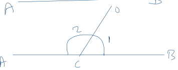
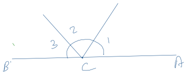
Lines and Angles

180° → Straight angle



$\angle 1 + \angle 2 = 180^\circ$

Linear pair

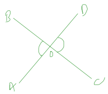


$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

BCA is a straight line

Theorem 6.1 : *If two lines intersect each other, then the vertically opposite angles are equal.*

Given: AD and BC are two lines intersecting each other at O.



To prove: $\angle AOB = \angle COD$

proof: AOD is a straight line

$\angle AOB + \angle BOD = 180^\circ$ (linear pair) (i)

BOC is a straight line

$\angle BOD + \angle COD = 180^\circ$ (linear pair) —(ii)

From (i) and (ii)

$\angle AOB + \cancel{\angle BOD} = \cancel{\angle BOD} + \angle COD$

$\boxed{\angle AOB = \angle COD}$

Similarly we can prove $\angle BOD = \angle AOC$

1. In Fig. 6.13, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$.

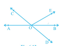


Fig. 6.13

Given: $\angle AOC + \angle BOE = 70^\circ$, $\angle BOD = 40^\circ$

To find: $\angle BOE$ & reflex $\angle COE$

$\angle AOC = \angle BOD$ (vertically opposite angles)

$\angle AOC = 40^\circ$ —(1)

$\angle AOC + \angle BOE = 70^\circ$ (given)

$40^\circ + \angle BOE = 70^\circ$ (From (1) $\angle AOC = 40^\circ$)

$\angle BOE = 70 - 40 = 30^\circ$

AB is a straight line.

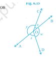
$\therefore \angle AOC + \angle COE + \angle BOE = 180^\circ$

$40^\circ + \angle COE + 30^\circ = 180^\circ$

$\angle COE = 180^\circ - 70^\circ = 110^\circ$

Reflex $\angle COE = 360^\circ - \angle COE = 360^\circ - 110^\circ = 250^\circ$

4. In Fig. 6.16, if $x + y = w + z$, then prove that AOB is a line.



Given: $x + y = w + z$.

To prove: AOB is a line

Proof: $x + y + z + w = 360^\circ$ (Complete angle)

$(x+y) + (x+y) = 360^\circ$ (Given $x+y = w+z$)

$\Rightarrow 2(x+y) = 360^\circ$

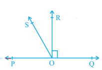
$x + y = 180^\circ$

but x & y are also linear pair,

$\therefore$ AOB is a line.

5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Given: POQ is a line, $\angle ROQ = 90^\circ$

To prove: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Proof:

$\angle QOS = \angle ROQ + \angle ROS$

$\angle QOS = 90^\circ + \angle ROS$

$\angle QOS - 90^\circ = \angle ROS$ —(i)

from the figure $\angle POR = \angle ROS + \angle POS$

$90^\circ = \angle ROS + \angle POS$

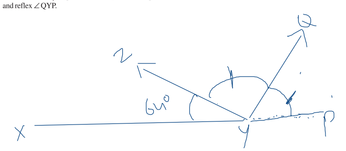
$90^\circ - \angle POS = \angle ROS$ (ii)

Add (i) & (ii)

$2\angle ROS = \angle QOS - \cancel{90} + \cancel{90} - \angle POS$

$\boxed{\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)}$

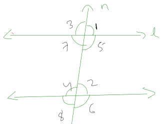
6. It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.



XYP is a str

$64^\circ + \angle ZYQ + \angle QYP = 180^\circ$

$2\angle ZYQ = 180^\circ - 64^\circ$



$l \parallel m$

① $\angle 1 = \angle 2$ (corresponding angles)

$\angle 3 = \angle 4$ (")

$\angle 5 = \angle 6$ (")

$\angle 7 = \angle 8$ (")

② $\angle 3 = \angle 5$ (V.O.A)

$\angle 1 = \angle 7$ (")

$\angle 2 = \angle 8$ (")

$\angle 4 = \angle 6$ (")

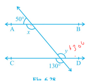
$\angle 3 = \angle 4$ (corresponding angles)

$\angle 3 = \angle 5$ (V.O.A)

$\angle 4 = \angle 5$ (alternate interior angles)

$\angle 2 = \angle 7$

1. In Fig. 6.28, find the values of x and y and then show that AB ∥ CD.



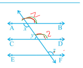
5:17

$x + 50 = 180°$ (linear)

$x = 130°$

$y = 130°$ ∴ AB ∥ CD

In Fig. 6.29, if AB ∥ CD, CD ∥ EF and $y : z = 3 : 7$, find x.



