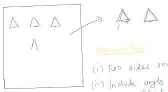
Triangles

1) Congruence – 2 NCERT Science book identical (same in shape, size)

Dairy Milk chocolate 

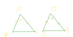
Activity for congruence for Δ



Observations:

(i) Two sides must be equal.

(ii) Include angle will be equal.



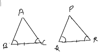
axiom

ΔBAC ≅ ΔEDF (SAS rule)

ΔCAB ≅ ΔFDE

Theorem

(ASA)



Given: ∠B = ∠Q, BC = QR, ∠C = ∠R

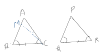
To prove: ΔABC ≅ ΔPQR

Proof: (i) Let AB = PQ

AB = PQ (construction)
∠B = ∠Q (given)
BC = QR (given)

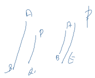
ΔABC ≅ ΔPQR (SAS rule).

(ii)



AB > PQ

PQ > AB



If AB > PQ,

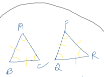
In ΔMBC & ΔPQR,

MB = PQ (by construction)
∠B = ∠Q (Given)
BC = QR (given)

ΔMBC ≅ ΔPQR (SAS rule).

∠MCB = ∠R (CPCT)

∠C = ∠R (Given)



ΔABC ≅ ΔPQR

CPCT (Corresponding parts of congruent triangles)

∠MCB = ∠C

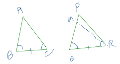
but this is not possible,

it's possible only when point M tends to poin A.

BM = BA

BA = QP

(iii)



If AB < PQ,

ΔABC and ΔMQR,

AB = MQ (By construction)
∠B = ∠Q (Given)
BC = QR (Given)

ΔABC ≅ ΔMQR (SAS rule).

∠ACB = ∠MRQ (CPCT)

∠C = ∠MRQ (CPCT)

∠C = ∠R (Given)

∠MRQ = ∠R.

This is not possible, only possible when point M tends to point P.

then, MQ = PQ

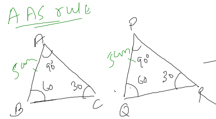
$\boxed{PQ = AB}$

Rules

(i) SAS rule (axiom). 

(ii) ASA rule (Thm – 3 cases)

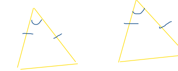
(iii) AAS rule

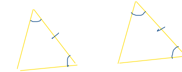


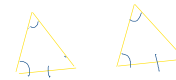
If two angles are equal, 3rd angle will be equal of Δ's

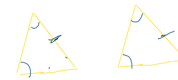
ΔABC ≅ ΔPQR (AAS rule)

1

 = SAS rule

 = ASA rule

 = AAS rule

 = AAS rule

ΔABC ≅ ΔDEF

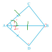
CPCT

AB = DE, ∠ACB = ∠DFE

BC = EF, ∠CAB = ∠FDE

AC = DF,

1)



Given :- $AC = AD$

$\angle 1 = \angle 2$,

To proof :- $\Delta ABC \cong \Delta ABD$

proof :- In ΔABC & ΔABD

$AC = AD$ (given)

$AB = AB$ (common side)

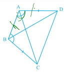
$\angle CAB = \angle DAB$ (given)

$\Delta ABC \cong \Delta ABD$ (SAS rule)

$BC = BD$ (CPCT)

---

2. ABCD is a quadrilateral in which AD = BC and ∠ DAB = ∠ CBA (see Fig. 7.17). Prove that
   (i) Δ ABD ≅ Δ BAC
   (ii) BD = AC
   (iii) ∠ ABD = ∠ BAC.



Given :- $AD = BC$, $\angle DAB = \angle CBA$.

To prove :- (i) $\Delta ABD \cong \Delta BAC$.

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$

proof :- In ΔABD & ΔBAC

$AD = BC$ (given)

$\angle DAB = \angle CBA$ (given)

$AB = BA$ (common)

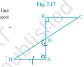
$\therefore \Delta ABD \cong \Delta BAC$ (SAS rule)

$BD = AC$ (CPCT)

$\angle ABD = \angle BAC$ (CPCT)

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

Fig. 7.17



Given :- $AD = BC$, $\angle A = \angle B = 90^\circ$.

