# BAN_CIS-5800-001 202310 Homework 1 - Part B (with code report)

Vaibhav Balkrishna Wanere

TOTAL POINTS

**45 / 60**

QUESTION 1

*1* Code Report **10 / 10**

✓ **- 0 pts** *Correct*

**- 3 pts** No description of est_homography implementation

**- 3 pts** No description of warp_pts implementation

**- 4 pts** Incorrect/no results of warped frames

QUESTION 2

*2* Projective Geometry(1a-j) **22.5 / 35**

1a

✓ **- 0 pts** *correct*

**- 2.5 pts** correct beginning equation but wrong final asnwer

**- 3.5 pts** incorrect

1b

**- 0 pts** correct

✓ **- 3.5 pts** *incorrect*

1c

✓ **- 0 pts** *correct*

**- 2.5 pts** Showed considerable work but incorrect answer

**- 3.5 pts** incorrect, little work or incorrect work shown

1d

**- 0 pts** correct

**- 1.5 pts** Did not show derivation of determinant, but correct answer

**- 3.5 pts** incorrect

**- 3.5 pts** incorrect

✓ **- 3.5 pts** *incorrect*

1e

✓ **- 0 pts** *Correct*

**- 3.5 pts** Does not specify the correct camera world setup that causes H_33 to vanish. Identify this corresponds to T_z and give an example of the case where this is 0.

1f

**- 0 pts** Correct

**+ 1.5 pts** Actually defines the inverse solution and gets it correct

**- 1.5 pts** defines the solution in terms of the 3x3 matrix A as specified on piazza

**- 3.5 pts** answer is incorrect

✓ **- 2 pts** *Correct methodology, although did not resolve to the write solution(such as solving for lambda or matrix inverses) or incorrect matrix setup*

1.g

✓ **- 3.5 pts** *Incorrect, does not specify the necessary* *equations, R(H^(-1)(101))+T or solution using lambda*

- **1 pts** Errors in the setup for calculating the final distance
- **0 pts** Correct

1h

- **3.5 pts** Incorrect solution, solution should resolve to Kr_1.

✓ **- 0 pts** *Correct*

- **2 pts** Incorrect K

1i

- **3.5 pts** Incorrect solution, solution should resolve to Kr_3.

✓ **- 0 pts** *Correct*

- **2 pts** Incorrect K

1j

- **2 pts** Derivation correct but mistakes along the way to get to the final answer
- **3.5 pts** Incorrect derivation or no work shown.

✓ **- 0 pts** *Correct*

1 dont forget negative sign

QUESTION 3

## 3 Lane Following(2a-c) 12.5 / 15

2a

✓ **- 0 pts** *Correct*

- **2 pts** Correc R and t, but the final homography is incorrect.
- **3 pts** The rotation matrix is not correct.
- **2 pts** The translation matrix is not correct.
- **1 pts** The translation matrix is reversed
- **5 pts** Wrong procedure, or no answer provided.

2b

- **0 pts** Correct (ignore flipped signs from 2a).

✓ **- 1 pts** *Correct procedure, but incorrect final results.*

- **2.5 pts** Small mistakes in procedure, and incorrect final results.
- **4 pts** Mistakes in procedure, and no intermediate results.
- **5 pts** Wrong procedure, or no answer provided.

2c

- **0 pts** Correct (ignore flipped signs)3.

✓ **- 1 pts** *Correct procedure, but incorrect final results.*

- **2.5 pts** Small mistakes in procedure, and incorrect final results.
- **4 pts** Mistakes in procedure, and no intermediate results.
- **5 pts** Wrong procedure, or no answer provided.

**- 0.5** *Point adjustment*

For 2(a), there's a sign flipped in the R, but not in the final results.

