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1 - Algoritmo Genético (GA)

O Algoritmo Genético (GA) foi a meta-heurística selecionada para implementação neste trabalho. Os GAs são algoritmos matemáticos inspirados nos mecanismos de evolução natural e recombinação genética. Esse algoritmo fornece um mecanismo de busca adaptativa que se baseia no princípio Darwiniano de reprodução e sobrevivência dos mais aptos. O pseudocódigo do algoritmo implementado pode ser vasto abaixo.

Algorithm 1 Algoritmo Genético

- 1: **Hiper-parâmetros**: tamanho da população, taxa de elitismo, taxa de mutação, quantidade de gerações executar, método de seleção, eventuais parâmetros do método de seleção.
- 2: Entrada: Instância contendo coordenadas euclidianas dos nós.
- 3: Saída: Solução de menor custo da última geração e o seu custo.
- 4: Inicializa população de indivíduos.
- 5: while geração ≤ max_gerações do
- 6: Avalie a população.
- 7: Selecione os pais.
- 8: *Crossover* entre os pais selecionados.
- 9: Etapa de mutação nos filhos gerados no passo anterior.
- 10: end while
- 11: return Solução de custo mínimo da população e seu custo.

Para os testes realizados utilizou-se dois modelos de GA:

- GA 1 Algoritmo utilizando torneio como método de seleção. Esse método consiste em escolher aleatoriamente k indivíduos para um torneio. O vencedor de cada torneio (aquele com melhor *fitness*) é selecionado para realizar o *crossover*. Nessa implementação considerou-se k=3.
- GA 2 Algoritmo utilizando roulette wheel como método de seleção. Nesse tipo de seleção a probabilidade de escolha de um indivíduo para reprodução da próxima geração é proporcional ao seu fitness, quanto melhor o fitness, maior a chance desse indivíduo ser escolhido.

Para ambos os modelos considerou-se um elitismo de 15% e uma taxa de mutação adaptativa. Se o indivíduo tem um *fitness* melhor que a média da população a chance dele sofrer mutação é menor (0.2), caso contrário é maior (0.4). Tanto para o GA 1 quanto para o GA 2 o tamanho da população foi 50 e a população inicial de indivíduos foi gerada através do algoritmo do vizinho mais próximo. Por fim, os algoritmos foram executados por 3000 gerações.

2 - Resultados: Os algoritmos foram implementado em Python (3.9). E os resultados estão expressos na forma "*média +/- desvio padrão*". O número de testes foi fixado em 10. Os testes foram executados em um notebook com processador i7 7th Gen., 8 Gb de RAM e sistema Linux.

Table 1: Custo obtido por cada algoritmo em cada teste.

Arquivo	GA 1	GA 2
kroA150.tsp	29087 +/- 405	30029 +/- 427
kroB100.tsp	23340 +/- 513	23823 +/- 386
pr107.tsp	45174 +/- 47	45184 +/- 170
kroC100.tsp	21777 +/- 230	22614 +/- 361
rat99.tsp	1266 +/- 11	1296 +/- 18
st70.tsp	710 +/- 6	711 +/- 11
kroB150.tsp	28732 +/- 603	30256 +/- 1187
kroB200.tsp	32669 +/- 527	34632 +/- 689
pr136.tsp	106259 +/- 942	107674 +/- 2208
pr144.tsp	60765 +/- 16	60782 +/- 56
pr124.tsp	62153 +/- 1195	62986 +/- 1459
pr76.tsp	114029 +/- 1695	118220 +/- 3335
kroD100.tsp	23372 +/- 183	23945 +/- 600
kroA200.tsp	31877 +/- 472	34430 +/- 623
kroE100.tsp	23301 +/- 285	23617 +/- 289
lin105.tsp	15893 +/- 330	16250 +/- 377
rat195.tsp	2479 +/- 24	2734 +/- 108
berlin52.tsp	8156 +/- 39	8228 +/- 71
kroA100.tsp	22572 +/- 314	22952 +/- 825
att48.tsp	11028 +/- 135	10829 +/- 123
pr152.tsp	76920 +/- 1028	78085 +/- 1818

Table 2: Tempo de execução (ms).

Arquivo GA 1 GA 2 kroA150.tsp 134040.0 +/- 8178.2 139908.4 +/- 4517.9 kroB100.tsp 65387.0 +/- 2487.4 69732.9 +/- 3363.7 pr107.tsp 67056.3 +/- 3012.5 71435.5 +/- 1988.2 kroC100.tsp 58097.6 +/- 625.4 63398.3 +/- 652.5 rat99.tsp 58947.7 +/- 3166.4 64392.5 +/- 3982.2 st70.tsp 34739.5 +/- 2612.9 40618.3 +/- 3346.9 kroB150.tsp 134949.6 +/- 6182.9 141340.9 +/- 5887.1 kroB200.tsp 216338.2 +/- 16518.6 223231.0 +/- 17543.5 pr136.tsp 100255.2 +/- 1386.4 106167.4 +/- 1285.0 pr144.tsp 109537.8 +/- 1847.6 115373.4 +/- 2381.8 pr124.tsp 83776.4 +/- 1527.4 89298.0 +/- 1998.9 pr6.tsp 34640.3 +/- 592.8 39162.0 +/- 415.6 kroD100.tsp 55290.0 +/- 136.6 60287.2 +/- 258.0 kroA200.tsp 198952.8 +/- 2936.9 207792.0 +/- 3703.9 kroE100.tsp 57509.3 +/- 1109.8 62880.9 +/- 964.2 lin105.tsp 62769.6 +/- 1863.4 67637.8 +/- 1586.2 rat195.tsp <t< th=""><th colspan="4">included and throughout (ma).</th></t<>	included and throughout (ma).			
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	pr152.tsp	136481.6 +/- 11074.5	140757.1 +/- 9252.4	