

CSCI B505 Spring 20: Written assignment 1

Due date: February 8 23:59 PM, File upload

The preferred format is PDF. Handwriting is allowed, but you should write clearly and neatly: ambiguities will be treated not in your favor.

For all answers, you should show their derivation: answers without an explanation will get 0 points.

1. (15) Suppose that algorithm A has a running time of exactly $3 \cdot 2^n - n$ CPU cycles and algorithm B has a running time of exactly $n^2 + n + 100$ CPU cycles for inputs of size n . What is the smallest integer input size n for which algorithm B runs faster than algorithm A? You should perform required calculations by hand.
2. (15) Using the formal definition of big-Oh, show that big-Oh is transitive: for any functions f, g, h , if $f = O(g)$ and $g = O(h)$, then $f = O(h)$.
3. (15) Is $\log(n^3) = O(\log n)$? Is $\log^3(n) = O(\log n)$? Prove using the formal definition of big-Oh.
4. (15) Let $f(n), g(n)$ be positive-valued functions. Using the formal definition of big-Oh, show that:

$$\min(f(n), g(n))^2 = O(f(n) \cdot g(n))$$

5. (20) Find the running time of the algorithm in Θ -notation (recall that $f = \Theta(g)$ iff $f = O(g)$ and $f = \Omega(g)$):

```
int count = 0;
for (int i = 0; i < n; i++)
    for (int j = 1; j < n; j++)
        for (int k = j; k < i; k *= 2)
            count++;
return count;
```

6. (20) Find the running time of the algorithm in Θ -notation:

```
int count = 0;
for (int i = 1; i < n; i *= 2)
    for (int j = 0; j < n; j += i)
        count++;
return count;
```