To prove :- CD bisects AB ($OA = OB$)

proof :- In ΔOAD and ΔOBC,

$AD = BC$ (given)

$\angle OAD = \angle OBC$ (90° each)

$\angle AOD = \angle BOC$ (V.O.A)

$\Delta OAD \cong \Delta OBC$ (AAS rule)

$OA = OB$ (CPCT) $\therefore$ CD bisects AB.

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that Δ ABC ≅ Δ CDA.

Fig. 7.18

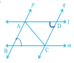


Fig. 7.19

Given :- $l \parallel m$, $p \parallel q$.

Show :- $\Delta ABC \cong \Delta CDA$

proof :- $l \parallel m$ ($AD \parallel BC$), $p \parallel q$ ($AB \parallel CD$)

$\therefore ABCD$ is a ||gm.

In ΔABC & ΔCDA

$AB = CD$ (opp. sides of ||gm.)

$BC = DA$ (opp sides of ||gm)

$\angle B = \angle D$ (" angle of ||gm)

$\Delta ABC \cong \Delta CDA$ (SAS rule)

5. Line l is the bisector of an angle ∠ A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠ A (see Fig. 7.20). Show that:
   (i) Δ APB ≅ Δ AQB
   (ii) BP = BQ or B is equidistant from the arms of ∠ A.

CPCT

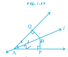


Fig. 7.20

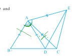
$\Delta APB \cong \Delta AQB$ (AAS rule)

$\angle 1 = \angle 2$ (given)

$\angle P = \angle Q$ (90°)

$AB = AB$ (common)

6. In Fig. 7.21, AC = AE, AB = AD and ∠ BAD = ∠ EAC. Show that BC = DE.



Given :- $AC = AE$, $AB = AD$, $\angle BAD = \angle EAC$.

To show :- $BC = DE$

To proof :- In ΔABC and ΔADE

$AC = AE$ (given)

$AB = AD$ (given)

$\angle BAD = \angle EAC$ (given)

adding $\angle DAC$ both sides -

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\angle BAC = \angle EAD$

$\Delta ABC \cong \Delta ADE$ (SAS rule)

$BC = DE$ (CPCT)

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠ BAD = ∠ ABE and ∠ EPA = ∠ DPB (see Fig. 7.22). Show that
   (i) Δ DAP ≅ Δ EBP
   (ii) AD = BE

Fig. 7.21

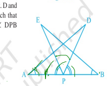


Fig. 7.22

Given :- $AP = PB$ (P is mid point),

$\angle BAD = \angle ABE$, & $\angle EPA = \angle DPB$.

To show :- (i) $\Delta DAP \cong \Delta EBP$,

(ii) $AD = BE$

proof :- In ΔDAP and ΔEBP

$\angle PAD = \angle PBE$ (Given $\angle BAD = \angle ABE$)

$AP = BP$ (P is mid point)

$\angle EPA = \angle DPB$ (given).

Add $\angle EPD$ both sides.

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\angle DPA = \angle EPB$

$\therefore \Delta DAP \cong \Delta EBP$ (ASA rule)

$AD = BE$ (CPCT)

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:
   (i) Δ AMC ≅ Δ BMD
   (ii) ∠ DBC is a right angle.
   (iii) Δ DBC ≅ Δ ACB
   (iv) $CM = \frac{1}{2} AB$

Fig. 7.22

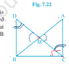


Fig. 7.23

Given :- $\angle C = 90^\circ$, $BM = AM$, $DM = CM$

To show :- (i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is 90°

(iii) $\Delta DBC \cong \Delta ACB$.

(iv) $CM = \frac{1}{2} AB$

proof :- (i) In ΔAMC and ΔBMD

$AM = BM$ (M is mid point)

$CM = DM$ (Given)

$\angle CMA = \angle DMB$ (V.O.A)

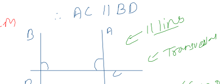
$\Delta AMC \cong \Delta BMD$ (SAS rule)

$AC = BD$ (CPCT)

$\angle ACM = \angle BDM$ (CPCT)

but these are alternate interior angles.

