Treatment effect estimation using Inverse probability of treatment weighting

Abstract

Studies of estimation of treatment effect are being the interest of many researchers. Inverse probability of treatment weighting is used in calculation of treatment effect. Inverse probability of treatment weighting is a useful method which is based on propensity scores methods. In this project studies about methods which use propensity cores, effect of treatment estimation were made. Also, there was created iptw and regression functions. Created functions were used to estimate treatment effect of the weekdays and weekends on the income of the movies in the domestic theaters from the period of June to December 2018. Data was taken from the site that tracks box office revenue https://www.boxofficemojo.com.

Introduction

Observational studies to estimate the effects of treatments, exposures, and interventions on health outcomes are increasingly used by researchers. Randomization in randomized controlled trials ensures that, on average, treated subjects will not differ systematically from control subjects in both measured and unmeasured baseline characteristics. Thus, the effect of treatment is estimated by directly comparing outcomes between the treatment groups. [1] Propensity score methods are being used with increasing frequency to estimate treatment effects using observational data. The propensity score is defined as the probability of treatment assignment conditional on measured baseline covariates.[1] Inverse probability of treatment weighting (IPTW) is the one of the method of using propensity score. The objectives of this project firstly, is to study how IPTW works and which methods are used to estimate treatment effect. Secondly, to create functions which will be used in treatment effect estimation on the real data.

Literature Review

Rubin considers the setting in which there is a binary or dichotomous exposure. Thus, it assumed that there are two possible treatments (e.g., active treatment vs. control treatment). The potential outcomes framework assumes that each subject has a pair of potential outcomes: $Y_i(0)$ and $Y_i(1)$, the outcomes under the control treatment and the active treatment, respectively, when received under identical circumstances. However, each subject receives only one of the control treatment or the active treatment. Let Z denote an indicator variable denoting the treatment received (Z = 0 for control treatment vs. Z = 1 for active treatment). Thus, only one outcome, Y_i, is observed for each subject: the outcome under the actual treatment received. The observed outcome is equal to $Y_i = Z_i Y_i(1) + (1 - Z_i)Y_i(0)$. Thus, Y_i is defined to be equal to $Y_i(0)$ if $Z_i = 0$, and to be equal to $Y_i(1)$ if $Z_i = 1$. [2] If we let X denote a vector of observed baseline covariates. Propensity

score is defined as the probability of treatment assignment conditional on measured baseline covariates: e = P(Z = 1|X). The inverse probability of treatment weight is defined as $w = \frac{Z}{e} + \frac{1-Z}{1-e}$. Each subject's weight is equal to the inverse of the probability of receiving the treatment that the subject received.[1]
Rubin explains that intuitively, the causal effect of one treatment, E, over another, C, for a particular unit and an interval of time from t1 to t2 is the difference between what would have happened at time t2 if the unit had been exposed to E initiated at t1 and what would have happened at t2 if the unit had been exposed to C initiated at t1: "If an hour ago I had taken two aspirins instead of just a glass of water, my headache would now be gone," or "Because an hour ago I took two aspirins instead of just a glass of water, my headache is now gone." [2] Rubin defines causal effect as the following: y(E) — y(C) That is the causal effect of the E versus C treatment on Y for that trial, that is, for that particular unit and the times t_1, t_2. Where, y(E) - the value of Y measured at t_2 on the unit, given that the unit received the experimental Treatment E initiated at t_1; y(C) - the value of Y measured at t_2 on the unit given that the unit received the control Treatment C initiated at t1. [2]
Cole and Hernan state that inverse probability weighting can be used to estimate exposure effects. Unlike standard statistical methods, weighting can appropriately adjust for confounding and selection bias due to measured time-varying covariates affected by prior exposure they point in the paper. The pseudo-population is the result of assigning to each participant a weight that is, informally, proportional to the participant's probability of receiving her own exposure history. In such a pseudo-population, one can regress the outcome on the exposure using a conventional regression model that does not include the measured confounders as covariates. Fitting a model in the pseudo-population is equivalent to fitting a weighted model in the study population. The parameters of such weighted regression models, which equal the parameters of marginal structural models, can be used to estimate the average causal effect of exposure in the original study population. [3] Each unit is exposed to a single treatment; $Z_i = 0$ if unit i receives the control treatment, and $Z_i = 1$ if unit i receives the active treatment. We therefore observe for each unit the triple (Z_i, Y_i, X_i), where Y_i is the realized outcome:

$$Y_i(Z_i) = \begin{cases} Y_i(0), Z_i = 0 \\ Y_i(1), Z_i = 1 \end{cases}$$

the propensity score (Rosenbaum and Rubin, 1983a) is defined as the conditional probability of receiving the treatment,

$$e(x) \equiv Pra(Z = 1 \,|\, X = x) = E[Z|X = x]$$

Population-average treatment effect (PATE):

$$\tau^p = E[Y(1) - Y(0)]$$

Since in the work by Rosenbaum and Rubin (1983a) there has been considerable interest in methods that avoid adjusting directly for all covariates, and instead focus on adjusting for differences in the propensity score, the conditional probability of receiving the treatment. This can be implemented in a number of different ways. One can weight the

observations using the propensity score (and indirectly also in terms of the covariates) to create balance between treated and control units in the weighted sample. Alternatively, one can divide the sample into subsamples with approximately the same value of the propensity score, a technique known as blocking. Finally, one can directly use the propensity score as a regressor in a regression approach.[4]
Weighting: The first set of propensity-score estimators use the propensity scores as weights to create a balanced sample of treated and control observations. Simply taking the difference in average outcomes for treated and controls. With the propensity score known one can directly implement this estimator as

$$\tau = \frac{1}{N}\sum_{i=1}^{N}(\frac{Z_i Y_i}{e(X_i} - \frac{(1-Z_i)Y_i}{1-e(X_i)}$$

There are also represented mixed method such as weighting and regression. One can rewrite the weighting estimator as the estimating following regression function by weighted least squares:

$$Y_i = \alpha + \tau * W_i + \varepsilon_i$$

With weights equal to

$$\lambda_i = \sqrt{\frac{W_i}{e(X_i)} + \frac{1-W_i}{1-e(X_i)}}$$

Without the weights the least squares estimator would not be consistent for the average treatment effect; the weights ensure that the covariates are uncorrelated with the treatment indicator and hence the weighted estimator is consistent points Guido in his review paper. [4] In the programming part of this project regression and iptw functions were created to implement the last described method.

Statistical programming

There was replicated IPTW function which calculates inverse probability of treatment weighting, which helps to show the effect of the treatments that was made. According to the authors [6] for binary treatments, the IPTW methods first needs to calculate propensity scores. Which will be used to calculate inverse probability treatment weights. The results will be used in the linear regression function GLR, which was also programmed from scratch to find estimators of the linear regression. Programmed new IPTW function accepts following variables:

```
IPTW = function(treatment,output,covs,data)
```

The main goal of this function is to calculate inverse probability of treatment weighting using propensity score methods. This function first calculates propensity score using

formuals described in the literature review part of the report. After calculating the propensity scores it calculated the inver probability weights, which are then saved in the new column of the input dataset.
The return object of the function is the existing dataset with a new columns like ps(propensity scores), iptw(weights). Programmed GLR function accepts the model and weights as the parameters and tries to estimate beta coefficients of the model that was submitted to a function. First it balances the model using given weights. After balancing the model it calculates the beta coefficients solving the matrix equation for b coefficients.

```
GLR = function(x,y,weights)
```