# CIS 5800 Machine Perception HW-1

Vaibhav Wanere

Feb 09 2023

## 1 Calculating Homography

1. Given matrices $X$ and $Y$ we can calculate homography matrix $H$.
2. Calculate matrix $A$ using $x$ and $Y$.
3. Calculate SVD of $A$.
4. Last row of $V$ is my $H$ matrix in vector form.
5. reshape $H$ to $3 \times 3$ form.
6. Calculate warped points by applying projective tranformation on interior points, this is done by multiplying interior points matrix by $H$. We get warped points.
7. Calculating inverse projective transformation from points on logo to points in video is done in the code provided in the question.
8. Results in the form of six frames are as below:


Figure 1: Frame 0


Figure 2: Frame 25


Figure 3: Frame 50


Figure 4: Frame 75


Figure 5: Frame 100

# 2 Perspective Projection and Projective Geometry

**Setup:** The camera and the world co-ordinates are related to each other by the equation:


Figure 6: Frame 125

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T \tag{1}$$

(**a**) What is the origin of the camera coordinate system in the world frame?
**Solution:-**
We know that the co-ordinates of the camera origin are:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

The equation 1 can also be written as:

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = R^{-1} \left( \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} + T \right) \tag{3}$$

As $R$ is an orthogonal matrix, $R^{-1} = R^T$

$$\therefore \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = R^T \left( \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} + T \right) \tag{4}$$

$$\text{for } \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

## *1* Code Report **10 / 10**

✓ **- 0 pts** *Correct*

**- 3 pts** No description of est_homography implementation

**- 3 pts** No description of warp_pts implementation

**- 4 pts** Incorrect/no results of warped frames


Figure 6: Frame 125

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T \tag{1}$$

**(a)** What is the origin of the camera coordinate system in the world frame?
**Solution:-**
We know that the co-ordinates of the camera origin are:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

The equation 1 can also be written as:

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = R^{-1} \left( \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} + T \right) \tag{3}$$

As $R$ is an orthogonal matrix, $R^{-1} = R^T$

$$\therefore \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = R^T \left( \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} + T \right) \tag{4}$$

$$\text{for} \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = R^T T \tag{6}$$

This is in terms of $R$ and $T$.

(**b**) Distance from camera origin to the plane $Y_w = 0$:
**Solution:-**
Now this really depends upon how is my plane $Y_w = 0$ *i.e.* $XZ$ plane oriented with respect to my camera frame.
We can imagine three simple cases for our satisfaction:
1. If it is normal to $X_c$ axis, then the distance is simply the $x$ component of our translation vector *i.e.* $T_x$.
2. If it is normal to $Y_c$ axis, then the distance is simply the $y$ component of our translation vector *i.e.* $T_y$.
3. If it is normal to $Z_c$ axis, then the distance is simply the $z$ component of our translation vector *i.e.* $T_z$.
And if it is parallel to none of the axes of the camera frame, then the distance will be equal to **'the distance between camera centre and a point of intersection ($(X_w, 0, Z_w)$) of the plane $Y_w = 0$. and a perpendicular drawn from the camera centre to $Y_w = 0$ plane'**.

(**c**) i. Projection of a world point $(X_w, Y_w, Z_w)$ to pixel coordinates in $(u, v)$.
ii. Projective transformation $H$ from the plane $Y_w = 0$ to the pixel plane.
**Solution:-**
We know the projection of world to camera:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T \tag{7}$$

from the given relation between $(u, v)$ and $(X_c, Y_c, Z_c)$, we can establish:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \begin{pmatrix} X_c/Z_c \\ Y_c/Z_c \end{pmatrix} \tag{8}$$

$$\therefore \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \begin{pmatrix} X_c \\ Y_c \end{pmatrix} \cdot \frac{1}{Z_c} \tag{9}$$

$$\therefore Z_c \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \begin{pmatrix} X_c \\ Y_c \end{pmatrix} \tag{10}$$

In homogeneous coordinates in $\mathbb{P}^2$ :

$$Z_c \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} \tag{11}$$

From the equations (7) and (11), we can write:

$$Z_c \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \left( R \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T \right) \tag{12}$$

Combining $R$ and $T$ as taught in class:

$$Z_c \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} R & T \\ 000 & 1 \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \tag{13}$$

$$\therefore Z_c \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \tag{14}$$

For a general case: $Z_c = \lambda$

$$\therefore \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \tag{15}$$

$$\therefore \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f.r_{11} & f.r_{12} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{22} & f.r_{23} & f.T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \tag{16}$$

The transformation matrix is:

$$H = \begin{pmatrix} f.r_{11} & f.r_{12} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{22} & f.r_{23} & f.T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{pmatrix} \tag{17}$$

Now for $Y_w = 0$ : We have seen for $Z_w = 0$ in class, after performing the same calculations, we get:

$$H = \begin{pmatrix} f.r_{11} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{23} & f.T_y \\ r_{31} & r_{33} & T_z \end{pmatrix} \tag{18}$$

Which is the required H for $Y_w = 0$.

