Rethinking Distributions Cheat Sheet

VB & VAAN

Last updated: Jan 5 2022

```r
set.seed(1159)
```

Intention

The intention for this doc is to update it with useful notes about the statistical distributions that show up in the Statistical Rethinking textbook, including probability density / mass functions, some moments when relevant, and, especially, comments on why one might choose specific distributions for different uses.

Distributions

Binomial

Rethinking notation:

$$X \sim \text{Binomial}(N, p)$$

Probability mass function:

$$P(X = x|N, p) = \binom{N}{x} p^x (1-p)^{N-x}$$

Where $N \in \{1, 2, ...\}$ is the number of trials, $x \in \{1, 2, ..., N\}$ is the number of successes, $p \in [0, 1]$ is the probability of success, and $\binom{N}{x}$ is "N choose x", aka the binomial coefficient (see the relevant section for more: The binomial coefficient).

Usefulness

This distribution is very useful as a **likelihood** when we want to model the probability of success, i.e., of one of the options of a binary response.

R commands

Random draw `rbinom(n, size, prob)` generates a vector with the number of successes (we called that x) in each of `n` draws from a binomial distribution where N equals `size` and p equals `prob`.

```r
example.rbinom <- rbinom(
  n = 5,
  size = 10,
  prob = .8
)

example.rbinom
```

```
## [1] 7 9 8 8 9
```

In this example, there were 7 successes out of 10 attempts in the first draw, 9 successes out of 10 attempts in the second draw, and so on.

Density `dbinom(x, size, prob)` computes the probability mass function $P(X = x|N, p)$, where x equals `x`, N equals `size`, and p equals `prob`. So, for example, the probability of observing 4 successes out of 10 attempts, if each has a probability of 0.7 of occurring is:

```r
dbinom(
  x = 4,
  size = 10,
  prob = .7
)
```

```
## [1] 0.03675691
```

Gaussian (normal)

Rethinking notation:

$$y_i \sim \text{Normal}(\mu, \sigma)$$

This is what is meant elsewhere by

$$y_i \sim \mathcal{N}(\mu, \sigma)$$

or even

$$y_i \overset{iid}{\sim} \mathcal{N}(\mu, \sigma)$$

(iid = independent and identically distributed, meaning each value h_i is independent of the others, and all come from the same distribution function)

Probability density function:

$$p(y|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)$$

The PDF can also be parameterized with $\tau = 1/\sigma^2$, as such:

$$p(y|\mu, \tau) = \sqrt{\frac{\tau}{2\pi}} \exp\left(-\frac{1}{2}\tau(y-\mu)^2\right)$$

Other useful things

Combinatorics

The binomial coefficient

The binomial coefficient, also known as "N choose x" allows us to calculate the number of *combinations* of size x of N different elements that are not repeated. A combination does not take order into account. Thus, for example, there are only three ways to combine two elements out of the set $\{A, B, C\}$, namely $\{A, B\}$, $\{A, C\}$, and $\{B, C\}$.

$$C_N^x = {}^N C_x = \binom{N}{x} = \frac{N!}{x!(N-x)!}$$

Online, you mostly find n and k instead of N and x, respectively.

You can use `choose(n, k)` to compute them.

```
# how many ways to pick exactly 7 diferent elements out of 10 total elements?
choose(10, 7)
```

```
## [1] 120
```

[Note: could be good to include here a good explainer for the formula?]