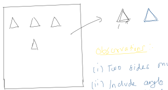
Triangles

1) Congruence – 2 NCERT Science book (identical: same in shape, size)

Dairy Milk chocolate

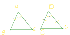
Activity for congruence for Δ



Observations:

(i) Two sides must be equal.

(ii) Include angle will be equal.



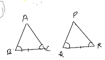
axiom

ΔBAC ≅ ΔEDF (SAS rule)

ΔCAB ≅ ΔFDE

Theorem

ASA



Given: ∠B = ∠Q,
BC = QR
∠C = ∠R

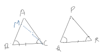
To prove: ΔABC ≅ ΔPQR

Proof: (i) Let AB = PQ

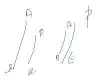
AB = PQ (construction)
∠B = ∠Q (given)
BC = QR (given)

ΔABC ≅ ΔPQR (SAS rule).

(ii)



AB > PQ
PQ > AB



If AB > PQ,
In ΔMBC & ΔPQR,
MB = PQ (by construction)
∠B = ∠Q (Given)
BC = QR (given)

ΔMBC ≅ ΔPQR (SAS rule).

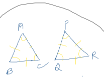
∠MCB = ∠R (CPCT)

∠C = ∠R (Given)

∠MCB = ∠C

but this is not possible,
it's possible only when point M tends to point A.

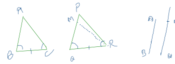
BM = BA
BA = QP



ΔABC ≅ ΔPQR

CPCT (Corresponding parts of congruent triangles)

(iii)



If AB < PQ,
ΔABC and ΔMQR,
AB = MQ (By construction)
∠B = ∠Q (Given).
BC = QR (Given)

ΔABC ≅ ΔMQR (SAS rule).

∠ACB = ∠MRQ (CPCT)
∠C = ∠MRQ (CPCT)
∠C = ∠R (Given)

∠MRQ = ∠R.

This is not possible, only possible when point M tends to point P.
then, MQ = PQ

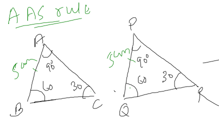
$\boxed{PQ = AB}$

Rules

(i) SAS rule (axiom).

(ii) ASA rule (Thm – 3 cases)

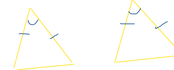
(iii) AAS rule



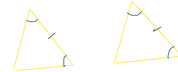
If two angles are equal, 3rd angle will be equal of Δ's

ΔABC ≅ ΔPQR (AAS rule)

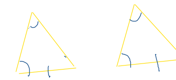
1)



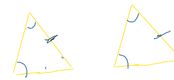
= SAS rule



= ASA rule



= AAS rule



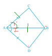
= AAS rule.

ΔABC ≅ ΔDEF

CPCT

AB = DE, ∠ACB = ∠DFE
BC = EF, ∠CAB = ∠FDE
AC = DF,

1)



Given :- AL = AD

$\angle 1 = \angle 2$,

To proof :- $\Delta ABC \cong \Delta ABD$

Proof :- In ΔABC & ΔABD

AC = AD (given)

AB = AB (Common side)

$\angle CAB = \angle DAB$ (given)

$\Delta ABC \cong \Delta ABD$ (SAS rule)

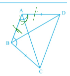
BC = BD (CPCT)

2. ABCD is a quadrilateral in which AD = BC and $\angle$ DAB = $\angle$ CBA (see Fig. 7.17). Prove that

(i) Δ ABD $\cong$ Δ BAC

(ii) BD = AC

(iii) $\angle$ ABD = $\angle$ BAC.



Given :- AD = BC , $\angle DAB = \angle CBA$.

To prove :- (i) $\Delta ABD \cong \Delta BAC$.

(ii) BD = AC

(iii) $\angle ABD = \angle BAC$

Proof :- In ΔABD & ΔBAC

AD = BC (given)

$\angle DAB = \angle CBA$ (given)

AB = BA (Common)

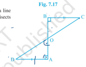
$\therefore \Delta ABD \cong \Delta BAC$ (SAS rule)

BD = AC (CPCT)

$\angle ABD = \angle BAC$ (CPCT)

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

Fig. 7.17



Given :- AD = BC , $\angle A = \angle B = 90^\circ$.

To prove :- CD bisects AB (OA = OB)

Proof :- In ΔOAD and ΔOBC,

AD = BC (given)

$\angle OAD = \angle OBC$ (90° each)

$\angle AOD = \angle BOC$ (V.O.A)

$\Delta OAD \cong \Delta OBC$ (AAS rule)

OA = OB (CPCT) $\therefore$ CD bisects AB.

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that Δ ABC $\cong$ Δ CDA.

Fig. 7.18

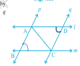


Fig. 7.19

Given :- $l \parallel m$, $p \parallel q$.

Show :- $\Delta ABC \cong \Delta CDA$

Proof :- $l \parallel m$ (AD $\parallel$ BC) $p \parallel q$ (AB $\parallel$ CD)

$\therefore$ ABCD is a $\parallel$gm.

In ΔABC & ΔCDA

AB = CD (opp. sides of $\parallel$gm.)

BC = DA (opp sides of $\parallel$gm)

$\angle B = \angle D$ (" angle of $\parallel$gm)

$\Delta ABC \cong \Delta CDA$ (SAS rule)

5. Line l is the bisector of an angle $\angle$ A and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle$ A (see Fig. 7.20). Show that:

(i) Δ APB $\cong$ Δ AQB

(ii) BP = BQ or B is equidistant from the arms of $\angle$ A.