(d) **Solution:-**
The resulting matrix $H$ can not always be a projective transformation. For a given transformation matrix to be projective, it must be invertible, it means that its determinant should be non-zero.
Thus, matrix $H$ will be a projective transformation if and only if, $det(H) \neq 0$.
For $det(H) = 0$,

(e) **Solution:-** For $h_{33}$ to vanish, $T_z = 0$. This is possible only if the origin of the world frame lies in the $X_cY_c$ plane.

(f) **Solution:-**
As $Y_w = 0$, we can use $H$ given by the equation (18):

$$\therefore \lambda \begin{pmatrix} f \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} f.r_{11} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{23} & f.T_y \\ r_{31} & r_{33} & T_z \end{pmatrix} \begin{pmatrix} X_w \\ Z_w \\ 1 \end{pmatrix} \tag{19}$$

$$\therefore \begin{pmatrix} X_w \\ Z_w \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} f.r_{11} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{23} & f.T_y \\ r_{31} & r_{33} & T_z \end{pmatrix}^{-1} \begin{pmatrix} f \\ 0 \\ 1 \end{pmatrix} \tag{20}$$

As $H$ is invertible:

$$\therefore \begin{pmatrix} X_w \\ Z_w \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} f.r_{11} & f.r_{21} & r_{31} \\ f.r_{13} & f.r_{23} & r_{33} \\ f.T_x & f.T_y & T_z \end{pmatrix} \begin{pmatrix} f \\ 0 \\ 1 \end{pmatrix} \tag{21}$$

As $\lambda = Z_c$ for this problem:

$$\therefore \begin{pmatrix} X_w \\ Z_w \\ 1 \end{pmatrix} = \begin{pmatrix} Z_c(f^2.r_{11} + r_{31}) \\ Z_c(f^2.r_{13} + r_{33}) \\ Z_c(f^2.T_x + T_z) \end{pmatrix} \tag{22}$$

$$\therefore X_w = Z_c(f^2.r_{11} + r_{31}) \tag{23}$$

and

$$Z_w = Z_c(f^2.r_{13} + r_{33}) \tag{24}$$

$$Z_w = Z_c(f^2.r_{13} + r_{33}) \tag{25}$$

This is the required answer.

(g) **Solution:-**
We have $u = f$ and $v = 0$ Let the world coordinates of point $F$ are $(X_w, Y_w, Z_w)$ and its camera co-ordinates are $(X_c, Y_c, Z_c)$. Now the projection $(f, 0)$ of this point on the image plane can be related to its camera coordinates using the following relations:

$$u = f\frac{X_c}{Z_c} \tag{26}$$

and

$$v = f\frac{Y_c}{Z_c} \tag{27}$$

Using the values of $u = f$ and $v = 0$, we are left with $X_c = Z_c$ and $Y_c = 0$ as $f$ and $Z_c$ are non zero as we have a non zero value of the focal length.
If we think more, we will understand that the distance of the point F from the center of projection is nothing but the magnitude of the vector $(X_c, Y_c, Z_c)$ which can be calculated as:

$$d = \sqrt{X_c^2 + Y_c^2 + Z_c^2} \tag{28}$$

$$\therefore d = \sqrt{X_c^2 + X_c^2} \tag{29}$$

$$\therefore d = X_c\sqrt{2} \tag{30}$$

**(h) Solution:-**

The vanishing point arising out of a set a parallel lines parallel to $X_w$ axis can be represented in world coordinates as $(1, 0, 0, 0)^T$.

