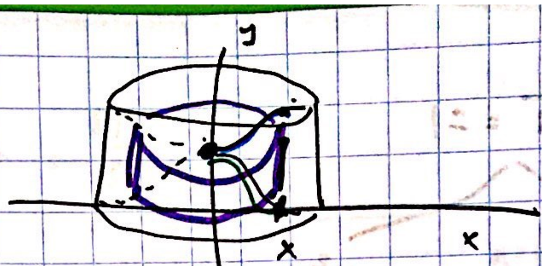
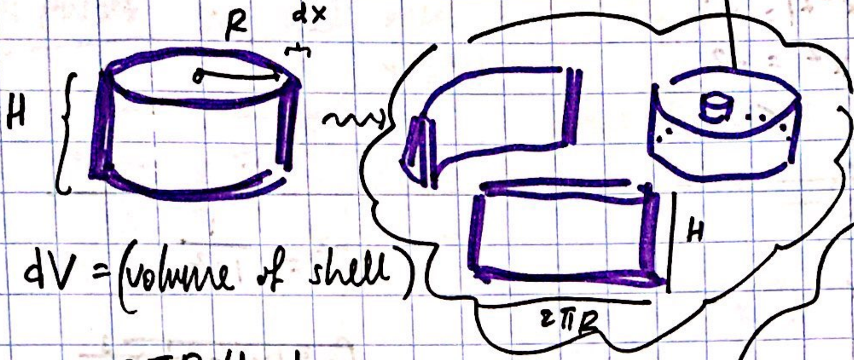
$\boxed{y_1 = (x^2+1)^{-2}}$
$\boxed{y_2 = 2-(x^2+1)^{-2}}$
$x=3$.



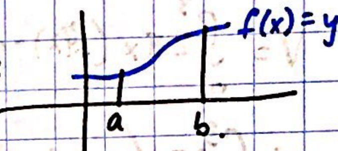
Rolos.



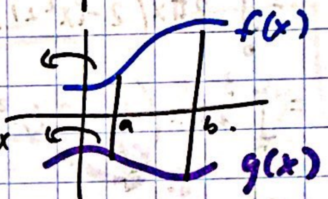
· washer method
· shell method

case in book:

single curve: $2\pi\int_a^b x f(x)\,dx$



2 curves: $2\pi\int_a^b x(f(x)-g(x))\,dx$

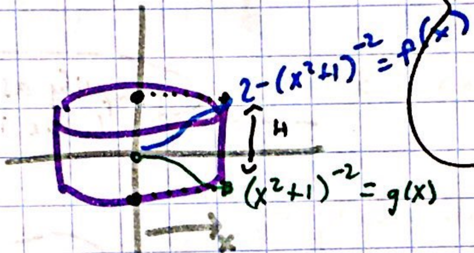


$dV = $ (volume of shell)

$= 2\pi R \cdot H \cdot dx$

(i) (ii)

(i) $R = x$
(ii) $H = f(x) - g(x)$

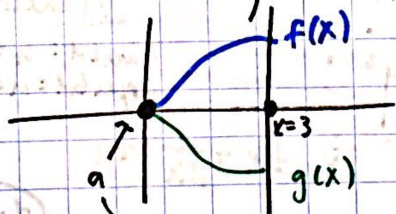


⇝ Put all together to get

$$V = \int dV = \int 2\pi RH\,dx$$
$$= \int_{x=a}^{b} 2\pi\, x\,(f(x)-g(x))\,dx$$

What about bounds? Bounds represent where the smallest & largest shells are. In our case, we want this region...

If your integral has a dy:
$V = \int_{y=a}^{c} (\dots)\,dy$



We want to find the lower bound a. This is the point where $f(x) = g(x)$ ... ie. $f(a) = g(a)$

$$2-(a^2+1)^{-2} = (a^2+1)^{-2} \quad \rightsquigarrow \text{(solve for } a)$$
$$+(a^2+1)^{-2} \qquad +(a^2+1)^{-2}$$

$X^2 = X\cdot X$
$X^{-2} = \frac{1}{X\cdot X}$

$$2 = 2(a^2+1)^{-2}$$
$$1 = (a^2+1)^{-2} = \frac{1}{(a^2+1)^2}$$

$$(a^2+1)^2 = 1$$
$$x^2+1 = \pm 1$$

$x^2+1 = \sqrt{1} = \pm 1$

$x^2+1 = 1$, $x^2 = 0$, $x = 0$

$x^2+1 = -1$, $x^2 = -2$, ✗

---

so $a = 0$

$10^{-2} = \frac{1}{10^2}$

$$V = \int_0^3 2\pi x\,(f(x) - g(x))\,dx$$

$\cancel{\int x^2 dx = \frac{1}{3}x^3}$

$$= \int_0^3 2\pi x\left(2 - (x^2+1)^{-2} - (x^2+1)^{-2}\right) dx$$

$$= \int_0^3 2\pi x\left(2 - 2(x^2+1)^{-2}\right) dx$$

simplify as much as you can before integrating.

