$ (alt interior angles)
$\angle 3 = \angle 4$ (")

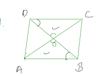
$\therefore \triangle AOD \cong \triangle COB$ (ASA rule)

AO = CO (CPCT)
OD = OB (CPCT)

Theorem 8.7 : *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

Given: ABCD is a quadrilateral.
AO = CO, OB = OD.

To proove: ABCD is a ||gm



proof: In $\triangle AOB$ and $\triangle COD$

AO = CO (given)
$\angle AOB = \angle COD$ (V.O.A).
OB = OD (given

$\therefore \triangle AOB \cong \triangle COD$ (SAS)

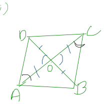
$\angle ABO = \angle CDO$ (CPCT)

but those are alt interior angles.

AB || DC -(i)

Now, In $\triangle AOD$ and $\triangle BOC$

AO = CO (given).
$\angle AOD = \angle BOC$ (V.O.A).
OD = OB (given).



$\therefore \triangle AOD \cong \triangle BOC$ (SAS rule)

$\angle DAO = \angle BCO$ (CPCT)

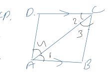
but these are alt interior angles

$\therefore$ AD || BC -(ii)

$\therefore$ from (i) and (ii)
ABCD is a ||gm

Theorem 8.8 : *A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.*

Given: In a quadrilateral ABCD.
AB || DC, AB = DC



To proof: ABCD is a ||gm.
AD || BC,

Const: Join A to C

proof: In $\triangle ABC$ and $\triangle CDA$.

AB = CD (given)
$\angle 1 = \angle 2$ (alt. interior angles)
AC = AC (Common.)

$\therefore \triangle ABC \cong \triangle CDA$ (SAS)
$\angle 3 = \angle 4$ (CPCT)

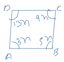
but these are alt. interior angles

$\therefore$ AD || BC

$\therefore$ ABCD is a ||gm

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

let the angles are $3n, 5n, 9n$ and $13n$.



$\angle A + \angle B + \angle C + \angle D = 360°$

(Angle sum property of quadrilateral)

$3n + 5n + 9n + 13n = 360°$

$30n = 360$

$n = 12$

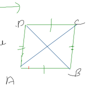
$\angle A = 36°$

$\angle B = 60°$

$\angle C = 108°$

$\angle D = 156°$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Given: ABCD is a ||gm, $AC = BD$.

To prove: ABCD is a rectangle

oppo sides equal & 1 angle is 90°

Proof: In $\triangle ABD$ and $\triangle BAC$

$AD = BC$ (oppos sides of ||gm are equal)

$AB = BA$ (common)

$BD = AC$ (Given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle BAD = \angle ABC$ (CPCT)

$\angle BAD + \angle ABC = 180°$ (co interior angles).

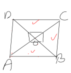
$\angle BAD + \angle BAD = 180°$

$2\angle BAD = 180°$

$\boxed{\angle BAD = 90}$

$\therefore$ ABCD is a rectangle

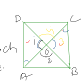
3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.



Given: $OA = OC$, $OB = OD$, Diagonal bisect at right angle

To prove: ABCD is a rhombus, $AB = BC = CD = AD$

4. Show that the diagonals of a square are equal and bisect each other at right angles.



Given: ABCD is a Square.

To prove: (i) $AC = BD$.

(ii) AC and BD bisect each other at right angle.

Proof: In $\triangle ABD$ and $\triangle BAC$

$AD = BC$ (given)

$AB = AB$ (Common)

$\angle A = \angle B$ (90°)

$\therefore \triangle ABD \cong \triangle BAC$ (SAS rule)

$AC = BD$ (CPCT)

(ii) In $\triangle AOD$ and $\triangle COB$

$AD = BC$ (given)

$\angle OAD = \angle OCB$ (alt interi angles)

$\angle AOD = \angle COB$ (V.O.A)

$\therefore \triangle AOD \cong \triangle COB$ (AAS rule)

$OD = OB$, $OA = OC$ (CPCT)

In $\triangle AOD$ and $\triangle AOB$

$AD = AB$ (Given)

$AO = AO$ (Common)

$OD = OB$ (proved above)

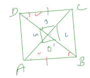
$\triangle AOD \cong \triangle AOB$ (SSS rule), $\angle AOD = \angle AOB$ (CPCT)

$\angle 1 = \angle 2$

$\angle 1 + \angle 2 = 180°$ (linear pair)

$\angle 1 = \angle 2 = 90° = \angle 3 = \angle 4$

5) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Given: $AC = BD$, $OD = OB$, $OA = OC$, $\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90°$

To prove: ABCD is a square, $AB = BC = CD = AD$, $\angle A = \angle B = \angle C = \angle D = 90°$

Proof: In $\triangle AOB$ and $\triangle COD$

$OA = OC$, $OB = OD$ (Given)

$\angle AOB = \angle COD$ (V.O.A)

$\triangle AOB \cong \triangle COD$ (SAS rule)

$AB = CD$ (CPCT)

$\angle OBA = \angle ODC$ (CPCT)

but those are alternate interior angles

$AB \parallel CD$

ABCD is a ||gm (oppo. sides equal and parallel).

