Quadrilaterals

point → A point is that which determines location

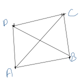
→ line segment

→ line.

Collinear points – Points lying on same line.

Quadrilaterals

Any enclosed figure made with 4 non-collinear points is known as quadrilateral



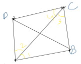
Vertices – A, B, C, D

Sides – AB, BC, CD, DA.

Angles – $\angle A, \angle B, \angle C, \angle D$

Diagonals – AC and BD

Thm **8.2 Angle Sum Property of a Quadrilateral**



Given: ABCD is a quadrilateral

To prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

Proof: In $\triangle ABC$

$\angle 1 + \angle B + \angle 3 = 180^\circ$ (Angle sum Property of a $\triangle$) – (i)

In $\triangle ADC$

$\angle 2 + \angle D + \angle 4 = 180^\circ$ (") – (ii)

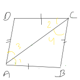
$(\angle 1 + \angle B + \angle 3) + (\angle 2 + \angle D + \angle 4) = 360^\circ$

$(\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^\circ$

$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$

Theorem 8.1 : *A diagonal of a parallelogram divides it into two congruent triangles.*

||gm – Quadrilateral whose opposite sides are ||.



Given – ||gm ABCD.

To prove: $\triangle ABC \cong \triangle ADC$

Proof: In $\triangle ABC$ and $\triangle ADC$

AB || DC, AD || BC, AC is a transversal.

$\angle 1 = \angle 2$ (alter interior angles)

$\angle 4 = \angle 3$ (")

AC = AC (Common)

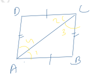
$\therefore \triangle ABC \cong \triangle ADC$ (ASA rule).

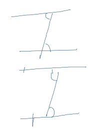
Theorem 8.2 : *In a parallelogram, opposite sides are equal.*

$\triangle ABC \cong \triangle DCA$

AB = DC, AD = BC (CPCT)

Theorem 8.3 : *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*





Given: quadrilateral ABCD, AB = DC, AD = BC

To prove: ABCD is a ||gm

Constr: Join A to C

Proof: In $\triangle ABC$ and $\triangle CDA$

AB = DC (given)

BC = AD (given)

AC = AC (common)

$\therefore \triangle ABC \cong \triangle CDA$ (SSS rule)

$\angle 1 = \angle 2$ (CPCT)

$\angle BAC = \angle DCA$

$\angle 3 = \angle 4$ (CPCT)

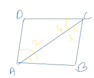
but these are alternate interior angles

$\therefore$ AB || DC

AD || BC

Which implies ABCD is a ||gm.

Theorem 8.4 : *In a parallelogram, opposite angles are equal.*



Given: ABCD is a ||gm, AB || DC & AD || BC

To proof: $\angle A = \angle C$ and $\angle B = \angle D$

Proof: AB || CD and AC is transversal

$\angle 1 = \angle 4$ – (i) (alt interior angles)

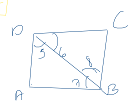
AD || BC and AC is transversal

$\angle 2 = \angle 3$ – (ii) (alt interior angles).

Add (i) and (ii)

$\angle 1 + \angle 2 = \angle 4 + \angle 3$

$\boxed{\angle A = \angle C}$



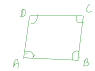
$\angle 5 = \angle 8$ (alt Interior)

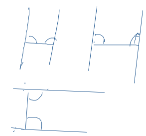
$\angle 6 = \angle 7$ (")

add

$\boxed{\angle D = \angle B}$

Theorem 8.5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*





Given: $\angle A = \angle C$ and $\angle B = \angle D$

To proof: ABCD is a ||gm

Proof: $\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Angle sum property of a quadrilateral)

$\angle A + \angle B + \angle A + \angle B = 360^\circ$

$2\angle A + 2\angle B = 360^\circ$

$\angle A + \angle B = 180^\circ$

but these are co-interior angles

$\therefore$ AD || BC – (i)

Similarly

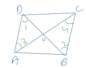
$\angle A + \angle B = 180^\circ$

$\angle A + \angle D = 180^\circ$

$\therefore$ AB || CD – (ii)

$\therefore$ From (i) and (ii) ABCD is a ||gm

Theorem 8.6 : *The diagonals of a parallelogram bisect each other.*



Given: ||gm ABCD.

