Vaibhav Wanere

- Given: pixel co-ordinates of world points.

Find : 3D co-ordinates of pixels in world co-ordinate system →

Consider the equation

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K\left(R \cdot W + t\right) \quad \text{—— (1)}$$

$R = R_w^c$ - rotation of world w.r.t. camera

~~$t = t_w^c$ - rotation of camera wrt to~~

$t = t_w^c$ - translation of world wrt to camera.

we also, have:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R_w^c \cdot \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + t_w^c$$

to know where is camera in the world, we have ⇒

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = \left(R_w^c\right)^{-1}\left(\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} - t_w^c\right)$$

to know camera origin in the world, we have,

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = R_c^w\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - t_w^c\right)$$

$$\therefore \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = -R_c^w \cdot t_w^c$$

but $\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix}$ is nothing but translation vector from world to came, so by defn.

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = t_c^w = -R_c^w \cdot t_w^c \quad \text{--- (2)}$$

going back to equation - (1)

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}_{3\times N} = K\left(RW + t\right)$$

$$= K(R\ t) \cdot W_{4\times N} \rightarrow \text{Homogeneous world co-ordinates}$$

$$= K\left(R_w^c \cdot t_w^c\right) \cdot W_{4\times N}$$

$R_w^c = \left(R_c^w\right)^{-1}$ → find from equation (2)

find ← R_w^c; R_c^w → Known

$$\boxed{t_w^c = -R_w^c \cdot t_c^w}$$

t_c^w → known

find → put here → R_w^c

as $Z_w = 0$ for world, R_w^c becomes 3X2 matrix as 3rd column can be ommited, we the append to its third column our t_w^c, which makes it (3X3)

$$\therefore \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}_{3\times N} = \underbrace{K \cdot (R\ t)}_{\text{A } 3\times 3 \text{ matrix}} \cdot W_{3\times N}$$

$$\therefore \frac{1}{\lambda} \cdot A^{-1} \cdot W = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

$$\therefore \frac{1}{\lambda} \cdot W = A^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

$$\therefore \frac{1}{\lambda} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}_{3\times N} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_{3\times N}$$

$$\therefore \frac{X_{w_i}}{\lambda} = A_{11} u_i + A_{12} v_i + A_{13} \quad (i = 1 \ldots N)$$

$$\frac{Y_{w_i}}{\lambda} = A_{21} \cdot u_i + A_{22} v_i + A_{23} \quad (i = 1 \ldots N)$$

$$\frac{1}{\lambda} = A_{31} \cdot u_i + A_{32} \cdot v_i + A_{33}$$

can be done using matrix for all N at once.

$$\therefore \boxed{X_{w_i} = \frac{A_{11} u_i + A_{12} v_i + A_{13}}{A_{31} u_i + A_{32} v_i + A_{33}}} \quad \text{— for } i = 1 \ldots N$$

$$\boxed{Y_{w_i} = \frac{A_{21} u_i + A_{22} v_i + A_{23}}{A_{31} u_i + A_{33} v_i + A_{33}}} \quad \text{— for } i = 1 \ldots N.$$

$$\lambda' \underbrace{\begin{bmatrix} X_{wi} & \cdots & X_{wN} \\ Y_{wi} & \cdots & Y_{wN} \\ \gamma & \cdots & 1 \end{bmatrix}} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \cdot \begin{bmatrix} u_i & \cdots & u_N \\ v_i & \cdots & v_N \\ 1 & \cdots & 1 \end{bmatrix}$$

Divide all rows by the last row. and ommit the last row to get the co-ordinates of world.

- We can append 3rd row with '0' to get 3D-co-ordinates.

My code passed gradescope but couldn't place object in the image.