

UTRGV

# TREATMENT EFFECT ESTIMATION USING INVERSE PROBABILITY TREATMENT WEIGHTING

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# Main Topics

## Points to talk about

Objectives

Theory

Methods

Results and simulation

Challenges

# The Project's Objectives

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TO STUDY HOW IPTW IS ESTIMATED

TO STUDY METHODS THAT ARE USED IN TREATMENT  
EFFECT ESTIMATION

TO WRITE FUNCTION WHICH CALCULATES IPTW

# Theory

## Treatment effect

The term 'treatment effect' refers to the causal effect of a binary (0-1) variable on an outcome variable of scientific or policy interest.

The term 'treatment effect' originates in a medical literature concerned with the causal effects of binary, yes-or-no 'treatments', such as an experimental drug or a new surgical procedure.

Treatment effects can be estimated using social experiments, regression models, matching estimators, and instrumental variables.

# Theory

## Propensity score

is defined as the probability of treatment assignment conditional on measured baseline covariates.

Propensity score methods are being used with increasing frequency to estimate treatment effects using observational data.

$$e = P(Z = 1|X)$$

# Theory

## Inverse probability of treatment weighting

is the one of the method of using propensity score.

$$w = Z/e + (1-Z)/(1-e)$$

Each subject's weight is equal to the inverse of the probability of receiving the treatment that the subject received.



# METHODS OF USING PROPENSITY SCORE TO ESTIMATE TREATMENT EFFECT

1

## Weighting

The first set of propensity-score estimators use the propensity scores as weights to create a balanced sample of treated and control observations.

$$\tilde{\tau} = \frac{1}{N} \sum_{i=1}^N \left( \frac{Z_i Y_i}{e(X_i)} - \frac{(1 - Z_i) Y_i}{1 - e(X_i)} \right)$$

2

## Conditional expectation

estimate the conditional expectation of Y given W and e(X). Treatment effect can be estimated by

$$v_w(e) = \mathbb{E}[Y(Z) | e(X) = e]$$
$$\widetilde{\tau}_{reg} = \frac{1}{N} \sum_{i=1}^N (\widehat{v}_1(e(X_i)) - \widehat{v}_0(e(X_i)))$$

3

## Weighting + Regression

One can rewrite the weighting estimator as the estimating following regression function by weighted least squares

$$Y_i = \alpha + \tau * W_i + \varepsilon_i$$

# BUILDING FUNCTIONS TO ESTIMATE TREATMENT EFFECT

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## **GLR (x,y,w)**

Linear regression model function which accepts treatment, output and weights as an input

Returns the estimated coefficients, which is a treatment effect estimation

## **IPTW(treatment, output, covs, data)**

Inverse probability treatment weighting function accepts as a parameters treatment column name, output column name, co-varities column name and data

Output: data frame with a new column like ps, ps\_recieved, and IPTW weights.



# BUILDING FUNCTIONS TO ESTIMATE TREATMENT EFFECT

```
#### IPTW calculator function
```

```
```{r}
IPTW = function(treatment,output,covs,data)
{
#ps and iptw estimation
ps = data %>%
group_by_(covs) %>%
summarise(ps = n()/sum(data[treatment]))

data_ps = data %>%
#this merges the propensity score into the bigger dataset by levels
left_join(ps, by = c(covs)) %>%
#if exposed, assign the propensity score. if unexposed assign 1-the
mutate(ps_received = case_when(
data[treatment]==1 ~ ps,
data[treatment]==0 ~ 1-ps)) %>%
#and the weights (for IPTW) are 1 over the ps_received
mutate(IPTW = weekend/ps_received + (1-weekend)/(1-ps_received))

return(data_ps)
}
```
```

```
#### Linear regression model function
```

```
```{r}
GLR = function(x,y,weights){
one = rep(1,length(y))
weight = sqrt(weights)
x = x*weight
y = y*weight
X = cbind(one,x)
b = solve(t(X) %*% X) %*% t(X) %*% y
J = matrix(rep(1,length(y)*length(y)),ncol = length(y),nrow = length(y))
SST0 = t(y) %*% y - 1/(length(y)) * t(y) %*% J %*% y
SSE = t(y) %*% y - t(b) %*% t(X) %*% y
SSR = t(b) %*% t(X) %*% y - ((1/length(y))*t(y)) %*% J %*% y
MSR = SSR/(length(b) - 1)
MSE = SSE/(length(y) - length(b))
F_S = MSR/MSE
R_S = SSR/SST0
res = y - X%*%b
predicted = X%*%b
residual_st_error = sqrt(sum(res^2)/(length(y) - length(b)))
resid_results = summary(res)
b_hat = solve(t(X) %*% X)
SE = sqrt(diag(b_hat))*residual_st_error
tb1 = b/SE
pvalue1 = 2 - 2*pt(abs(tb1), df = length(y)- length(b))
coefficient_results = cbind(b=b,SE=SE,tb1=tb1,pvalue1=pvalue1)
p_value_F = pf(F_S, length(b) - 1, length(y) - length(b), lower.tail = FALSE)
newList = list("coefficients" = coefficient_results, "residuals" = resid_results ,df = length(y) - length(b),
residual_st_error, "F*" = F_S, "R^2" = R_S, "p_value" = p_value_F)
return(list(newList=newList,res=res,predicted=predicted))
}
```
```