To prove: AO = CO, OB = OD

Proof: In $\triangle AOD$ and $\triangle COB$

AD = BC (opposite sides of ||gm)

$\angle 1 = \angle 2$ (alt interior angles)

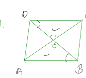
$\angle 3 = \angle 4$ (")

$\therefore \triangle AOD \cong \triangle COB$ (ASA rule)

AO = CO (CPCT)

OD = OB (CPCT)

Theorem 8.7 : *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*



Given: ABCD is a quadrilateral. AO = CO, OB = OD.

To prove: ABCD is a ||gm

Proof: In $\triangle AOB$ and $\triangle COD$

AO = CO (given)

$\angle AOB = \angle COD$ (V.O.A).

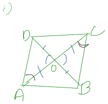
OB = OD (given)

$\therefore \triangle AOB \cong \triangle COD$ (SAS)

$\angle ABO = \angle CDO$ (CPCT)

but these are alt interior angles

AB || DC – (i)



Now, In $\triangle AOD$ and $\triangle BOC$

AO = CO (given).

$\angle AOD = \angle BOC$ (V.O.A).

OD = OB (given).

$\therefore \triangle AOD \cong \triangle BOC$ (SAS rule)

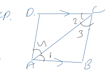
$\angle DAO = \angle BCO$ (CPCT)

but these are alt interior angles

$\therefore$ AD || BC – (ii)

$\therefore$ From (i) and (ii) ABCD is a ||gm

Theorem 8.8 : *A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.*



Given: In a quadrilateral ABCD. AB || DC, AB = DC

To proof: ABCD is a ||gm. AD || BC,

Const: Join A to C

Proof: In $\triangle ABC$ and $\triangle CDA$.

AB = CD (given)

$\angle 1 = \angle 2$ (alt. interior angles)

AC = AC (Common)

$\therefore \triangle ABC \cong \triangle CDA$ (SAS)

$\angle 3 = \angle 4$ (CPCT)

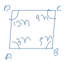
but these are alt. interior angles

$\therefore$ AD || BC

$\therefore$ ABCD is a ||gm

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

let the angles are $3x, 5x,$ $9x$ and $13x$.



$\angle A + \angle B + \angle C + \angle D = 360^\circ$

(Angle sum property of quadrilateral)

$3x + 5x + 9x + 13x = 360^\circ$

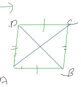
$30x = 360$

$x = 12$

$\angle A = 36^\circ$
$\angle B = 60^\circ$
$\angle C = 108^\circ$
$\angle D = 156^\circ$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Given: ABCD is a llgm, AC = BD.



To prove: ABCD is a rectangle

oppo sides equal & 1 angle 90°

Proof: In $\triangle ABD$ and $\triangle BAC$

AD = BC (oppo sides of llgm are equal)
AB = BA (common)
BD = AC (Given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle BAD = \angle ABC$ (CPCT)

$\angle BAD + \angle ABC = 180^\circ$ (co interior angles).

$\angle BAD + \angle BAD = 180^\circ$

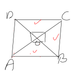
$2\angle BAD = 180^\circ$

$\boxed{\angle BAD = 90}$

$\therefore$ ABCD is a rectangle

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Given: OA = OC, OB = OD, Diagonal bisect at right angle

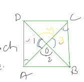


To prove: ABCD is a rhombus,

AB = BC = CD = AD

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Given: ABCD is a Square.



To prove: (i) AC = BD.
(ii) AC and BD bisect each other at right angle.

Proof: In $\triangle ABD$ and $\triangle BAC$

AD = BC (Given)
AB = AB (Common)
$\angle A = \angle B$ (90°)

$\therefore \triangle ABD \cong \triangle BAC$ (SAS rule)

AC = BD (CPCT)

(ii) In $\triangle AOD$ and $\triangle COB$

AD = BC (given)
$\angle OAD = \angle OCB$ (alt interi angles)
$\angle AOD = \angle COB$ (V.O.A)

$\therefore \triangle AOD \cong \triangle COB$ (AAS rule)

OD = OB, OA = OC (CPCT)

In $\triangle AOD$ and $\triangle AOB$

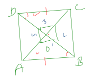
AD = AB (Given)
AO = AO (Common)
OD = OB (proved above)

