

Exercise 3.3 – GRE scores, Part I – Valerie Briot

Sophia has the following scores for GRE:

Verbal reasoning score = 160

Quantitative reasoning score = 157

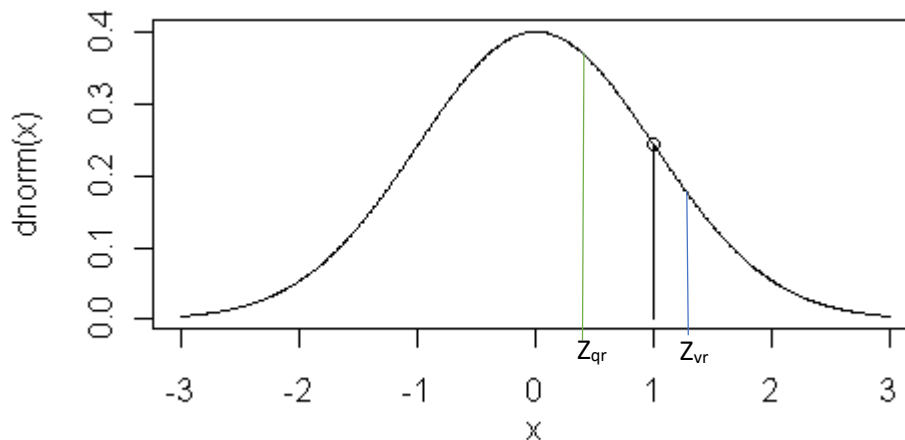
Verbal reasoning distribution for all test takers is nearly normal with $\mu_{vr} = 151$ with $\sigma_{vr} = 7$

Quantitative reasoning distribution for all test takers is nearly normal with $\mu_{qr} = 153$ with $\sigma_{qr} = 7.67$

- a) Write the short-hand for these 2 distributions:
 $N_{vr} (\mu=151, \sigma=7)$ and $N_{qr} (\mu=153, \sigma=7.67)$
- b) What is Sophia's Z-score on Verbal reasoning section? On the Quantitative Reasoning section?
Draw a standard normal distribution curve and mark these 2 Z-scores.

$$\text{Verbal reasoning Z-score} = Z = \frac{x - \mu}{\sigma} = \frac{160 - 151}{7} = \frac{9}{7} = 1.29$$

$$\text{Quantitative reasoning Z-score} = Z = \frac{x - \mu}{\sigma} = \frac{157 - 153}{7.67} = \frac{4}{7.67} = 0.52$$



- c) What does Z-scores mean?
Sophia's Verbal reasoning score is 1.29 standard deviation above the mean and Sophia's Quantitative reasoning score is 0.52 standard deviation above the mean.
- d) Relative to others, which section did she do better in?
Since her Z-score for Verbal reasoning is further away from the mean on positive side, she did better in this section of GRE.

- e) Find her percentile score for her 2 exams?

Looking up her respective z-scores in the table we have the following:

$$\text{Perc}_{\text{vr}} = 0.9015 = 90\%$$

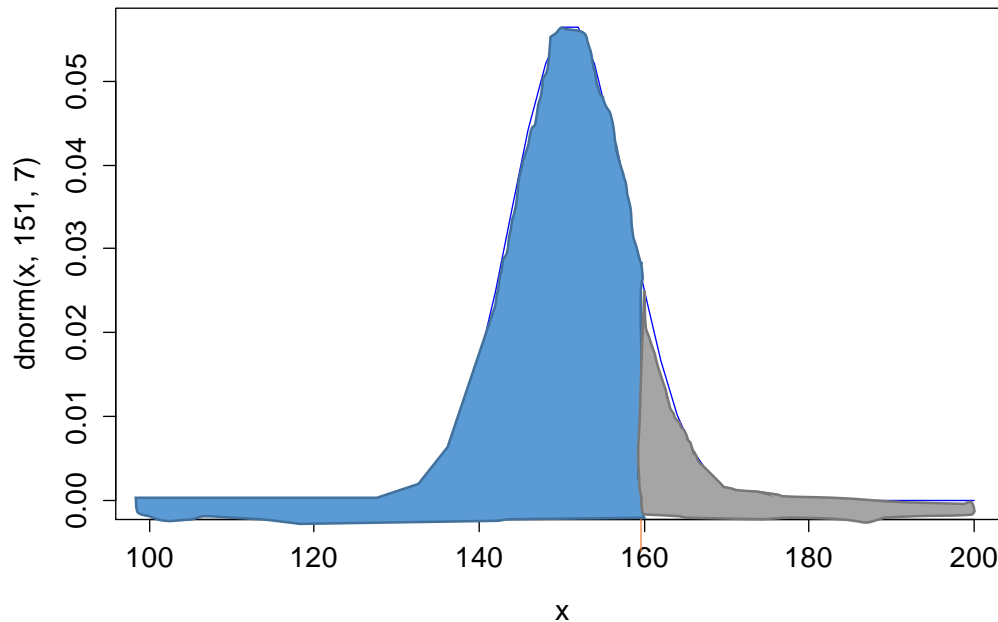
$$\text{Perc}_{\text{qr}} = 0.6985 = 70\%$$

Alternatively, we could use the pnorm function in R

$$\text{Perc}_{\text{vr}} = \text{pnorm}(160, \text{mean}=151, \text{sd} = 7) = 0.9007286$$

$$\text{Perc}_{\text{qr}} = \text{pnorm}(157, \text{mean}=153, \text{sd} = 7.67) = 0.6989951$$

- f) What percent of the test taker did better than her on the Verbal reasoning section?
On the Quantitative reasoning section?



The percentage of test takers that score higher than Sophia's is represented in gray. The answer to the previous question is represented in blue. Hence,

$$P_{\text{vr}}(x > 160) = 1 - 90 = 10\%$$

$$\text{Similarly for } P_{\text{qr}}(x > 153) = 1 - 70 = 30\%$$

These could be computed directly in R, by the following:

$$P_{\text{vr}}(x > 160) = \text{pnorm}(160, \text{mean}=151, \text{sd} = 7, \text{lower.tail} = \text{FALSE}) = 0.0992714$$

$$P_{\text{qr}}(x > 157) = \text{pnorm}(157, \text{mean}=153, \text{sd} = 7.67, \text{lower.tail} = \text{FALSE}) = 0.3010049$$

- g) If we compare the raw score, it would appear that Sophia scored higher on the Quantitative reasoning section

since her score is higher there. However, this proves to be incorrect since the 2 scores are on different scale as shown by having different distribution.

h) If distributions for the scores are not normal, can we answer b) – f)?

The z-score calculation is independent of the type of distribution and can be evaluated as long as the mean and standard deviation is known. However, question d) – g) assume normal distribution.