$BC = AD$ -(i) $AB = CD$ -(ii)

In $\triangle AOB$ and $\triangle BOC$

$OA = OC$ (Given)

$\angle AOB = \angle BOC$ (each 90°)

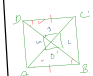
$OB = OB$ (Common)

$\therefore \triangle AOB \cong \triangle BOC$ (SAS rule)

$AB = BC$ (CPCT) - (iii)

From (i), (ii) & (iii)

$\boxed{AB = CD = AD = BC}$



In $\triangle ABD$ and $\triangle BAC$

$AD = BC$ (oppo sides of ||gm)

$AB = BA$ (Common)

$BD = AC$ (given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle A = \angle B$ (CPCT) - (v)

$\angle A + \angle B = 180°$ (co-interior angles)

$\angle A + \angle A = 180°$

$2\angle A = 180°$

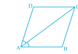
$\angle A = 90°$

$\therefore$ ABCD is a square.

6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig. 8.19). Show that

(i) it bisects $\angle C$ also,

(ii) ABCD is a rhombus.



6–10

Fig. 8.21

11. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22). Show that

(i) quadrilateral ABED is a parallelogram

(ii) quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$

(iv) quadrilateral ACFD is a parallelogram

(v) $AC = DF$

(vi) $\triangle ABC \cong \triangle DEF$.

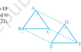


Fig. 8.22

Given: $AB = DE$, $AB \parallel DE$, $BC = EF$, $BC \parallel EF$.

(i) In quad ADED, $AB = DE$, also $AB \parallel DE$ (given)

$\therefore$ ABED is a ||gm (opposide are equal and ||)

(ii) In quad BEFC, $BC = EF$, $BE \parallel EF$ (given)

$\therefore$ BEFC is a ||gm (oppo sides are equal & ||)

(iii) If ABED is a ||gm,

$AD \parallel BE$ -(iii) $AD = BE$ -(iv)

BEFC is a ||gm,

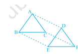
$CF \parallel BE$ -(v) $CF = BE$ (vi)

From (iii) and (v)

$AD \parallel CF$

From (iv) and (vi)

$AD = CF$



(iv) In quad ACFD,

$AD \parallel CF$ and $AD = CF$ (proved above)

$\therefore$ ACFD is a ||gm

(v) $AC = DF$ (oppo sides of ||gm ACFD.

(vi) In $\triangle ABC$ and $\triangle DEF$.

$AB = DE$, $BC = EF$ (Given)

$AC = DF$ (proved above)

$\therefore \triangle ABC \cong \triangle DEF$ (SSS rule)

12. ABCD is a trapezium in which AB ∥ CD and AD = BC (see Fig. 8.23). Show that
(i) ∠A = ∠B
(ii) ∠C = ∠D
(iii) Δ ABC ≅ Δ BAD
(iv) diagonal AC = diagonal BD
[*Hint*: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

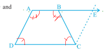
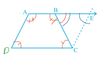


Fig. 8.23

Given :- ABCD is a trapezium, AB ∥ CD, AD = BC
Const :- extend AB to E, Join C to E.
CE ∥ DA and CE = DA

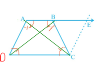
Sol :- CE ∥ DA, & AE is transversal
$\angle A + \angle E = 180^\circ$ – (i) (Co interior angles)
$\angle CBA + \angle CBE = 180^\circ$ – (ii) (linear pair)



As CE = DA and CE ∥ DA
∴ AECD is a ∥gm (opp sides equal and ∥).
AD = CE (opp sides of ∥gm) –(iii)
AD = BC (given) –(iv)
from (iii) & (iv)
∴ BC = CE
∴ ∠CBE = ∠CEB (angle oppo to equal sides)
$\angle CBA + \angle CBE = 180^\circ$ (using (ii))
$\angle CBA + \angle CEB = 180^\circ$ –(vi)
$\angle A + \angle E = 180^\circ$ – (using (i))
$\angle A + \angle E = \angle CEB + \angle CBA$.
$\angle A + \cancel{\angle E} = \cancel{\angle E} + \angle B$
$\boxed{\angle A = \angle B}$

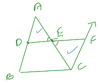
(ii) ABCD is a trapezium, AB ∥ CD, AD is transversal
$\angle A + \angle D = 180^\circ$ (Co-interior angles)
AB ∥ CD, BC is transversal,
$\angle B + \angle C = 180^\circ$ (co-interior angles)

$\cancel{\angle A} + \angle D = \cancel{\angle B} + \angle C$. (∠A = ∠B proved above)
∠D = ∠C



(iii) In ΔABC and ΔBAD
∠A = ∠B (proved above)
BC = AD (given)
AB = AB (common)
ΔABC ≅ ΔBAD (SAS rule)
(iv) AC = BD (CPCT)

Theorem 8.9 : *The line segment joining the mid-points of two sides of a triangle is parallel to the third side.*



Given :- ΔABC, D is midpoint of AB, E is midpoint of AC
To prove :- DE ∥ BC, DE = $\frac{1}{2}$ BC
Const :- CF ∥ AB
Proof :- In ΔAED and ΔCEF.
∠AED = ∠CEF (VOA)
∠DAE = ∠ECF (alt interior angles)
AE = EC (E is midpoint)
∴ ΔAED ≅ ΔCEF (ASA rule).
DE = EF (CPCT)
AD = CF
but AD = DB (D is midpoint).
∴ DB = CF also DB ∥ CF
∴ DBCF is a ∥gm (∵ opp sides equal and ∥)
$\boxed{\therefore BC \parallel DE}$

also BC = DF
BC = DE + EF
BC = DE + DE.
BC = 2DE
$\boxed{DE = \frac{1}{2} BC}$

If D & E are mid points, DE ∥ BC, DE = $\frac{1}{2}$ BC

DE ∥ BC
DE = $\frac{1}{2}$ BC

Converse

If D is mid point & DE ∥ BC → E is mid point