$\triangle AOD \cong \triangle AOB$ (SSS rule), $\angle AOD = \angle AOB$ (CPCT)

$\angle 1 = \angle 2$
$\angle 1 + \angle 2 = 180^\circ$ (linear pair)
$\angle 1 = \angle 2 = 90^\circ = \angle 3 = \angle 4$

5) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Given: AC = BD,
OD = OB, OA = OC.
$\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90^\circ$



To prove: ABCD is a Square,
AB = BC = CD = AD, $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Proof: In $\triangle AOB$ and $\triangle COD$

OA = OC
OB = OD (Given)
$\angle AOB = \angle COD$ (V.O.A)

$\triangle AOB \cong \triangle COD$ (SAS rule)

AB = CD (CPCT)
$\angle OBA = \angle ODC$ (CPCT)

but these are alternate interior angles

AB ∥ CD

ABCD is a llgm (oppo. sides equal and parallel).

BC = AD –(i) AB = CD –(ii)

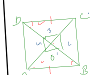
In $\triangle AOB$ and $\triangle BOC$

OA = OC (Given)
$\angle AOB = \angle BOC$ (each 90°)
OB = OB (Common)

$\therefore \triangle AOB \cong \triangle BOC$ (SAS rule)

AB = BC (CPCT) – (iii)

From (i), (ii) & (iii) $\boxed{AB = CD = AD = BC}$



In $\triangle ABD$ and $\triangle BAC$

AD = BC (oppo sides of llgm)
AB = BA (Common)
BD = AC (given)

$\therefore \triangle ABD \cong \triangle BAC$ (SSS rule)

$\angle A = \angle B$ (CPCT) –(V)

$\angle A + \angle B = 180^\circ$ (Co-interior angles)

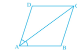
$\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = 90^\circ$

$\therefore$ ABCD is a Square.

6. Diagonal AC of a parallelogram ABCD bisects $\angle$A (see Fig. 8.19). Show that
(i) it bisects $\angle$C also,
(ii) ABCD is a rhombus.



6–10

Fig. 8.21

11. In ΔABC and ΔDEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22). Show that
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD ∥ CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) $\Delta ABC \cong \Delta DEF$.

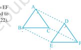


Fig. 8.22

Given: AB = DE, AB ∥ DE,
BC = EF, BC ∥ EF.

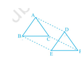
(i) In quad ADED, AD = DE, also AB ∥ DE (given)
$\therefore$ ABED is a llgm (opposide are equal and ∥)

(ii) In quad BEFC, BC = EF, BE ∥ EF (given)
$\therefore$ BEFC is a llgm (oppo sides are equal & ∥)

(iii) If ABED is a llgm,
AD ∥ BE –(ii) AD = BE –(iv)

BEFC is a llgm,
CF ∥ BE –(V) CF = BE (vi)

From (iii) and (v)
AD ∥ CF
From (iv) and (vi)
AD = CF



(iv) In quad ACFD,
AD ∥ CF and AD = CF (proved above)
$\therefore$ ACFD is a llgm

(V) AC = DF (oppo sides of llgm ACFD.

(vi) In $\triangle ABC$ and $\triangle DEF$.

AB = DE
BC = EF (Given)
AC = DF (proved above)

$\therefore \triangle ABC \cong \triangle DEF$ (SSS rule)

12. ABCD is a trapezium in which AB ∥ CD and AD = BC (see Fig. 8.23). Show that

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) Δ ABC ≅ Δ BAD

(iv) diagonal AC = diagonal BD

[*Hint*: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

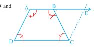
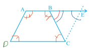


Fig. 8.23

Given:- ABCD is a trapezium, AB ∥ CD, AD = BC

Const:- extend AB to E, Join C to E.
CE ∥ DA and CE = DA

Sol:- CE ∥ DA, & AE is transversal
∠A + ∠E = 180° – (i) (Co interior angles)
∠CBA + ∠CBE = 180° – (ii) (linear pair)