Its coordinates can be found out by taking its projection on our image plane. Which can be done as given below using the equation 16:

$$\therefore \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f.r_{11} & f.r_{12} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{22} & f.r_{23} & f.T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{31}$$

$$\therefore \lambda u = f.r_{11} \tag{32}$$

$$\lambda v = f.r_{21} \tag{33}$$

$$\lambda = r_{31} \tag{34}$$

$$\therefore u = f\frac{r_{11}}{r_{31}} \tag{35}$$

$$\therefore v = f\frac{r_{21}}{r_{31}} \tag{36}$$

**(i) Solution:-**

For lines parallel to $Z_w$ axis, we have:

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f.r_{11} & f.r_{12} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{22} & f.r_{23} & f.T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{37}$$

$$\therefore \lambda u = f.r_{13} \tag{38}$$

$$\lambda v = f.r_{23} \tag{39}$$

$$\lambda = r_{33} \tag{40}$$

$$\therefore u = f\frac{r_{13}}{r_{33}} \tag{41}$$

$$\therefore v = f\frac{r_{23}}{r_{33}} \tag{42}$$

**(j) Solution:-**

Line equation of horizon arising out of all the sets of parallel lines in the $X_wZ_w$ plane will be the equation of line passing through the their respective vanishing points.

We are aware that the equation of a line passing through two points can be calculated by doing the cross product of these point vectors.

We can indeed use the vanishing points obtained above because they belong to the lines parallel to $X_w$ and $Z_w$ axes.
Let these vanishing points are $v_x$ and $v_z$. Thus,

$$\vec{v_x} \times \vec{v_z} = \begin{vmatrix} u & v & 1 \\ f\frac{r_{11}}{r_{31}} & f\frac{r_{21}}{r_{31}} & 1 \\ f\frac{r_{13}}{r_{33}} & f\frac{r_{23}}{r_{33}} & 1 \end{vmatrix} \tag{43}$$

$$\vec{v_x} \times \vec{v_z} = u(f\frac{r_{21}}{r_{31}} - f\frac{r_{23}}{r_{33}}) - v(f\frac{r_{11}}{r_{31}} - f\frac{r_{13}}{r_{33}}) + ((f\frac{r_{11}}{r_{31}}.f\frac{r_{23}}{r_{33}}) - (f\frac{r_{21}}{r_{31}} - f\frac{r_{13}}{r_{33}})) \tag{44}$$

$$\vec{v_x} \times \vec{v_z} = f(\frac{r_{21}}{r_{31}} - \frac{r_{23}}{r_{33}})u - f(\frac{r_{11}}{r_{31}} - \frac{r_{13}}{r_{33}})v + f^2(\frac{r_{11}}{r_{31}}.\frac{r_{23}}{r_{33}} - \frac{r_{21}}{r_{31}}.\frac{r_{13}}{r_{33}}) \tag{45}$$

$$\vec{v_x} \times \vec{v_z} = f(\frac{r_{21}}{r_{31}} - \frac{r_{23}}{r_{33}})u - f(\frac{r_{11}}{r_{31}} - \frac{r_{13}}{r_{33}})v + \frac{f^2}{r_{31}.r_{33}}(r_{11}r_{23} - r_{21}r_{13}) \tag{46}$$

This is the required equation of the horizon!

# 3 Lane Following

(a) **Solution:-**
From equation 18 we have the transformation matrix for $Y_w = 0$:

$$H = \begin{pmatrix} f.r_{11} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{23} & f.T_y \\ r_{31} & r_{33} & T_z \end{pmatrix} \tag{47}$$

The rotation matrix for a rotation angle $\beta$ about $Y$ axis is given by:

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{pmatrix} \tag{48}$$

From right hand thumb rule about $Y$ axis $\beta$ is negative, also $T_x = 0,\ T_y = h,\ T_z = 0$, putting all this relations in $H$ we have:

## *2* Projective Geometry(1a-j) **22.5 / 35**

1a

✓ **- 0 pts** *correct*

**- 2.5 pts** correct beginning equation but wrong final asnwer

**- 3.5 pts** incorrect

1b

**- 0 pts** correct

✓ **- 3.5 pts** *incorrect*

1c

✓ **- 0 pts** *correct*

**- 2.5 pts** Showed considerable work but incorrect answer

**- 3.5 pts** incorrect, little work or incorrect work shown

1d

**- 0 pts** correct

**- 1.5 pts** Did not show derivation of determinant, but correct answer

**- 3.5 pts** incorrect

**- 3.5 pts** incorrect

✓ **- 3.5 pts** *incorrect*

1e

✓ **- 0 pts** *Correct*

**- 3.5 pts** Does not specify the correct camera world setup that causes H_33 to vanish. Identify this corresponds to T_z and give an example of the case where this is 0.