$$= \int_0^3 2\pi x \cdot 2\left(1 - (x^2+1)^{-2}\right) dx$$

$$= 4\pi \int_0^3 x\left(1 - (x^2+1)^{-2}\right) dx$$

$$\int (A+B)\,dx = \int A\,dx + \int B\,dx$$

$$= 4\pi \int_0^3 \left[x - x(x^2+1)^{-2}\right] dx$$

$$= 4\pi \int_{x=0}^3 x\,dx - 4\pi \int_{x=0}^3 x(x^2+1)^{-2}\,dx$$

check bounds.
lower bd: $u = 0^2+1 = 1$
upper bd: $u = 3^2+1 = 10$

inside function
$\boxed{u = x^2+1}$
$du = 2x\,dx \Rightarrow dx = \frac{1}{2x}\,du$

$= 4\pi\left(\frac{1}{2}x^2\right)\Big|_0^3$

$= \left(4\pi \cdot \frac{1}{2}\right)(3^2 - 0^2)$

$= 4\pi \cdot \frac{9}{2}$

$= \frac{36\pi}{2}$

$= 18\pi$

$$= 4\pi \int_{u=1}^{10} x \cdot u^{-2} \cdot \frac{1}{2x}\,du$$

shouldn't be any of the old variable.

$x \cdot \frac{1}{2x} = \frac{x}{1} \cdot \frac{1}{2x} = \frac{x}{2x}$

$\cancel{\frac{1}{2x} = \frac{1}{2} \cdot \frac{1}{x}}$

$$= 4\pi \int_{u=1}^{10} u^{-2} \cdot \frac{1}{2} \cdot du$$

$$= \underbrace{4\pi \cdot \frac{1}{2}}_{2\pi} \int_{u=1}^{10} u^{-2}\,du$$

$\frac{u^{-1}}{-1}$ check: $\frac{d}{du}\left(\frac{u^{-1}}{-1}\right) = \left(\frac{1}{-1}\right)\frac{d}{du}(u^{-1}) = (-1)(-u^{-2}) = u^{-2}$.

$$= 2\pi\left(-u^{-1}\right)\Big|_{u=1}^{10}$$

$$= 2\pi\left(-(10)^{-1} - (-1)(1)^{-1}\right)$$

$-(10)^{-1} = -\frac{1}{10}$

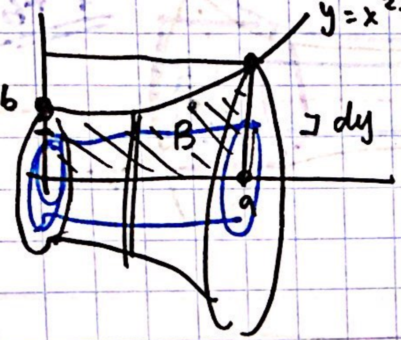
$2\pi\left(-\frac{1}{10} + 1\right) = 2\pi\left(\frac{9}{10}\right) = \frac{18\pi}{10} = \frac{9\pi}{5}$

$$18\pi - \frac{9\pi}{5}$$

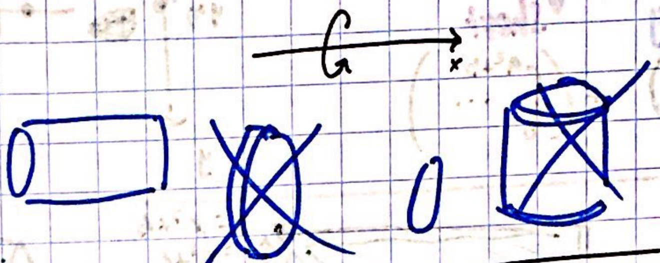
$$\frac{90\pi}{5} - \frac{9\pi}{5} = \frac{90\pi - 9\pi}{5} = \frac{81\pi}{5}$$

6.4 HW #6. Use shell method to calculate:

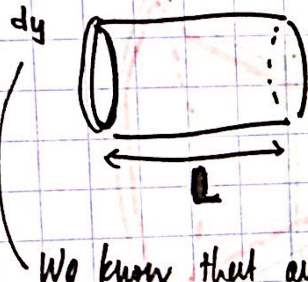
$a=4$
$b=5$



rotate B around x-axis.



shell



$\uparrow R = y = x^2+b$

$dV = 2\pi R \cdot L \cdot dy$ ⇝ $\boxed{V_{total} = \int_{?}^{??} 2\pi R \cdot L \cdot dy}$