# Simulation

**Dataset:** Movies domestic gross income from Jun-Dec 2018

Real data from <https://www.boxofficemojo.com/>  
was partially changed to get the weekend column(which is a treatment)

| Date<br><int> | Day<br><int> | DayNu...<br><int> | Top10.Gross<br><fctr> | YD<br><fctr> | LW<br><fctr> | Releases<br><int> | Film<br><fctr> | Gross<br><fctr> | GrossNew<br><fctr> |
|---------------|--------------|-------------------|-----------------------|--------------|--------------|-------------------|----------------|-----------------|--------------------|
| 43465         | 1            | 365               | \$36,240,441          | -19.60%      | -14.20%      | 53                | Aquaman        | \$10,011,638    | \$10011638         |
| 43464         | 7            | 364               | \$50,932,176          | -12.40%      | 2.90%        | 51                | Aquaman        | \$16,440,551    | \$16440551         |
| 43463         | 6            | 363               | \$58,118,460          | 2.60%        | 4.60%        | 51                | Aquaman        | \$18,632,907    | \$18632907         |
| 43462         | 5            | 362               | \$56,667,767          | 9.70%        | -2.90%       | 51                | Aquaman        | \$17,041,113    | \$17041113         |
| 43461         | 4            | 361               | \$51,671,321          | -7%          | 299.20%      | 53                | Aquaman        | \$14,622,228    | \$14622228         |
| 43460         | 3            | 360               | \$55,579,761          | -21.60%      | 261.80%      | 52                | Aquaman        | \$16,903,518    | \$16903518         |
| 43459         | 2            | 359               | \$70,898,743          | -3.40%       | 30%          | 53                | Aquaman        | \$21,982,419    | \$21982419         |
| 43458         | 1            | 358               | \$31,752,644          | -35.80%      | 184.50%      | 49                | Aquaman        | \$10,851,928    | \$10851928         |
| 43457         | 7            | 357               | \$49,473,471          | -10.90%      | 72.70%       | 53                | Aquaman        | \$18,793,733    | \$18793733         |
| 43456         | 6            | 356               | \$55,554,266          | -4.80%       | 29.80%       | 52                | Aquaman        | \$21,346,634    | \$21346634         |

Goal: To estimate effect of the weekend on the gross revenue

If it is weekday treatment = 0 if its weekend treatment = 1

Covariates: Film

Output: Revenue per day



# Treatment effect estimation

# Results

```
```{r}
iptw_data = IPTW('weekend','GrossRevenue','Film',new_df)
outDatWeight = data.frame(outcome = iptw_data$GrossRevenue, treatment = iptw_data$weekend, wt = iptw_data$IPTW)
x = outDatWeight$treatment
y = outDatWeight$outcome
w = outDatWeight$wt
head(iptw_data)
fit = GLR(x, y, w)
fit$newList$coefficients
```
```

| data | day | dayof | Gross    | ps        | SE        |
|------|-----|-------|----------|-----------|-----------|
| 1    | 1   | 1     | 10011638 | 0.2291667 | 11.524740 |
| 2    | 1   | 1     | 16440551 | 0.2291667 | 11.524740 |
| 3    | 1   | 1     | 18632907 | 0.2291667 | 11.524740 |
| 4    | 1   | 1     | 17041113 | 0.2291667 | 11.524740 |
| 5    | 1   | 1     | 14622228 | 0.2291667 | 11.524740 |
| 6    | 1   | 1     | 16903518 | 0.2291667 | 11.524740 |