1f

**- 0 pts** Correct

**+ 1.5 pts** Actually defines the inverse solution and gets it correct

**- 1.5 pts** defines the solution in terms of the 3x3 matrix A as specified on piazza

**- 3.5 pts** answer is incorrect

✓ **- 2 pts** *Correct methodology, although did not resolve to the write solution(such as solving for lambda or matrix inverses) or incorrect matrix setup*

1.g

✓ **- 3.5 pts** *Incorrect, does not specify the necessary equations, R(H^(-1)(101))+T or solution using lambda*

**- 1 pts** Errors in the setup for calculating the final distance

**- 0 pts** Correct

1h

**- 3.5 pts** Incorrect solution, solution should resolve to Kr_1.

✓ **- 0 pts** *Correct*

**- 2 pts** Incorrect K

1i

**- 3.5 pts** Incorrect solution, solution should resolve to Kr_3.

✓ **- 0 pts** *Correct*

**- 2 pts** Incorrect K

1j

**- 2 pts** Derivation correct but mistakes along the way to get to the final answer

**- 3.5 pts** Incorrect derivation or no work shown.

✓ **- 0 pts** *Correct*

1 dont forget negative sign

We can indeed use the vanishing points obtained above because they belong to the lines parallel to $X_w$ and $Z_w$ axes.
Let these vanishing points are $v_x$ and $v_z$. Thus,

$$\vec{v_x} \times \vec{v_z} = \begin{vmatrix} u & v & 1 \\ f\frac{r_{11}}{r_{31}} & f\frac{r_{21}}{r_{31}} & 1 \\ f\frac{r_{13}}{r_{33}} & f\frac{r_{23}}{r_{33}} & 1 \end{vmatrix} \tag{43}$$

$$\vec{v_x} \times \vec{v_z} = u(f\frac{r_{21}}{r_{31}} - f\frac{r_{23}}{r_{33}}) - v(f\frac{r_{11}}{r_{31}} - f\frac{r_{13}}{r_{33}}) + ((f\frac{r_{11}}{r_{31}}.f\frac{r_{23}}{r_{33}}) - (f\frac{r_{21}}{r_{31}} - f\frac{r_{13}}{r_{33}})) \tag{44}$$

$$\vec{v_x} \times \vec{v_z} = f(\frac{r_{21}}{r_{31}} - \frac{r_{23}}{r_{33}})u - f(\frac{r_{11}}{r_{31}} - \frac{r_{13}}{r_{33}})v + f^2(\frac{r_{11}}{r_{31}}.\frac{r_{23}}{r_{33}} - \frac{r_{21}}{r_{31}}.\frac{r_{13}}{r_{33}}) \tag{45}$$

$$\vec{v_x} \times \vec{v_z} = f(\frac{r_{21}}{r_{31}} - \frac{r_{23}}{r_{33}})u - f(\frac{r_{11}}{r_{31}} - \frac{r_{13}}{r_{33}})v + \frac{f^2}{r_{31}.r_{33}}(r_{11}r_{23} - r_{21}r_{13}) \tag{46}$$

This is the required equation of the horizon!

# 3 Lane Following

(a) **Solution:-**
From equation 18 we have the transformation matrix for $Y_w = 0$:

$$H = \begin{pmatrix} f.r_{11} & f.r_{13} & f.T_x \\ f.r_{21} & f.r_{23} & f.T_y \\ r_{31} & r_{33} & T_z \end{pmatrix} \tag{47}$$

The rotation matrix for a rotation angle $\beta$ about $Y$ axis is given by:

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{pmatrix} \tag{48}$$

From right hand thumb rule about $Y$ axis $\beta$ is negative, also $T_x = 0,\ T_y = h,\ T_z = 0$, putting all this relations in $H$ we have:

$$H = \begin{pmatrix} fcos\beta & -fsin\beta & 0 \\ 0 & 0 & fh \\ sin\beta & cos\beta & 0 \end{pmatrix} \tag{49}$$

This is indeed the required projective transform.