volume of shell

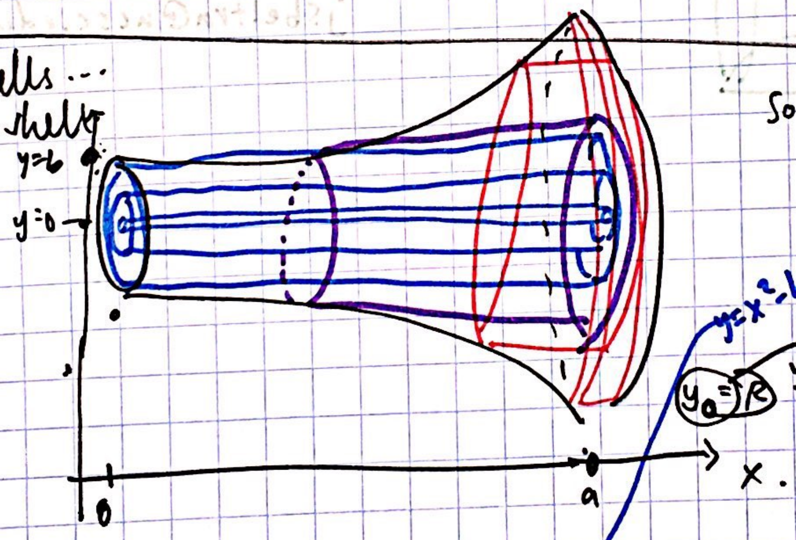
we're integrating "with respect to" y. i.e. R & L should be in terms of y.

We know that our integral $V = \int (\ldots)\, dy$ in terms of y.

· $\boxed{R = y}$

· $L = a - \sqrt{y-b}$

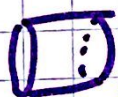
Creating shells... The first shell

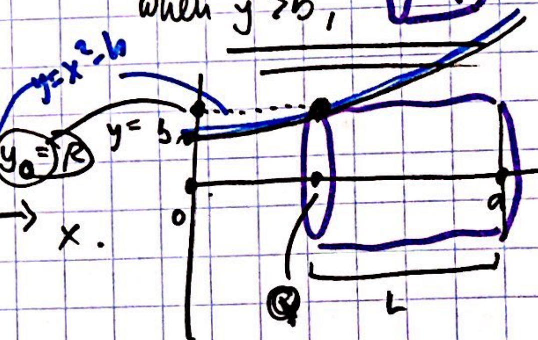


So... when y is between 0 & b.

$L = a$

when $y > b$,





$L = a - Q$

$= a - \sqrt{y-b}$

$L = 4 - \sqrt{y-5}$

So Q is the point on the x-axis where if you plug in Q in to the $\boxed{\text{curve}}$ it gives us $y_0 = R$

$y = x^2+b$

$y_0 \quad Q$

$y_0 = Q^2 + b$

$\vdots$

$Q^2 = y_0 - b$

$Q = \sqrt{y_0 - b}$

$$V_{total} = \int_{lb.}^{ub.} 2\pi R L \, dy$$

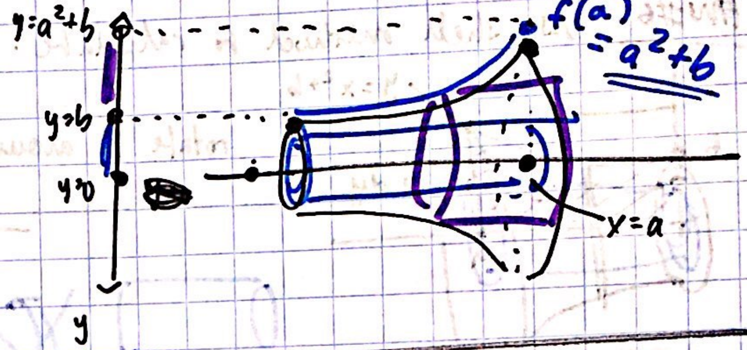
$R = y$

$L = \begin{cases} a & \text{~~when y between 0 \& b~~} \\ a - \sqrt{y-b} & \text{~~when y > b~~} \end{cases}$

What about bounds??.

lower bd $= 0$

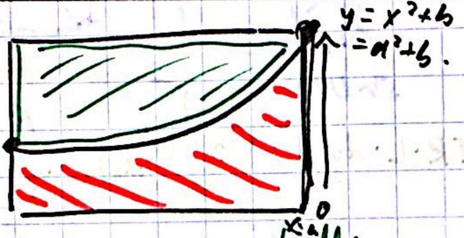
upper bd $= a^2+b$

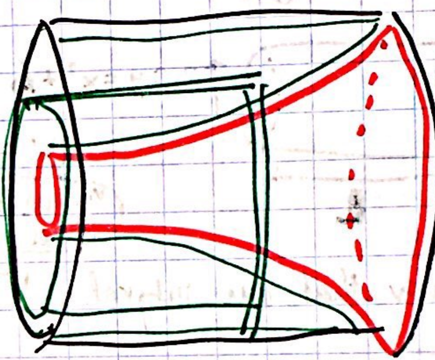


$$V_{total} = V_{long} + V_{short}$$

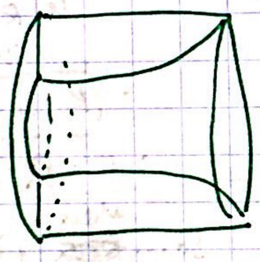
$$\left(\int_0^b \cdots\right) + \left(\int_b^{a^2+b} \cdots\right)$$

Another way... rotate the entire box





solve for green region using shells.



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