$\therefore AC \parallel BD$



$\angle DBC + \angle ACB = 180^\circ$ (sum of interior angles)

$\angle DBC + 90^\circ = 180^\circ$

$\angle DBC = 90^\circ$

(iii) In ΔDBC and ΔACB.

$DB = AC$ (proved above)

$\angle B = \angle C$ (each 90°)

$BC = CB$ (common)

$\Delta DBC \cong \Delta ACB$ (SAS rule)

(iv) $CD = BA$ (CPCT)

$CM + DM = AB$

$CM + CM = AB$ ($CM = DM$ given)

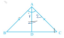
$2CM = AB$

$\boxed{CM = \frac{1}{2} AB}$

Theorem 7.2 : *Angles opposite to equal sides of an isosceles triangle are equal.*

Given :- $AB = AC$

To prove :- $\angle B = \angle C$



Construction :- AD is angle bisector of $\angle A$

Sol :- In ΔBAD and ΔCAD.

$AB = AC$ (given)

$\angle 1 = \angle 2$ (By construction)

$AD = AD$ (common)

$\Delta BAD \cong \Delta CAD$ (SAS rule)

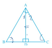
$\angle B = \angle C$ (CPCT)

Theorem 7.3 : *The sides opposite to equal angles of a triangle are equal.*

Given : $\angle B = \angle C$

To prove : $AB = AC$

Construction : AD is angle bisector of $\angle A$ ($\angle 1 = \angle 2$)



Proof :- ΔBAD and ΔCAD

$\angle B = \angle C$ (given)

$\angle 1 = \angle 2$ (By construction)

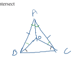
$AD = AD$ (common)

$\Delta BAD \cong \Delta CAD$ (AAS rule)

$AB = AC$ (CPCT)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠ B and ∠ C intersect each other at O. Join A to O. Show that :

(i) OB = OC (ii) AO bisects ∠ A



Sol : ΔABC, $AB = AC$

$\angle B = \angle C$ (Angle opposite to equal sides are equal.)

$\frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\angle OBC = \angle OCB$

In ΔOBC, $\angle OBC = \angle OCB$

$OB = OC$ (sides opposite to equal angles are equal.)

In ΔAOB and ΔAOC

$AB = AC$ (given)

$\angle ABO = \angle ACO$ ($\frac{1}{2}\angle B = \frac{1}{2}\angle C$)

$OB = OC$ (proved above)

$\therefore \Delta AOB \cong \Delta AOC$ (SAS rule)

$\angle BAO = \angle CAO$ (CPCT)

$\therefore$ AO bisects $\angle A$

2. In Δ ABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that Δ ABC is an isosceles triangle in which AB = AC.

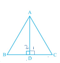


Fig. 7.30

Given : $\angle 1 = \angle 2 = 90°$ & $BD = DC$

To prove : $AB = AC$

Proof : ΔABD and ΔACD

$BD = CD$ (given)

$\angle 2 = \angle 1$ (90°)

$AD = AD$ (common)

$\Delta ABD \cong \Delta ACD$ (SAS rule)

$AB = AC$ (CPCT)

$\therefore \Delta ABC$ is an isosceles Δ

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

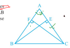
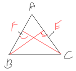


Fig. 7.31



Given : $AB = AC$

To prove : $CF = BE$

Proof : In ΔACF and ΔBAE

$\angle CAF = \angle BAE$ (common)

$\angle F = \angle E$ (90°)

$CA = BA$ (given)

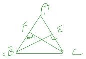
$\therefore \Delta ACF \cong \Delta BAE$ (AAS rule)

$BE = CF$ (CPCT)

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) Δ ABE ≅ Δ ACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.



In ΔABE and ΔACF

→ $\angle A = \angle A$ (common)

$\angle E = \angle F$ (90°)

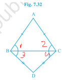
$BE = CF$ (given)

$\therefore \Delta ABE \cong \Delta ACF$ (AAS rule)

$AB = AC$ (CPCT)

Fig. 7.32

5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ ABD = ∠ ACD.