NOA

= 126

$18 \times 7 = 126°$

Given: AB ∥ CD, CD ∥ EF

To find: x

let y and z be $3a$ & $7a$

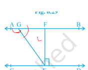
$\angle\ y + z = 180°$ (linear pair)

$3a + 7a = 180$

$10a = 180$

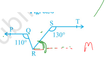
$a = 18°$

In Fig. 6.30, if AB ∥ CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



(linear Pair)

In Fig. 6.31, if PQ ∥ ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.
[**Hint** : Draw a line parallel to ST through point R.]



Given, PQ ∥ ST,

$\angle PQR = 110°$

$\angle RST = 130°$

To find $\angle QRS$

Const: RM ∥ ST ∥ PQ

RM ∥ ST (By construction)

$\angle MRS + \angle RST = 180°$ (sum of interior angles)

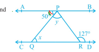
$\angle MRS + 130° = 180°$ (Given $\angle RST = 130°$)

$\angle MRS = 180° - 130°$

$= 50°$

As RM ∥ PQ

In Fig. 6.32, if AB ∥ CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.



$\angle APQ = x$ (Alternate interior angles)

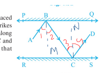
In ΔPQR

$\angle PRD = x + y$ (exterior angle property)

$127 = 50 + y$

$y = 127 - 50° = 77$

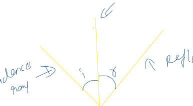
6. In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB ∥ CD.



Given:

To prove:

Const - Construct normal on PQ mirror BM



$\angle i = \angle r$

let $\angle BAM = 1$

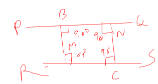
$\angle 1 = \angle 2$ (law of reflection or BM is angle bisector).

$\angle 3 = \angle 4$ (" or CN is angle bisector)

PQ ∥ RS (Given)

BM ⊥ PQ, & CN ⊥ RS

∴ BM ∥ CN, & BC is a transversal



so, $\angle 2 = \angle 3$ (i) (Alternate interior angles)

$\angle 1 = \angle 4$ (ii) (law of reflection $\angle 2 = \angle 1$ & $\angle 3 = \angle 4$)

Add (i) & (ii)

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\angle ABC = \angle BCD$

but these are alternate interior angles

AB ∥ CD

$ab(x^2+y^2)-xy(a^2+b^2)$

$= abx^2+aby^2-a^2xy-b^2xy$

$= abx^2-b^2xy+aby^2-a^2xy$

$= bx(ax-by)-ay(by-ax)$

$(bx-ay)(ax-by)$

$x^3-x^2+ax+x-a-1$

$= x^2(x-1)+ax-a+x-1$

$= x^2(x-1)+a(x-1)+1(x-1)$

$= (x-1)(x^2+a+1)$

$(3a-1)^2-6a+2$

$(3a-1)^2-2(3a-1)$

$(3a-1)((3a-1)-2)$

$(3a-1)(3a-3)$

$x(a+b)-6(a+b)^2$

$2(a+b)(2-3(a+b))$

$2(a+b)(2-3a-3b)$

$(2x-3)^2-8x+12$

$(2x-3)(2x-3-4)$ $-4(2x-3)$

i) $(x+a)$ is a factor (x^n+a^n) for any odd positive integer.

$F(x) = x^n+a^n$

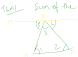
$x = -a$ 1 3 5

$F(-a) = (-a)^n+a^n$

$= -a^n+a^n = 0$

$\therefore$ by factor thm, $(x+a)$ is a factor of (x^n+a^n) for any odd +ve int.

Thm1 Sum of the angles of a Δ is 180°



Given – ΔPQR.

