

1. Preliminary experiments for Sevilla and Sioux Falls networks

1.1. for (MC)

| Network | Formulation using (??)-(??) | | Formulation using (??)-(??) | |
|---------|-----------------------------|--------|-----------------------------|--------|
| | t | LP gap | t | LP gap |
| Sioux | 48.81 | 2.96 | 177.39 | 7.4 |
| | gap | LP gap | gap | LP gap |
| Sevilla | 3.02 | 1.26 | $+\infty$ | 2.31 |

Table 1: Comparing the performance of the two different types of mode choice and capacity constraints for (MC) within a time limit of 1 hour for Sevilla and Sioux Falls instances.

| Network | BD_Norm1 | | BD_Norm2 | | BD_Norm3 | |
|---------|----------|------|----------|------|----------|-------|
| | t | cuts | t | cuts | t | cuts |
| Sioux | 24.67 | 2352 | 1186.46 | 4761 | 53.06 | 19480 |
| Sevilla | 341.61 | 7479 | - | | 526.56 | 68203 |

Table 2: Comparing the performance of the three dual normalizations within a time limit of 1 hour for (MC) for Sevilla and Sioux Falls instances..

| Network | BD_CW | | Algorithm ??+BD_CW | |
|---------|--------|------|--------------------|------|
| | t | cuts | t | cuts |
| Sioux | 30.22 | 2959 | 34.08 | 3254 |
| Sevilla | 438.39 | 6322 | 706.78 | 8824 |

Table 3: Comparing the performance of the Algorithm ?? for (MC) for Sevilla and Sioux Falls instances.

2. Time performance

3. Branch-and-Benders-cut performance for Germany instances

Para la instancia de Germany50 se han utilizado los datos reales proporcionados a excepción del coste de los nodos y la utilidad privada.

Para la instancia de Ta2 se han utilizado los datos reales proporcionados a excepción del coste de los nodos, coste de las aristas y la utilidad privada.

| | | BD_Trđ | | BD_Norm | | BD_CW | |
|-----------|------------|----------|----------|----------|----------|----------|----------|
| | | t_master | t_subpbs | t_master | t_subpbs | t_master | t_subpbs |
| <i>MC</i> | without CS | 1040.49 | 54.76 | 501.53 | 39.5 | 445.91 | 11.9 |
| | +CS | 607.52 | 29.97 | 546.50 | 29.37 | 267.76 | 4.63 |
| <i>PC</i> | without CS | 467.26 | 36.8 | 485.61 | 28.81 | 824.85 | 12.56 |
| | +CS | 223.79 | 37.95 | 278.61 | 44.6 | 187.08 | 10.47 |

Table 4: Comparing the performance of the master and the subproblems in the three algorithms for (*MC*) and (*PC*).

| Network | | CPLEX | Auto_BD | | BD_Trđ | | BD_Norm | | BD_CW | |
|------------|-----|-----------|--------------|--------------|--------------|---------------|--------------|---------------|--------------|---------------|
| without CS | G50 | gap +∞ | gap 26.31 | cuts 4274 | gap 37.34 | cuts 9788 | gap 30.06 | cuts 11842 | gap 31.51 | cuts 11472 |
| | Ta2 | t - | t - | cuts 6455 | t 1109.98 | cuts 12969 | t 471.03 | cuts 13560 | t 215.13 | cuts 12628 |
| | G50 | gap | gap 20.85 | cuts 3176 | gap 36.27 | cuts 11856 | gap 40.74 | cuts 15847 | gap 19.22 | cuts 8836 |
| +CS | Ta2 | t | t - | cuts 6364 | t 486.52 | cuts 8890 | t 280.52 | cuts 7704 | t 156.11 | cuts 11117 |

Table 5: Comparing the performance of the three algorithms for (*MC*).