(CPCT)

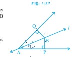


Fig. 7.20

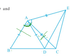
$\Delta APB \cong \Delta AQB$ (AAS rule)

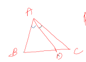
$\angle 1 = \angle 2$ (Given)

$\angle P = \angle Q$ (90°)

AB = AB (Common)

6. In Fig. 7.21, AC = AE, AB = AD and $\angle$ BAD = $\angle$ EAC. Show that BC = DE.





Given :- AC = AE , AB = AD , $\angle BAD = \angle EAC$.

To show :- BC = DE

To proof :- In ΔABC and ΔADE.

AC = AE (given)

AB = AD (given)

$\angle BAD = \angle EAC$ (given)

adding $\angle DAC$ both sides.

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\angle BAC = \angle EAD$

$\Delta ABC \cong \Delta ADE$ (SAS rule)

BC = DE (CPCT)

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle$ BAD = $\angle$ ABE and $\angle$ EPA = $\angle$ DPB (see Fig. 7.22). Show that

(i) Δ DAP $\cong$ Δ EBP

(ii) AD = BE

Fig. 7.21

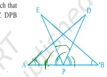


Fig. 7.22

Given :- AP = PB (P is mid point),

$\angle BAD = \angle ABE$, & $\angle EPA = \angle DPB$.

To show :- (i) $\Delta DAP \cong \Delta EBP$,

(ii) AD = BE

Proof :- In ΔDAP and ΔEBP

$\angle PAD = \angle PBE$ (Given $\angle BAD = \angle ABE$)

AP = BP (P is mid point)

$\angle EPA = \angle DPB$ (given).

Add $\angle EPD$ both sides.

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\angle DPA = \angle EPB$

$\therefore \Delta DAP \cong \Delta EBP$ (ASA rule)

AD = BE (CPCT)

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:

(i) Δ AMC $\cong$ Δ BMD

(ii) $\angle$ DBC is a right angle.

(iii) Δ DBC $\cong$ Δ ACB

(iv) CM = $\frac{1}{2}$ AB

Fig. 7.22

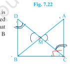


Fig. 7.23

Given :- $\angle C = 90^\circ$, BM = AM , DM = CM

To show :- (i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is 90°

(iii) $\Delta DBC \cong \Delta ACB$.

(iv) $CM = \frac{1}{2} AB$

Proof :- (i) In ΔAMC and ΔBMD

AM = BM (M is mid point)

CM = DM (Given)

$\angle CMA = \angle DMB$ (V.O.A)

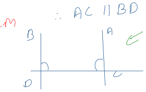
$\Delta AMC \cong \Delta BMD$ (SAS rule)

AC = BD (CPCT)

$\angle ACM = \angle BDM$ (CPCT)

but these are alternate interior angles.

$\therefore AC \parallel BD$



$\angle DBC + \angle ACB = 180^\circ$ (Sum of interior angles)

$\angle DBC + 90^\circ = 180^\circ$

$\angle DBC = 90^\circ$

(iii) In ΔDBC and ΔACB.

DB = AC (proved above)

$\angle B = \angle C$ (each 90°)

BC = CB (Common)

$\Delta DBC \cong \Delta ACB$ (SAS rule)

(iv) CD = BA (CPCT)

CM + DM = AB

CM + CM = AB (CM = DM given)

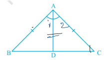
2CM = AB

$\boxed{CM = \frac{1}{2} AB}$

Theorem 7.2 : *Angles opposite to equal sides of an isosceles triangle are equal.*

Given :- $AB = AC$

To prove :- $\angle B = \angle C$



Construction :- AD is angle bisector of $\angle A$.

Sol :- ΔBAD and ΔCAD.

$AB = AC$ (given)

$\angle 1 = \angle 2$ (By construction)

$AD = AD$ (common)

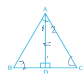
$\Delta BAD \cong \Delta CAD$ (SAS rule)

$\angle B = \angle C$ (CPCT)

Theorem 7.3 : *The sides opposite to equal angles of a triangle are equal.*

Given :- $\angle B = \angle C$

To prove :- $AB = AC$



Construct :- AD is angle bisector of $\angle A$ ($\angle 1 = \angle 2$)

Proof :- ΔBAD and ΔCAD

$\angle B = \angle C$ (given)

$\angle 1 = \angle 2$ (By construction)

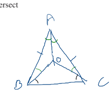
$AD = AD$ (common).

$\Delta BAD \cong \Delta CAD$ (AAS rule)

$AB = AC$ (CPCT)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠ B and ∠ C intersect each other at O. Join A to O. Show that :

(i) OB = OC (ii) AO bisects ∠ A



Given

To show

Sol :- ΔABC, $AB = AC$

$\angle B = \angle C$ (Angle opposite to equal sides are equal.)

$\frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\angle OBC = \angle OCB$

In ΔOBC

if $\angle OBC = \angle OCB$

$OB = OC$ (sides opposite to equal angles are equal.

In ΔAOB and ΔAOC

$AB = AC$ (given)

$\angle ABO = \angle ACO$ ($\frac{1}{2}\angle B = \frac{1}{2}\angle C$)

$OB = OC$ (proved above)

$\therefore \Delta AOB \cong \Delta AOC$ (SAS rule)

$\angle BAO = \angle CAO$ (CPCT)

$\therefore$ AO bisects $\angle A$