

# CSCI B505 Spring 20: Written Assignment 2

Due date: Friday, February 28, 11:59pm

Submit your solution on Canvas. Handwriting is allowed, but you should write clearly and neatly: ambiguities will be treated not in your favor.

**Problem 1 (20)** Given an array  $a$  consisting of  $n$  distinct elements, please describe the algorithm which finds  $k$  smallest elements of  $a$  (in any order). E.g. for  $a = [10, 3, 7, 5, 1]$  and  $k = 2$ , you should return  $[3, 1]$ . We impose the following limitations:

- Array  $a$  is read-only, meaning that you can read its elements but can't change the array in any way.
- You are allowed to use only  $O(k)$  additional memory, which you can use for any purpose. In particular, it means that you can't create a copy of the array in case when  $n \gg k$ .
- Your algorithm must run in  $O(n \log k)$  time.

There is no partial credit if any of these conditions is violated.

**Problem 2 (60)** Solve the following recurrences using the recurrence tree method. For all recurrences, the base case is  $T(n) = 1$  for  $0 \leq n \leq 1$ . Your answer must be in  $\Theta$ -notation. Bounds on the number of operations per level and recursion depth (if you use them in your estimation) must be shown formally.

1. (20)  $T(n) = 2T(n/4) + 1$
2. (20)  $T(n) = 2T(n/2) + n^2$
3. (20)  $T(n) = T(n/3) + T(2n/3) + n$

**Problem 3 (20)** Consider the following sorting algorithm. Given an array  $a$  consisting of  $n$  distinct elements, you do the following:

- Randomly shuffle elements in  $a$ . E.g. array  $[1, 2, 3]$  becomes one of  $[1, 2, 3]$ ,  $[1, 3, 2]$ ,  $[2, 1, 3]$ ,  $[2, 3, 1]$ ,  $[3, 1, 2]$  and  $[3, 2, 1]$  with equal probability.
- Run **deterministic** QUICKSORT on  $a$ .

Prove formally that the expected running time of this algorithm is  $O(n \log n)$ . You may use as a fact that the expected running time of randomized QUICKSORT is  $O(n \log n)$ .

Hints:

1. Consider PARTITION function. Show that PARTITION for our algorithm behaves very similarly to PARTITION of randomized QUICKSORT.
2. Consider the first element of a randomly shuffled array. What can you say about it?