| | | Network | CPLEX | Auto_BD | | BD_Trđ | | BD_Norm | | BD_CW | |
|---------------|-----|---------|-------|---------|-------|--------|-------|---------|-------|-------|--|
| without CS | G50 | gap | gap | cuts | gap | cuts | gap | cuts | gap | cuts | |
| | | 72.44 | 11.38 | 7450 | 28.57 | 14176 | 21.78 | 9977 | 20.68 | 7602 | |
| | Ta2 | t | t | cuts | t | cuts | t | cuts | t | cuts | |
| - | | - | 2822 | 257.44 | 6950 | - | 6436 | 190.80 | 6386 | | |
| +CS | G50 | gap | gap | cuts | gap | cuts | gap | cuts | gap | cuts | |
| | | | 10.04 | 5459 | 18.76 | 8185 | 23.96 | 11915 | 14.22 | 9819 | |
| | Ta2 | t | t | cuts | t | cuts | t | cuts | t | cuts | |
| - | | - | 1117 | 138.94 | 2763 | 127.19 | 2645 | 75.78 | 2702 | | |

Table 6: Comparing the performance of the three algorithms for (*PC*).

| | | Auto_BD | | BD_Trđ | | BD_Norm | | BD_CW | |
|----------------------|--|---------|------|--------|-------|---------|-------|-------|-------|
| | | gap | cuts | gap | cuts | gap | cuts | gap | cuts |
| without{CS, IS, RNC} | | 26.31 | 4274 | 37.34 | 9788 | 30.06 | 11842 | 31.51 | 11472 |
| +CS | | 20.85 | 3176 | 36.27 | 11856 | 40.74 | 15847 | 19.22 | 8836 |
| +CS+IS | | 56.12 | 3492 | 27.28 | 11635 | 29.74 | 11707 | 17.83 | 8206 |
| +CS+IS+RNC | | - | - | 31.33 | 10150 | 32.27 | 11538 | 14.14 | 9064 |

Table 7: Computing gaps to solve (*MC*) for Germany50 instance comparing the performance of three families of Benders cuts.

| | Auto_BD | | BD_TrD | | BD_Norm | | BD_CW | |
|----------------------|---------|------|--------|-------|---------|-------|-------|------|
| | gap | cuts | gap | cuts | gap | cuts | gap | cuts |
| without{CS, IS, RNC} | 11.38 | 7450 | 28.57 | 14176 | 21.78 | 9977 | 20.68 | 7602 |
| +CS | 10.04 | 5459 | 18.76 | 8185 | 23.96 | 11915 | 14.22 | 9819 |
| +CS+IS | 11.65 | 3446 | 21.32 | 9552 | 21.14 | 9087 | 18.54 | 5075 |
| +CS+IS+RNC | - | - | 23.85 | 9771 | 19.93 | 8326 | 13.87 | 7955 |

Table 8: Computing gaps to solve (PC) for Germany50 instance comparing the performance of three families of Benders cuts.

| | Auto_BD | | BD_TrD | | BD_Norm | | BD_CW | |
|----------------------|---------|------|---------|-------|---------|-------|--------|-------|
| | t | cuts | t | cuts | t | cuts | t | cuts |
| without{CS, IS, RNC} | - | 6455 | 1109.98 | 12969 | 471.03 | 13560 | 215.13 | 12628 |
| +CS | - | 6364 | 486.52 | 8890 | 280.52 | 7704 | 156.11 | 11117 |
| +CS+IS | - | 191 | 587.41 | 13452 | 884.95 | 12530 | 161.14 | 8893 |
| +CS+IS+RNC | - | - | 500.14 | 10302 | 530.34 | 8238 | 136.13 | 8307 |

Table 9: Computing gaps to solve (MC) for Ta2 instance comparing the performance of three families of Benders cuts.

| | Auto_BD | | BD_TrD | | BD_Norm | | BD_CW | |
|----------------------|---------|------|--------|------|---------|------|--------|------|
| | t | cuts | t | cuts | t | cuts | t | cuts |
| without{CS, IS, RNC} | - | 2822 | 257.44 | 6950 | - | 6436 | 190.80 | 6386 |
| +CS | - | 1117 | 138.94 | 2763 | 127.19 | 2645 | 75.78 | 2702 |
| +CS+IS | 1456.67 | 254 | 153.48 | 3322 | 141.69 | 3535 | 57.88 | 2411 |
| +CS+IS+RNC | - | - | 196.55 | 3286 | 136.94 | 2096 | 68.39 | 2512 |