**(b) Solution:-**

Take two points on the line designating the right lane: $P_1 = (X_{w_1}, Y_{w_1}, Z_{w_1}) = (d, 0, 0)$ and $P_2 = (X_{w_2}, Y_{w_2}, Z_{w_2}) = (d, 0, 1)$.

Projections of these points on image plane are points $p_1$ and $p_2$ respectively:

Using the projection equation:

$$\lambda.p_1 = \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} fcos\beta & -fsin\beta & 0 \\ 0 & 0 & fh \\ sin\beta & cos\beta & 0 \end{pmatrix} \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} \tag{50}$$

$$\therefore \lambda u = f.d.cos\beta \tag{51}$$

$$\lambda v = 0 \tag{52}$$

$$\therefore v = 0 \tag{53}$$

$$\lambda = d.sin\beta \tag{54}$$

$$u = \frac{f}{tan\beta} \tag{55}$$

$$\therefore p_1 = (\frac{f}{tan\beta}, 0, 1) \tag{56}$$

For $p_2$

$$\lambda.p_2 = \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} fcos\beta & -fsin\beta & 0 \\ 0 & 0 & fh \\ sin\beta & cos\beta & 0 \end{pmatrix} \begin{pmatrix} d \\ 0 \\ 1 \end{pmatrix} \tag{57}$$

$$\therefore u = \frac{f}{tan\beta} \tag{58}$$

$$v = \frac{fh}{d.sin\beta} \tag{59}$$

$$\therefore p_2 = (\frac{f}{tan\beta}, \frac{fh}{d.sin\beta}, 1) \tag{60}$$

Now, $p_1 \times p_2$ will give me the equation of line passing through this points:

$$p_1 \times p_2 = \begin{vmatrix} u & v & 1 \\ \frac{f}{tan\beta} & 0 & 1 \\ \frac{f}{tan\beta} & \frac{fh}{d.sin\beta} & 1 \end{vmatrix} \tag{61}$$

The equation of line is then:

$$(-\frac{fh}{d.sin\beta})u + (0)v + \frac{f}{tan\beta}.\frac{fh}{d.sin\beta} = 0 \tag{62}$$

Which gives:

$$A = -\frac{fh}{d.sin\beta} \tag{63}$$

$$B = 0 \tag{64}$$

$$C = \frac{f}{tan\beta}.\frac{fh}{d.sin\beta} \tag{65}$$

(c) **Solution:-**
Let $(A, B, C)$ upto a scale is $(kA, kB, kC)$. We can compute:

From equation 63:

$$d.sin\beta = -\frac{fh}{kA} \tag{66}$$

Put this result in equation 65, we have:

$$tan\beta = -\frac{fA}{C} \tag{67}$$

From above two relations we can obtain $d$ and $\beta$ given $f$, $h$ and $(A, B, C)$ upto some scale $k$.

---

END OF REPORT

---

## 3 Lane Following(2a-c) 12.5 / 15

2a

✓ **- 0 pts** *Correct*

- **- 2 pts** Correc R and t, but the final homography is incorrect.
- **- 3 pts** The rotation matrix is not correct.
- **- 2 pts** The translation matrix is not correct.
- **- 1 pts** The translation matrix is reversed
- **- 5 pts** Wrong procedure, or no answer provided.

2b

- **- 0 pts** Correct (ignore flipped signs from 2a).

✓ **- 1 pts** *Correct procedure, but incorrect final results.*

- **- 2.5 pts** Small mistakes in procedure, and incorrect final results.
- **- 4 pts** Mistakes in procedure, and no intermediate results.
- **- 5 pts** Wrong procedure, or no answer provided.

2c

- **- 0 pts** Correct (ignore flipped signs)3.

✓ **- 1 pts** *Correct procedure, but incorrect final results.*

- **- 2.5 pts** Small mistakes in procedure, and incorrect final results.
- **- 4 pts** Mistakes in procedure, and no intermediate results.
- **- 5 pts** Wrong procedure, or no answer provided.

**- 0.5** *Point adjustment*

For 2(a), there's a sign flipped in the R, but not in the final results.