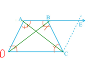


As CE = DA and CE ∥ DA
∴ AECD is a llm / opp sides equal and ∥). (iii)
AD = CE (oppo sides of llgm)
AD = BC (given) – (iv)
From (iii) & (iv)
∴ BC = CE
∴ ∠CBE = ∠CEB (angle oppo to equal sides)
∠CBA + ∠CBE = 180° (using (ii))
∠CBA + ∠CEB = 180° (vi)
∠A + ∠E = 180° – (Using (i))
∠A + ∠E = ∠CEB + ∠CBA.
∠A + $\cancel{\angle E}$ = $\cancel{\angle E}$ + ∠B
$\boxed{\angle A = \angle B}$

(ii) ABCD is a trapezium, AB ∥ CD, AD is transversal
∠A + ∠D = 180° (Co-interior angles)

AB ∥ CD, BC is transversal,
∠B + ∠C = 180° (Co-interior angles)

~~∠A~~ + ∠D = ~~∠B~~ + ∠C . (∠A = ∠B proved above)
∠D = ∠C

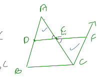


(iii)
In ΔABC and ΔBAD
BC = AD (given)
∠A = ∠B (proved above)
AB = AB (common)
ΔABC ≅ ΔBAD (SSS rule)
(iv) AC = BD (CPCT)

Theorem 8.9 : *The line segment joining the mid-points of two sides of a triangle is parallel to the third side.*

Given:- ΔABC, D is midpoint of AB, E is midpoint of AC

To prove:- DE ∥ BC, DE = ½ BC



Const:- CF ∥ AB

Proof:- In ΔAED and ΔCEF.
∠AED = ∠CEF (VOA)
∠DAE = ∠FCE (Alt interior angles)
AE = EC (E is mid point)
∴ ΔAED ≅ ΔCEF (ASA rule).
DE = EF (CPCT)
AD = CF
but AD = DB (D is midpoint).
∴ DB = CF also DB ∥ CF
∴ DBCF is a llgm (∵ opp sides equal and ∥)
∴ BC ∥ DE
also BC = DF
BC = DE + EF
BC = DE + DE
BC = 2DE
$\boxed{DE = \frac{1}{2} BC}$

If D & E are mid points, DE ∥ BC, DE = ½ BC

DE ∥ BC
DE = ½ BC

Converse

If D is mid point & DE ∥ BC → E is mid point

EXERCISE 8.2

1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that :

(i) SR ∥ AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

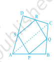


Fig. 8.29

Given:- ABCD is a quad. P, Q, R and S are mid points. AC is diagonal.

To prove (i), (ii), (iii)

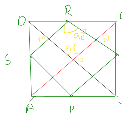
Proof:- In ΔDAC,
S is midpoint of AD,
R is midpoint of DC
SR ∥ AC. SR = ½ AC (By mid point theorem) – (i)

(ii) In ΔBAC,
P is midpoint of AB
Q is midpoint of BC
PQ ∥ AC, PQ = ½ AC (By MPT) – (ii)

From (i) and (ii)
SR ∥ PQ, SR = PQ
∴ PQRS is a llgm (A pair of opposite side is equal and ∥)

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

PQRS → similar to 1st Qn is a llgm



Const :- Join D to B as (SR ∥ AC)

In ΔBDC,
Q is the midpoint of BC
R is the " of DC
QR ∥ BD (Using MPT)
∴ RN ∥ MO

SR ∥ AC (proved above)
∴ MR ∥ ON

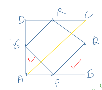
∴ MORN is a llgm (opposite sides of a quadrilateral are ∥).

ABCD is a rhombus (given)
rhombus diagonal bisect each other at 90°

∠O = 90°
∠O = ∠R (opp angles of llgm)
∴ PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Given : ABCD is a ▭
To prove : PQRS is a rhombus
all sides equal & diagonal bisect each other at 90°



In ΔSAP and ΔQBP
AP = BP
CP is mid point
∠SAP = ∠QBP (90°)
AD = BC (opposite sides of rectangle)
½AD = ½BC
AS = BQ
∴ ΔSAP ≅ ΔQBP (SAS rule)
SP = QP (CPCT)
∴ PQRS is a rhombus

Proof:- PQRS is a llgm –

In ΔADC,
S is mid pt AD
R is " DC
∴ SR ∥ AC, SR = ½ C

In ΔABC
P is mid point of AB
Q is " of BC
∴ PQ ∥ AC, PQ = ½ AC (MPT)