Application to Data

To estimate treatment effect estimation of the weekend and weekdays on the movie's income, the real data were taken from the box office revenue site https://www.boxofficemojo.com. As the treatment there were created new column for the existing dataset assigning 0 if it was a weekday and assigning 1 if it was a weekend. As the output gross income of the data were taken. To illustrate functionality of IPTW and GLR described dataset was used.

```
##    Date Day DayNumb Top10.Gross      YD      LW Releases    Film
Gross
## 1 43465   1     365 $36,240,441 -19.60% -14.20%       53 Aquaman
$10,011,638
## 2 43464   7     364 $50,932,176 -12.40%   2.90%       51 Aquaman
$16,440,551
## 3 43463   6     363 $58,118,460   2.60%   4.60%       51 Aquaman
$18,632,907
## 4 43462   5     362 $56,667,767   9.70%  -2.90%       51 Aquaman
$17,041,113
## 5 43461   4     361 $51,671,321     -7% 299.20%       53 Aquaman
$14,622,228
## 6 43460   3     360 $55,579,761 -21.60% 261.80%       52 Aquaman
$16,903,518
##    GrossNew GrossRevenue weekend
## 1 $10011638     10011638       0
## 2 $16440551     16440551       1
## 3 $18632907     18632907       1
## 4 $17041113     17041113       0
## 5 $14622228     14622228       0
## 6 $16903518     16903518       0
```

Below you can see the example of calling the IPTW function to calculate the inverse probability treatment weights.

```
iptw_data = IPTW('weekend','GrossRevenue','Film',new_df)

## Warning: group_by_() is deprecated.
## Please use group_by() instead
##
```

```
## The 'programming' vignette or the tidyeval book can help you
## to program with group_by() : https://tidyeval.tidyverse.org
## This warning is displayed once per session.
```

Below you can see the new dataset with estimated weights was created.

```
##    Date Day DayNumb Top10.Gross      YD      LW Releases    Film
Gross
## 1 43465   1     365 $36,240,441 -19.60% -14.20%       53 Aquaman
$10,011,638
## 2 43464   7     364 $50,932,176 -12.40%   2.90%       51 Aquaman
$16,440,551
## 3 43463   6     363 $58,118,460   2.60%   4.60%       51 Aquaman
$18,632,907
## 4 43462   5     362 $56,667,767   9.70%  -2.90%       51 Aquaman
$17,041,113
## 5 43461   4     361 $51,671,321     -7% 299.20%       53 Aquaman
$14,622,228
## 6 43460   3     360 $55,579,761 -21.60% 261.80%       52 Aquaman
$16,903,518
##    GrossNew GrossRevenue weekend        ps ps_received     IPTW
## 1 $10011638     10011638       0 0.2291667   0.7708333 4.363636
## 2 $16440551     16440551       1 0.2291667   0.2291667 4.363636
## 3 $18632907     18632907       1 0.2291667   0.2291667 4.363636
## 4 $17041113     17041113       0 0.2291667   0.7708333 4.363636
## 5 $14622228     14622228       0 0.2291667   0.7708333 4.363636
## 6 $16903518     16903518       0 0.2291667   0.7708333 4.363636
```