5:33

Given : $AB = AC$, $BD = CD$

To show : $\angle ABD = \angle ACD$

In ΔABC

If $AB = AC$ (given)

$\angle 1 = \angle 2$ (Angle opposite to equal sides are equal) (1)

If ΔDBC

$BD = CD$ (given)

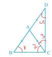
$\angle 3 = \angle 4$ (" ") (11)

Add (1) and (11)

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle ABD = \angle ACD$

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠ BCD is a right angle.



Given : $AB = AC$, $AB = AD$

To prove : $\angle BCD$ is 90°

Proof : In ΔABC,

$AB = AC$ (given) — (i)

$\angle 1 = \angle 2$ (angle opposite to equal sides are equal)

$AD = AB$ — (ii)

From (i) and (ii)

$AD = AC$

$\angle 3 = \angle 4$ (" ")

In ΔBCD.

$\angle B + \angle C + \angle D = 180°$ (Angle sum property)

$\angle 1 + \angle 4 + \angle 2 + \angle 3 = 180°$

$(\angle 1 + \angle 4) + \angle 2 + \angle 3 = 180°$

$\angle 2 + \angle 3 + \angle 2 + \angle 3 = 180°$

$2(\angle 2 + \angle 3) = 180°$

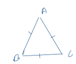
$\angle 2 + \angle 3 = 90°$

$\angle DCB = 90°$

$\therefore \angle BCD$ is a right angle.

7. ABC is a right angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C. → Angle sum property

8. Show that the angles of an equilateral triangle are 60° each.



Given : $AB = AC = BC$

To prove : $\angle B = \angle C = \angle A = 60°$

Proof : $AB = BC$

$\angle B = \angle C$ (angle opposite to equal sides) — (i)

If $BC = AC$

$\angle B = \angle A$ (") — (ii)

From (i) and (ii)

$\angle A = \angle B = \angle C$

In ΔABC

$\angle A + \angle B + \angle C = 180°$ (Angle sum property)

$\angle A + \angle A + \angle A = 180°$

$3\angle A = 180°$

$\angle A = 60°$, $\angle B = \angle C = 60°$

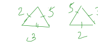
(i) SAS rule

(ii) ASA rule

(iii) AAS rule

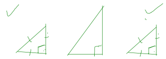
(Thm)

iv)



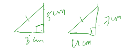
(SSS Rule)

(v)



(RHS rule)

R H S: Right angle, Hypotenuse, Side



EXERCISE 7.3

1. Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i) Δ ABD ≅ Δ ACD

(ii) Δ ABP ≅ Δ ACP

(iii) AP bisects ∠ A as well as ∠ D.

(iv) AP is the perpendicular bisector of BC.

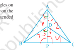


Fig. 7.39

Given : $AB = AC$, $DB = DC$

To prove :

Proof : (i) In ΔABD and ΔACD

$AB = AC$ (given)

$BD = CD$ (given)

$AD = AD$ (common)

$\therefore \Delta ABD \cong \Delta ACD$ (SSS rule)

$\angle DAB = \angle CAD$ (CPCT)

$\angle 1 = \angle 2$

$\therefore$ AP bisects $\angle A$.

In ΔBPD and ΔCPD

$BP = CP$ (proved above)

$DP = DP$ (common)

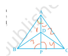
$BD = CD$ (given)

$\therefore \Delta BPD \cong \Delta CPD$ (SSS rule)

$\angle BDP = \angle CDP$ (CPCT)

$\angle 5 = \angle 6$ (CPCT)

$\therefore$ AP bisects $\angle D$



(ii) In ΔABP and ΔACP

$AB = AC$ (given)

$\angle 1 = \angle 2$ (proved above)

$AP = AP$ (common side)

$\therefore \Delta ABP \cong \Delta ACP$ (SAS rule)

$BP = CP$ (CPCT)

AP bisects BC

$\angle 3 = \angle 4$ (CPCT)

$\angle 3 + \angle 4 = 180°$ (linear pair)

$\angle 3 + \angle 3 = 180°$

$2\angle 3 = 180°$

$\angle 3 = 90°$

$\angle 4 = 90°$

$\therefore$ AP is ⊥ bisector to BC