Table 10: Computing gaps to solve (PC) for Ta2 instance comparing the performance of three families of Benders cuts.

| C_{max} | BD_Norm+CS | | BD_CW+CS | |
|-----------|------------|-------|----------|-------|
| | gap | cuts | gap | cuts |
| 0.3 TC | 74.40 | 5026 | 48.01 | 5806 |
| 0.5 TC | 56.84 | 15490 | 18.91 | 10121 |
| 0.7 TC | 19.56 | 19193 | 11.53 | 14858 |

a.

| u | BD_Norm+CS | | BD_CW+CS | |
|-------------|------------|-------|----------|-------|
| | gap | cuts | gap | cuts |
| 1.5 $SPath$ | 67.57 | 13531 | 11.95 | 7545 |
| 2 $SPath$ | 56.84 | 15490 | 18.91 | 10121 |
| 3 $SPath$ | 29.53 | 11875 | 18.22 | 8559 |

b.

Table 11: Sensitivity analysis for (MC) with G50 instance

| β | BD_Norm+CS | | BD_CW+CS | |
|---------|------------|-------|----------|-------|
| | gap | cuts | gap | cuts |
| 0.3 | 21.14 | 9174 | 18.22 | 4154 |
| 0.5 | 24.01 | 11915 | 17.44 | 6678 |
| 0.7 | 17.47 | 15498 | 17.80 | 12736 |

a.

| u | BD_Norm+CS | | BD_CW+CS | |
|-------------|------------|-------|----------|------|
| | t | cuts | t | cuts |
| 1.5 $SPath$ | 23.91 | 12491 | 10.14 | 6651 |
| 2 $SPath$ | 24.01 | 11915 | 17.44 | 6678 |
| 3 $SPath$ | 15.95 | 11761 | 13.02 | 6744 |

b.

Table 12: Sensitivity analysis for (PC) with G50 instance

| C_{max} | BD_Norm+CS | | BD_CW+CS | |
|---------------|------------|-------|----------|-------|
| | t | cuts | t | cuts |
| 0.3 <i>TC</i> | 1124.30 | 7746 | 329.89 | 7124 |
| 0.5 <i>TC</i> | 827.92 | 12435 | 194.53 | 10257 |
| 0.7 <i>TC</i> | 84.25 | 3220 | 23.30 | 3936 |

a.

| u | BD_Norm+CS | | BD_CW+CS | |
|------------------|------------|-------|----------|-------|
| | t | cuts | t | cuts |
| 1.5 <i>SPath</i> | - | | 457.70 | 11266 |
| 2 <i>SPath</i> | 827.92 | 12435 | 194.53 | 10257 |
| 3 <i>SPath</i> | 458.73 | 8775 | 109.36 | 9604 |

b.

Table 13: Sensitivity analysis for (*MC*) with Ta2 instance

| β | BD_Norm+CS | | BD_CW+CS | |
|---------|------------|------|----------|------|
| | t | cuts | t | cuts |
| 0.3 | 214.30 | 1177 | 162.83 | 2715 |
| 0.5 | 127.44 | 2645 | 78.61 | 2811 |
| 0.7 | 2689.41 | 6668 | 461.44 | 5573 |

a.

| u | BD_Norm+CS | | BD_CW+CS | |
|------------------|------------|------|----------|------|
| | t | cuts | t | cuts |
| 1.5 <i>SPath</i> | 327.73 | 4821 | 130.81 | 3251 |
| 2 <i>SPath</i> | 127.44 | 2645 | 78.61 | 2811 |
| 3 <i>SPath</i> | 130.03 | 2745 | 65.98 | 2277 |

b.

Table 14: Sensitivity analysis for (*PC*) with Ta2 instance