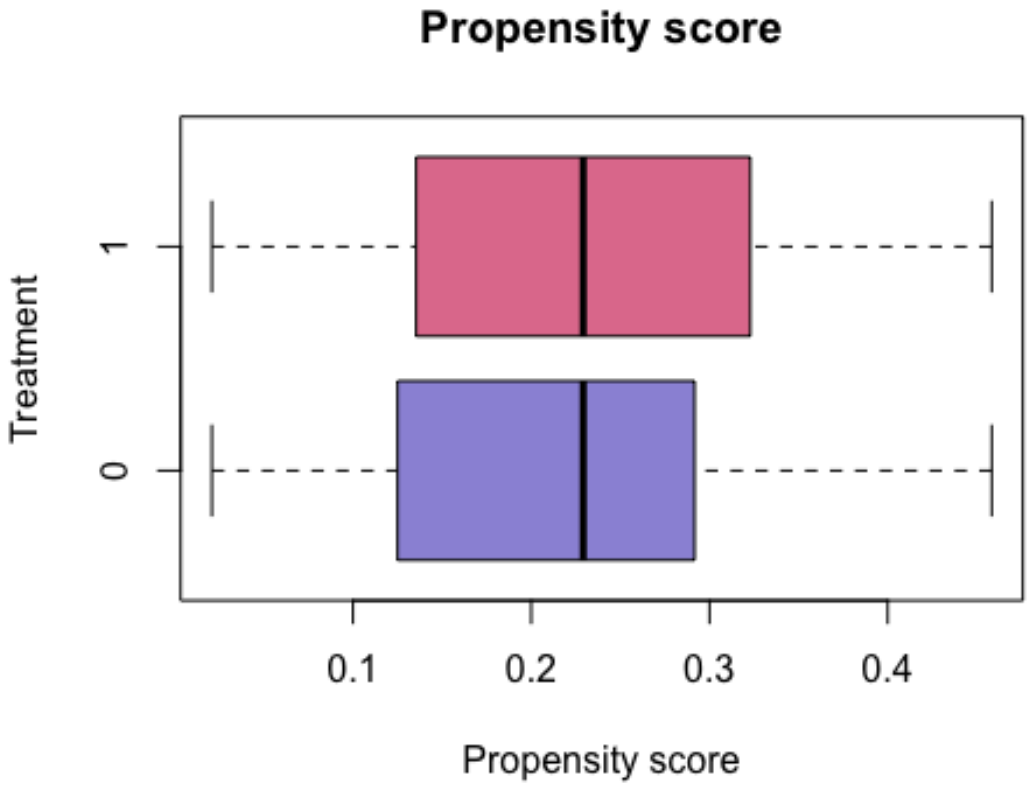
Those weights were passed to the GLR function which will calculate the treatment effect of the weekends on the income of the film.

```
outDatWeight = data.frame(outcome = iptw_data$GrossRevenue, treatment =
iptw_data$weekend, wt = iptw_data$IPTW)
x = outDatWeight$treatment
y = outDatWeight$outcome
w = outDatWeight$wt
fit = GLR(x, y, w)
fit$newList$coefficients
```

```
##                      SE
## one 15465369 1341927.8 11.524740 0.000000e+00
## x    7425855  959891.3  7.736141 9.654499e-13
```

On the next plot you can observe the propensity scores of the two treatments that were used to calculate the treatment effect.

```
boxplot(ps~weekend,col = c(rgb(0.1,0.1,0.7,0.5) ,
rgb(0.8,0.1,0.3,0.6)),data=iptw_data, main="Propensity score",
   xlab="Propensity score", ylab="Treatment",horizontal=TRUE)
```



Conclusion

Inverse probability of treatment weighting (IPTW) is the one of the method of using propensity score. Using IPTW one can estimate the treatment. It can be very handy for researchers not only in medicine, but also in other sphere where causal effect of a binary needs to be estimated. There were made studies about IPTW and propensity scores estimation. IPTW and GLR functions were created using R programming language. Created functions were used on the real data to estimate the treatment effect.

References

1. Peter C. Austin Elizabeth A. Stuart, Moving towards best practice when using inverse probability of treatment weighting (IPTW) using the propensity score to estimate causal treatment effects in observational studies, Statistics in Medicine Volume 34, Issue 28, 03 August 2015

2. Rubin DB. Estimating causal effects of treatments in randomized and nonrandomized studies. Journal of Educational Psychology 1974; 66:688–701.

3. Stephen R. Cole and Miguel A. Hernån Constructing Inverse Probability Weights for Marginal Structural Models

4. Guido W. Imbens NONPARAMETRIC ESTIMATION OF AVERAGE TREATMENT EFFECTS UNDER EXOGENEITY: A REVIEW*