data.frame  
6 x 15

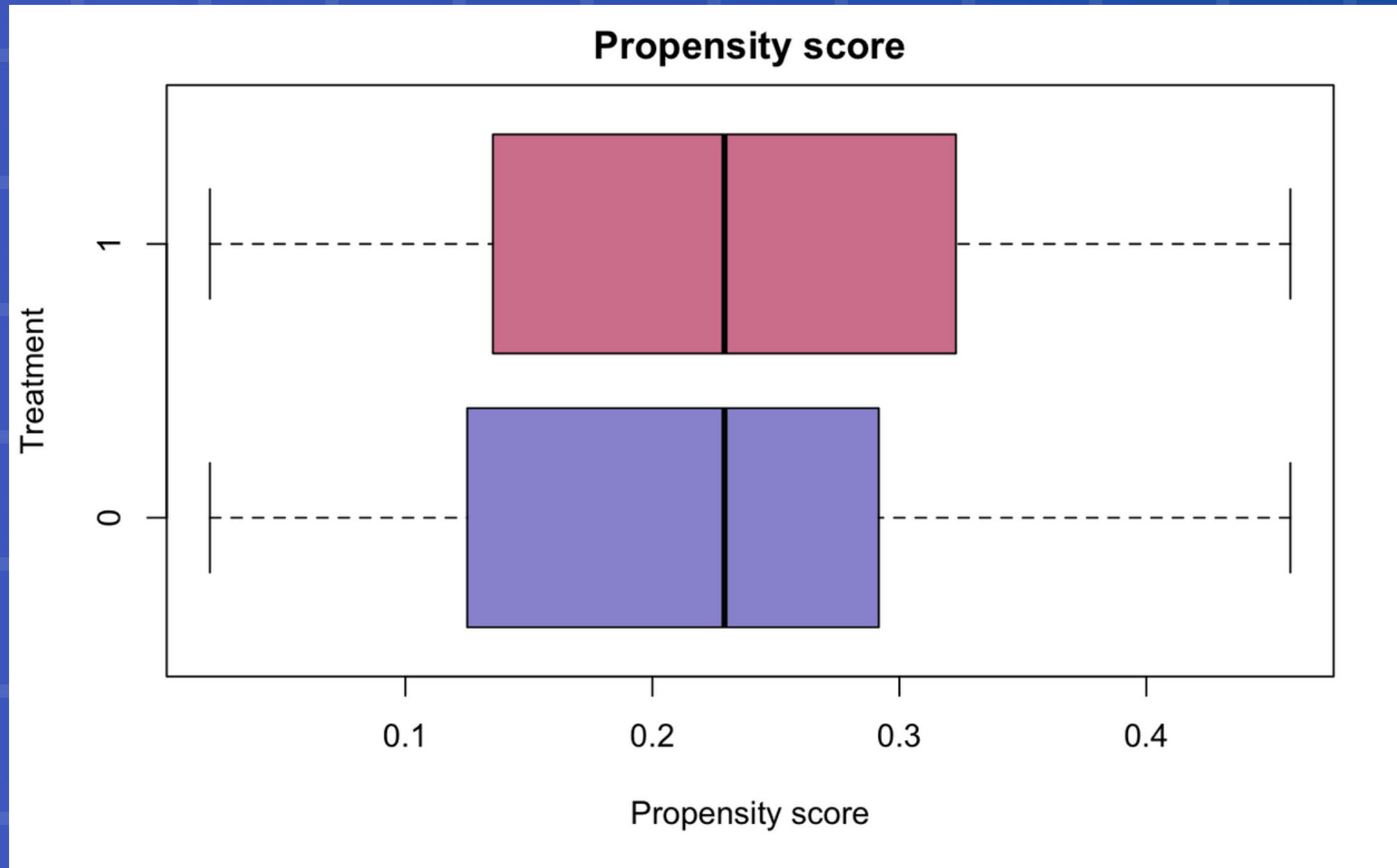
R Console

| LW<br><fctr> | Releases<br><int> | Film<br><fctr> | Gross<br><fctr> | GrossNew<br><fctr> | GrossRevenue<br><int> | weekend<br><dbl> | ps<br><dbl> | ps_received<br><dbl> | IPTW<br><dbl> |
|--------------|-------------------|----------------|-----------------|--------------------|-----------------------|------------------|-------------|----------------------|---------------|
| -14.20%      | 53                | Aquaman        | \$10,011,638    | \$10011638         | 10011638              | 0                | 0.2291667   | 0.7708333            | 4.363636      |
| 2.90%        | 51                | Aquaman        | \$16,440,551    | \$16440551         | 16440551              | 1                | 0.2291667   | 0.2291667            | 4.363636      |
| 4.60%        | 51                | Aquaman        | \$18,632,907    | \$18632907         | 18632907              | 1                | 0.2291667   | 0.2291667            | 4.363636      |
| -2.90%       | 51                | Aquaman        | \$17,041,113    | \$17041113         | 17041113              | 0                | 0.2291667   | 0.7708333            | 4.363636      |
| 299.20%      | 53                | Aquaman        | \$14,622,228    | \$14622228         | 14622228              | 0                | 0.2291667   | 0.7708333            | 4.363636      |
| 261.80%      | 52                | Aquaman        | \$16,903,518    | \$16903518         | 16903518              | 0                | 0.2291667   | 0.7708333            | 4.363636      |

| SE  |          |           |           |              |
|-----|----------|-----------|-----------|--------------|
| one | 15465369 | 1341927.8 | 11.524740 | 0.000000e+00 |
| x   | 7425855  | 959891.3  | 7.736141  | 9.654499e-13 |

# Results

## Propensity scores



# Challenges

FORMATTING INPUT  
PARAMETERS FOR  
IPTW FUNCTION

IPTW IS USED TO  
CALCULATE ONLY  
AVERAGE  
TREATMENT EFFECT

TREATMENT EFFECT  
ESTIMATION  
METHODS NEED TO  
BE STUDIED MORE

**Thank you!**  
**Any questions?**