To prove :- $\angle P+\angle Q+\angle R = 180^\circ$

Const : $MN \parallel QR$

Proof : $MN \parallel QR$ (construction)

$\angle 1 = \angle 5$ –(i) (alternate interior angles)

$\angle 2 = \angle 6$ –(ii) (")

MPN is a straight line,

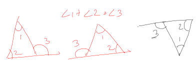
$\therefore \angle 5+\angle 6+\angle 3 = 180^\circ$

$\angle 1+\angle 2+\angle 3 = 180^\circ$ (from (i) & (ii))

$\boxed{\angle Q+\angle P+\angle R = 180^\circ}$

1. In Fig. 6.39, sides QP and RQ of Δ PQR are produced to points S and T respectively. If ∠ SPR = 135° and ∠ PQT = 110°, find ∠ PRQ.

$\angle 1+\angle 2 = \angle 3$



To find.. $\angle PRQ$

Soln : $\angle SPR+\angle QPR = 180^\circ$ (linear pair)

$135^\circ+\angle QPR = 180^\circ$

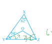
$\angle QPR = 180^\circ-135^\circ = 45^\circ$

$\angle PQT = \angle QPR+\angle PRQ$ (Ext angle pro)

$110^\circ = 45^\circ+\angle PRQ$

$\angle PRQ = 65^\circ$

In Fig. 6.40, ∠ X = 62°, ∠ XYZ = 54°. If YO and ZO are the bisectors of ∠ XYZ and ∠ XZY respectively of Δ XYZ, find ∠ OZY and ∠ YOZ.

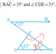


64

$180^\circ-(62^\circ+54^\circ)$

$= 64$

In Fig. 6.41, if AB ∥ DE, ∠ BAC = 35° and ∠ CDE = 53°, find ∠ DCE.



6. In Fig. 6.44, the side QR of Δ PQR is produced to a point S. If the bisectors of ∠ PQR and ∠ PRS meet at point T, then prove that ∠ QTR = $\frac{1}{2}$ ∠ QPR.

$\angle PRS$ is exterior angle

$\angle PRS = \angle PQR+\angle QPR$ (Ext angle pro)

$\angle QPR = \angle PRS-\angle PQR$

$\angle QPR = 2\angle TRS-2\angle TQR$ (QT is a bisector, TR is a bisector)

Proof :-

$\angle TRS$ is an exterior angle

$\angle TRS = \angle TQR+\angle QTR$ (Ext angle property)

$\angle QTR = \angle TRS-\angle TQR$ –(i)

$\angle QPR = 2(\angle TRS-\angle TQR)$

$\angle QPR = 2(\angle QTR)$ from (i)

$\boxed{\angle QTR = \frac{1}{2}\angle QPR}$

Doubts

i) Find the values of a and b so that $(2x^3+ax^2+x+b)$ has $(x+2)$ and $(2x-1)$ as factors

(ii) $(4x^4-2x^3-6x^2+2x+6)$

$(2x^2+x-1)$

let say $(ax+b)$

$p(x) = (4x^4-2x^3-6x^2+2x+6)-(ax+b)$

$g(x) = (2x^2+x-1)$

$g(x) = 0$, $(2x^2+x-1)$

$= 2x^2+2x-x-1$

$= 2x(x+1)-1(x+1)$

$= (2x-1)(x+1)$

$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^4-2\left(\frac{1}{2}\right)^3-6\times\left(\frac{1}{2}\right)^2+\left(2\times\frac{1}{2}\right)+6-a\times\frac{1}{2}-b$

$= 4\times\frac{1}{16}-\left(2\times\frac{1}{8}\right)-\left(6\times\frac{1}{4}\right)+1-\frac{a}{2}-b+6$

$= \frac{1}{4}-\frac{1}{4}-\frac{3}{2}+1-\frac{a}{2}-b+6 = 0$

$= -\frac{3}{2}+1+6 = \frac{a}{2}+b$

$\frac{-3+2+12}{2} = \frac{a}{2}+b$

$\frac{a}{2}+b = \frac{11}{2}$

$\frac{a+2b}{2} = \frac{11}{2}$

$a+2b = 11$ (ii)

$p(-1) = 0$

$= 4\times(-1)^4-2(-1)^3-6\times(-1)^2+(2\times-1)+6-(-a+b) = 0$

$= 4+2-6-2+6+a-b$

$= 6-2+a+b = 0$

$a+b = -4$ (ii)

$a+b = 4$ –i

$a+2b = 11$ – subtract

$b = 5$

$a = 2$